

Isolating Kondo anyons for topological quantum computation

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We propose to use residual anyons of the overscreened Kondo effect for topological quantum computation. A superconducting proximity gap of $\Delta < T_K$ can be utilized to isolate the anyon from the continuum of excitations and stabilize the nontrivial fixed point. We use weak-coupling renormalization group, dynamical large- N technique, and bosonization to show that the residual entropy of multichannel Kondo impurities survives in a superconductor. We find that while (in agreement with recent numerical studies) the nontrivial fixed point is unstable against intrachannel pairing, it is robust in the presence of a finite interchannel pairing. Based on this observation, we suggest a superconducting charge Kondo setup for isolating and detecting the Majorana fermion in the two-channel Kondo system.

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I. INTRODUCTION

The quest for realizing non-Abelian anyons, like Majorana zero modes (MZMs) and para-fermions, has led to an extensive research due to their application in topological quantum computation. [1,2] However, in spite of a decade of active research and considerable progress, an unequivocal demonstration of MZMs in noninteracting systems is yet to be seen. Moreover, the currently pursued Ising anyons are insufficient for an all-topological quantum computation, which requires Fibonacci anyons. The main option for realizing parafermions is the edge states of fractional quantum Hall systems in proximity to a superconductor (SC) [3,4]. Even so, an elaborate technique is required to isolate the Fibonacci sector [5]. Here we propose an alternative route of using the fractionalization inherent to the Kondo effect to realize MZMs and Fibonacci anyons.

The Kondo effect arises when the electrons in a metal screen a magnetic impurity spin [Fig. 1(a)] so that the spin effectively disappears at low temperatures [6]. When various channels compete in screening a magnetic impurity, the spin is overscreened and this typically leads to a fractionalization of the spin and a residual degree of freedom at low temperatures [7–12]. In the simplest case of two-channel Kondo (2CK) model, the infrared (IR) fixed point (FP) contains a decoupled MZM with ground-state degeneracy of $g_{2CK} = \sqrt{2}$, similar to the edge mode of the one-dimensional (1D) Kitaev model. The three-channel Kondo (3CK) has $g_{3CK} = (1 + \sqrt{5})/2$, corresponding to a Fibonacci anyon. Can these anyons, e.g., the 2CK MZM, be utilized for quantum computation?

Nowadays multichannel Kondo systems are not as out-of-reach as before. Experiments on semiconducting quantum dots [13,14] and charge Kondo effect [15–17] in the quantum Hall regime [18,19] demonstrated 2CK and 3CK physics and

there are also other proposals based on Majorana boxes [20], Floquet-driven Anderson impurity [21], or magnetically frustrated Kondo systems [22–24]. Even though a MZM can be achieved using simpler noninteracting setups, a 2CK realization paves the way for producing Fibonacci anyons using 3CK physics. However, there are various obstacles:

(i) It is unclear if the residual MZM in the 2CK model is localized at the position of the impurity spin or delocalized throughout the system.

(ii) Local relevant spin/channel symmetry-breaking perturbations destabilize the nontrivial fixed point [25].

(iii) The gapless spectrum of the conduction band prohibits singling-out the topological sector and braiding.

(iv) The coupling between two spin-impurities mediated by the conduction band destabilizes individual 2CK FPs, driving them to a Fermi liquid at the IR [26].

To solve these problems, we suggest to gap out the conduction band at low energies by a superconductor with a gap $\Delta < T_K$. The low-energy effective theory of such a system is only in terms of fractionalized degrees of freedom γ_j localized at the position of spin impurities, suitable for braiding. For example, in the 2CK case, $H_{\text{eff}} = i \sum_{m < n} M_{mn} \gamma_m \gamma_n + \mathcal{O}(\gamma^4)$. The spin impurities do not need to move in real space and a braiding in configuration space of M_{mn} is sufficient for computation.

For simplicity and practicality we consider spin-singlet s-wave pairing. For an infinitesimal Kondo coupling, the local moment remains decoupled and it remains so, as long as $T_K < \Delta$. [28,29] On the other hand, the $T_K > \Delta$ regime is expected to connect smoothly to the $\Delta \rightarrow 0$ limit. Indeed, for a single-channel Kondo impurity at the strong-coupling FP one can “pair” Kramers pairs of $\pi/2$ phase-shifted IR quasiparticles.

However, at the non-Fermi liquid ground state of overscreened Kondo systems, an in-going single-particle state is scattered into out-going many-body states [10] (no single-particle amplitude in the 2CK case). In the absence of well-defined quasiparticles, it is unclear whether it is possible at all to open up a gap and how the nontrivial FP can smoothly

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connect to a superconducting state far away from the impurity. In fact, recent numerical renormalization group (NRG) studies indicated that the 2CK FP is unstable against pairing [30].

Considering the potential application, we revisit the problem here. After a brief discussion of the weak-coupling regime, we use dynamical large- N technique to show that the residual entropy survives in the limit of a finite $\Delta < T_K$. To gain insight about the more relevant SU(2) spin, we use bosonization. We find that although the 2CK FP is unstable against intrachannel pairing, it is robust against interchannel pairing, and we propose a setup to isolate the MZM.

II. MODEL

The model consists of K channels of noninteracting spinful electrons, proximity-paired to a SC and Kondo coupled to an impurity spin. The Hamiltonian is $H = H_0 + H_\Delta + H_K$, where

$$H_0 = \sum_{k,a,\alpha} \epsilon_k c_{ka\alpha}^\dagger c_{k\alpha}, \quad H_K = J_K \sum_{kk',a,\alpha\beta} \vec{S} \cdot c_{ka\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{k'\alpha\beta}. \quad (1)$$

Here $a = 1 \dots K$ is the channel index. In the SU(2) case \vec{S} is a $S = 1/2$ spin operator, $\vec{\sigma}$ are the Pauli matrices, and $\alpha, \beta = \uparrow, \downarrow$. We consider a singlet proximity pairing

$$H_\Delta = \Delta \sum_{k,a} [c_{k,a,\uparrow}^\dagger c_{-k,a,\downarrow}^\dagger + \text{H.c.}], \quad (2)$$

which for a wide conduction band of electrons, $|\epsilon_k| < D_0$, results in the local Green's function (without Kondo)

$$g_c(z) = -2\rho \frac{z\tau^0 + \Delta\tau^x}{\sqrt{z^2 - \Delta^2}} \log \frac{D_0 - \sqrt{z^2 - \Delta^2}}{-D_0 - \sqrt{z^2 - \Delta^2}}, \quad (3)$$

with $\tau^{0/x}$ being Pauli matrices in the Nambu space.

III. WEAK-COUPLING

At the weak-coupling limit (small J_K/D_0) for any K , the Kondo coupling evolves as $dJ_K/d\ell = \rho(D)J_K^2$ where $d\ell = -dD/D$ and $\rho(\epsilon) = g_{c,ee}'(\epsilon - i0^+)$. This density of states is shown in the inset of Fig. 1(b). As the cutoff D is reduced $J_K(D)$ increases. $T_K(\Delta)$ is defined as the D at which J_K diverges. If this happens before D is reduced to Δ , the Kondo coupling has already renormalized to its infrared value and the moment is fully/over-screened. Otherwise, the ground state is an unscreened local moment, separated from the screened phase with a first-order transition [28]. $T_K(\Delta)$ as a function of $T_K^0 = D_0 e^{-2D_0/J_K}$ is shown in Fig. 1(b). The two phases are separated by $T_K^0/\Delta = 1/2$. To explicitly study S_{IR} at $T \ll \Delta$, we use the large- N limit.

IV. DYNAMICAL LARGE- N

We use Schwinger bosons [31] to represent SU(N) spins as $S_{\alpha\beta} = b_\alpha^\dagger b_\beta$ with $\alpha, \beta = 1 \dots N$. The bosons have larger Hilbert space than the original spin and a constraint $b_\alpha^\dagger b_\alpha = 2S$ has to be imposed to stay within the physical subspace. Plugging this into H_K and using a Hubbard-Stratonovich

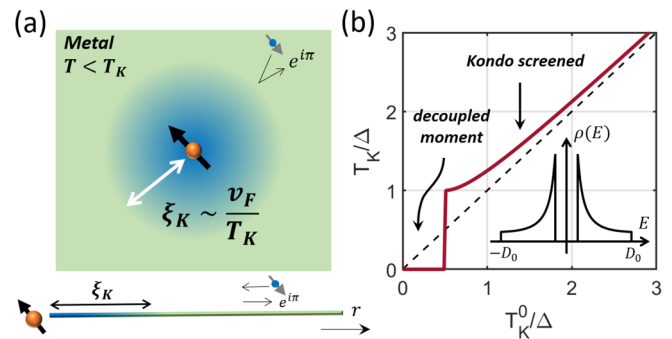


FIG. 1. (a) A Kondo impurity in a metal is screened within a length scale of Kondo cloud [27] so that other electrons only experience a $e^{2i\delta_c}$ with $\delta_c = \pi/2$ phase shift. Bottom: The problem can be reduced to a 1D problem in the radial direction. (b) $T_K(\Delta)$ as a function of T_K^0 . There is a threshold $T_K^0 = \Delta/2$ below which the system remains in the local moment $S = \log(2)$ phase. The inset shows the local density of states in the superconducting region.

transformation, H_K reduces to

$$H_K = \sum_a \frac{\chi_a^\dagger \chi_a}{J_K} + \sum_{\alpha\alpha'} [\chi_a^\dagger b_{\alpha\alpha'} c_{\alpha\alpha'}^\dagger + \text{H.c.}], \quad (4)$$

where Grassmannian “holons” χ are introduced. Keeping the ratios finite, but taking the $N, K, 2S \rightarrow \infty$ limit, the Green’s functions and self-energies of bosons and holons obey simple forms as follows:

$$\Sigma_B(\tau) = -\frac{K}{N} g_{c,ee}(\tau) G_\chi(\tau), \quad \Sigma_\chi(\tau) = g_{c,ee}(-\tau) G_B(\tau), \quad (5)$$

which can be solved self-consistently together with the Dyson equations $G_B^{-1}(z) = z - \lambda - \Sigma_B(z)$ and $G_\chi^{-1}(z) = -J_K^{-1} - \Sigma_\chi(z)$. See Refs. [31–34] for details.

We first study the fully screened case $K = 2S$. The phase shift $N\delta_c$, the residual entropy S , and the effective moment T_χ are shown in Fig. 2(a) as a function of T/T_K^0 . The local moment phase at $T_K^0/\Delta < 1$ (red) and the screened phase at $T_K^0/\Delta > 1$ (blue) are clearly visible. Figure 2(b) shows the same quantities in the overscreened case. While for $T_K^0/\Delta > 1$ (blue), the moment disappears at low temperature, the residual entropy (same value as the gapless system [8,10]) survives. However, it remains unclear whether this large- N result holds for $N = K = 2$ and $2S = 1$. Therefore, to better examine the SU(2) case, we use field-theory techniques.

V. BOSONIZATION

The Hamiltonian H in two dimensions can be reduced to a sum of 1D wires (in the radial direction) terminated at the position of the impurity (Fig. 3, Appendix A 1). We first linearize the spectrum using $c_{\alpha\alpha} = e^{ik_F x} \psi_{R\alpha\alpha} + e^{-ik_F x} \psi_{L\alpha\alpha}$ and use the recipe $\psi_{L/R,\alpha\alpha} = F_{\alpha\alpha L/R} \exp[i\sqrt{\pi}(\phi_{\alpha\alpha} \pm \theta_{\alpha\alpha})]$ to express the left/right-moving fermions in terms of conjugate bosons with commutation relations $[\phi_{\alpha\alpha}(x), \theta_{\beta\beta}(y)] = -\frac{i}{2} \text{sign}(x-y) \delta_{\alpha\beta} \delta_{\alpha\beta}$ and mutually anticommute charge-lowering operators $F_{\alpha\alpha}$, referred to as Klein factors [35]. The UV open boundary condition at the position of the impurity corresponds to $\theta_{\alpha\alpha}(0) = 0$ and $F_{\alpha\alpha R} = F_{\alpha\alpha L} = F_{\alpha\alpha}$.

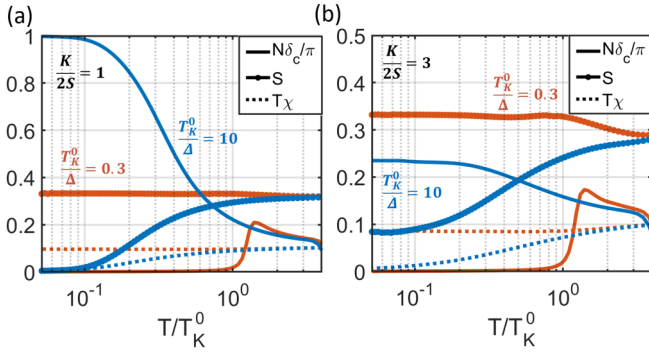


FIG. 2. Conduction electron phase shift $N\delta_c/\pi$, thermodynamical entropy S , and effective moment $T\chi$ as function of T/T_K^0 for (a) fully screened case $K/2S = 1$ and (b) overscreened case $K/2S = 3$. Blue curves correspond to $T_K^0/\Delta = 10$ and red curves correspond to $T_K^0/\Delta = 0.3$. The calculation was done for $2S/N = 0.1$.

VI. 1CK

In the single-channel case ($K = 1$) we define charge/spin bosons according to $\phi_{c/s} = (\phi_\uparrow \pm \phi_\downarrow)/\sqrt{2}$ (and same for $\theta_{c/s}$) in terms of which

$$H_0 = \frac{v_F}{2} \sum_{\mu=c,s} \int_0^\infty dx [(\partial_x \phi_\mu)^2 + (\partial_x \theta_\mu)^2], \quad (6)$$

and the proximity pairing becomes ($\mathcal{I}^+ \equiv F_\uparrow^\dagger F_\downarrow^\dagger$)

$$H_\Delta \sim \frac{\Delta}{\pi} \int_0^\infty dx \cos \sqrt{2\pi} \theta_s(x) [\mathcal{I}^+ e^{-i\sqrt{2\pi} \phi_c} + \text{H.c.}]. \quad (7)$$

The accompanying $e^{-i\sqrt{2\pi} \phi_c}$ factor ensures that \mathcal{I}^+ always appears with \mathcal{I}^- in the partition function. Whenever this is the case, bosonic combination of Klein factors, e.g., \mathcal{I}^+ can be replaced by 1 and fermionic ones by a corresponding Majorana fermion. Thus, Eq. (7) leads to the pinning $\phi_c = 0$, $\theta_s = \sqrt{\pi}/2$ or vice versa at the IR. An anisotropic Kondo interaction bosonizes to

$$H_K = \frac{J_K^\perp}{2} [S^+ S^- e^{i\sqrt{2\pi} \phi_s(0)} + \text{H.c.}] - \frac{J_K^z}{\sqrt{2\pi}} S^z \partial_x \theta_s(0), \quad (8)$$

where $S^- \equiv F_\downarrow^\dagger F_\uparrow^\dagger$. We use the unitary transformation $U = \exp[i\mu S^z \phi_s(0)]$ to tune the system $H \rightarrow U^\dagger H U$ to the Toulouse II [36] or *decoupling* [37] point. For any K , the decoupling point is defined as the choice of μ which maximizes the scaling dimension for the transverse Kondo coupling J_K^\perp [38]. For the single-channel case, the bosonic factor $e^{i\sqrt{2\pi} \phi_s}$ can be eliminated by $\mu = \sqrt{2\pi}$. Moreover, $\tilde{\phi}(x) = \phi(x)$ but

$$\tilde{\theta}(x) = \theta(x) + \mu \sqrt{2} S^z \text{sign}(x), \quad (9)$$

and the Hamiltonian becomes

$$U^\dagger (H_K + H_0) U = H_0 + \frac{J_K^\perp}{2} [S^+ S^- + \text{H.c.}] - \frac{J_K^z}{\sqrt{2\pi}} \partial_x \theta_s(0) S^z.$$

At the decoupling point, J_K^\perp is highly relevant whereas the term proportional to $J_K^z = J_K^z - 2\mu\sqrt{\pi}v_F$ is (marginally) irrelevant. Via this term the conduction electrons “observe” the state of the spin (along the z -direction) and entangle to it, resulting in its decoherence. The strong coupling FP

corresponds to $J_K^\perp \rightarrow \infty$ and $J_K^z \rightarrow 0$, i.e., a singlet between the dressed spin S and the Klein-factor composite S .

Now we turn to coexisting Kondo and pairing terms. Due to Eq. (9) the pairing term is modified by the unitary transformation $U^\dagger H_\Delta U$, and the pinning of $\theta_s(x)$ depends on the state of the spin. An infinitesimal $J_K^\perp \vec{S}_\perp \cdot \vec{S}_\perp$ cannot flip the spin. However, a large J_K^\perp can “melt” the bosonic solid of $\theta_s(x)$ in an area ξ_K . Near the impurity, J_K^\perp is large and the spin precession is negligible. But at long distances $J_K^\perp \rightarrow 0$ and the spin precession is set by J_K^\perp which grows at low energies, i.e., $S^z(\tau) = e^{\tau J^\perp \vec{S}_\perp \cdot \vec{S}_\perp} S^z e^{-\tau J^\perp \vec{S}_\perp \cdot \vec{S}_\perp}$ is fluctuating in time, with a rate given by J^\perp . For $\theta_s(x)$ to follow this evolution, it costs an energy $\frac{v_F}{2} [\partial_x \phi_s]^2 = \frac{v_F}{2} [\partial_x \theta_s]^2 \propto [\partial_x S^z(\tau)]^2$ per unit length. Hence beyond a certain distance, $\theta_s(x)$ can no longer follow the spin and it is pinned to $\langle S^z \rangle = 0$. This defines a characteristic distance $\xi_K \sim v_F/T_K(\Delta)$ [27].

VII. 2CK

In the two-channel case ($K = 2$), we follow [9] and define collective bosons according to

$$\begin{pmatrix} \phi_c \\ \phi_s \\ \phi_f \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} \phi_{1\uparrow} \\ \phi_{1\downarrow} \\ \phi_{2\uparrow} \\ \phi_{2\downarrow} \end{pmatrix}, \quad (10)$$

and similarly for θ_μ . Again $U = \exp[i\mu S^z \phi_s(0)]$ with $\mu = \sqrt{\pi}$ transforms H_K to

$$J_K^\perp [S^+ \mathcal{F}_s (\mathcal{F}_{sf} e^{i\sqrt{\pi} \phi_{sf}(0)} + \text{H.c.}) + \text{H.c.}] - \frac{J_K^z}{\sqrt{\pi}} S^z \partial_x \theta_s(0).$$

We defined four new Klein factors (Appendix A 2) in accord with Eq. (10) and identified $S_1^- = \mathcal{F}_s \mathcal{F}_{sf}$ and $S_2^- = \mathcal{F}_s \mathcal{F}_{sf}^\dagger$. Note that since S^+ always accompanies \mathcal{F}_s , the total spin fluctuates only mildly. Representing $S^+ \mathcal{F}_s = d^\dagger$ with a Dirac fermion $d \equiv \gamma + i\gamma'$, and re-fermionizing the dim-1/2 operator $\psi_{L/R,sf} \equiv \mathcal{F}_{sf} e^{i\sqrt{\pi}[\phi_{sf}(x) \pm \theta_{sf}(x)]}$, the Hamiltonian becomes a resonant Andreev scattering

$$U^\dagger H_K U \rightarrow 2iJ_K^\perp \gamma' \eta_{sf}(0) - \frac{J_K^z}{\sqrt{\pi}} i\gamma \gamma' \partial_x \theta_s(0), \quad (11)$$

where $\eta_{sf}(x) \equiv \frac{1}{2}(\psi_{sf} + \psi_{sf}^\dagger)$. It is safe to represent \mathcal{F}_{sf} with a Majorana fermion $\rightarrow \Gamma_{sf}$, here. The ground state of J_K^\perp term is thus a Schrödinger’s cat state [39] between the spin and the pinning of the boson $\phi_{sf}(0) = 0, \sqrt{\pi}$. This means the difference of the spin in the two channels $\theta_{sf}(0)$, which is the only nonconserved charge, strongly fluctuates [40]. The first term of Eq. (11) hybridizes γ' with the conduction Majoranas $\eta_{sf}(0)$, and provides it with the scaling dimension 1/2. Near this IR FP, the originally marginal interaction J_K^z , coupling γ and γ' , becomes a dim-3/2 irrelevant operator (the leading [41]). At the IR FP, γ is entirely decoupled [9] consistent with the $S = \frac{1}{2} \log 2$ residual entropy [8,10], realizing a MZM.

Can this MZM survive in a SC? The simplest form of SC is an induced intrachannel pairing [cf. Eq. (7)],

$$H_\Delta^{\text{intra}} = \frac{4\Delta}{\pi} \int_0^\infty dx \{ \cos \sqrt{2\pi} \theta_{s1}(x) \cos \sqrt{2\pi} \phi_{c1} + \cos \sqrt{2\pi} \theta_{s2}(x) \cos \sqrt{2\pi} \phi_{c2}, \quad (12)$$

where we set $\mathcal{I}_j^\pm \rightarrow 1$, as before. $U^\dagger H_\Delta^{\text{intra}} U$ has the effect of $\theta_s \rightarrow \tilde{\theta}_s$ as in Eq. (9). Far from the impurity the two lines can be minimized independently. Since both θ_{s1} and θ_{s2} are pinned, $\theta_{sf} = (\theta_{s1} - \theta_{s2})/\sqrt{2}$ is also pinned. This means that ϕ_{sf} is strongly fluctuating and the term $J_K^\perp i\gamma' \Gamma_{sf} \cos \sqrt{\pi} \phi_{sf}(0)$ becomes highly irrelevant. Thus inclusion of a small Δ destabilizes the 2CK FP, in agreement with NRG results [30].

On the other hand, a singlet/triplet interchannel pairing has the form

$$\begin{aligned} H_\Delta^{\text{inter}} &= \Delta \int_0^\infty dx [(c_{1\uparrow}^\dagger c_{2\downarrow}^\dagger \pm c_{2\uparrow}^\dagger c_{1\downarrow}^\dagger) + \text{H.c.}] \\ &\sim \frac{\Delta}{\pi} \{ \mathcal{F}_c^\dagger e^{-i\sqrt{\pi}\phi_c} [\mathcal{F}_{sf}^\dagger e^{-i\sqrt{\pi}\phi_f} \cos \sqrt{\pi}(\theta_s + \theta_f) \\ &\quad \pm \mathcal{F}_{sf} e^{i\sqrt{\pi}\phi_f} \cos \sqrt{\pi}(\theta_f - \theta_s)] + \text{H.c.} \} \end{aligned} \quad (13)$$

using $F_{1\uparrow}^\dagger F_{2\downarrow}^\dagger = \mathcal{F}_c^\dagger \mathcal{F}_{sf}^\dagger$ and $F_{1\downarrow}^\dagger F_{2\uparrow}^\dagger = \mathcal{F}_c^\dagger \mathcal{F}_{sf}$. Again, we can safely replace Klein factors with Majorana fermions $\mathcal{F}_\mu \rightarrow \Gamma_\mu$ because of the exponential factors and $U^\dagger H_\Delta^{\text{intra}} U$ has the effect of $\theta_s \rightarrow \tilde{\theta}_s$ as in Eq. (9). Note that there is no θ_{sf} here! This interaction tends to pin the ϕ_{sf} value and is benign to the 2CK FP as the bulk pinning can smoothly connect to the boundary value. The harmlessness of the interchannel pairing can also be seen in a two-site problem, where the addition/removal of a pair of interchannel electrons maps the strong coupling to weak-coupling or vice versa without affecting the channel isotropy. The possibility of coexistence of interchannel pairing and 2CK FP seen here, can be verified in future NRG studies.

VIII. MULTIPLE IMPURITIES

The case of two 2CK impurities coupled to the same bath was discussed previously [26]. The double 2CK FP is transformed to a line of FPs by the RKKY interaction $J_H \vec{S}_1 \cdot \vec{S}_2$ which becomes a marginal operator $iJ'_H \gamma_1 \gamma_2 \partial_x \Phi(0)$ at the nontrivial FP. Here J'_H is the renormalized coupling and Φ is a linear combination of the spin bosons of each impurity [26]; the two decoupled Majoranas form a nonlocal charge qubit whose state is dynamically measured (and decohered) by the gapless $\partial_x \Phi$ mode. The presence of a gap in the spectrum has the additional feature of suppressing such decoherence effects and reducing it to a static $iM_{12} \gamma_1 \gamma_2$ interaction discussed before.

IX. EXPERIMENTAL REALIZATION

Based on the above discussion, we propose a modified version of the charge Kondo setup [18,19] at zero magnetic field to isolate the MZM in the 2CK case. In the simplest charge Kondo effect, a spinless single electron transistor (SET) with large charging energy is coupled to a spinless conduction bath. The SET is tuned to a charge degeneracy point $\Delta Q = 0$, $1e$ so that the *charge parity* plays the role of the pseudospin. The location of the electron, either in SET or the conduction bath, plays the role of the conduction electron pseudospin [15]. The spinful case provides the simplest realization of a two-channel charge Kondo effect. Incidentally, this is ideally suitable for combining with the previous discussion. A proximity pairing

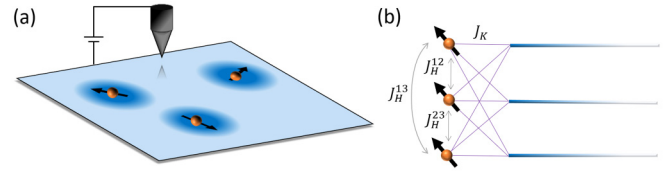


FIG. 3. (a) The problem of three (multichannel) Kondo impurities in 2D can be mapped to (b) the problem of three Kondo impurities at the end of semi-infinite wires. This is so even in the presence of proximity pairing of the host metal. Also various potential scatterings (e.g., a scanning gate microscope) only modify the mutual Kondo coupling and RKKY interactions among the impurities.

of the SET (or SET made of SC) and conduction bath with singlet SC leads to purely interchannel pairing. Since the role of the spin and channel are reversed, a singlet pairing in the lead and the SET correspond to T_\pm interchannel spin-triplet pairing in the original basis, whereas Eq. (10) shows that a T_0 triplet pairing is benign to 2CK physics. Considering $c_{1\uparrow}^\dagger c_{2\uparrow}^\dagger - c_{1\downarrow}^\dagger c_{2\downarrow}^\dagger = c_{1\rightarrow}^\dagger c_{2\leftarrow}^\dagger - c_{1\leftarrow}^\dagger c_{2\rightarrow}^\dagger$, we expect the latter to hold for all interchannel triplet pairings albeit with a π phase difference, although this cannot be seen in abelian bosonization. Such Coulomb-blockaded superconducting islands are common in topological Kondo effect [20,42]. However, the MZM here is produced by the 2CK, rather than by the band topology.

Realization of the charge Kondo effect requires $\delta E \ll k_B T \ll T_K \ll E_C$ where δE is the mean-level spacing, $T_K \sim E_C e^{-\pi^2/2\mathcal{T}}$ is the Kondo temperature expressed in terms of transmission \mathcal{T} , and $E_C = e^2/C$ is the charging energy. This condition can be met in small metallic grains, e.g., the hybrid metal-semiconductor setup of Iftikhar *et al.* [18,19]. Alternatively, a purely proximity-induced superconductivity in semiconductor heterostructures with large carrier mass can be used. Since carrier mobility is unimportant, one possible option is a dot with a large charging energy defined using in-plane gates in a shallow 2D valence band hole gas [43,44].

X. DETECTION

The presence of the MZM has to be inferred indirectly; we consider an additional *normal* lead (e.g., a scanning tunneling microscope) *weakly coupled* to the SET to measure the conductance between the two leads. At the 2CK FP, the coupling of the probe channel is irrelevant [16,17] and a conductance of $G \propto T$ is expected on resonance at $T > \Delta$ [16,17]. For $T < \Delta$, the capacitive coupling to the SET, with another scanning SET might be a possibility, but its feasibility requires further studies. Alternatively, entropy measurements along those is Ref. [45] can be envisioned.

XI. CONCLUSION

We proposed to use Kondo-based anyons in proximity with superconductivity for quantum computation. We found that the residual entropy survives a gap in the spectrum, particularly if the gap is produced by an interchannel proximity pairing. The presence of the gap in the spectrum protects the Majorana fermion and the ground-state degeneracy against

small symmetry-breaking perturbations. We suggested a superconducting version of the charge Kondo setup for isolating the MZM in the 2CK model.

Note added. After posting this paper we became aware of a recent paper [46] that appeared online a few days before and also seeks to use multichannel Kondo anyons for computation, but in gapless systems. The two papers, however, do not have any significant overlap.

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APPENDIX:

In these Appendixes we provide supporting documents for some of the statements in the paper. Appendix A 1 shows that the problem of N multichannel Kondo impurities immersed in two and three dimensions can be reduced to a set of one-dimensional problems, including the superconducting proximity pairing term. Appendix A 2 summarizes our convention for the Klein factors that are used in bosonization.

1. Dimensional reduction

For completeness, here we show how a problem of M impurities in two dimensions can be reduced to a sum of 1D metallic systems terminated at the spin impurities, appropriate for field theory analysis. This is a generalization of the $M = 2$ case from Ref. [22]. We start from a two-dimensional system described by $H = H_0 + H_K + H_\Delta$ defined in Eqs. (1) and (2), and generalize it to the case that there are M Kondo impurities located at positions \vec{d}_n coupled to the conduction band.

a. Kondo coupling

We consider M impurities located on a 2D plane at positions \vec{d}_n (shown in Fig. 3). We have

$$H_K = \int \frac{d^2k d^2k'}{(2\pi)^4} \psi_{\vec{k}}^\dagger \frac{\vec{\sigma}}{2} \psi_{\vec{k}'} \cdot \sum_{n=1}^M J_n \vec{S}_n e^{-i(\vec{k}-\vec{k}')\cdot\vec{d}_n}. \quad (\text{A1})$$

We start by defining the fermions

$$\psi_{nE} = \int \frac{d^2k}{(2\pi)^2} \delta(\epsilon_k - E) e^{i\vec{k}\cdot\vec{d}_n} \psi_{\vec{k}}, \quad (\text{A2})$$

in terms of which the Kondo Hamiltonian is

$$H_K = \int dE dE' \sum_{n=1}^M J_n \psi_{nE}^\dagger \frac{\vec{\sigma}}{2} \psi_{nE'} \cdot \vec{S}_n. \quad (\text{A3})$$

The problem is that ψ_{nE} do not obey standard anticommutation relations. Rather

$$\{\psi_{nE}, \psi_{mE'}^\dagger\} = g_{nm}(E) \delta(E - E'), \quad (\text{A4})$$

where

$$g_{nm}(E) = \int \frac{d^2k}{(2\pi)^2} \delta(\epsilon_k - E) e^{i\vec{k}\cdot(\vec{d}_n - \vec{d}_m)}. \quad (\text{A5})$$

Representing the vector $\vec{d}_{nm} \equiv \vec{d}_n - \vec{d}_m$ in polar coordinates by $\vec{d}_{nm} = d_{nm}(\cos \phi_{nm}, \sin \phi_{nm})$ we can write

$$\begin{aligned} g_{nm}(E) &= \frac{k_E}{2\pi \partial_k \epsilon_E} \int_0^{2\pi} \frac{d\phi}{2\pi} e^{ik_E d_{nm} \cos(\phi - \phi_{nm})} \\ &= \frac{k_E}{2\pi \partial_k \epsilon_E} \sum_p e^{-ip\phi_{nm}} J_p(k_E d_{nm}) i^p \int_0^{2\pi} \frac{d\phi}{2\pi} e^{ip\phi} \\ &= g_E J_0(k_E d_{nm}), \end{aligned} \quad (\text{A6})$$

where

$$g_E = \frac{k_E}{2\pi \partial_k \epsilon_E}. \quad (\text{A7})$$

Remarkably, g_{nm} depends only on mutual distances of the impurities, measured by the corresponding wavelength $k_E d_{nm}$. The matrix g_{nm} is real and symmetric and it has real eigenvalues λ_n and can be diagonalized by orthogonal eigenvectors \vec{u}_n where $g\vec{u}_n = \lambda_n \vec{u}_n$. So we can write $g_{nm} = \sum_p u_{np} \lambda_p u_{mp}^*$ and the orthogonality is $\sum_p u_{np} u_{mp}^* = \delta_{nm}$. Defining new operators with

$$\psi_{nE} = \sum_i u_{ni}(E) \sqrt{\lambda_i(E)} \tilde{\psi}_{iE}, \quad (\text{A8})$$

and

$$\tilde{\psi}_{iE} \equiv \frac{1}{\sqrt{\lambda_i(E)}} \sum_n u_{ni}^*(E) \psi_{nE} \quad (\text{A9})$$

we find that they are orthonormal

$$\{\tilde{\psi}_{iE}, \tilde{\psi}_{jE'}\} = \delta_{ij} \delta(E - E'), \quad (\text{A10})$$

and the Kondo Hamiltonian becomes

$$H_K = \int dE dE' \sum_{nij} J_{nij}(E, E') \vec{S}_n \cdot \tilde{\psi}_{iE}^\dagger \frac{\vec{\sigma}}{2} \tilde{\psi}_{jE'}, \quad (\text{A11})$$

where

$$J_{nij}(E, E') = J_n \sqrt{\lambda_i(E) \lambda_j(E')} u_{ni}(E) u_{nj}^*(E'). \quad (\text{A12})$$

Although the J_{nij} couplings are complex in general, the relation $J_{nij}^*(E, E') = J_{nji}(E', E)$ ensures the hermiticity of the Hamiltonian. Whether or not a spin n couples the channels i and j depend on the product of wave functions of i and j at the site n , which can be tuned by moving potential scatterings induced by scanning tips (Fig. 3). Next, we do a Taylor expansion of $J_{nij}(E, E')$ function around Fermi energy and keep only the leading relevant term. H_K can be written in the matrix form $[J_n]_{ij} = J_{nij}$,

$$H_K = \sum_n \Psi^\dagger(0) \left[J_n \vec{S}_n \cdot \frac{\vec{\sigma}}{2} \right] \Psi(0), \quad (\text{A13})$$

where Ψ is a vector in $m = 1 \dots M$ index and the spin is implicit. As we see the impurities talk to all the channels and scatter electron between all the channels. A result of the

Taylor expansion of $J_{nij}(E, E')$ is that RKKY interactions will be induced between the spins

$$H_K \rightarrow H_K + \sum_{ij} J_H^{ij} \vec{S}_i \cdot \vec{S}_j. \quad (\text{A14})$$

b. Mapping to 1D

The mapping to (unfolded) left-movers follows Ref. [22]

$$\psi_L(x, t) = \frac{1}{\sqrt{v}} \int_{-D}^D dE e^{-iE(t+x/v)} \psi_E, \quad (\text{A15})$$

for $x \in (-\infty, +\infty)$ with commutation relations

$$\{\psi_L^\dagger(x), \psi_L(y)\} = 2\pi \delta(x - y). \quad (\text{A16})$$

Alternatively, we can work in the folded space of left and right-movers defined as

$$\psi_R(x, t) = \psi_L(-x, t), \quad (x > 0). \quad (\text{A17})$$

c. Kinetic term

As we saw above, the Kondo interaction involves M 1D conduction bands pulled out of the 2D conduction band. The natural guess for their kinetic energy is

$$\begin{aligned} H_0(\vec{d}) &\equiv \int dE E \psi_{nE}^\dagger \psi_{nE} \\ &= \int \frac{d^2k d^2k'}{(2\pi)^4} \epsilon_k \delta(\epsilon_k - \epsilon_{k'}) e^{i(\vec{k}-\vec{k}')\cdot\vec{d}} \psi_{\vec{k}}^\dagger \psi_{\vec{k}'}. \end{aligned} \quad (\text{A18})$$

Summing over all the positions we have

$$\int d^2r H_0(\vec{r}) = \delta(0) \int \frac{d^2k d^2k'}{(2\pi)^2} \epsilon_k \psi_{\vec{k}}^\dagger \psi_{\vec{k}'}, \quad (\text{A19})$$

which is the total Hamiltonian up to a $\delta(0)$, i.e., the total volume. Transformation from ψ_{nE} to $\tilde{\psi}_{iE}$ is a unitary transformation at each energy and it does not change the kinetic part. Therefore, we have

$$H_0 \rightarrow \sum_{n=1}^M \int dE E \tilde{\psi}_{nE}^\dagger \tilde{\psi}_{nE}. \quad (\text{A20})$$

d. Pairing term

Without loss of generality, we consider a channel-diagonal s-wave singlet pairing

$$H_\Delta = \int \frac{d^2k}{(2\pi)^2} (\Delta \psi_{k\uparrow}^\dagger \psi_{-k\downarrow}^\dagger + \text{H.c.}). \quad (\text{A21})$$

Let us look at

$$\begin{aligned} H_\Delta^+(\vec{d}_n) &\equiv \Delta \int dE \psi_{nE\uparrow}^\dagger \psi_{nE\downarrow}^\dagger \\ &= \Delta \int dE \int \frac{d^2k d^2k'}{(2\pi)^4} \delta(\epsilon_k - E) \\ &\quad \times \delta(\epsilon_{k'} - E) \psi_{k\uparrow}^\dagger \psi_{q\downarrow}^\dagger e^{-i(\vec{k}+\vec{k}')\cdot\vec{d}_n}. \end{aligned}$$

Summing over all the points gives a $(2\pi)^2 \delta(\vec{k} + \vec{k}')$ term that gives

$$\int d^2r H_\Delta^+(\vec{r}) = \delta(0) \Delta \int \frac{d^2k}{(2\pi)^2} \psi_{k\uparrow}^\dagger \psi_{-k\downarrow}^\dagger, \quad (\text{A22})$$

which is our initial Hamiltonian up to the system volume. Therefore,

$$\begin{aligned} H_\Delta &\rightarrow \Delta \sum_{n=1}^M \int dE (\psi_{nE\uparrow}^\dagger \psi_{nE\downarrow}^\dagger + \text{H.c.}) \\ &= \Delta \sum_{ij} \int dE (\tilde{\psi}_{iE\uparrow}^\dagger \tilde{\psi}_{jE\downarrow}^\dagger + \text{H.c.}). \end{aligned} \quad (\text{A23})$$

2. Klein factors

Here we briefly review the treatment of Klein-factors following Ref. [35]. Denoting the total number of fermions μ with

$$N_\mu = \int_0^\infty dx c_\mu^\dagger(x) c_\mu(x), \quad \mu \in \{1 \uparrow, 1 \downarrow, 2 \uparrow, 2 \downarrow\}, \quad (\text{A24})$$

the Klein factors have the property (no summation):

$$F_\mu F_\mu^\dagger = F_\mu^\dagger F_\mu = 1, \quad (\text{A25})$$

$$\{F_\mu, F_\nu^\dagger\} = 2\delta_{\mu\nu}, \quad (\text{A26})$$

$$\{F_\mu, F_\nu\} = 0, \quad \forall \mu \neq \nu, \quad (\text{A27})$$

$$[F_\mu, N_\nu] = \delta_{\mu\nu} F_\nu, \quad (\text{A28})$$

$$[F_\mu, \phi_\nu(x)] = [F_\mu, \theta_\nu(x)] = 0. \quad (\text{A29})$$

In the paper we transform into the collective boson basis:

$$\begin{pmatrix} \mathcal{N}_c \\ \mathcal{N}_s \\ \mathcal{N}_f \\ \mathcal{N}_{sf} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} N_{1\uparrow} \\ N_{1\downarrow} \\ N_{2\uparrow} \\ N_{2\downarrow} \end{pmatrix}. \quad (\text{A30})$$

In accord with this transformation, we define new Klein factors \mathcal{F}_μ for $\mu = c, s, f, sf$ that satisfy

$$\mathcal{F}_\mu \mathcal{F}_\mu^\dagger = \mathcal{F}_\mu^\dagger \mathcal{F}_\mu = 1, \quad (\text{A31})$$

$$\{\mathcal{F}_\mu, \mathcal{F}_\nu^\dagger\} = 2\delta_{\mu\nu}, \quad (\text{A32})$$

$$\{\mathcal{F}_\mu, \mathcal{F}_\nu\} = 0, \quad \forall \mu \neq \nu, \quad (\text{A33})$$

$$[\mathcal{F}_\mu, \mathcal{N}_\nu] = \delta_{\mu\nu} \mathcal{F}_\nu, \quad (\text{A34})$$

$$[\mathcal{F}_\mu, \phi_\nu(x)] = [\mathcal{F}_\mu, \theta_\nu(x)] = 0. \quad (\text{A35})$$

It can be seen that these any off-diagonal relation combination, like $F_\mu^\dagger F_\nu$ change the population of the collective modes \mathcal{N}_ξ by integers and thus can be represented by a combination of new Klein factors [35]. For example,

$$F_{1\uparrow}^\dagger F_{1\downarrow} = \mathcal{F}_{sf}^\dagger \mathcal{F}_s^\dagger, \quad F_{2\uparrow}^\dagger F_{2\downarrow} = \mathcal{F}_{sf} \mathcal{F}_s. \quad (\text{A36})$$

In the paper, we repeatedly use these and similar relations.

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