Editors' Suggestion

Anomalous low-frequency conductivity in easy-plane XXZ spin chains

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Using the framework of generalized hydrodynamics, we compute the low-frequency spin conductivity $\sigma(\omega)$ of XXZ spin chains with easy-plane anisotropy. We find that for almost all values of the anisotropy $-1 < \Delta < 1$, the low-frequency conductivity scales anomalously with frequency, as $\sigma(\omega) \sim 1/\sqrt{\omega}$. We interpret this anomalous response as a consequence of quasiparticles undergoing Lévy flights. For special values of the anisotropy, the divergence is cut off at low frequencies, so $\sigma(\omega)$ has a finite dc limit. These results reveal a hitherto unknown mechanism for anomalous response in integrable systems and also provide a physical explanation of the discontinuous behavior of the spin Drude weight. We use our approach to recover that at the isotropic point $\Delta = 1$, $\sigma(\omega) \sim \omega^{-1/3}$. We support our results with extensive numerical studies using matrix-product operator methods.

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I. INTRODUCTION

Integrable models play a crucial part in quantum many-body physics: On one hand, they are among the few strongly interacting quantum systems for which exact results exist; on the other, their dynamics are special by virtue of their integrability. Integrable systems have extensively many conserved quantities [1–4] and stable ballistically propagating quasiparticles, unlike chaotic systems. Although integrability is technically a fine-tuned property, many experimentally relevant one-dimensional models—such as the Hubbard, Heisenberg, and Lieb-Liniger models—are either exactly or approximately integrable [5]. The dynamics of integrable and nearly integrable models have been extensively studied, both theoretically [1–4,6–16] and experimentally in ultracold gases [17–24] and low-dimensional quantum magnets (see Ref. [25] and references therein).

Although the exact dynamics of large integrable systems remains challenging, the recently developed framework of generalized hydrodynamics (GHD) has shed considerable light on their coarse-grained properties [26–29,29–47]. The picture of dynamics that emerges from GHD (as well as complementary methods, such as exact bounds [1,48–54] and numerical studies [55–64]) is rich and counterintuitive. Although integrable systems host ballistic quasiparticles, transport is not necessarily ballistic [65–67]. Some conserved quantities spread through regular or anomalous diffusion [41,42,68,69], and even when ballistic transport is present, local autocorrelation functions can decay with anomalous exponents [43,44].

The present work addresses the ac conductivity of the XXZ spin-$\frac{1}{2}$ chain, governed by the Hamiltonian

$$H = J\sum_i \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z\right). \qquad (1)$$

Here, $S_i^\alpha = \sigma_i^\alpha/2$ are spin-$\frac{1}{2}$ operators with σ_i^α the Pauli matrices on site i, the parameter Δ is the anisotropy, and J is an overall coupling scale that we will set to unity. We consider the "easy-plane" regime $-1 < \Delta < 1$, so we can parametrize $\Delta \equiv \cos(\pi\lambda)$. For concreteness we assume the system is at infinite temperature and in the thermodynamic limit (though the physics is presumably qualitatively similar at any $T > 0$ [70]). Spin transport in this model is ballistic, so the spin conductivity takes the form $\sigma(\omega) = \mathcal{D}_\lambda \delta(\omega) + \sigma_\lambda^{\text{reg.}}(\omega)$. Much is known, through exact bounds as well as GHD [29,37,48,49,54,70,71], about the behavior of $\mathcal{D}_\lambda$ (which is called the Drude weight); however, the finite-frequency part has only been studied numerically [55,72–75]. The apparent behavior of $\mathcal{D}_\lambda$ is remarkable: It appears to be discontinuous and fractal as a function of λ. When $\lambda = p/q$ is rational, several distinct methods [29,37,48,49,54,70,71] lead to the conclusion that

$$\mathcal{D}_\lambda = \frac{1}{12}(1-\Delta^2) f\left(\frac{\pi}{q}\right), \quad f(x) = \frac{3}{2}\left[\frac{1-\frac{\sin(2x)}{2x}}{\sin^2 x}\right]. \qquad (2)$$

Equation (2) is a rigorous lower bound on $\mathcal{D}$, which GHD [29,31,37,38] predicts is saturated. Remarkably, Eq. (2) allows the Drude weight to jump by $\mathcal{O}(1)$ as Δ changes infinitesimally: $\lim_{x\to 0} f(x) = 1$ for any irrational number λ but is higher by an $\mathcal{O}(1)$ amount at an arbitrarily close small-denominator rational. These jumps in the zero-frequency spectral weight strongly suggest that the finite-frequency behavior must also be nontrivial, motivating our study.

We combine GHD with exact constraints on the low-frequency behavior to arrive at a unified picture of transport in this unusual regime. For generic λ, the quasiparticles responsible for spin transport undergo a Lévy flight in addition to their ballistic motion; this leads to the scaling $\sigma(\omega) \sim 1/\sqrt{\omega}$.

For rational $\lambda = p/q$, this behavior is cut off at frequencies $\omega_q^* \sim 1/q^4$, giving rise to a finite dc conductivity $\sim q^2$. The "missing" finite-frequency spectral weight then appears as the additional Drude weight peak in Eq. (2). This is a nontrivial consistency check on our results. Finally we compare our results with extensive simulations using matrix-product operators; the numerical results are consistent with a power-law divergence of $\sigma(\omega)$, although we cannot access late enough times to fix the exponent. Our results also explain recent numerical observations on the λ dependence of finite-time response functions [76].

II. CONSTRAINTS ON $\sigma(\omega)$

The high-temperature limit of $\sigma(\omega)$ is given by the Kubo formula

$$\sigma_\lambda(\omega) = \beta \int_0^\infty dt \sum_x C_{jj}(x,t) e^{i\omega t} = \pi \mathcal{D}_\lambda \delta(\omega) + \sigma_\lambda^{\text{reg}}(\omega), \quad (3)$$

in terms of the autocorrelator $C_{jj}(x,t)$ of the current $j(x) \equiv -i(S_x^+ S_{x+1}^- - \text{H.c.})$:

$$C_{jj}(x,t;\lambda) \equiv Z^{-1}\text{Tr}\big[e^{iH_\lambda t} j(x) e^{-iH_\lambda t} j(0) e^{-\beta H_\lambda}\big]. \quad (4)$$

Here, Z is the partition function and β is the inverse temperature. In what follows we will suppress the subscript (since we discuss only one correlation function) and define $C_\lambda(t) \equiv \sum_x C_{jj}(x,t;\lambda)$. We will take the $\beta \to 0$ limit; in this limit, all response functions including $\sigma(\omega)$ vanish, but the autocorrelation function (4) is well behaved, and therefore so is the quantity $\sigma(\omega)/\beta$ (3): In the following, we will absorb this factor of $1/\beta$ in the definition of $\sigma(\omega)$. The Drude weight is defined as $\mathcal{D}_\lambda \equiv \lim_{t\to\infty} C_\lambda(t)$, and the dc conductivity is defined as $\sigma_\lambda^{\text{dc}} \equiv \lim_{\omega\to 0} \sigma_\lambda^{\text{reg}}(\omega)$.

The autocorrelator $C_\lambda(t)$, at any finite t, must be a continuous function of Δ and thus of λ: by the Lieb-Robinson theorem [77], one can truncate the infinite system on these timescales to a finite system of size $\propto t$, and all properties of finite systems evolve continuously with Δ. For some small ε, Eq. (2) implies that one can find nearby values $\lambda, \lambda + \varepsilon$ such that $\mathcal{D}_\lambda$ and $\mathcal{D}_{\lambda+\varepsilon}$ differ by a large amount. Even so, locality implies that $|C_\lambda(t) - C_{\lambda+\varepsilon}(t)|$ remains small until some late time t^*. One can easily show that $t^* \gtrsim 1/\varepsilon$. Equivalently, in the frequency domain,

$$\int_0^\Omega d\omega |\sigma_\lambda(\omega) - \sigma_{\lambda+\epsilon}(\omega)| w \lesssim C\frac{\varepsilon}{\Omega}, \quad (5)$$

where $\sigma_\lambda(\omega)$ is the *full* conductivity (3) at anisotropy λ, $\Omega > \varepsilon$ is generic, and C is a constant of order unity. Changing λ by ε can only shift spectral weight over frequencies $\sim\varepsilon$. Thus there is a characteristic frequency $\omega^*(\varepsilon) \lesssim \varepsilon$ such that for $\omega \gtrsim \omega^*(\varepsilon)$ the conductivity is essentially ε independent. The drastic rearrangement of spectral weight that gives rise to the fractal structure of $\mathcal{D}_\lambda$ (2) must happen below this frequency (Fig. 1).

We now discuss how this constraint relates the ac conductivity of an irrational λ to the dc conductivity of rational approximants. We approximate the irrational value, denoted λ_∞, by a sequence of rationals $\{\lambda_q = p/q\}$ with increasing denominators q. We assume that $C_\lambda(t)$ decays monotonically at sufficiently late times for all λ; within GHD this assumption certainly holds. By the reasoning above, until some late time t_q^*, $C_{\lambda_\infty}(t) \approx C_{\lambda_q}(t) > \mathcal{D}_{\lambda_q}$. Assuming monotonicity, therefore, $C_{\lambda_\infty}(t) - \mathcal{D}_{\lambda_\infty} > \delta\mathcal{D}_q \sim 1/q^2$, for all such large q, with $\delta\mathcal{D}_q \equiv \mathcal{D}_{\lambda_q} - \mathcal{D}_{\lambda_\infty}$.

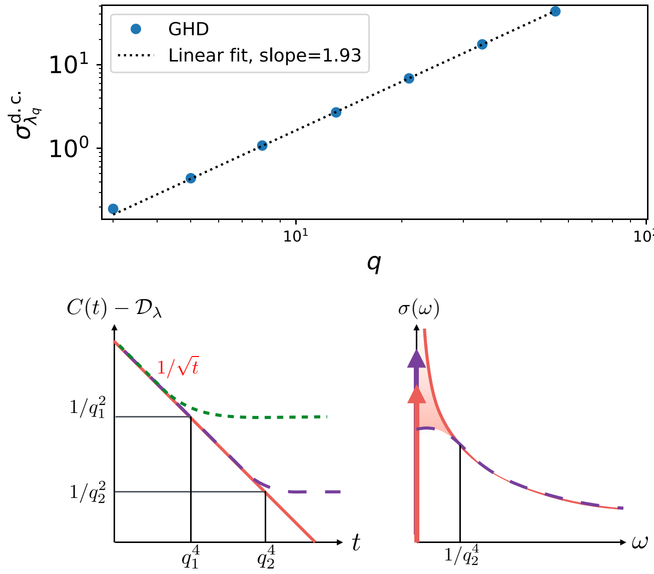

FIG. 1. Upper panel: dc conductivity for rational Fibonacci approximants $\lambda_q = F_{n-1}/F_{n+1}$ vs $q = F_{n+1}$ to the generic irrational anisotropy $\lambda_\infty \equiv 1/\varphi^2$ where φ is the golden ratio. We find $\sigma_{\lambda_q}^{\text{dc}} \sim q^\beta$ with $\beta \approx 1.93$, corresponding to $\alpha \simeq 0.49$ in Eq. (7). Lower panel: relationship between the dc conductivity for approximants, the crossover timescale, and the ac conductivity for λ_∞. Left: the autocorrelation function $C(t)$ for λ_∞ must follow that of a rational approximant with a given denominator q_i until a crossover timescale $t_{q_i}^* \sim q_i^4$ (derived in the text). This forces $C(t) \sim 1/\sqrt{t}$ for λ_∞. Right: in the frequency domain, the "excess Drude weight" at the rational approximant must precisely match the missing spectral weight in $\sigma(\omega)$ for $\omega < \omega_q^* \sim q^{-4}$.

We make the general ansatz $C_{\lambda_\infty}(t) - \mathcal{D}_{\lambda_\infty} \sim 1/t^{1-\alpha}$ [i.e., $\sigma_{\lambda_\infty}(\omega) \sim \omega^{-\alpha}$]. This ansatz fixes the crossover timescale t_q^* for large q, as follows. For $t \lesssim t_q^*$, $C_{\lambda_q}(t) \approx C_{\lambda_\infty}(t)$, whereas for $t \gtrsim t_q^*$, $C_{\lambda_q}(t) \approx \mathcal{D}_{\lambda_q}$. Equating the two forms at $t \sim t_q^*$ we find that $(t_q^*)^{1-\alpha} \sim \delta\mathcal{D}_q^{-1} \sim q^2$, so

$$t_q^* \sim q^{2/(1-\alpha)}. \quad (6)$$

Finally, we relate this to the dc conductivity $\sigma_{\lambda_q}^{\text{dc}}$ at λ_q. This is the integral of $C_{\lambda_q}(t) - \mathcal{D}_{\lambda_q}$, which follows the power-law $1/t^{1-\alpha}$ and is cut off at time t_q^*. Combining this result with Eq. (6) we find that

$$\sigma_{\lambda_q}^{\text{dc}} \sim q^{2\alpha/(1-\alpha)}, \quad \sigma_{\lambda_\infty}(\omega) \sim \omega^{-\alpha}, \quad (7)$$

where $\alpha \geqslant 0$. Indeed this reasoning can be used to show that $\sigma(\omega)$ diverges, even without invoking GHD. By Dirichlet's approximation theorem, $|\lambda_q - \lambda_\infty| \lesssim 1/q^2$. Therefore, $t_q^* \gtrsim q^2$, so $C_{\lambda_\infty}(t) - \mathcal{D}_{\lambda_\infty} \gtrsim 1/t$. Fourier transforming gives $\sigma(\omega) \gtrsim |\log \omega|$ at low frequencies, establishing a divergence. (This divergence had previously been predicted using GHD [68].)

Equation (7) can be derived directly in frequency space, as follows. Suppose $\sigma_{\lambda_\infty}(\omega) \sim \omega^{-\alpha}$. Then by Eq. (5), $\int_0^{\omega_q^*} d\omega[\sigma^{\rm reg}_{\lambda_\infty}(\omega) - \sigma^{\rm dc}_{\lambda_q}] \simeq \delta\mathcal{D}_q \sim 1/q^2$: The extra Drude weight at the commensurate point must precisely match the missing part of the regular spectral weight (Fig. 1). Thus, $[\omega_q^*]^{1-\alpha} \sim 1/q^2$, so $\omega_q^* \sim q^{-2/(1-\alpha)}$, consistent with Eq. (6) and $\omega_q^* \sim 1/t_q^*$.

III. GENERALIZED HYDRODYNAMICS

The argument above shows that $\sigma(\omega)$ must diverge at low frequencies for irrational λ. However, Eq. (7) does not determine the exponent α. To do this we adopt the framework of generalized hydrodynamics (GHD) [26,27], which was recently extended to incorporate diffusion [39–41]. GHD describes the response of integrable systems in the hydrodynamic regime, i.e., when the system is locally in equilibrium in a generalized Gibbs ensemble [3,4,7,12,78–82]. In this regime, the dynamics of an integrable system maps onto that of a classical soliton gas [34]. Solitons (i.e., the quasiparticles of the integrable system) propagate ballistically but acquire time delays when they scatter elastically off each other [34,40,83,84]. Gaussian fluctuations of the quasiparticle densities lead to fluctuations of the distance traveled by each quasiparticle [40]. Each quasiparticle thus follows a biased random walk. Since the quasiparticles carry conserved charges, such as spin, these charges also pick up subleading diffusive corrections to ballistic transport: The variance of the spin current is related to the variance of quasiparticle velocities due to collisions. For the dc conductivity $\sigma^{\rm dc}_{\lambda_q}$, we have the relation [39,41]

$$\sigma^{\rm dc}_{\lambda_q} = \frac{1}{4}\sum_{kl}\int d\theta_1 d\theta_2 \rho_k(\theta_1)\rho_l(\theta_2) f_k f_l |v_k(\theta_1) - v_l(\theta_2)| \times \left[\mathcal{K}^{\rm dr}_{kl}(\theta_1-\theta_2)\left(\frac{m_k^{\rm dr}}{\rho_k^{\rm tot}(\theta_1)\sigma_k} - \frac{m_l^{\rm dr}}{\rho_l^{\rm tot}(\theta_2)\sigma_l}\right)\right]^2 \quad (8)$$

in terms of data from the thermodynamic Bethe ansatz (TBA) [5]. In this expression, k, l label quasiparticle species and θ_i label rapidities, and the other symbols denote properties (within TBA) of quasiparticles with labels k, θ: $\rho_k(\theta)$ is the density of quasiparticles, $f_k = 1 - \rho_k(\theta)/\rho_k^{\rm tot}(\theta)$ is related to their filling factor (independent from θ at infinite temperature), $\rho_k^{\rm tot}(\theta)$ is the total density of states, $v_k(\theta)$ and $m_k^{\rm dr}$ are, respectively, the dressed velocity—derived from the dressed dispersion relation—and dressed magnetization, and $\sigma_k = \pm 1$ is the so-called σ *parity* of quasiparticle species k. The dressed kernel $\mathcal{K}^{\rm dr}$ is the solution to an integral equation that has to be solved numerically [85]. For a brief overview of the TBA formalism as it applies here see Ref. [85]; for more details we refer to Ref. [5].

As a generic irrational number, we choose $\lambda_\infty = 1/\varphi^2$ where φ is the golden ratio. This number is generic in the sense that it is poorly approximable by rationals; in this it resembles almost all real numbers [86]. The TBA for this number has the advantage of being tractable, with a simple quasiparticle hierarchy. We have checked other irrationals in Ref. [85]. The continued fraction expansion of $\varphi^2 = 1/(2 + 1/(1 + 1/(\ldots)))$. Truncating this expansion by replacing the last term with 2 gives the series $\lambda_n = F_{n-1}/F_{n+1}$, where F_n is the nth Fibonacci number. The Bethe ansatz solution for λ_n involves n quasiparticle species. At zero field, the first $n-2$ quasiparticle species carry no dressed magnetization; the last two quasiparticle species each carry a magnetization $\sim F_{n+1} = q$ and are responsible for the spin Drude weight. We refer to quasiparticles with larger values of n as being "larger," which is true at lattice scale; however, GHD treats all quasiparticles as pointlike. Spin transport is dominated by charged quasiparticles; the other, "neutral" quasiparticles affect spin transport by scattering elastically off the charged quasiparticles and causing them to diffuse.

GHD yields the following conclusions for spin transport. Charged quasiparticles move with a characteristic velocity which saturates to an $\mathcal{O}(1)$ value as $q \to \infty$, and as they move they scatter off neutral quasiparticles. Large neutral quasiparticles are rare $\rho_{n-2} \sim q^{-2}$ but also have an outsized influence, because their scattering phase shifts are large. Figure 2 separates out the contributions to $\sigma^{\rm dc}_{\lambda_q}$ by quasiparticle index/size: Evidently the dominant contribution comes from scattering off the largest neutral quasiparticle. Explicitly evaluating Eq. (8) with the appropriate TBA data we find that $\sigma_{\rm dc}(\lambda_q) \sim q^2$. This asymptotics can be derived analytically [85] and is consistent with numerical evaluation of Eq. (8) (Fig. 1). Using Eq. (7) this means that

$$\sigma_{\lambda_\infty}(\omega) \sim 1/\sqrt{\omega}, \quad (9)$$

and therefore that $t_q^* \sim q^4$.

IV. SOLITON GAS PICTURE

This long crossover timescale has a physical interpretation in terms of the semiclassical soliton gas framework [34,40]. The dressed kernel $\mathcal{K}^{\rm dr}(\theta)$ is peaked at $\theta = 0$, with a peak height that scales as q and a peak width that scales as $1/q$ (Fig. 2). The dominant scattering events that a charged particle experiences are those with large neutral quasiparticles which have almost the same rapidity and therefore almost the same velocity (up to $\sim 1/q$). At large q the heaviest neutral quasiparticle has density $1/q^2$; fixing its rapidity to a window of size $1/q$ reduces the density of dominant scatterers to $1/q^3$. Since the two quasiparticles start out spaced at a distance q^3 and have a relative velocity $\sim 1/q$, they collide on a timescale $t_q^* \sim q^4$. At much shorter timescales, the system is not in local equilibrium and the asymptotic result (8) does not apply.

One can derive further physical insight by applying the soliton-gas framework to the motion of the charged quasiparticle at very large q but for $t \ll t_q^*$. In this regime, as time passes, the charged quasiparticle encounters increasingly large neutral quasiparticles and therefore picks up increasingly large displacements. On timescale t, the largest collision will involve a quasiparticle for which $q(t) \sim t^{1/4}$. This quasiparticle gives a (dressed) displacement [31,34,87] of order $\Delta x^{\rm dr} = \mathcal{K}^{\rm dr}/p'(\theta) \sim q^3 \sim t^{3/4}$ where we have used the fact that the dressed momentum scales as $p'(\theta) \sim \rho^{\rm tot}(\theta) \sim q^{-2}$. Therefore, the variance of the position of the charged quasiparticle scales as $t^{3/2}$, consistent with our exponent for the conductivity. Since the charged quasiparticle spreads through kicks of power-law increasing strength, whose probability also falls off as a power law, it is a Lévy flight with dynamical

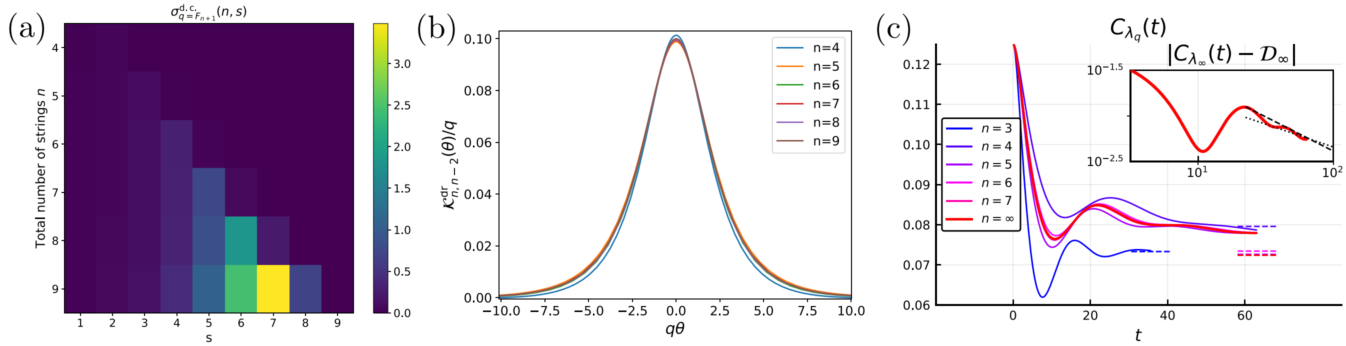

FIG. 2. (a) Contributions to the dc conductivity of the charged quasiparticle from scattering off each species of neutral quasiparticle. For any given n, the dominant source of diffusion is the heaviest neutral quasiparticle $n-2$. (b) Rapidity dependence of the dressed kernel for scattering between the charged quasiparticle and the largest neutral quasiparticle; we find that $K^{\mathrm{dr}}_{n,n-2}(\theta)$ has a peak of height q_n and width $1/q_n$, as shown by the data collapse. (c) TEBD numerics for the current-current correlator for various n; plots for the larger n stay close to the $n=\infty$ value at the accessible times. Inset: Power-law decay of $C_{\lambda_\infty}(t) - \mathcal{D}_{\lambda_\infty}$: Although our time range is limited, our data is consistent with an exponent $1-\alpha \in (\frac{1}{2}, \frac{3}{4})$ (dashed lines).

exponent $z = 4/3$ [88]. It would be interesting to compare the spin structure factor to known scaling forms for Lévy flights.

V. TEBD SIMULATIONS

To check our analysis, we have also explicitly evaluated $C(t)$ (4). At infinite temperature Eq. (4) takes the form $C(x,t) = 2^{-L}\mathrm{Tr}[j(x,t/2)j(0,-t/2)]$. By translation invariance, the operator $j(x,t/2)$ is just a translated version of the operator $j(0,t/2) = -j^{\dagger}(0,-t/2)$. Thus to evaluate $C(x,t)$ (4) it suffices to time evolve a *single* local operator. This can be done using the time-evolving block decimation method [89–91] for matrix-product operators [92,93]. This simplification allows us to save considerable computational overhead and study systems in the thermodynamic limit for times up to $t \approx 60$ working at a fixed bond dimension $\chi = 512$. At even longer times, errors accumulate and give unphysical results [85], but for $t \lesssim 60$ (Fig. 2) the errors remain small.

Our results are shown in Fig. 2. At the accessible timescales, data for the larger n stay close to that for $n = 4$ and far from their asymptotic values (dashed lines). This confirms our picture that the decay is a slow process. The inset shows the evolution of $C_{\lambda_\infty}(t) - \mathcal{D}_{\lambda_\infty}$ on a log-log plot; although our dynamic range is limited, the data support a power law, with an exponent $1-\alpha \in (\frac{1}{2}, \frac{3}{4})$, roughly consistent with our predictions. Note that at times $t \sim 100$ one can only reach the crossover timescale for $q \sim 3$, so we are far from the asymptotic behavior.

Finally, our results also explain the numerical observation [76] that $C_\lambda(t)$ at a fixed time t has peaks in λ whose width scales as $t^{-1/2}$. The crossover timescale for resolving that the system is $\delta\lambda$ from the peak is $t \sim 1/(\delta\lambda)^2$, implying this result.

VI. DISCUSSION

In closing, we apply our approach to the isotropic point $\Delta = 1$ using the rational series $\lambda_q = 1/q$ as $q \to \infty$. In that limit, the Drude weight vanishes $\mathcal{D}_{\lambda_\infty} = 0$, but we still have $\delta\mathcal{D}_q \sim q^{-2}$ from (2). Therefore, Eqs. (7) still hold, but Eq. (8) now predicts $\sigma^{\mathrm{dc}}_q \sim q$, corresponding to $\alpha = 1/3$. This yields $\sigma_{\lambda_\infty}(\omega) \sim \omega^{-1/3}$ at the isotropic point, consistent with earlier results [42,60,94]. The corresponding time scale $t^*_q \sim q^3$ was also identified approaching the isotropic point from $\Delta > 1$ [42]. The Lévy flight mechanism discussed here is different from the apparent Kardar-Parisi-Zhang mechanism for superdiffusion at $\Delta = 1$ [44,61,69], in terms of both the dynamical exponents and scaling function. This raises the question of whether there are "universality classes" of integrable dynamics and of what types of dynamical scaling are possible in integrable models. (Even in the present case, for special irrational numbers that are closer to their approximants, we expect longer crossover timescales and thus parametrically steeper divergences in $\sigma(\omega)$; in the specific instance of Liouville numbers, our arguments imply that $\sigma(\omega) \sim 1/\omega$ up to sub-power-law corrections.)

Within GHD the ultimate significance of the value $\alpha = 1/2$ is not obvious. However, the crossover scale $t^*_q \sim q^4 \sim 1/|\lambda_\infty - \lambda_q|^2$ resembles a golden rule rate, suggesting the following interpretation: If one starts in an eigenstate of H_{λ_∞} and turns on the perturbation $\delta H \equiv H_{\lambda_q} - H_{\lambda_\infty}$, the quasiparticle structure changes considerably. The quasiparticles at H_{λ_∞} are no longer stable, and it is natural to suppose that their decay rates scale as $|\lambda_q - \lambda_\infty|^2$, yielding $t^*_q \sim q^4$. Developing a golden rule analysis [95] of such quenches between integrable systems is an interesting topic for future work.

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