

Locating the pseudogap closing point in cuprate superconductors: Absence of entrant or reentrant behavior

Jeffery L. Tallon ^{1,*} James G. Storey ¹ John R. Cooper ² and John W. Loram ^{2,†}

¹Robinson Research Institute, Victoria University of Wellington, P.O. Box 33436, Lower Hutt 5046, New Zealand

²Cavendish Laboratory, Cambridge University, Cambridge CB3 0HE, United Kingdom



(Received 30 July 2019; revised manuscript received 20 April 2020; accepted 28 April 2020; published 21 May 2020)

Many current descriptions of the pseudogap in underdoped cuprates envision a doping-dependent transition line $T^*(p)$ which descends monotonically toward zero just beyond optimal doping. There is much debate as to the location of the terminal point p^* where $T^*(p)$ vanishes, whether or not there is a phase transition at T^* and exactly how $T^*(p)$ behaves below T_c within the superconducting dome. One perspective sees $T^*(p)$ cutting the dome and continuing to descend monotonically to zero at $p^* \approx 0.19$ holes/Cu—referred to here as entrant behavior. Another perspective derived from photoemission studies is that $T^*(p)$ intersects the dome near $p \approx 0.23$ holes/Cu then turns back below T_c , falling to zero again around $p^* \approx 0.19$ —referred to here as reentrant behavior. By examining field-dependent thermodynamic data for $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ we show that neither entrant nor reentrant behavior is supported. Rather, p^* sharply delimits the pseudogap regime: For $p < 0.19$ the pseudogap is always present, independent of T . Similar results are found for $\text{Y}_{0.8}\text{Ca}_{0.2}\text{Ba}_2\text{Cu}_3\text{O}_{7-\delta}$. For both materials, $T^*(p)$ is not a temperature but a crossover scale, $\approx E^*(p)/2k_B$, reflecting instead the underlying pseudogap energy $E^*(p)$ which vanishes as $p \rightarrow p^* \approx 0.19$.

DOI: [10.1103/PhysRevB.101.174512](https://doi.org/10.1103/PhysRevB.101.174512)

I. INTRODUCTION

Hole-doped cuprate superconductors, at and below optimal doping, are characterized by the opening of a partial gap in the electronic density of states (DOS), the so-called pseudogap [1,2], which profoundly affects all spectroscopic properties and, below the transition temperature T_c , results in an abrupt crossover from “strong” to “weak” superconductivity [3–5]. Recently, it has become evident that the underlying behavior involves a change of the Fermi surface from large, with area $1 + p$, to either Fermi arcs [6] or small hole pockets on the zone diagonal near the antiferromagnetic (AF) zone boundary and having area p [7–11]. Evidence for this change can be found in angle-resolved photoelectron spectroscopy (ARPES) [12,13], quasiparticle interference in scanning tunneling spectroscopy [9,10], and the reported crossover observed in the normal state at very high magnetic field from Hall number $n_H = 1 + p$ to $n_H = p$ [7,11]. (However, recent high-field Hall measurements on $\text{Tl}_2\text{Ba}_2\text{CuO}_6$ and $\text{Bi}_2\text{Sr}_2\text{CuO}_6$ do not support this interpretation [14]—see Appendix C).

It has long been known that the *apparent* characteristic pseudogap temperature, T^* , below which pseudogap effects are often reported, falls with increasing hole concentration, p , and vanishes at a critical doping, $p^* \approx 0.19$ holes/Cu [15,16]. Despite an intensive search, no specific heat anomaly has been reported at T^* [3,17], thus implying that $T^*(p)$ is not a thermodynamic phase transition line. Indeed, this line was

originally reported as an *energy scale*, $E^*(p)$, which descends to zero at p^* , not a *temperature scale* [18–21]. Later reports, however, suggest that there is indeed some kind of mean-field transition occurring in the vicinity of T^* with various order-parameter-like properties observed to vanish there. The measurements include polarized neutron scattering [22], polar Kerr effect [23], ARPES [24], time-resolved reflectivity [25], resonant ultrasound spectroscopy [26], and susceptibility nematicity [27]. The last two techniques were presented as clear evidence of a thermodynamic transition [26,27], although in the former case we have questioned this interpretation [17] and any thermodynamic effects are evidently extremely weak. In contrast, the pseudogap effects reported from the specific heat are very strong, involving a large suppression of entropy and a consequent suppression of the superconducting jump at T_c , the condensation energy, and the superfluid density [3,4]. This necessarily raises the question as to whether these obviously disparate results are at all related. Also of special relevance to the present paper, several prominent studies on $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ (Bi2212) identified the termination point of the pseudogap as located at a much higher doping of $p^* \approx 0.23$, at or near a proposed Lifshitz transition from a holelike to electronlike Fermi surface [28]. These include ARPES [29,30], Raman [31], and transport [32] studies.

The problem stated

It is obvious from the above that there is a clear contradiction between these more recent studies and the earlier thermodynamic, spectroscopic, and transport studies. In the present paper, we seek to address and resolve this contradiction.

*jeff.tallon@vuw.ac.nz

†Deceased.

Additionally, we examine the thermodynamic behavior in the neighborhood of p^* within the superconducting dome to search for what we call entrant or reentrant behavior. If T^* delineates the opening of the pseudogap, as claimed, and if the pseudogap is responsible for a large loss of spectral weight as is obvious from thermodynamic, NMR [33,34], infrared [35], and superfluid density [4] measurements, then there should be radical changes in the superconducting state when the temperature falls below $T^* < T_c$. On cooling below T_c , one would expect the condensation free energy, critical fields, and superfluid density to initially grow as if there were no pseudogap, consistent with the strong superconductivity seen in the overdoped region. Then, on crossing the T^* line, the mooted opening of the pseudogap would deplete this spectral weight such that these thermodynamic parameters would grow much more slowly on further cooling and perhaps even reduce in magnitude. Indeed, if there is a mean-field phase transition at T^* , then the T -dependent slope of these properties will change discontinuously. We refer to this general behavior as entrant, where the slope of $T^*(p)$ below T_c remains negative as depicted in the inset to Fig. 2(b). In the following, we present a search for such behavior.

Further, we also test the converse of this, namely, if the slope of $T^*(p)$ below T_c were positive as depicted by the gray line in Fig. 2(a). We refer to this as reentrant behavior. The motivation for this scenario is found in the ARPES study by Vishik *et al.* [29]. These authors claim to observe the pseudogap out to $p \approx 0.23$ just above T_c . $T^*(p)$ is then suggested to “back-bend,” adopting a positive slope and falling to zero at $p^* = 0.19$. (This proposed back-bending phase diagram is also invoked by He *et al.* [30]). If this were the case, then between $0.19 < p < 0.23$, cooling below T_c means that the initial onset of superconductivity just below T_c is already within the pseudogap state and all measures (critical fields, condensation energy, and superfluid density) should then indicate weak superconductivity. However, on crossing the reentrant $T^*(p)$ line, the superconductor will exit the pseudogap state into a strong superconductivity regime in which these measures are no longer suppressed and they will grow much more rapidly with further decreasing temperature. Such reentrant behavior has both theoretical support [36] and is experimentally observed in $\text{Ba}(\text{Fe}_{1-x}\text{Co}_x)_2\text{As}_2$ [37].

The anticipated thermodynamic behavior is illustrated in Fig. 1. Figure 1(a) shows our calculation (black dashed curve) of the weak-coupling d -wave condensation free energy, $\Delta F (= \Delta F_{\text{ns}} = F_n - F_s)$, as fitted to our experimental data for $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ ($\text{Bi}2212$) just above p^* , i.e., without a pseudogap (see Fig. 2 and also Won and Maki [38]). The second black dashed curve is ΔF with the same temperature dependence but rescaled down in magnitude to give the same ground-state condensation energy observed in our $\text{Bi}2212$ samples at optimal doping, $p = 0.16$, where $T^* \approx T_c$, i.e., with a pseudogap present. We use a $\tanh[(T - 30)/T_0]$ crossover function between the two $\Delta F(T)$ curves with $T_0 = 5, 10,$ and 15 K, as shown by the red, green, and blue curves, respectively. These curves depict the effect of reentrant behavior on the condensation energy, depending on how abrupt the crossover is. Literature examples, illustrative of this kind of behavior, include a low-temperature boost in superfluid density seen in $\text{Ba}_{1-x}\text{K}_x\text{Fe}_2\text{As}_2$ [39], and a boost in critical

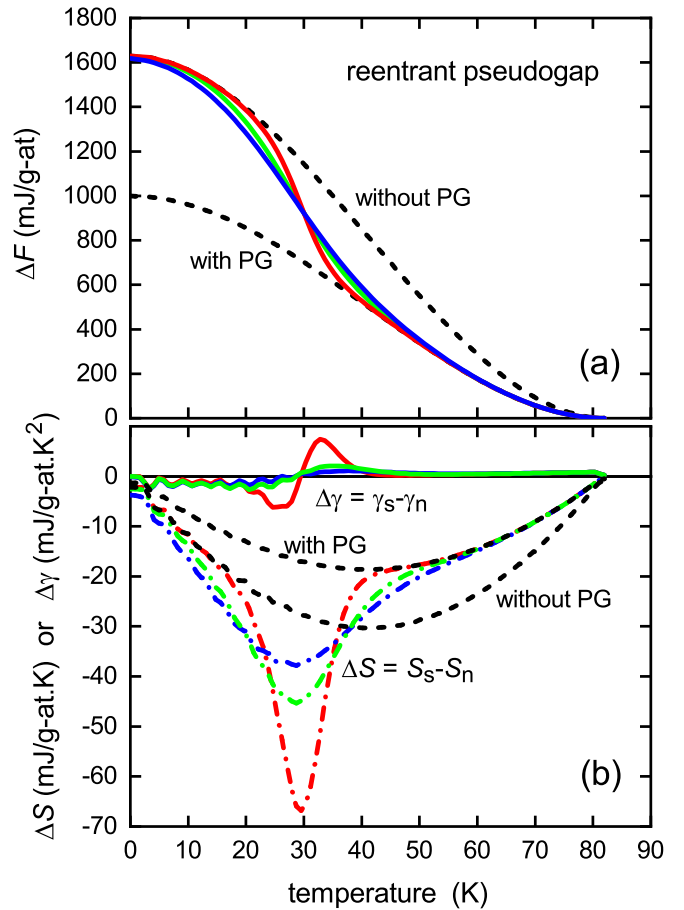


FIG. 1. The anticipated thermodynamic effects of reentrant crossover into the pseudogap on cooling in the superconducting state (a) condensation free energy, (b) entropy, $\Delta S(T)$ (dash-dot curves), and specific heat coefficient, $\Delta\gamma(T)$ (solid curves). The dashed curves are the weak-coupling d -wave behavior, with $\Delta F(0)$ reduced by the pseudogap. The crossover is implemented using a $\tanh[(T - 30)/T_0]$ crossover function with $T_0 = 5, 10,$ and 15 K as shown by the red, green, and blue curves, respectively. In such a crossover, $\Delta S(T)$ is characterized by two inflexion points and $\Delta\gamma(T)$ by an additional large anomaly near 30 K.

current density and superfluid density seen in $\text{PrOs}_4\text{Sb}_{12}$ [40,41].

If such a boost does occur, then Fig. 1(b) shows the effect on $\Delta S = -\partial\Delta F/\partial T$ (dash-dot curves) and $\Delta\gamma = \partial\Delta S/\partial T$ (solid curves), with red, green, and blue having the same meaning as in Fig. 1(a). This illustrates quite generically a reasonably model-free expectation for a reentrant crossover, namely, a double inflexion of $\Delta S(T)$ and an additional anomaly in $\Delta\gamma(T)$ below T_c that can be very large, depending on the narrowness of the crossover. The calculated $\Delta\gamma$ anomaly values of 7.4, 2.1, or 1.1 mJ/g.at.K² (for $T_0 = 5, 10,$ or 15 K, respectively) are easily detectable by our differential measurements which are sensitive to ± 0.05 mJ/g.at.K².

For an entrant pseudogap, there will be a corresponding crossover from the upper dashed curve in Fig. 1(a) to the lower dashed curve, with consequent marked anomalies in ΔS and $\Delta\gamma$, including the possibility of a positive excursion in ΔS for a narrow crossover.

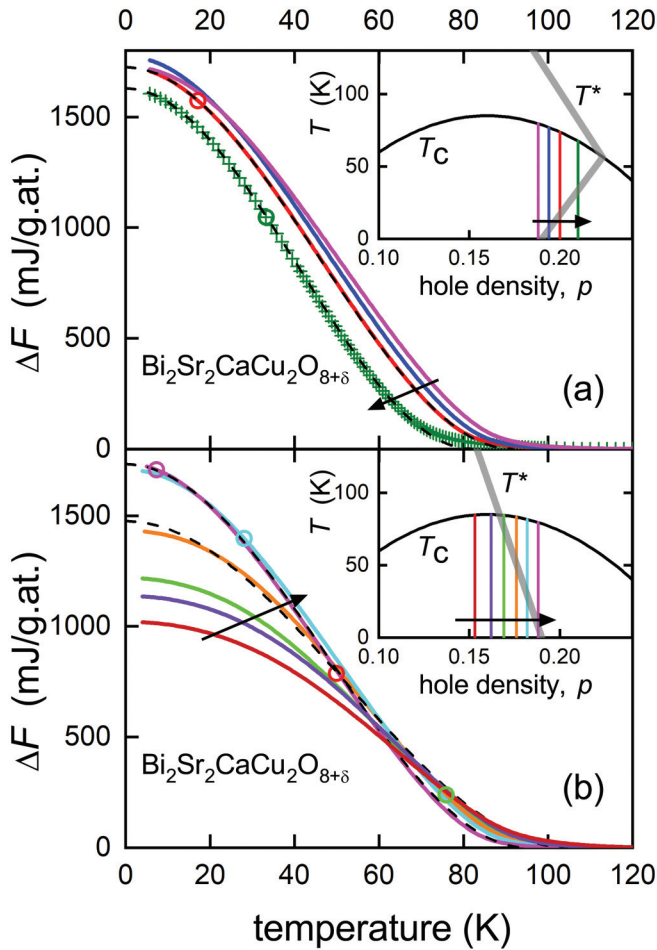


FIG. 2. The two $T^*(p)$ scenarios tested in this work (grey lines in insets): (a) reentrant, where the putative pseudogap line meets the $T_c(p)$ phase curve then backbends to fall to zero at critical doping $p^* = 0.19$ and (b) entrant, where the putative pseudogap line $T^*(p)$ falls monotonically to zero at $p^* = 0.19$ with positive slope. Colored curves in the main panels show the measured condensation free energy, $\Delta F(T)$, obtained by integrating the electronic condensation entropy, $\Delta S(T)$. The curves in each main panel are at doping levels indicated by the color-coded vertical lines on the phase diagrams shown in the insets. The excellent agreement with near-weak-coupling d -wave BCS free-energy calculations (four dashed curves) rule out both the reentrant and entrant scenarios. Small deviations near T_c are due to superconducting fluctuations.

As noted, Vishik *et al.* [29] are not alone in proposing that the normal-state pseudogap persists to $p \approx 0.23$. Raman scattering in B_{1g} symmetry has been interpreted to suggest that the pseudogap just above T_c extends to $p \approx 0.23$, the putative location of the Van Hove singularity (VHS), but not beyond [31]. Legros *et al.* [32], using high-field transport studies, promoted the same picture. These would give additional apparent support to a reentrant phase diagram.

Either way, entrant or reentrant behavior should give a complex, non-BCS-like T dependence of critical fields, condensation energy, and superfluid density providing either a downturn or a boost, respectively, to these properties on traversing the T^* line. If there is a mean-field thermodynamic transition at T^* , then these changes will be abrupt. Moreover,

as the pseudogap is effective in causing a large reduction in electronic entropy [3], these effects should be substantial. Here we report the complete absence of such anomalous thermodynamic features. This includes the absence of inflexion points in the electronic entropy, in the entropy difference between normal and superconducting states, and in the entropy change in applied field. Neither do we observe any anomaly in the various free energies investigated, including the field-dependent free energy.

In Appendix B, we summarize the extensive evidence from many different spectroscopies and transport measurements showing that the pseudogap closes at $p \approx 0.19$. Appendix C briefly argues that this result is also applicable to a wider group of what we call *canonical* cuprates.

II. EXPERIMENTAL DETAILS

The ARPES [29], Raman [31], and high-field transport [32] studies, whose conclusions we dispute, all concern the generic system $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ (Bi2212). As a consequence, we focus solely on this system so we are discussing precisely the same materials. However, we note that almost identical results were obtained for $\text{Y}_{0.8}\text{Ca}_{0.2}\text{Ba}_2\text{Cu}_3\text{O}_{7-\delta}$ (Y-123) which, like Bi2212, can also be overdoped.

Differential specific heat measurements on Bi2212 samples were originally made in 1998–2000 and the basic results were reported in two publications [3,42]. However, we present here additional details, analysis, and insights. We synthesised and investigated three distinct polycrystalline samples: $\text{Bi}_{2.1}\text{Sr}_{1.9}\text{CaCu}_2\text{O}_{8+\delta}$, $\text{Bi}_{2.1}\text{Sr}_{1.9}\text{Ca}_{0.7}\text{Y}_{0.3}\text{Cu}_2\text{O}_{8+\delta}$, and $\text{Bi}_{1.8}\text{Pb}_{0.3}\text{Sr}_{1.9}\text{CaCu}_2\text{O}_{8+\delta}$, where the second material allows lower doping and the third allows higher hole doping than the parent material. The 0.1 excess Bi is found to reside on the Sr site [43] and is necessary to achieve single-phase materials. The samples were synthesised by repeated solid-state reaction in stoichiometric quantities and each, including the reference, was approximately 0.9 g in weight. The samples were fully oxygenated (overdoped) then subjected to a series of anneals progressively lowering the oxygen content and hence the doping state. Temperature- and field-dependent specific heat measurements were carried out for each successive doping level.

As described previously [42], the measurements use a high-resolution differential technique where the specific heat difference between the sample and a reference is measured to a precision of 1 part in 10^5 . The reference was chosen to be 4% cobalt-doped $\text{Bi}_{2.1}\text{Sr}_{1.9}\text{CaCu}_2\text{O}_{8+\delta}$, where the Co substitutes on the Cu sites, reducing T_c rapidly due to strong scattering in combination with a nodal d -wave order parameter. While in other cuprates the chosen reference has been a Zn-substituted sample [3], the solubility of Zn in Bi2212 is low, while that of Co is up to at least 10% [44]. With 4% Co substitution, T_c can be reduced to zero by deoxygenation so there is no superconducting anomaly observed in the specific heat coefficient of the reference sample.

The mass of the reference is chosen so as to have as close as possible the same number of atoms as the sample. The differential technique thus allows most of the phonon term to be backed off, enabling the much smaller electronic specific heat to be separated from the lattice term [45]. This still leaves

a small residual phonon term which peaks at 16.5 K, but this is found to scale precisely with the change in oxygen content of the sample and so can be identified and eliminated [42]. Then, because a sequence of 11 doping states were studied per sample, the deduced electronic specific heat coefficient, $\gamma(T)$, is differenced relative to the end doping state, thus automatically removing any possible residual contribution from the reference.

Most significantly, it was found that, for the same doping state, the three Bi2212 compositions yielded essentially identical thermodynamic parameters (see Fig. 3 of Ref. [3]), so we conclude that the thermodynamic properties are independent of cation cross substitution (and associated minor disorder), they are essentially dependent only on doping and thus represent the generic thermodynamic properties of this system. The extracted data is thus applicable to, and directly comparable with, the various Bi2212 samples used in the above-noted ARPES, Raman, and high-field transport measurements. The calculation of various thermodynamic functions from the data, including the superfluid density and the relationship between electronic entropy and spin susceptibility, is described in Appendix A. Henceforth, all references to thermodynamic functions concern the electronic term only and so we now drop the descriptor “electronic.”

Finally, for consistency of comparison, we evaluate doping, p , in the same manner as used in the other spectroscopic studies which we discuss, namely, from the ratio of T_c/T_c^{\max} by inverting the empirical parabolic phase curve $T_c = T_c^{\max}[1 - 82.6(p - 0.16)^2]$ [46]. In addition, we also measured the room temperature thermopower to determine p [47] and this gave completely consistent results. Further, the residual phonon term in $\gamma(T)$ scales precisely with changing oxygen content, thus enabling very precise measures of increments in δ in the chemical formulas. As shown previously [46,48], we find $\Delta\delta = \Delta p$ as expected if the two doped holes per additional oxygen are shared between the two CuO_2 planes per formula unit.

Using these characterizations we may be more precise in our definition of the pseudogap closing point, described above as $p^* \approx 0.19$, by the rather narrow range $p^* = 0.19 \pm 0.005$.

III. RESULTS AND ANALYSIS

A. Condensation free energy

Figure 2 shows the condensation free energy, $\Delta F(T) = \Delta F_{\text{ns}}$, calculated from the experimental data by integrating $\Delta S_{\text{ns}} = S_n - S_n^{\text{expt}}$ for nine different doping states. As described in Appendix A 2, the normal-state entropy S_n (and γ_n) are calculated from a rigid ARPES-derived dispersion with our previously reported pseudogap model [49], including ungapped Fermi arcs. (Note that the $\Delta F(T)$ data reported elsewhere [50] were for a nodal pseudogap model. The impact of the two differing models on $\Delta F(T)$ is compared in Appendix D).

The magenta curves in Figs. 2(a) and 2(b) are at critical doping $p = p^* = 0.188$ while (a) shows three other curves for higher doping ($p = 0.194, 0.20, 0.21$) which straddle the putative reentrant $T^*(p)$ line of Vishik *et al.* [29], and (b) shows five other curves for lower doping ($p = 0.182, 0.176, 0.169,$

0.162, 0.153) which straddle the putative entrant $T^*(p)$ line reported, for example, by Zaki *et al.* [51] or Naqib *et al.* [16]. The data curves are plotted as linear segments (rather than smooth spline curves) but the quality of the data is illustrated in panel (a) for the most heavily doped sample (olive green curve) where just every fourth data point is plotted using the green crosses.

The black dashed curves in both panels are our calculated mean-field near-weak-coupling d -wave temperature dependence of ΔF . The data is fitted simply by setting the magnitude of the mean-field transition temperature, T_c^{mf} , and the value of the ground-state condensation energy, $\Delta F(0) \equiv \Delta U(0)$. We used the Padamsee α model [52] recalculated for d -wave superconductivity with the value $2\alpha = 2\Delta/(k_B T_c^{\text{mf}}) = 4.5$, as inferred previously [50,53]. (Note that the weak-coupling value is 4.288 [38]). There is an excellent match between the simple mean-field behavior and the observed condensation free energy across the entire T range except close to T_c where superconducting fluctuations are the cause of the small discrepancies. In the underdoped samples, the discrepancy due to fluctuations is larger, mainly due to the fact that T_c^{mf} is so much greater than T_c here, but also because our Padamsee model does not include an antinodal pseudogap which also affects the T dependence near T_c . However, it is important to note that these small fluctuation-induced deviations seen here in the underdoped region below p_{crit} are the opposite to what would be expected in the entrant scenario. On cooling toward and below T_c we see that $\Delta F(T)$ actually rises more slowly at first (due to fluctuations), then quickly develops its full weight, with the T dependence over most of the range below T_c following the simple mean-field behavior.

Lastly, the color-coded circles on each curve in both panels show where the putative T^* is expected from the two insert figures. Notably, there are no knees or kinks observed in $\Delta F(T)$ at these points.

The excellent match between mean-field behavior and the observed condensation energy is significant in light of the mooted reentrant (a) or entrant (b) behavior in Fig. 2. In the former case, $\Delta F(T)$ should rise more slowly at first as though heading for a small ground-state value (reduced due to the pseudogap), then abruptly upturn at T^* (see circles) as the superconductor moves out of the reentrant pseudogap state. Instead, all of these overdoped samples follow the canonical behavior for a single order parameter, all exhibiting strong superconductivity with a similar ground-state condensation energy. Indeed, it has been shown that, across this overdoped region, the BCS ratio $\Delta F(0)/(\gamma_n k_B T_{c,\text{mf}}^2)$ adopts a constant value of ≈ 0.17 , as expected for near weak-coupling d -wave superconductivity [50]. The small decrease in $\Delta F(0)$ at the highest doping is therefore simply due to the fall in T_c on the overdoped side. In contrast, Fig. 2(b) shows a rapid fall in $\Delta F(0)$ as doping falls below p^* . This is due to the abrupt opening of the pseudogap at p^* removing antinodal states that would otherwise be available for superconductivity. It is clear that these states are completely removed at all temperatures below T_c and not just at a putative T^* (circles) below T_c . Despite this rapid fall in $\Delta F(0)$, each of the curves rises monotonically, free of any semblance of a knee and consistent with a single-order-parameter mean-field behavior as shown by the dashed curves.

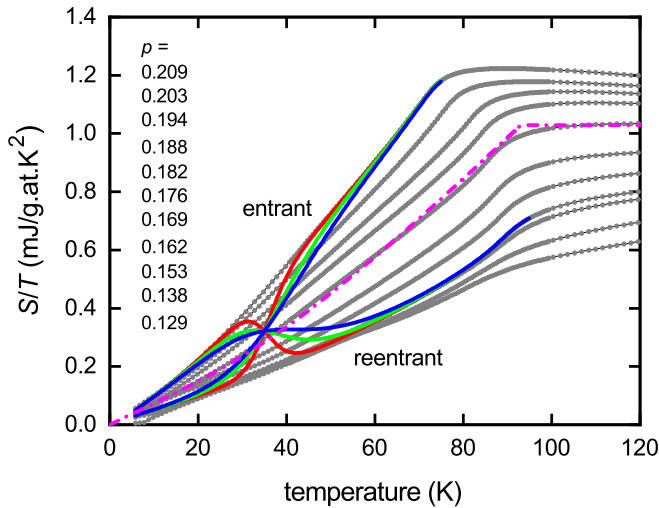


FIG. 3. The temperature dependence of S/T for pure Bi2212. Gray data points: experimental entropy data (every fourth data point shown). Magenta dash/dot curve: Our calculation of S/T using the Padamsee α -model for a d -wave order parameter in the near-weak-coupling case where $2\Delta/k_B T_c = 4.5$. The expected entrant or reentrant crossover behavior into, and out of, the pseudogapped state is shown by the red, green, and blue curves. The crossover is implemented using a $\tanh[(T - 35)/T_0]$ function with $T_0 = 5, 10,$ and 15 K as shown by the red, green, and blue curves, respectively. No such crossover is evident, even weakly so, in the experimental data.

Crucially, this conclusion is independent of the details of the normal-state entropy fits, noted above, which are necessary to construct the condensation energy. To see this, we show in Fig. 3 the *as-measured* entropy data from Loram *et al.* [3], plotted as $S(T)/T$, for the full 11 doping states measured for pure Bi2212 (gray data points). This is obtained by integrating the experimental $\gamma(T)$ data. For clarity, the plotted data show every fourth data point. It is evident that the measured data is free of any obvious entrant or reentrant anomaly. The magenta dash/dot curve shows our calculation of $S(T)/T$ for a d -wave order parameter where $2\Delta/k_B T_c = 4.5$, i.e., near weak coupling, as calculated using the Padamsee α -model [52]. Evidently, any deviation in the experimental data from this canonical behavior is very small except near T_c where fluctuations cause a rounding of the transition. Also plotted is the expected entrant or reentrant behavior if there should occur a crossover into, or out of, the pseudogap state. Again we have used crossover ranges of $T_0 = 5, 10,$ and 15 K (red, green, and blue curves). There is no suggestion in the data of even a weak crossover into, or out of, the pseudogap state. For the optimally doped sample (the end point of the crossover), we infer $E^* = 16$ meV. A crossover anomaly corresponding to a pseudogap one-tenth this size would probably be observable in our data. Thus we suggest that if there is an entrant or reentrant pseudogap near p^* its magnitude is less than 2 meV.

We note in passing that the low- T slope of S/T is proportional to Δ_0^{-1} . The convergence of the four underdoped curves at low temperature shows that the gap magnitude saturates at low doping while, with increasing doping, the gap magnitude

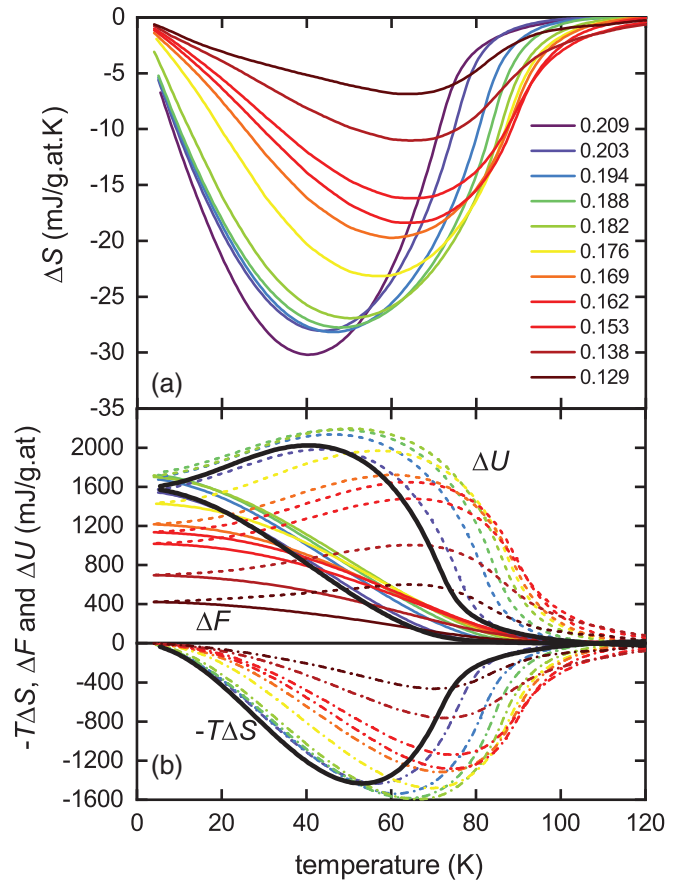


FIG. 4. (a) The T dependence of the electronic condensation entropy $\Delta S = S_s - S_n$ for Bi2212 at the various doping levels shown, from $p = 0.162$ to 0.209 . All curves are totally free of the inflexions shown in Fig. 1 characteristic of reentrant behavior. The curvature near T_c arises from strong superconducting fluctuations. (b) The T dependence of ΔF and its component terms ΔU and $-T\Delta S$. The color coding is the same as in panel (a). Notably, the strong fluctuation terms seen in both ΔU and $T\Delta S$ are almost completely absent in ΔF , where the respective curves for $p = 0.209$ are highlighted for clarity.

is seen to fall increasingly out to the highest overdoped state. A similar result is reported in ARPES measurements [29].

B. Condensation entropy

The benefit of investigating $\Delta F_{ns}(T)$ for entrant or reentrant behavior is that it is otherwise a simple monotonic function. The entropy $\Delta S(T)$ is nonmonotonic but it is, as shown in Fig. 1, more sensitive to anomalous behavior because of the derivative (and $\Delta\gamma(T)$ even more so). Figure 4(a) shows the condensation entropy $\Delta S(T) = S_n(T) - S_s(T)$ for Bi2212 where, as noted, $S_n(T)$ is calculated from the ARPES-derived dispersion as described in Appendix A. The entropy extrapolates to zero at $T = 0$, as required, and the curvature extending well above T_c arises from strong superconducting fluctuations [53]. (These are strong in the sense that the ratio T_c^{mf}/T_c can be as large as 1.6. Note that this conclusion does depend on correctly identifying the mean-field behavior of $\gamma(T)$ below the fluctuation region below T_c [50,53]). Apart from these

features, the entropy is totally devoid of any inflexion or flattening as might be indicative of reentrant (refer to Fig. 1) or entrant behavior, respectively. The condensation free energy is shown in Fig. 4(b) along with its components $\Delta U(T)$ and $T\Delta S(T)$. Given the form of Eq. (A1), it is clear that for any reasonably sharp anomaly in $\gamma(T)$ (whose width is less than 10–20% of T_c), there will always be significant cancellation in $\Delta U - T\Delta S$. But note that the nonmonotonic behavior of each of these components exactly cancel so as to leave a simple monotonic mean-field behavior in $\Delta F(T)$. Note also that the strong superconducting fluctuations evident in these component terms largely offset each other so as to almost totally suppress the fluctuation term in the free energy. This is an important warning that some thermodynamic and spectroscopic features are more sensitive to fluctuations than others. Those that are very sensitive to fluctuations (like the entropy) include the spin susceptibility (see Appendix A and its consequent relevance to NMR studies), the superconducting gap function which extends well above T_c and is proportional to $2\Delta F(T) + T\Delta S(T)$ [50], and the Raman scattering cross section [7]. As a consequence, it is easy to confuse a normal-state partial gap arising from superconducting fluctuations with the pseudogap and it is necessary to additionally use magnetic fields to suppress one and not the other to definitively distinguish them [16,54].

C. Magnetic entropy

Figure 7(c) shows the T dependence of $\Delta\gamma(13, T) = \gamma(13, T) - \gamma(0, T)$, the change in γ due to increasing field from 0 to 13 tesla. (This will be discussed later relative to the other panels in the figure with respect to fluctuations). This data is integrated with respect to temperature, as in Eq. (A1) of Appendix A, to obtain the field-dependent free energy difference, $\Delta F(13, T)$, and its internal energy and entropy components. These are plotted in Fig. 5. $\Delta F(13, T)$ is related to the superfluid density, $\rho_s(T) = \lambda^{-2}(T)$, as detailed in Appendix A 3. Considering $\Delta S(13, T)$ first, we note that the inflexion associated with putative reentrant behavior, as illustrated in Fig. 1(b), is absent on the high-temperature side of the extremum. There is a significant inflexion on the low-temperature side but this is expected under the London model and is present in all samples. The strong superconducting fluctuation feature seen in $\Delta U(13, T)$ and $-T\Delta S(13, T)$ is again nearly completely canceled out in $\Delta F(13, T)$.

A normalized plot of $\Delta F(13, T, p)/\Delta F(0, T, p)$ versus T/T_c reveals that all doping states have very similar temperature dependencies. None shows a downturn or a boost relative to the others that might reflect entrant or reentrant behavior. These are not expected to have identical T dependencies. Both the opening of the antinodal pseudogap and the London model for $\Delta F(13, T, p)$ will produce small systematic variations in the scaled behavior depending, e.g., on the amplitude of the superfluid density. But there is no substantial deviation that would signal the opening or closing of the pseudogap below T_c .

Figure 6 summarizes the doping dependence of a number of key thermodynamic amplitudes. Figure 6(a) shows the previously reported jump, $\Delta\gamma(T_c, p)$, in the specific heat coefficient at T_c for pure Bi2212, 0.15 Y-substituted Bi2212

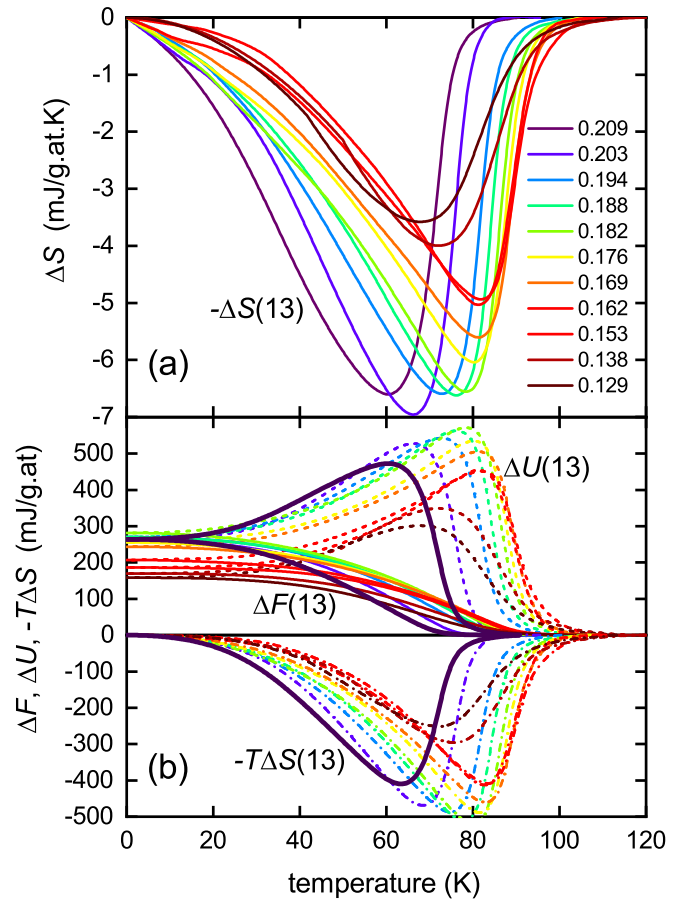


FIG. 5. (a) The T dependence of the magnetic entropy difference, $\Delta S(13) = S(13) - S(0)$, obtained in changing field from 0 to 13 tesla, obtained by integrating the $\Delta\gamma(13) = \gamma(13) - \gamma(0)$ data for Bi2212 shown in Fig. 7(c). Doping levels are given in the legend. (b) The magnetic free energy difference $\Delta F(13)$ and its internal energy and entropy components obtained by integrating the same $\Delta\gamma(13)$ data using Eq. (A1). As in Fig. 4, the curvature near T_c arises from strong superconducting fluctuations, though less marked here. The color coding is the same as in panel (a). As in Fig. 4(b), the strong fluctuations seen in both $\Delta U(13)$ and $T\Delta S(13)$ are almost completely absent in $\Delta F(13)$. The respective curves for $p = 0.218$ are highlighted for clarity.

and 0.2Pb substituted Bi2212 [3]. We earlier showed that this jump comprised an apparent mean-field step and a fluctuation step. Using an entropy balance analysis of the fluctuation specific heat, we were able to deduce the “true” mean-field step, $\Delta\gamma^{\text{mf}}(p) = \Delta\gamma(T_c^{\text{mf}}, p)$, that would have occurred at the significantly higher T_c^{mf} if fluctuations were suppressed [53]. $\Delta\gamma^{\text{mf}}$ is plotted in the figure by the green stars. These track the observed total $\Delta\gamma$ values rather closely. Also shown is the difference in jump height, $\Delta\gamma(13, p)$, at 0 and 13 tesla (gold symbols). The vertical green-shaded band shows the interval $p = 0.190 \pm 0.005$ around critical doping. These quantities all reveal an abrupt fall in amplitude beginning at $p \approx 0.19$ with the opening of the pseudogap. Note that these include properties evaluated at T_c or at T_c^{mf} which is up to 40 K higher. While $\Delta\gamma(T_c, p)$ is not necessarily a true measure of the weight of the jump (and hence of the density of Cooper pairs)

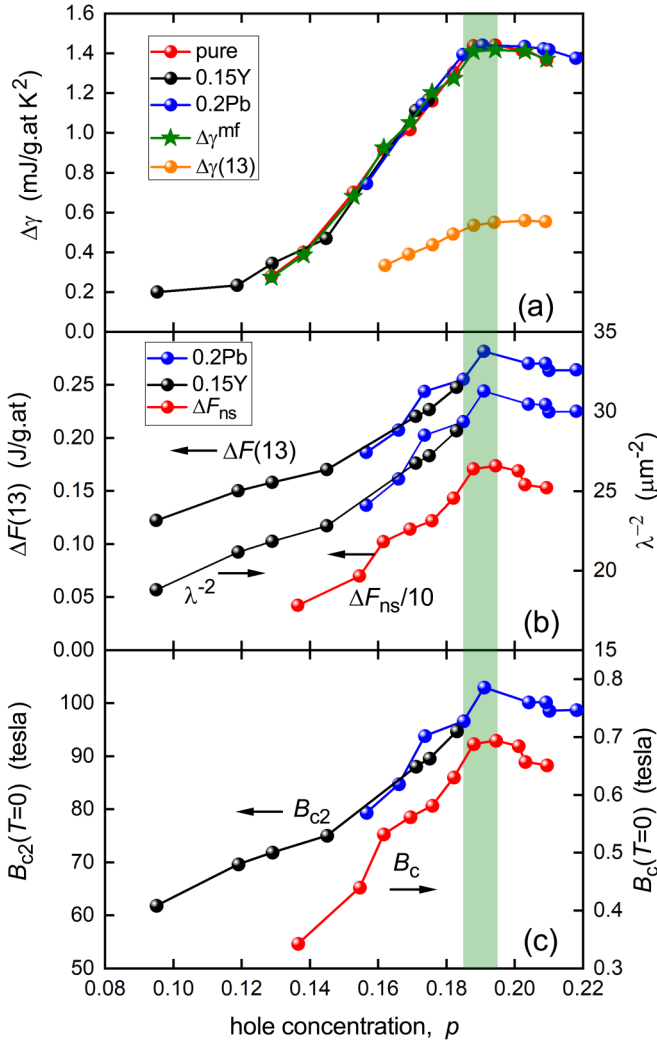


FIG. 6. Summary of the doping dependence of various thermodynamic quantities: (a) shows the jump, $\Delta\gamma(T_c, p)$, in specific heat coefficient at T_c for pure Bi2212, 0.15 Y-substituted Bi2212 and 0.2Pb substituted Bi2212. Also shown is the previously reported true mean-field jump, $\Delta\gamma^{\text{mf}}(p) = \Delta\gamma(T_c^{\text{mf}}, p)$, deduced from an entropy balance analysis of the fluctuation specific heat [53], and the difference in jump height, $\Delta\gamma(13, p)$ between 0 and 13 tesla. (b) shows the ground-state amplitude of the field-dependent change in free energy, $\Delta F(13, T=0, p)$ and ΔF_{ns} in zero field, and the superfluid density, $\lambda_0^{-2}(p)$ deduced from Eq. (A4). (c) shows $B_{c2}(0)$ determined from $\Delta F(13, 0)$ and $B_c(0)$ from $\Delta F_{\text{ns}}(0)$.

because transition widths may (and do) vary with doping, the deduced true mean-field step $\Delta\gamma^{\text{mf}}(p)$ is an integrated measure and is thus largely independent of any variations in transition width. These data collectively reveal a sudden collapse of pair density on reducing the doping below $p \approx 0.19$, clearly signaling the opening of the pseudogap there.

Figure 6(b) summarizes the doping dependence of the ground-state values of the condensation free energy, $\Delta F_{\text{ns}} = F_n - F_s$, and the field-induced free energy change $\Delta F(13)$. These are $T = 0$ properties and they both also show an abrupt fall on reducing the doping below $p \approx 0.19$ simultaneous with the abrupt fall in properties at T_c and T_c^{mf} . The former, ΔF_{ns} ,

defines the thermodynamic critical field, $B_c = \phi_0/[2\sqrt{2}\pi\xi\lambda]$, while the latter, $\Delta F(13)$, defines the ground-state superfluid density, $\lambda^{-2}(0)$, using the London model, Eq. (A4). (The detailed calculation of the full T dependence of $\lambda^{-2}(T)$ is described elsewhere). These parameters are all plotted in Figs. 6(b) and 6(c). Finally, having determined $B_c(0)$ and $\lambda^{-2}(0)$, we calculate $B_{c2}(0)$ which is plotted in Fig. 6(c). The peak value of this parameter at p^* of 103 tesla is very reasonable and the implied value of $\xi(0) = 1.79$ nm, consistent with various reported measurements [55].

It is evident from Fig. 6 that all key parameters associated with the strength of the condensate commence their pseudogap-induced collapse at $p \approx 0.19$ whether at $T = 0$, $T = T_c$, or $T = T_c^{\text{mf}}$ —a range of some 140 K. The onset of the pseudogap does not therefore follow a doping-dependent $T^*(p)$ line but is fixed at $p = p_{\text{crit}} \approx 0.19$, independent of temperature. Combining this with the repeated observation that the normal-state entropy above T_c never recovers from lost states up to 300 K and higher [3,18,19], we may conclude that the opening of the pseudogap occurs only at $p = p_{\text{crit}} \approx 0.19$, independent of temperature from $T = 0$ to well over 300 K. This is consistent with our claim that the pseudogap line is an energy scale, not a temperature scale [20,21]. The inference is that the very weak but abrupt mean-field transitions observed near a monotonically descending $T^*(p)$ line in the underdoped regime occur *within* the preexisting pseudogap state which extends to very high temperatures. They are not transitions *into* the pseudogap state, and possibly not even associated with the pseudogap. The pseudogap may simply create conditions conducive to this correlation. Thus, for example, a pseudogap-induced change of the Fermi surface can allow nesting q -vectors that induce charge ordering [56].

IV. DISCUSSION

A. Fluctuations

Despite our extensive results showing that the pseudogap closes at $p^* \approx 0.19$, independent of temperature, and the large amount of thermodynamic and spectroscopic data that supports this (see Appendix B), there persists the lingering view that there truly exists a pseudogap $T^*(p)$ line that extends deep into the overdoped region meeting the superconducting phase curve $T_c(p)$ around $p = p^* \approx 0.23$ [29–32] and projecting to zero just where $T_c \rightarrow 0$. How can two such different pictures still persist in such a mature field? We suggest that the confusion has two primary causes. First, the existence of a partial gap in the DOS arising from superconducting fluctuations, and extending well above T_c , is easily confused with the pseudogap. And second, the literature actually confuses a spectroscopically determined energy scale, $E^*(p)$, with a temperature scale, $E^*(p)/k_B$. In this section, we discuss this situation in some detail.

In many cases, nominal T^* values are obtained by identifying “kinks” or downturns in the T dependence of various physical properties. The problem is that, above T_c , one must take superconducting fluctuations into account. Experimentally, these are seen most easily from a field-dependent downturn in the in-plane electrical resistivity and a diamagnetic contribution to the static susceptibility which is larger for magnetic fields perpendicular to the CuO_2 planes. It is not as

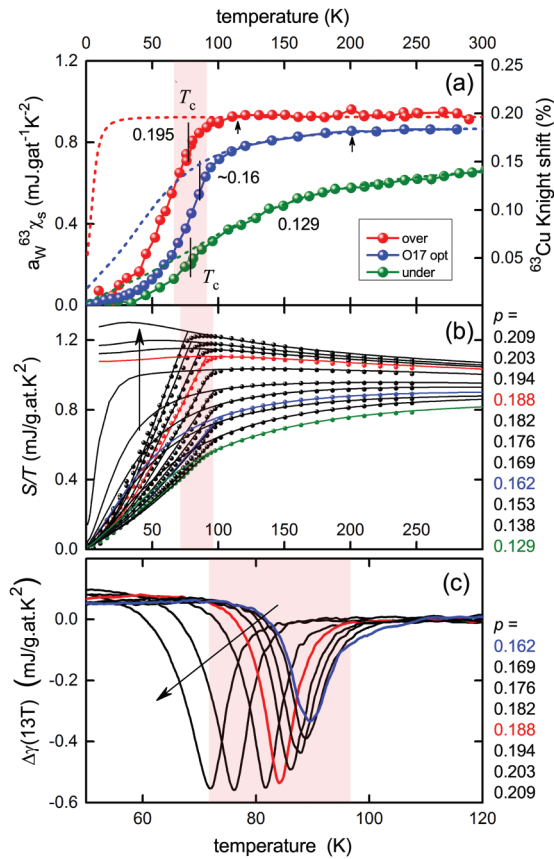


FIG. 7. (a) ^{63}Cu Knight shift for Bi2212 [60] expressed in % shift and in entropy units using $a_W \chi_s$; doping, p , is annotated. The ^{17}O shift for $p = 0.16$ is reproduced from Ref. [61]. Dashed curves: Normal-state pseudogap fits with $E^* = 1.4, 13.4,$ and 27.6 meV. (b) Electronic entropy coefficient S/T for 11 doping states from Ref. [3]. Curves for critical doping where the pseudogap closes, optimal doping, and most underdoped are shown by red, blue, and green, respectively, and correspond to the color coding in (a). All curves are calculated [49] using the full ARPES-derived dispersion involving Fermi arcs as applicable to both normal and superconducting states, while those in Fig. 1 use the non-weak-coupling Padamsee calculation for $\Delta F(T)$ (see text). (c) The change, $\Delta\gamma(13T) = \gamma(13T) - \gamma(0)$, in specific heat coefficient, γ , between 0 and 13 T for doping levels listed at right.

widely recognized that in overdoped Bi2212 superconducting fluctuations can also reduce the electronic DOS at the Fermi level [57–59] and hence decrease the spin susceptibility and affect all other properties which depend on the DOS. Although we are not aware of an explicit theoretical treatment, in our work a decrease in the measured electronic entropy, as T_c is approached from above, must mean that there is a decrease in the electronic DOS. This must be true because there is no other source of entropy once the phonon part has been correctly subtracted.

The point is illustrated in some detail in Fig. 7(a) where the ^{63}Cu Knight shift data of Ishida *et al.* [60] is reproduced and compared with the thermodynamic functions in Figs. 7(b) and 7(c). The intimate relationship between the spin susceptibility and entropy is detailed in Appendix A 4. This NMR data was used by Legros *et al.* [32] to augment their “phase diagram”

(see their Supplementary Information) which was largely adapted from Vishik *et al.* [29]. Also shown is the ^{17}O Knight shift data from Ishida’s Fig. 3 as earlier reported by Takigawa and Mitzi [61]. Spin shifts are thus presented for overdoped ($p = 0.195$), optimally doped ($p \approx 0.16$), and underdoped ($p = 0.129$), as annotated. The scale on the right-hand side shows the Knight shift referenced to the T -independent orbital shift. Note that a spin shift of $^{63}\text{K} = 0.2$ (%) corresponds to $^{63}\text{K} = 0.177$ (%). The small vertical arrows show the T^* values where Ishida *et al.* identified the opening of the pseudogap (in our opinion, erroneously). The dashed curves are our pseudogap fits to their normal-state data. As before [53], we have assumed a otherwise flat band with a pseudogap represented by a simple V-shaped electronic DOS centered on E_F and remaining finite there, thus reflecting the presence of residual Fermi arcs. The susceptibility is calculated using the standard formula for weakly interacting fermions,

$$\chi_s = -2\mu_B^2 \int_{-\infty}^{\infty} \frac{\partial f}{\partial E} N(E) dE, \quad (1)$$

where $f(E)$ is the Fermi function. This, with the V-shaped DOS, gives the analytical formula,

$$\chi_s(T) = 2N_0\mu_B^2 \times \left[1 - \frac{N_0 - \nu_0}{N_0} \left(\frac{E^*}{2k_B T} \right)^{-1} \ln \left[\cosh \left(\frac{E^*}{2k_B T} \right) \right] \right], \quad (2)$$

where N_0 is the background DOS, ν_0 is the residual DOS at E_F (arising from the Fermi arcs), and E^* is the gap edge of the pseudogap V.

A more realistic model involving the ARPES-derived dispersion, as in the calculation [49] of the normal-state entropy in Fig. 7(b), gives almost identical results and does not alter our conclusions. In this model, significant deviations only occur closer to the VHS, proximity to which is evidenced in the low-temperature upturn in S/T in the most overdoped samples in Fig. 7(b). The three NMR data sets are far enough away from the VHS to not be affected.

The three dashed curves in Fig. 7(a) for over-, optimally, and underdoped Bi2212 are the fits with pseudogap energies of $E^* = 1.4, 13.4,$ and 27.6 meV, respectively. (In the first of these, E^* has probably already vanished at this doping but we retain the small value of $E^* = 1.4$ meV to illustrate below that large curvature, as seen in this instance, can only occur at low temperature when E^* is very small. So the large curvature observed in NMR data near 90 K [60] cannot be ascribed to the pseudogap).

The left-hand scale shows the spin susceptibility in entropy units, $a_W \chi_s$, obtained from multiplying χ_s by the Wilson ratio for weakly interacting fermions, $a_W = (\pi k_B / \mu_B)^2 / (3\mu_0)$ —see Appendix A 4.

As already noted, Fig. 7(b) shows the experimental [3] and modelled [49] electronic entropy coefficient calculated for Bi2212 from the ARPES-derived dispersion [28]. To better expose the detail, this plot shows only every 20th experimental data point. Notably, the absolute magnitudes in Figs. 7(a) and 7(b) are very similar, underscoring the previous observation [18] that S/T and χ_s are much the same in Fermi

units, consistent with the elementary excitations being weakly interacting fermions. Note that this is also true for many strongly correlated heavy fermion systems [62].

As mentioned, the two small arrows in Fig. 7(a) indicate T^* values inferred by Ishida *et al.* [60] for the optimal and overdoped samples. The two points are reproduced in the plot of $T^*(p)$ reported by Legros *et al.* [32] with the overdoped sample showing $T^* \approx 110$ K, well above $T_c \approx 79$ K. However, the normal-state fits (dashed curves) combined with the corresponding entropy data [red data and curves in Fig. 7(b)] show that the downturn just above T_c is associated with superconducting fluctuations which reduce both the spin susceptibility and the measured entropy via their effect on the electronic DOS. It is unrelated to the pseudogap. This is even more evident in the specific heat coefficient, $\gamma(T)$, where the fluctuation contribution is symmetric about T_c [50,53]—see Fig. 7(c). A lingering criticism of the electronic specific heat data is the question as to whether the differential technique really has succeeded in accurately backing off the much larger phonon contribution. To meet this concern, we show in Fig. 7(c) the change, $\Delta\gamma(H)$, in γ between zero external field and $\mu_0H = 13$ T as reported by Loram *et al.* [3]. This difference automatically eliminates any residual phonon contribution and we see in the field-dependent anomaly the effect of a magnetic field in suppressing fluctuations above and below T_c . The fluctuation range for the sample at critical doping (red curve) is shown by the pink shading and this fluctuation range is reproduced by the shading in Figs. 7(a) and 7(b). (Note that despite the same doping state, the T_c values are somewhat different for the single crystal in Fig. 7(a) with $T_c = 79$ K and the polycrystal in Fig. 7(b) with $T_c = 83.9$ K. Such differences between crystals and polycrystals are not uncommon. They probably arise from the presence of slightly more in-plane defects in state-of-the-art single crystals). In summary, the similarities between the downturns in $^{63}\text{K}(T)$ and the measured entropy just above T_c , coinciding as they do with the pink-shaded fluctuation range, together with the evidence from tunneling studies in overdoped Bi2212 [58,59] lead us to conclude that these downturns arise not from the pseudogap but from superconducting fluctuations which reduce the electronic DOS at E_F , and that the pseudogap has already closed at this critical doping level or is very small, i.e., $E^*/k_B \approx 0$ (and certainly $T^* \neq 110$ K).

The vanishing of E^* at this doping may not be immediately evident from the Knight shift but the benefit of placing χ_s alongside its complementary thermodynamic variable S/T is that any small loss of normal-state electronic entropy at low T (due to a small residual pseudogap) must also be reflected in the actual measured entropy near T_c due to entropy conservation. This is more easily seen in $\gamma = \partial S/\partial T$, as the area under a $\gamma(T)$ curve is entropy. Thus any (small) gapping in the DOS will produce a (small) reduction in the low- T normal-state specific heat coefficient, γ_n , and therefore, by entropy conservation, the peak in specific heat at T_c will also be reduced. Figure 6(a) shows that the jump, $\Delta\gamma$, at T_c remains constant in the overdoped region and then only below critical doping (red curve) does it begin to collapse, and it does so rapidly. In principle, the same effect might be seen by differentiating the spin susceptibility, $\partial(\chi_s T)/\partial T \propto \chi$, but this would require very good data and there is no

spin analog of the third law of thermodynamics that leads to “susceptibility conservation”. The pseudogap has closed at this critical doping state.

To pursue this in even more detail, we consider the evolution of S/T with doping shown in Fig. 7(b). For the most underdoped sample (green data and curves), the pseudogap is large, with $E^*/k_B = 322$ K, and S/T is broadly curved over a comparable temperature range—see also the fits to the green NMR data points in Fig. 7(a). With increasing doping, E^* falls and the curvature increases, but as it does so the region of high curvature falls toward $T = 0$, as also shown in the fits in Fig. 7(a). It simply is not possible with any realistic pseudogap model for there to be a pseudogap region of high curvature at 100 K as seen in the red data points in Fig. 7(a). The downward curvature would necessarily begin above 180 K. Such a narrow region of downturn can only be found at low temperature when E^* is small, as seen in the black $p = 0.182$ curve in Fig. 7(b) where $E^*/k_B = 51$ K, or in the red dashed curve in Fig. 7(a) where $E^*/k_B = 8$ K. Typically, the region of maximum curvature lies near a temperature of about E^*/k_B and the downward curvature extends up to $2E^*/k_B$. Thus the range is narrow only when E^* is small. As noted, the identification of a T^* simply from *any* downward curvature above T_c is easily confused with superconducting fluctuations.

We make two further comments in relation to two prominent literature reports. First, the Raman group of Sacuto *et al.* [63] very recently concluded that the pseudogap $T^*(p)$ line collapses vertically in the superconducting state but not until the strongly overdoped range $p = 0.224 \pm 0.002$ —just at the proposed location of the Van Hove point. However, it is clear from the thermodynamic and superfluid density [48] measurements described above that the strongly entropy-depleting pseudogap is completely absent beyond $p^* = 0.19$ and these authors must be observing a spectral feature in the Raman response other than the pseudogap, with this feature disappearing at the Lifshitz transition. The discussion in the present paper strongly suggests that the Raman response for $p > 0.19$ is influenced either by this proposed Lifshitz transition or by a partial gap at E_F arising from superconducting fluctuations that is also seen in intrinsic tunneling data [58,59].

Second, we have mentioned the backbending pseudogap phase curve of He *et al.* [30]. These authors report a depletion of the integrated antinodal spectral weight just above T_c that extends out to $p \approx 0.22$ and is attributed by them to the pseudogap. At first sight, this is consistent with the Sacuto Raman data [63]. However, it is clear from all of the above that this is likely to be associated with loss of spectral weight arising from superconducting fluctuations just above T_c [53,57]. Field-dependent measurements (or impurity substitution) would clarify this situation.

B. The pseudogap “phase diagram”

As a final topic, we wish to discuss the individual data points in the pseudogap $T - p$ “phase diagram” of Vishik *et al.* [29] and Legros *et al.* [32], as widely used by others. The T^* data points from SIS tunneling in both figures are not temperatures but energy gaps divided by k_B . Further, these gaps in the overdoped region are superconducting gap magnitudes [15] and therefore unrelated to the pseudogap. We

have already noted that the $T^* \approx 110$ K NMR data point in Ref. [60] should instead be close to 0 K. Resistivity-derived T^* values in Legros *et al.* are evaluated in the usual way from downturns in resistivity data taken from Oda *et al.* [64]. However, closer scrutiny shows that the downturn for optimal doping should be lower (if treated consistently over all dopings) and the overdoped downturn is, again, attributable solely to superconducting fluctuations as described above.

The c -axis resistivity data points are due to Watanabe *et al.* [65] who find a semiconductinglike upturn on cooling below a certain $T^*(\rho_c)$ value. However, in an earlier publication, these authors concluded in similar studies, “[W]e find that the onset of the semiconducting $\rho_c(T)$ does not coincide with the opening of the spin gap seen in the $\rho_a(T)$ ” [58]. Note that within a tunneling model for c -axis transport [58,66], such an upturn is in fact anticipated with the onset of superconducting fluctuations due to the reduction in the electronic DOS. Moreover, the highest doping value of T^* was explicitly identified by these authors [65] as lying below the onset temperature for superconducting fluctuations, just as we have asserted.

Finally, the scanning-tunneling-spectroscopy (STS)-derived T^* data points in both Vishik and Legros are due to Gomes *et al.* [67] and these simply map out the onset of the depression in DOS caused by superconducting fluctuations, not by the closure of the pseudogap. These data points map nicely onto the superconducting “pairing temperature” inferred by Kondo *et al.* [68] and onto the doping-dependent mean-field transition temperature $T_c^{\text{mf}}(p)$ determined from an entropy conservation treatment of the fluctuation specific heat [53]. In short, we believe these various reported $T^*(p)$ phase diagrams which merge with the $T_c(p)$ phase curve in the heavily overdoped region are incorrect and should be abandoned.

There are relatively few systematic studies which distinguish between the pseudogap and superconducting fluctuations, using, for example, an applied magnetic field, impurity substitution, or by using a suitable fitting procedure for the overall T dependence. Kokanović *et al.* [69] implement the latter, taking advantage of the broad temperature scale for the pseudogap compared with the relatively narrow domain of superconducting fluctuations. More precisely, plotting $\chi_c(T) - \chi_{ab}(T)$ versus T (Fig. 3(a) of Ref. [69]) eliminates an isotropic Curie term, C/T , and allows the diamagnetic fluctuation contribution, which is pronounced in $\chi_c(T)$, to be seen more clearly. Alloul *et al.* [54] and Naqib *et al.* [16] apply a field to identify and suppress fluctuations. Both studies show that the $T^*(p)$ line cuts through the fluctuation pairing temperature above T_c , trending toward zero as $p \rightarrow 0.19$. However, the latter study, on epitaxial thin films of $\text{Y}_{0.8}\text{Ca}_{0.2}\text{Ba}_2\text{Cu}_3\text{O}_{7-\delta}$, was able to track T^* below T_c by combining progressive Zn substitution with their field studies. $T^*(p)$ was thereby found to cut the Zn-free $T_c(p)$ phase curve and continue undeflected toward zero as $p \rightarrow 0.19$. Even so, we consider this as reflecting an underlying p -dependent energy scale which vanishes, rather than a closing temperature for the pseudogap.

A similar approach has been used by Usui *et al.* [70] to study separate pseudogap and superconducting fluctuation effects in single-crystal Bi2212, using both in-plane and out-of-plane resistivity. They use the same method as we do to

establish doping state. Using applied fields to identify the onset of a strong magnetoresistance, they obtain a fluctuation range similar to what we have reported. They identify T^* values from the usual downturn in resistivity up to a doping state of 0.16, but not at $p = 0.20$ where, already, the T -dependent resistivity is slightly superlinear. All this is consistent with the picture we have presented. However, they also extract T^* values from the upturn in the c -axis resistivity, $\rho_c(T)$, which are ≈ 100 K higher and which persist at least out to $p = 0.22$ and project to persist far beyond this. In our view, and as noted earlier, in the view of Watanabe *et al.* [58], the upturn does not signal the opening of the pseudogap seen in the $a - b$ -plane transport. Moreover, it is at odds with the later data reported by Watanabe [65]. And, in the recent ARPES study by the Z.-X. Shen group [71] the pseudogap is only observed in the “incoherent strange metal state” which opens abruptly below $p \approx 0.19$, thereby effectively abandoning their earlier reentrant scenario. This upturn in $\rho_c(T)$ which projects to persist outside of the superconducting dome therefore cannot be associated with the pseudogap.

A final, and central, question to consider is whether $p^* = 0.19$ is a generic feature of the cuprates. The question is considered briefly in Appendix C and our conclusion is that, for *canonical cuprates*, it probably is general.

V. CONCLUSIONS

In summary, in a search for either entrant or reentrant pseudogap behavior, we have examined the condensation free energy, $\Delta F(T, p)$, the magnetization free energy, $\Delta F(H, T, p)$ and their entropy counterparts for $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ in closely spaced doping states either side of critical doping, $p^* \approx 0.19$, where we find that the pseudogap closes. In every case, we observe $\Delta F(T)$ to closely follow the near-weak-coupling T dependence for a single unperturbed d -wave order parameter. There is no obvious enhancement in $\Delta F(T)$ on crossing a putative backbending $T^*(p)$ line in the reentrant scenario, nor is there any obvious suppression in $\Delta F(T)$ on crossing a putative monotonically decreasing $T^*(p)$ line in the entrant scenario. One simply observes a strong reduction in the overall amplitude of the entire $\Delta F(T)$ curve once the pseudogap opens as p falls below 0.19, with no change in its mean-field-like, single-order-parameter *shape*. Neither do we observe an associated inflexion anomaly in the entropy $\Delta S(T)$. Note that our main conclusions do not depend on the specific mean-field model used here for $\Delta F(T)$. For example, an alternative interpretation [59] involving pair-breaking below T_c and Gaussian fluctuations above T_c would also give smooth behavior in $\Delta F(T)$, i.e., no anomalies in the measured entropy, allowing us to rule out both entrant and reentrant behavior. In this alternative interpretation, the values of T_c^{mf} are much smaller, typically of order 1.1 and the remnant normal-state pseudogap proposed by others to extend to $p \approx 0.23$ [29,63] is ascribed to Gaussian superconducting fluctuations at higher T [59,72–74] that cross over to critical fluctuations as T_c is approached from above. Such a picture was proposed earlier [72] for the heat capacity of several cuprate families and may apply to the Bi2212 data discussed here. We conclude that $p^* \approx 0.19$ is the *temperature-independent* location where the pseudogap abruptly opens or closes, consistent with

the observed fact that the pseudogap-induced lost entropy for $p < 0.19$ is never recovered to well above room temperature. Paralleling this, the “lost susceptibility” is never recovered, even to 500 K. We reiterate that the pseudogap line often drawn on the phase diagram is actually the pseudogap energy scale (expressed as E^*/k_B) which falls with increasing doping and vanishes at $p^* = 0.19$. The pseudogap is still present and fully developed above this line to very high temperature. The thermodynamically weak mean-field nematic transition observed near a sloping $T^*(p)$ line [27] does not reflect the opening of the pseudogap but, rather, occurs within the preexisting pseudogap.

APPENDIX A: THERMODYNAMIC FUNCTIONS

1. Free energy

The differential specific heat technique allows the extraction of the electronic specific heat coefficient, $\gamma(T)$, from the much larger phonon term. From this, we may obtain the electronic entropy by integration and the electronic free energy by integration again of the entropy. We find it preferable to perform just a single integration and obtain the free energy using the following generic thermodynamic relation:

$$-\Delta F(T) = \int_{\tau}^T T \Delta\gamma(T) dT - T \int_{\tau}^T \Delta\gamma(T) dT, \quad (\text{A1})$$

where the first term is the electronic internal energy, $-\Delta U(T)$, and the second term is the electronic entropy term, $T\Delta S(T)$. The term $\Delta\gamma(T)$ may refer to the difference between the superconducting and normal states, i.e., $\Delta\gamma(T) = \gamma_n - \gamma_s$; $\Delta S(T) = S_n - S_s$, etc., or it may refer to the difference in γ between its value in field, H , and its value in zero field, i.e., $\Delta\gamma(H, T) = \gamma(H, T) - \gamma(0, T)$; $\Delta S(H, T) = S(H, T) - S(0, T)$, etc.. Ideally, the integrals in Eq. (A1) are from $\tau = T_c$ down to some value of $T < T_c$. However, the cuprates are distinguished by strong fluctuations over quite a broad range around T_c and, as a consequence, the integrals must be performed from some temperature, τ , well above T_c , sufficiently above the range of superconducting fluctuations, where $\Delta\gamma(\tau)$, $\Delta F(\tau)$ and $\Delta S(\tau)$ have fallen to zero.

2. Normal-state thermodynamic functions

To determine the condensation terms $\gamma_n - \gamma_s$, $S_n - S_s$ and $F_n - F_s$, we need to evaluate the normal-state terms $\gamma_n(T)$ and $S_n(T)$. The former can be determined within fairly strict bounds by a simple construction of $\gamma_n(T)$ that satisfies entropy balance. However, we place the additional strict requirement that $\gamma_n(T)$ must be consistent with the DOS obtained from the electronic dispersion. We take the ARPES-derived dispersion of Kaminski *et al.* [28] and calculate the DOS $N(E)$, assuming the dispersion shifts rigidly (relative to the Fermi level) with doping. From the electronic DOS, we calculate S_n as described earlier [3,49,50], using the standard formula for weakly interacting fermions,

$$S_n = -2k_B \int_{-\infty}^{\infty} [f \ln(f) + (1-f) \ln(1-f)] N(E) dE, \quad (\text{A2})$$

where $f(E)$ is the Fermi function.

TABLE I. Fit parameters [49] to $S_n(T)/T$ using a rigid ARPES-derived dispersion [28] showing doping state, p , distance from the Van Hove singularity, $E_{\text{VHS}} - E_F$, pseudogap energy E^* obtained from ARPES-derived DOS, E^* obtained from entropy [3], Fermi arc angle, θ_{FA} , which is a measure of the length of the Fermi arc, and T_c .

| (holes/Cu) | $E_{\text{VHS}} - E_F$ (meV) | E^* (meV) from DOS | E^* (meV) from entropy | θ_{FA} (deg) | T_c (K) |
|------------|------------------------------|----------------------|--------------------------|----------------------------|-----------|
| 0.2093 | 8 | 0 | 0 | 0 | 71.2 |
| 0.2030 | 14 | 0 | 0 | 0 | 75.6 |
| 0.1944 | 18 | 0 | 0 | 0 | 81.4 |
| 0.1879 | 23 | 0 | 1.3 | 0 | 83.9 |
| 0.1821 | 35 | 4 | 4.4 | 10 | 85.8 |
| 0.1758 | 55 | 12 | 8.8 | 22 | 87.4 |
| 0.1694 | 65 | 18 | 12.3 | 25 | 88.6 |
| 0.1616 | 76 | 22 | 15.8 | 28 | 89.3 |
| 0.1527 | 84 | 24 | 18.5 | 30 | 89.1 |
| 0.1382 | 91 | 30 | 23.8 | 34 | 85.6 |
| 0.1288 | 107 | 36 | 27.8 | 35 | 81.9 |

The pseudogap is treated as before, both nodal [75] and with Fermi arcs [49], and $S_n(T)$ is fitted to the normal-state experimental data, the only fitting parameters being the magnitude of the pseudogap, $E^*(p)$, the distance away from the VHS, $E_{\text{VHS}} - E_F$ and the Fermi arc angle θ_{FA} (when used). In the latter case, the fits are shown by the solid curves in Fig. 7(b) as reported earlier [49] and the fit parameters are listed in Table I. Also listed are the E^* values determined previously from the downward (parallel) shift of the entropy $S(T)$ (for $T \gg T_c$), assuming the same V-shaped gap in the DOS mentioned in Sec. IV A. The two value sets track each other very nicely despite the difference in the way in which the \mathbf{k} dependence of the gap is described.

To compare with other reported measures of the pseudogap energy, we reproduce in Fig. 8 the data points reported in Fig. 2 from the Raman study of Le Tacon *et al.* [76]. We

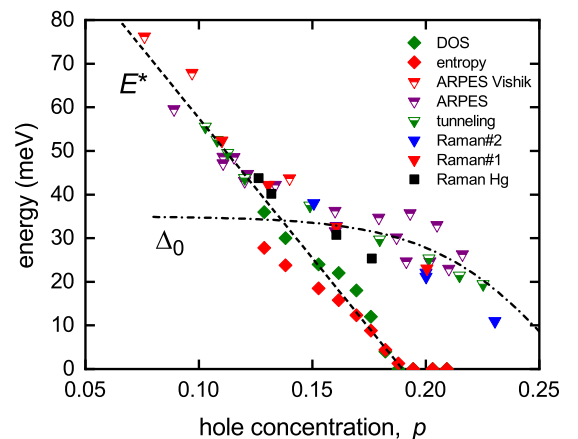


FIG. 8. The doping dependence of the antinodal gap as reported by Le Tacon *et al.* compared with our pseudogap fit values for E^* from Table I. We have added the gap values reported by Vishik *et al.* [29]. The data project to $E^*(p=0) = 122$ meV, comparable to the nearest-neighbor exchange energy, J .

use the same symbols and color schemes, but we omit the B_{2g} data points as these relate to the superconducting gap only. To these we add the three antinodal E^* gaps reported by Vishik *et al.* [29]. The data is low temperature, well below T_c , and therefore the antinodal gap also includes the superconducting gap when it is comparable to or greater than E^* . To assist differentiation, the dashed line shows the evolution of the pseudogap and the dash/dot curve that of Δ_0 . Our fit values are very consistent with this evolution. The pseudogap line projects to $E^*(p=0) = 122$ meV, consistent with the presumed underlying energy scale of J , the nearest-neighbor exchange interaction. We emphasize that measurements made above T_c (and its fluctuation range) reveal the pseudogap only and the data points retreat to the dashed line.

The resulting data fits give an excellent description of the experimental entropy above T_c and the values of $\gamma_n(T)$, obtained by differentiation, satisfy entropy conservation below T_c . (We note, however, that the precise location of the Van Hove point differs from one cuprate to the next and even for Bi2212 its exact location does not affect our conclusions).

3. Superfluid density

The field-dependent change of free energy $\Delta F(H, T)$ has been analyzed in terms of the London model for the field-dependent magnetization [77],

$$\mu_0 M(H, T) = \frac{\phi_0}{8\pi\lambda^2} \ln \left(2e^{2-2\gamma_E} \frac{H - H_{c1}}{H_{c2}} \right), \quad (\text{A3})$$

where γ_E is Euler's constant ($=0.5772$). By integration with respect to H , this yields

$$\Delta F(H, T) = -\frac{\phi_0(H - H_{c1})}{8\pi\lambda^2(T)} \ln \left(2e^{1-2\gamma_E} \frac{H - H_{c1}}{H_{c2}} \right). \quad (\text{A4})$$

Thus, by assuming $H \gg H_{c1}$ and plotting $\Delta F(H, T)/H$ versus $\ln(H)$, one expects linear behavior with a slope proportional to the superfluid density, $\rho_s = \lambda^{-2}$, and x -axis intercept giving $\ln(H_{c2})$.

4. Spin susceptibility

Equation (A2) may be regarded as an integral of the DOS weighted by a ‘‘Fermi window’’ function. The spin susceptibility is a similar thermodynamic function given by

$$\chi_s = -2\mu_B^2 \int_{-\infty}^{\infty} \frac{\partial f(E)}{\partial E} N(E) dE, \quad (\text{A5})$$

where the Fermi window is the function $\partial f/\partial E$. These two Fermi windows are essentially identical if the χ_s window is stretched in temperature by a factor of 1.19 [78]. Thus, if the electronic system is weakly interacting, S/T and χ_s will be related. In fact, it was found for a number of cuprates that the normal-state S/T is quantitatively very close to $a_W \chi_s$ over a very wide range of doping and temperature [18] consistent with the elementary excitations being weakly interacting fermions. Note that this is also true for many strongly correlated heavy-fermion systems [62]. Here a_W is the Wilson ratio for weakly interacting fermions, $a_W = (\pi k_B/\mu_B)^2/(3\mu_0)$.

This being the case, we may apply tests to the spin susceptibility similar to those that we apply to the entropy, and χ_s

may be expected to show superconducting fluctuations over a similar temperature range. We exploit this fact in Fig. 7.

APPENDIX B: PSEUDOGAP CLOSURE AT $p \approx 0.19$

We have claimed since 1994 that the pseudogap closes abruptly at $p = 0.19 \pm 0.05$ [18–20] for both Bi2212 and (Y,Ca)Ba₂Cu₃O_{7- δ} . The independent evidence for this has grown substantially since then and we summarize it here, both for convenience and to show the compelling nature of this extensive evidence. As doping is reduced below this value, we see an abrupt collapse in

- (i) the condensation energy [3];
- (ii) the jump $\Delta\gamma_c$ in specific heat coefficient at T_c [3]—see Fig. 6;
- (iii) the mean-field jump $\Delta\gamma^{\text{mf}}$ at T_c^{mf} [53]—see Fig. 6;
- (iv) the differential field-induced jump, $\Delta\gamma(H)$, at T_c [3]—see Figs. 6 and 7(c);
- (v) the ground-state superfluid density [4,48,79];
- (vi) the entropy at T_c [79];
- (vii) the Knight shift at T_c [34,75];
- (viii) the critical impurity concentration to suppress T_c to zero [79];

(ix) the critical fields, H_{c1} , H_c , and H_{c2} [5,80]; and
 (x) the self-field critical current, J_c^{sf} [80–82], all signifying an abrupt crossover from strong superconductivity to weak superconductivity associated with removal of states or spectral weight available for superconductivity (note this is not to be confused with ‘‘weak-coupling’’ superconductivity).

(xi) This crossover is not merely of theoretical interest—these dramatic changes all combine to impact the fine tuning of conductors for practical and commercial applications [41,80,81].

Below $p \approx 0.19$, there abruptly occurs

- (xii) a permanent (i.e., to very high temperature) loss of electronic entropy [3];
- (xiii) a permanent loss of spin susceptibility [3,73];
- (xiv) a large loss of spectral weight in the c -axis infrared conductivity extending to the highest temperatures investigated (300 K) [35];
- (xv) an abrupt loss of ground-state ($\pi,0$) ARPES quasiparticle peak intensity that correlates with the loss of superfluid density and condensation energy [83];

(xvi) a sudden change of the Fermi surface from large, with area $1+p$, to Fermi arcs or small hole pockets on the zone diagonal with area p as seen in STS quasiparticle scattering [9,10] and ARPES [12,13];

(xvii) a normal-state crossover in Hall number n_H from $1+p$ to p [7,11];

(xviii) a change in the sign of the Coulomb condensation energy occurring precisely at $p = 0.19$ [84,85];

(xix) a crossover from coherent to incoherent antinodal quasiparticles occurring precisely at $p = 0.19$ independent of temperature and coinciding with the opening of the EDC pseudogap [71,86];

(xx) a crossover from a near weak-coupling superconductor with $2\Delta_0/(k_B T_c) = 4.5$ across the overdoped region to a rapidly rising ratio in the underdoped region [71]. Already, by optimal doping, $p \approx 0.16$, the BCS ratio has doubled [71].

(xxi) The temperature-dependence of various normal-state properties at different doping states may be scaled onto a single curve using a doping-dependent scaling parameter $E^*(p)/k_B$ with the energy, $E^*(p)$ falling to zero at the critical doping $p \approx 0.19$. These include the in-plane thermoelectric power and the c -axis electrical resistivity [66]; and

(xxii) the doping dependence for $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ of the integrated weight of AF spin fluctuations $\int_0^{50\text{meV}} \chi''(\mathbf{q}_{\text{AF}}, \omega) d\omega$, as measured by inelastic neutron scattering, falls progressively with increasing doping to zero at $p = 0.19$ [15,49]. This strongly suggests that the pseudogap is associated with short-range AF correlations.

At $p = 0.19$, various authors report

(xxiii) T -linear normal state in-plane DC resistivity with sublinear below and superlinear above [86,87] and

(xxiv) very recently, Sterpetti *et al.* reported a quantum-critical-like cusp-shaped crossover to a pseudogapped “strange metal phase” in single-unit-cell Bi2212. They conclude “having found it to be centered at about 0.19 holes/Cu” [88].

APPENDIX C: CRITICAL DOPING IN OTHER CUPRATES

We have largely confined our discussion to Bi2212 because this was the subject material in the four key references [29–32] that we consider. Substantial evidence has been gathered in Appendix B for the rather abrupt closing of the pseudogap at $p^* = 0.19 \pm 0.005$ in both Bi2212 and Y123. Is this perhaps more general for all cuprates?

Turning then to other materials, determining the doping state of $\text{Bi}_2\text{Sr}_2\text{CuO}_6$ (Bi2201) has always been challenging. It does not follow the common relationship between doping and room-temperature thermopower [47]. Thus, see, e.g., Ando *et al.* [89]. The field has tended to use the method of Ando to specify doping for this system. However, it is noteworthy that the same crossover from a large Fermi surface to Fermi arcs is observed in STS quasiparticle scattering in both Bi2212 and Bi2201 (Refs. [9] and [10], respectively). For Bi2212, the doping of this crossover is determined by the Luttinger count to be $p \approx 0.19$. For Bi2201, the crossover occurs at $p \approx 0.15$ as determined by the Ando method but at $p \approx 0.19$ by the Luttinger count (Fig. 2(J) in Ref. [10]). If this is indeed the opening of the pseudogap, then this specific location would indeed seem to be more general.

We are not aware of many relevant studies in the case of $\text{HgBa}_2\text{CuO}_{4+\delta}$ (Hg1201), however, we note that the broad features of the dome-shaped cuprate $T_c(p)$ phase diagram are dominated by the pseudogap. The superconducting energy gap amplitude $\Delta_0(p)$ typically falls along with $T_c(p)$ on the overdoped side but remains largely constant on the underdoped side [3,29,30,35,90]. As noted, the ratio $2\Delta_0/(k_B T_c)$ for Bi2212 remains more or less constant on the overdoped side with the value of ≈ 4.5 [30] while the ratio $2\Delta_0/(k_B T_c^{\text{mf}})$ for both Bi2212 and (Y,Ca)123 is close to the weak-coupling value of 4.3 across the overdoped region [53]. It is thus the opening of the pseudogap in the lightly overdoped region and its growth with underdoping which accounts for the underdoped fall in T_c while $\Delta_0(p)$ remains fixed—see Fig. 4 of Ref. [79]. Hence the well-known dramatic rise in the ratio $2\Delta_0/(k_B T_c)$ with underdoping [30]. Now (Y,Ca)123 and

La214 have very similar $T_c(p)$ phase curves as a function of doping, differing only by an almost constant amplitude—see Fig. 2 of Ref. [91]. This implies the onset of the pseudogap close to $p \approx 0.19$ for La214 as in Y123. And, significantly, Hg1201 also has a $T_c(p)$ phase curve almost identical to that of (Y,Ca)123—see Fig. 5 of Ref. [92] and cf. Fig. 2 of Ref. [91]. This then suggests that the pseudogap opening for Hg1201 also lies near $p \approx 0.19$. Consistent with this, Yamamoto *et al.* [92] presented scaling plots of the resistivity and thermoelectric power which reveal a T^* line that descends with doping very similar to that for (Y,Ca)123, projecting to a p^* value close to 0.19, especially if the optimally doped data is ignored, where a T^* value is difficult to ascertain as it approaches the value of T_c . All in all, the data is not inconsistent with $p^* \approx 0.19$ for Hg1201.

It follows from the above that the same can be said for La214. Also, significantly, the c -axis infrared conductivity crosses over from insulating to metallic with a strong low-frequency Drude peak between doping states of 0.18 and 0.20 [93]. Moreover, the jump height $\Delta\gamma(T_c)$ begins to collapse rapidly below $p \approx 0.18$ [19], as it also does for $\text{Tl}_{0.5}\text{Pb}_{0.5}\text{Sr}_2\text{Ca}_{1-x}\text{Y}_x\text{Cu}_2\text{O}_7$ [94], in both cases indicating the abrupt opening of the pseudogap. Further, the phase behavior of $\text{Tl}_{0.5}\text{Pb}_{0.5}\text{Sr}_{2-x}\text{La}_x\text{CaCu}_2\text{O}_7$ is almost identical to that of $\text{Tl}_{0.5}\text{Pb}_{0.5}\text{Sr}_2\text{Ca}_{1-x}\text{Y}_x\text{Cu}_2\text{O}_7$ across the superconducting phase diagram [95], suggesting that the pseudogap opens identically in this compound.

While more comprehensive studies would be desirable, this broad suite of cuprates all seem to exhibit an abrupt pseudogap opening somewhere near 0.19 holes/Cu. We may refer to these as *canonical* cuprates. At the present time, it seems unlikely that $\text{Tl}_2\text{Ba}_2\text{CuO}_{6+\delta}$ (Tl2201) would be different. For example, its thermoelectric power [47], low-field Hall angle [96], and specific heat [97] show the usual canonical behavior. On the other hand, as noted in the Introduction, a crossover in Hall number from p to $1 + p$ was recently observed for Tl2201 [14] occurring as high as $p \approx 0.25$ and we have previously associated such a crossover with the closing of the pseudogap. That assignment is perhaps brought into question given the absence of a thermodynamic signature of the pseudogap in Tl2201 there. It has been shown theoretically [98] that a similar crossover in the Hall number is expected near a VHS if a Pomeranchuk-type nematic instability occurs, transforming a closed Fermi surface to open sheets. However, calculations (see Fig. S5 in Ref. [14]) suggest that in Tl2201 the VHS lies at a much higher value of $p = 0.55$.

The other potential outlier is the cuprate system extensively studied by the group of Taillefer [99,100], namely, $\text{La}_{1.6-x}\text{Nd}_{0.4}\text{Sr}_x\text{CuO}_4$. Drawing on transport and ARPES studies, these authors identified the closing of the pseudogap at $p = 0.23 \pm 0.01$, well beyond the value for the other cuprates discussed above. It may well be that this system is noncanonical in this respect, however, we note that the ARPES investigation [101] simply identified an antinodal gap “just above T_c ” and associated this with the pseudogap. As discussed above, it is now recognized that a pairing gap (or superconducting partial gap) persists well above T_c , in addition to the pseudogap over the lower doping range. The energy distribution curves reported for $\text{La}_{1.6-x}\text{Nd}_{0.4}\text{Sr}_x\text{CuO}_4$ just above T_c are very similar to those reported for Bi2212

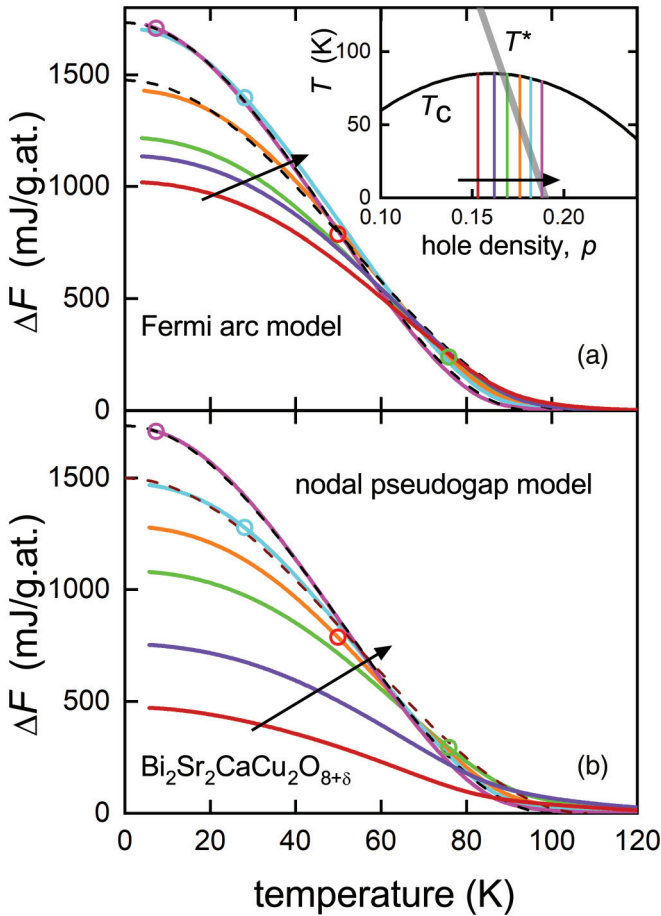


FIG. 9. Comparison of the effect on the deduced condensation energy, ΔF_{ns} , of the two models used for the pseudogap: (a) Fermi arc model [49] and (b) nodal pseudogap model [75]. All color coding as in Fig. 2. Dashed curves are the near-weak-coupling d -wave T dependence of $\Delta F_{\text{ns}}(T)$.

above T_c [68] in terms of their shape and spectral-weight redistribution, but of lower resolution. A more extensive doping and temperature-dependent study would be needed to truly ascertain that the observed partial gap is indeed the pseudogap.

The above is just a sketch of a crucially important topic which deserves much more detailed study. In principle, we would expect that $p^* \approx N_0 \times J$, where J is the nearest-neighbor exchange interaction [3] and N_0 is the DOS at the Fermi level. Inasmuch as both these quantities do not vary much from one cuprate to another, we would not expect p^* to vary much. This remains to be demonstrated as a canonical property of the cuprates and we encourage its investigation.

APPENDIX D: COMPARISON OF FERMI ARCS VERSUS NODAL PSEUDOGAP MODEL

We have used two models for the pseudogap in the past: a nodal pseudogap [75] on a background S_n calculated from the rigid ARPES-derived dispersion of Kaminski *et al.* [28] and a Fermi arc model [49] based on the same background S_n . Beyond p^* , there is, of course, no impact, so the calculated ΔF_{ns} values are as in Fig. 2. It is useful to note the difference obtained between the two models for $p < p^*$ when the pseudogap is open. Clearly, a nodal pseudogap strips away states available for superconductivity near the nodes so the condensation energy will be smaller than that arising in a Fermi arc picture. The two models are compared in Fig. 9 and this confirms expectations. The differences are quite substantial. However, in either case, a canonical near-weak-coupling T dependence is still obtained (black dashed curves). The ΔF_{ns} curves for the nodal pseudogap were those reported previously [50] for calculating the T -dependent superconducting gap amplitude. The Fermi-arc derived ΔF_{ns} curves are new.

- [1] M. R. Norman, D. Pines, and C. Kallin, The pseudogap: Friend or foe of high- T_c , *Adv. Phys.* **54**, 715 (2005).
- [2] T. Timusk and B. Statt, The pseudogap in high-temperature superconductors: An experimental survey, *Rep. Prog. Phys.* **62**, 61 (1999).
- [3] J. W. Loram, J. Luo, J. R. Cooper, W. Y. Liang and J. L. Tallon, Evidence on the pseudogap and condensate from the electronic specific heat, *J. Phys. Chem. Solids* **62**, 59 (2001).
- [4] C. Bernhard, J. L. Tallon, Th. Blasius, A. Golnik, and Ch. Niedermayer, Anomalous Peak in the Superconducting Condensate Density of Cuprate High- T_c Superconductors at a Unique Doping State, *Phys. Rev. Lett.* **86**, 1614 (2001).
- [5] G. Grissonnanche, O. Cyr-Choinière, F. Laliberté, S. René de Cotret, A. Juneau-Fecteau, S. Dufour-Beauséjour, M.-È. Delage, D. LeBoeuf, J. Chang, B. J. Ramshaw, D. A. Bonn, W. N. Hardy, R. Liang, S. Adachi, N. E. Hussey, B. Vignolle, C. Proust, M. Sutherland, S. Krämer, J.-H. Park, D. Graf, N. Doiron-Leyraud, and L. Taillefer, Direct measurement of the upper critical field in cuprate superconductors, *Nat. Commun.* **5**, 3280 (2014).
- [6] M. R. Norman, H. Ding, M. Randeria, J. C. Campuzano, T. Yokoya, T. Takeuchi, T. Takahashi, T. Mochiku, K. Kadowaki, P. Guptasarma, and D. G. Hinks, Destruction of the Fermi surface in underdoped high- T_c superconductors, *Nature (London)* **392**, 157 (1998).
- [7] J. G. Storey, Incoherent superconductivity well above T_c in high- T_c cuprates – harmonizing the spectroscopic and thermodynamic data, *New J. Phys.* **19**, 073026 (2017).
- [8] J. G. Storey, Hall effect and Fermi surface reconstruction via electron pockets in the high- T_c cuprates, *Europhys. Lett.* **113**, 27003 (2016).
- [9] K. Fujita, C. K. Kim, I. Lee, J. Lee, M. H. Hamidian, A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, and J. C. Davis, Simultaneous transitions in cuprate momentum-space topology and electronic symmetry breaking, *Science* **344**, 612 (2014).
- [10] Y. He, Y. Yin, M. Zech, A. Soumyanarayanan, M. M. Yee, T. Williams, M. C. Boyer, K. Chatterjee, W. D. Wise, I. Zeljkovic, T. Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, and J. E. Hoffman, Fermi surface and pseudogap evolution in a cuprate superconductor, *Science* **344**, 608 (2014).
- [11] S. Badoux, W. Tabis, F. Laliberté, G. Grissonnanche, B. Vignolle, D. Vignolles, J. Béard, D. A. Bonn, W. N. Hardy, R.

- Liang, N. Doiron-Leyraud, L. Taillefer, and C. Proust, Change of carrier density at the pseudogap critical point of a cuprate superconductor, *Nature (London)* **531**, 210 (2016).
- [12] H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu, P. D. Johnson, H. Claus, D. G. Hinks, and T. E. Kidd, Reconstructed Fermi Surface of Underdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ Cuprate Superconductors, *Phys. Rev. Lett.* **107**, 047003 (2011).
- [13] A. Damascelli, Z. Hussain, and Z.-X. Shen, Angle-resolved photoemission studies of the cuprate superconductors, *Rev. Mod. Phys.* **75**, 473 (2003).
- [14] C. Putzke, S. Benhabib, W. Tabis, J. Ayres, Z. Wang, L. Malone, S. Licciardello, J. Lu, T. Kondo, T. Takeuchi, N. E. Hussey, J. R. Cooper, and A. Carrington, Reduced Hall carrier density in the overdoped strange metal regime of cuprate superconductors, [arXiv:1909.08102v1](https://arxiv.org/abs/1909.08102v1).
- [15] J. L. Tallon and J. W. Loram, The doping dependence of T^* —what is the real high- T_c phase diagram? *Physica C* **349**, 53 (2001).
- [16] S. H. Naqib, J. R. Cooper, and J. L. Tallon, Doping phase diagram of $\text{Y}_{1-x}\text{Ca}_x\text{Ba}_2(\text{Cu}_{1-y}\text{Zn}_y)_3\text{O}_{7-\delta}$ from transport measurements: Tracking the pseudogap below T_c , *Phys. Rev. B* **71**, 054502 (2005).
- [17] J. R. Cooper, J. W. Loram, I. Kokanović, J. G. Storey, and J. L. Tallon, Pseudogap in $\text{YBa}_2\text{Cu}_3\text{O}_{6+\delta}$ is not bounded by a line of phase transitions: Thermodynamic evidence, *Phys. Rev. B* **89**, 201104(R) (2014).
- [18] J. W. Loram, K. A. Mirza, J. R. Cooper, W. Y. Liang, and J. M. Wade, Electronic specific heat of $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ from 1.8 to 300 K, *J. Supercond.* **7**, 243 (1994).
- [19] J. W. Loram, K. A. Mirza, J. M. Wade, J. R. Cooper, and W. Y. Liang, The electronic specific heat of cuprate superconductors, *Physica C* **235-240**, 134 (1994).
- [20] J. L. Tallon, G. V. M. Williams, M. P. Staines, and C. Bernhard, Energy and length scales in the superconducting phase diagram for HTSC cuprates, *Physica C* **235-240**, 1821 (1994).
- [21] G. V. M. Williams, J. L. Tallon, and J. W. Loram, Crossover temperatures in the normal-state phase diagram of high- T_c superconductors, *Phys. Rev. B* **58**, 15053 (1998).
- [22] B. Fauqué, Y. Sidis, V. Hinkov, S. Pailhès, C. T. Lin, X. Chaud, and P. Bourges, Magnetic Order in the Pseudogap Phase of High- T_c Superconductors, *Phys. Rev. Lett.* **96**, 197001 (2006).
- [23] J. Xia, E. Schemm, G. Deutscher, S. A. Kivelson, D. A. Bonn, W. N. Hardy, R. Liang, W. Siemons, G. Koster, M. M. Fejer, and A. Kapitulnik, Polar Kerr-Effect Measurements of the High-Temperature $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ Superconductor: Evidence for Broken Symmetry Near the Pseudogap Temperature, *Phys. Rev. Lett.* **100**, 127002 (2008).
- [24] M. Hashimoto, R.-H. He, K. Tanaka, J.-P. Testaud, W. Meevasana, R. G. Moore, D. Lu, H. Yao, Y. Yoshida, H. Eisaki, T. P. Devereaux, Z. Hussain, and Z.-X. Shen, Particle-hole symmetry breaking in the pseudogap state of $\text{Bi}2201$, *Nat. Phys.* **6**, 414 (2010).
- [25] R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S.-K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, From a single-band metal to a high-temperature superconductor via two thermal phase transitions, *Science* **331**, 1579 (2011).
- [26] A. Shekhter, B. J. Ramshaw, R. Liang, W. N. Hardy, D. A. Bonn, F. F. Balakirev, R. D. McDonald, J. B. Betts, S. C. Riggs, and A. Migliori, Bounding the pseudogap with a line of phase transitions in $\text{YBa}_2\text{Cu}_3\text{O}_{6+\delta}$, *Nature (London)* **498**, 75 (2013).
- [27] Y. Sato, S. Kasahara, H. Murayama, Y. Kasahara, E.-G. Moon, T. Nishizaki, T. Loew, J. Porras, B. Keimer, T. Shibauchi, and Y. Matsuda, Thermodynamic evidence for a nematic phase transition at the onset of the pseudogap in $\text{YBa}_2\text{Cu}_3\text{O}_y$, *Nat. Phys.* **13**, 1074 (2017).
- [28] A. Kaminski, S. Rosenkranz, H. M. Fretwell, M. R. Norman, M. Randeria, J. C. Campuzano, J. M. Park, Z. Z. Li, and H. Raffy, Change of Fermi-surface topology in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ with doping, *Phys. Rev. B* **73**, 174511 (2006).
- [29] I. M. Vishik, M. Hashimoto, R.-H. He, W.-S. Lee, F. Schmitt, D. Lu, R. G. Moore, C. Zhang, W. Meevasana, T. Sasagawa, S. Uchida, K. Fujita, S. Ishida, M. Ishikado, Y. Yoshida, H. Eisaki, Z. Hussain, T. P. Devereaux, and Z.-X. Shen, Phase competition in trisected superconducting dome, *Proc. Nat. Acad. Sci. U.S.A.* **109**, 18332 (2012).
- [30] Y. He, M. Hashimoto, D. Song, S.-D. Chen, J. He, I. M. Vishik, B. Moritz, D.-H. Lee, N. Nagaosa, J. Zaanen, T. P. Devereaux, Y. Yoshida, H. Eisaki, D. H. Lu, and Z.-X. Shen, Rapid change of superconductivity and electron-phonon coupling through critical doping in Bi-2212 , *Science* **362**, 62 (2018).
- [31] S. Benhabib, A. Sacuto, M. Civelli, I. Paul, M. Cazayous, Y. Gallais, M.-A. Méasson, R. D. Zhong, J. Schneeloch, G. D. Gu, D. Colson, and A. Forget, Collapse of the Normal-State Pseudogap at a Lifshitz Transition in the $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ Cuprate Superconductor, *Phys. Rev. Lett.* **114**, 147001 (2015).
- [32] A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, Universal T -linear resistivity and Planckian dissipation in overdoped cuprates, *Nat. Phys.* **15**, 142 (2018).
- [33] G. Balakrishnan, R. Dupree, I. Farnan, D. McK. Paul, and M. E. Smith, An upper bound for the density of states at the yttrium site in $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$, *J. Phys. C: Solid State Phys.* **21**, L847 (1988).
- [34] H. Alloul, T. Ohno, and P. Mendels, ^{89}Y NMR Evidence for a Fermi-Liquid Behavior in $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$, *Phys. Rev. Lett.* **63**, 1700 (1989).
- [35] L. Yu, D. Munzar, A. V. Boris, P. Yordanov, J. Chaloupka, Th. Wolf, C. T. Lin, B. Keimer, and C. Bernhard, Evidence for Two Separate Energy Gaps in Underdoped High-Temperature Cuprate Superconductors from Broadband Infrared Ellipsometry, *Phys. Rev. Lett.* **100**, 177004 (2008).
- [36] E. G. Moon and S. Sachdev, Quantum critical point shifts under superconductivity: Pnictides and cuprates, *Phys. Rev. B* **82**, 104516 (2010).
- [37] S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni, S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, and A. I. Goldman, Anomalous Suppression of the Orthorhombic Lattice Distortion in Superconducting $\text{Ba}(\text{Fe}_{1-x}\text{Co}_x)_2\text{As}_2$ Single Crystals, *Phys. Rev. Lett.* **104**, 057006 (2010).
- [38] H. Won and K. Maki, d -wave superconductor as a model of high- T_c superconductors, *Phys. Rev. B* **49**, 1397 (1994).

- [39] R. Khasanov, and Z. Guguchia, Probing the multi gap behavior within “11” and “122” families of iron based superconductors: the muon-spin rotation studies, *Supercond. Sci. Technol.* **28**, 034003 (2015).
- [40] T. Cichorek, A. C. Mota, F. Steglich, N. A. Frederick, W. M. Yuhasz, and M. B. Maple, Pronounced Enhancement of the Lower Critical Field and Critical Current Deep in the Superconducting State of $\text{PrOs}_4\text{Sb}_{12}$, *Phys. Rev. Lett.* **94**, 107002 (2005).
- [41] E. F. Talantsev and J. L. Tallon, Universal self-field critical current for thin-film superconductors, *Nat. Commun.* **6**, 7820 (2015).
- [42] J. W. Loram, J. L. Luo, J. R. Cooper, W. Y. Liang, and J. L. Tallon, The condensation energy and pseudogap energy scale of Bi:2212 from the electronic specific heat, *Physica C* **341-348**, 831 (2000).
- [43] J. L. Tallon, R. G. Buckley, P. W. Gilberd, M. R. Presland, I. W. M. Brown, M. E. Bowden, L. A. Christian, and R. Goguel, High- T_c superconducting phases in the series $\text{Bi}_{2.1}(\text{Ca}, \text{Sr})_{n+1}\text{Cu}_n\text{O}_{2n+4}$, *Nature* **333**, 153 (1988).
- [44] J. L. Tallon, Normal-state pseudogap in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ characterized by impurity scattering, *Phys. Rev. B* **58**, 5956(R) (1998).
- [45] J. W. Loram, K. A. Mirza, J. R. Cooper and W. Y. Liang, Electronic Specific Heat of $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ from 1.8 to 300 K, *Phys. Rev. Lett.* **71**, 1740 (1993).
- [46] M. R. Presland, J. L. Tallon, R. G. Buckley, R. S. Liu, and N. E. Flower, General trends in oxygen stoichiometry effects in Bi and Tl superconductors, *Physica C* **176**, 95 (1991).
- [47] S. D. Obertelli, J. R. Cooper, and J. L. Tallon, Systematics in the thermoelectric power of high- T_c oxides, *Phys. Rev. B* **46**, 14928 (1992).
- [48] W. Anukool, S. Barakat, C. Panagopoulos, and J. R. Cooper, Effect of hole doping on the London penetration depth in $\text{Bi}_{2.15}\text{Sr}_{1.85}\text{CaCu}_2\text{O}_{8+\delta}$ and $\text{Bi}_{2.1}\text{Sr}_{1.9}\text{Ca}_{0.85}\text{Y}_{0.15}\text{Cu}_2\text{O}_{8+\delta}$, *Phys. Rev. B* **80**, 024516 (2009).
- [49] J. G. Storey, J. L. Tallon, and G. V. M. Williams, Pseudogap ground state in high-temperature superconductors, *Phys. Rev. B* **78**, 140506(R) (2008).
- [50] J. L. Tallon, F. Barber, J. G. Storey, and J. W. Loram, Coexistence of the superconducting energy gap and pseudogap above and below the transition temperature of cuprate superconductors, *Phys. Rev. B* **87**, 140508(R) (2013).
- [51] N. Zaki, H.-B. Yang, J. D. Rameau, P. D. Johnson, H. Claus, and D. G. Hinks, Cuprate phase diagram and the influence of nanoscale inhomogeneities, *Phys. Rev. B* **96**, 195163 (2017).
- [52] H. Padamsee, J. E. Neighbor, and C. A. Shiffman, Quasiparticle phenomenology for thermodynamics of strong-coupling superconductors, *J. Low Temp. Phys.* **12**, 387 (1973).
- [53] J. L. Tallon, J. G. Storey, and J. W. Loram, Fluctuations and critical temperature reduction in cuprate superconductors, *Phys. Rev. B* **83**, 092502 (2011).
- [54] H. Alloul, F. Rullier-Albenque, B. Vignolle, D. Colson, and A. Forget, Superconducting fluctuations, pseudogap and phase diagram in cuprates, *Europhys. Lett.* **91**, 37005 (2010).
- [55] C. P. Poole, H. A. Farach, R. J. Creswick, and R. Prozorov, *Superconductivity* (Academic Press, London, 2007), p. 343.
- [56] R. Comin, A. Frano, M. M. Yee, Y. Yoshida, H. Eisaki, E. Schierle, E. Weschke, R. Sutarto, F. He, A. Soumyanarayanan, Y. He, M. Le Tacon, I. S. Elfimov, J. E. Hoffman, G. A. Sawatzky, B. Keimer, and A. Damascelli, Charge order driven by Fermi-arc instability in $\text{Bi}_2\text{Sr}_{2-x}\text{La}_x\text{CuO}_{6+\delta}$, *Science* **343**, 390 (2014).
- [57] A. Larkin and A. Varlamov, *Theory of Fluctuations in Superconductors* (Clarendon Press, Oxford, U.K., 2005), p. 264.
- [58] T. Watanabe, T. Fujii, and A. Matsuda, Anisotropic Resistivities of Precisely Oxygen Controlled Single-Crystal $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$: Systematic Study on “Spin Gap” Effect, *Phys. Rev. Lett.* **79**, 2113 (1997).
- [59] T. M. Benseman, J. R. Cooper, and G. Balakrishnan, Temperature and field dependence of the intrinsic tunneling structure in overdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$, *Phys. Rev. B* **98**, 014507 (2018).
- [60] K. Ishida, K. Yoshida, T. Mito, Y. Tokunaga, Y. Kitaoka, K. Asayama, A. Nakayama, J. Shimoyama, and K. Kishio, Pseudogap behavior in single-crystal $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ probed by Cu NMR, *Phys. Rev. B* **58**, R5960 (1998).
- [61] M. Takigawa, and D. B. Mitzi, NMR Studies of Spin Excitations in Superconducting $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ Single Crystals, *Phys. Rev. Lett.* **73**, 1287 (1994).
- [62] J. Jaklic, and P. Prelovsek, Finite-temperature properties of doped antiferromagnets, *Adv. Phys.* **49**, 1 (2000).
- [63] B. Loret, S. Sakai, S. Benhabib, Y. Gallais, M. Cazayous, M. A. Méasson, R. D. Zhong, J. Schneeloch, G. D. Gu, A. Forget, D. Colson, I. Paul, M. Civelli, and A. Sacuto, Vertical temperature boundary of the pseudogap under the superconducting dome in the phase diagram of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$, *Phys. Rev. B* **96**, 094525 (2017).
- [64] M. Oda, K. Hoya, R. Kubota, C. Manabe, N. Momono, T. Nakano, and M. Ido, Strong pairing interactions in the underdoped region of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\sigma}$, *Physica C* **281**, 135 (1997).
- [65] T. Watanabe, T. Fujii, and A. Matsuda, Pseudogap in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ Studied by Measuring Anisotropic Susceptibilities and Out-of-Plane Transport, *Phys. Rev. Lett.* **84**, 5848 (2000).
- [66] J. R. Cooper, H. Minami, V. W. Wittorff, D. Babić, and J. W. Loram, Effect of the normal state gap on the thermoelectric power, irreversibility line and c -axis resistivity of $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$, *Physica C* **341-348**, 855 (2000).
- [67] K. K. Gomes, A. N. Pasupathy, A. Pushp, S. Ono, Y. Ando, and A. Yazdani, Visualizing pair formation on the atomic scale in the high- T_c superconductor $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$, *Nature* **447**, 569 (2007).
- [68] T. Kondo, W. Malaeb, Y. Ishida, T. Sasagawa, H. Sakamoto, T. Takeuchi, T. Tohyama, and S. Shin, Point nodes persisting far beyond T_c in Bi2212 , *Nat. Commun.* **6**, 7699 (2015).
- [69] I. Kokanović, J. R. Cooper, and K. Iida, Changes in the in- and out-of-plane magnetic susceptibility of YBCO crystals with temperature and hole content, *Europhys. Lett.* **98**, 57011 (2012).
- [70] T. Usui, D. Fujiwara, S. Adachi, H. Kudo, K. Murata, H. Kushibiki, T. Watanabe, K. Kudo, T. Nishizaki, N. Kobayashi, S. Kimura, K. Yamada, T. Naito, T. Noji, and Y. Koike, Doping dependencies of onset temperatures for the pseudogap and superconductive fluctuation in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$, studied from both in-plane and out-of-plane magnetoresistance measurements, *J. Phys. Soc. Jpn.* **83**, 064713 (2014).
- [71] S.-D. Chen, M. Hashimoto, Y. He, D. Song, K.-J. Xu, J.-F. He, T. P. Devereaux, H. Eisaki, D.-H. Lu, J. Zaanen, and Z.-X.

- Shen, Incoherent strange metal sharply bounded by a critical doping in Bi2212, *Science* **366**, 1099 (2019).
- [72] J. W. Loram, J. R. Cooper, J. M. Wheatley, K. A. Mirza, and R. S. Liu, Critical and Gaussian fluctuation effects in the specific heat and conductivity of high- T_c superconductors, *Philos. Mag. B* **65**, 1405 (1992).
- [73] S. H. Naqib, J. R. Cooper, and J. W. Loram, Effects of Ca substitution and the pseudogap on the magnetic properties of $Y_{1-x}Ca_xBa_2Cu_3O_{7-\delta}$, *Phys. Rev. B* **79**, 104519 (2009).
- [74] I. Kokanović, D. J. Hills, M. L. Sutherland, and J. R. Cooper, Diamagnetism of $YBa_2Cu_3O_{6+x}$ crystals above T_c : Evidence for Gaussian fluctuations, *Phys. Rev. B* **88**, 060505 (2013).
- [75] J. G. Storey, J. L. Tallon, and G. V. M. Williams, Saddle-point van Hove singularity and the phase diagram of high- T_c cuprates, *Phys. Rev. B* **76**, 174522 (2007).
- [76] M. Le Tacon, A. Sacuto, A. Georges, G. Kotliar, Y. Gallais, D. Colson, and A. Forget, Two energy scales and two distinct quasiparticle dynamics in the superconducting state of underdoped cuprates, *Nat. Phys.* **2**, 537 (2006).
- [77] A. L. Fetter and P. C. Hohenberg, Theory of type II superconductors, in *Superconductivity*, edited by R. D. Parks (Marcel Dekker, New York, 1969), Vol. 2, pp. 838–843.
- [78] J. W. Loram, K. A. Mirza, and J. R. Cooper, Properties of the superconducting condensate and normal-state pseudogap in high- T_c superconductors derived from the electronic specific heat, *IRC in Superconductivity Research Review* (Cavendish Laboratory, Cambridge University, 1998).
- [79] J. L. Tallon, J. W. Loram, and J. R. Cooper, The superfluid density in cuprate high- T_c superconductors – a new paradigm, *Phys. Rev. B* **68**, 180501(R) (2003).
- [80] J. L. Tallon, Thermodynamics and critical current density in high- T_c superconductors, *IEEE Trans. Appl. Supercond.* **25**, 8000806 (2015).
- [81] E. F. Talantsev, N. M. Strickland, S. C. Wimbush, J. G. Storey, J. L. Tallon, and N. J. Long, Hole doping dependence of critical current density in $YBa_2Cu_3O_7$ conductors, *Appl. Phys. Lett.* **104**, 242601 (2014).
- [82] M. Naamneh, J. C. Campuzano, and A. Kanigel, Doping dependence of the critical current in $Bi_2Sr_2CaCu_2O_{8+\delta}$, *Phys. Rev. B* **90**, 224501 (2014).
- [83] D. L. Feng, D. H. Lu, K. M. Shen, C. Kim, H. Eisaki, A. Damascelli, R. Yoshizaki, J.-I. Shimoyama, K. Kishio, G. D. Gu, S. Oh, A. Andrus, J. O'Donnell, J. N. Eckstein, and Z.-X. Shen, Signature of superfluid density in the single-particle excitation spectrum of $Bi_2Sr_2CaCu_2O_{8+\delta}$, *Science* **289**, 277 (2000).
- [84] F. Carbone, A. B. Kuzmenko, H. J. A. Molegraaf, E. van Heumen, V. Lukovac, F. Marsiglio, D. van der Marel, K. Haule, G. Kotliar, H. Berger, S. Courjault, P. H. Kes, and M. Li, Doping dependence of the redistribution of optical spectral weight in $Bi_2Sr_2CaCu_2O_{8+\delta}$, *Phys. Rev. B* **74**, 064510 (2006).
- [85] J. Levallois, M. K. Tran, D. Pouliot, C. N. Presura, L. H. Greene, J. N. Eckstein, J. Uccelli, E. Giannini, G. D. Gu, A. J. Leggett, and D. van der Marel, Temperature-Dependent Ellipsometry Measurements of Partial Coulomb Energy in Superconducting Cuprates, *Phys. Rev. X* **6**, 031027 (2016).
- [86] J. L. Tallon, J. W. Loram, and C. Panagopoulos, Pseudogap and quantum-transition phenomenology in HTS cuprates, *J. Low Temp. Phys.* **131**, 387 (2003).
- [87] S. Oh, T. Di Luccio, and J. N. Eckstein, T -linearity of in-plane resistivity in $Bi_2Sr_2CaCu_2O_{8+\delta}$ thin films, *Phys. Rev. B* **71**, 052504 (2005).
- [88] E. Sterpetti, J. Biscaras, A. Erb, and A. Shukla, Crossover to strange metal phase: Quantum criticality in one unit cell $Bi_2Sr_2CaCu_2O_{8+x}$, *J. Phys.: Condens. Matter* **32**, 045601 (2020).
- [89] Y. Ando, Y. Hanaki, S. Ono, T. Murayama, K. Segawa, N. Miyamoto, and S. Komiya, Carrier concentrations in $Bi_2Sr_{2-z}La_zCu_2O_{6+\delta}$ single crystals and their relation to the Hall coefficient and thermopower, *Phys. Rev. B* **61**, 14956(R) (2000).
- [90] A. V. Pimenov, A. V. Boris, Li Yu, V. Hinkov, Th. Wolf, J. L. Tallon, B. Keimer, and C. Bernhard, Nickel Impurity-Induced Enhancement of the Pseudogap of Cuprate High- T_c Superconductors, *Phys. Rev. Lett.* **94**, 227003 (2005).
- [91] J. L. Tallon, G. V. M. Williams, N. E. Flower, and C. Bernhard, Phase separation, pseudogap and impurity scattering in the HTS cuprates, *Physica C* **282-287**, 236 (1997).
- [92] A. Yamamoto, W.-Z. Hu, and S. Tajima, Thermoelectric power and resistivity of $HgBa_2CuO_{4+\delta}$ over a wide doping range, *Phys. Rev. B* **63**, 024504 (2000).
- [93] R. Henn, A. Wittlin, M. Cardona, and S. Uchida, Dynamics of the c -polarized infrared-active modes in $La_{2-x}Sr_xCuO_4$, *Phys. Rev. B* **56**, 6295 (1997).
- [94] J. W. Loram, K. A. Mirza, and R. S. Liu, Specific heat anomalies in the 108 K superconducting system $Y_{1-x}Ca_xSr_2Cu_2Ti_{0.5}Pb_{0.5}O_7$, *Supercond. Sci. Technol.* **4**, S286 (1991).
- [95] M. R. Presland, and J. L. Tallon, Superconductivity at 105 K in $Tl_{0.5}Pb_{0.5}CaSr_{2-x}La_xCu_2O_7$, *Physica C* **177**, 1 (1991).
- [96] A. P. Mackenzie, S. R. Julian, D. C. Sinclair, and C. T. Lin, Normal-state magnetotransport in superconducting $Tl_2Ba_2CuO_{6+\delta}$ to millikelvin temperatures, *Phys. Rev. B* **53**, 5848 (1996).
- [97] J. M. Wade, J. W. Loram, K. A. Mirza, J. R. Cooper, and J. L. Tallon, Electronic specific heat of $Tl_2Ba_2CuO_{6+\delta}$ from 2 K to 300 K for $0 \leq \delta \leq 0.1$, *J. Supercond.* **7**, 261 (1994).
- [98] A. V. Maharaj, I. Esterlis, Y. Zhang, B. J. Ramshaw, and S. A. Kivelson, Hall number across a van Hove singularity, *Phys. Rev. B* **96**, 045132 (2017).
- [99] N. Doiron-Leyraud, O. Cyr-Choinière, S. Badoux, A. Ataei, C. Collignon, A. Gourgout, S. Dufour-Beauséjour, F. v F. Tafti, F. Laliberté, M.-E. Boulanger, M. Matusiak, D. Graf, M. Kim, J.-S. Zhou, N. Momono, T. Kurosawa, H. Takagi, and L. Taillefer, Pseudogap phase of cuprate superconductors confined by Fermi surface topology, *Nat. Commun.* **8**, 2044 (2017).
- [100] C. Collignon, S. Badoux, S. A. A. Afshar, O. Cyr-Choinière, J.-S. Zhou, S. Licciardello, S. Wiedmann, N. Doiron-Leyraud, and L. Taillefer, Fermi-surface transformation across the pseudogap critical point of the cuprate superconductor $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$, *Phys. Rev. B* **95**, 224517 (2017).
- [101] C. E. Matt, C. G. Fatuzzo, Y. Sassa, M. Mansson, S. Fatale, V. Bitetta, X. Shi, S. Pailhes, M. H. Berntsen, T. Kurosawa *et al.*, Electron scattering, charge order, and pseudogap physics in $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$: An angle-resolved photoemission spectroscopy study, *Phys. Rev. B* **92**, 134524 (2015).