Interplay between collective modes in hybrid electron-gas–superconductor structures

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We study hybridization of collective plasmon and Carlson-Goldman-Artemenko-Volkov modes in a hybrid system, consisting of two-dimensional layers of electron gas in the normal state and superconductor, coupled by long-range Coulomb forces. The interaction between these collective modes is not possible in a regular single-layer two-dimensional system since they exist in nonoverlapping domains of dimensionless parameter $\omega\tau$, where ω is the external electromagnetic field frequency and τ is electron-scattering time. Thus, in a single-layer structure, these modes are mutually exclusive. However, the coupling may become possible in a hybrid system consisting of two separated in space materials with different properties, in particular, the electron-scattering time. We investigate the electromagnetic power absorption by the hybrid system and reveal the conditions necessary for the hybridization of collective modes.

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I. INTRODUCTION

Mesoscopic systems in normal (not superconducting) state lodge elementary excitations, such as excitons and plasmons. They determine the response of the material to external perturbations. Plasmons represent collective modes of particle density fluctuations and play an essential role in the material response to external alternating electromagnetic (EM) fields. Their dispersion law and damping strongly depend on the dimensionality of the system. Indeed, in the case of three-dimensional (3D) electron plasma, the plasmon branch is with a good accuracy dispersionless, whereas two-dimensional (2D) plasmons are characterized by a square-root dispersion relation and vanish in the long-wavelength limit (at least, in the framework of the quasistatic approximation). Many-component 2D systems like an electron-hole 2D gas are characterized by two plasmon branches with linear (acoustic plasmon) and square-root (optical plasmon) dispersions.

If with a decrease of temperature the system undertakes a phase transition to another state like a Bose-Einstein or superconducting (SC) condensate, the ground state changes and new elementary excitations describing low-energy properties of the system arise. This can lead to drastic modification of the material response to external EM fields, which becomes especially important in view of recent progress in plasmonics, optoelectronics, and high-temperature condensation phenomena.

In the pioneering work of Bardeen, Cooper, and Schriffer (BCS) [1], only single-particle excitations characterized by the SC gap were studied. Later, Anderson [2], Bogoliubov *et al.* [3], and Vaks, Galitskii, and Larkin [4] developed a more extensive theory, which includes the collective excitations of Cooper pairs. They showed that in a neutral Fermi system with an attractive interaction between fermions collective modes possess a soundlike dispersion. A charged Fermi system, instead, demonstrates the collective modes with the frequency pushed towards the plasmon frequency of the 3D charged electron gas. This frequency is usually much larger than the BCS gap and consequently it does not play a role in conventional phenomena in superconductors.

These modes correspond to oscillations of the SC order parameter. More precisely, the Anderson-Bogoliubov mode represents the oscillation of the phase of the order parameter, whereas the Higgs mode corresponds to the oscillations of its amplitude. These collective modes play a crucial role in the gauge-invariant response of superconductors to external EM perturbations [5].

Later on, Carlson and Goldman [6–8] experimentally discovered another type of collective modes, which occur in the vicinity of the transition temperature $T_c - T \ll T_c$ in superconductors. This experimental finding stimulated theoretical studies. In the dirty-sample case, there was suggested the model of Schmid and Schön [9], whereas in the clean (low-disorder) limit there was developed the model by Artemenko and Volkov (AV) [10,11]. Qualitatively, the AV mode corresponds to the out-of-phase oscillations of normal and SC currents in such a way that it prevents the emergence of net charges in the system. The frequency of this mode is smaller than the plasmon frequency of normal 3D electrons, thus suppressing the interaction between them.

In the meanwhile, the coupling between single-particle and collective modes in hybrid systems consisting of a 2D electron layer and a 2D condensate of Bose particles represents actively developing areas of research recently [12–14]. In particular, there emerge such phenomena as the magnetoplasmon Fano resonances in uniform magnetic fields [15] and amplification of the incident light [16,17]. The progress in theoretical proposals and the lack of well-established experimental platforms motivate the study of the interplay of different collective modes in hybrid systems. In this paper,

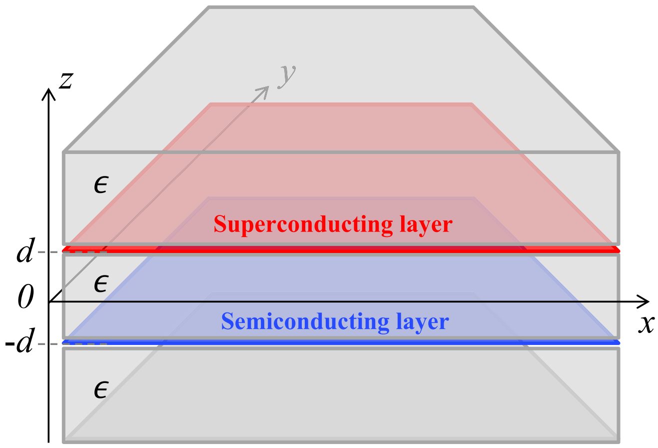

FIG. 1. System schematic. A hybrid structure consisting of a superconducting layer located at $z = d$ and a semiconductor layer at $z = -d$. The layers are thus separated by the distance $2d$ and surrounded by the environment with the effective dielectric constant ϵ.

we investigate the hybridization of collective plasmon and AV modes in a hybrid semiconductor-superconductor structure (where one of the subsystems is in the SC state). As we have already pointed out, the plasmon mode has a gapless dispersion in the 2D case. We will show that it opens a possibility of interaction between AV modes and plasmons.

At first sight, despite the gaplessness of the plasmon mode, the interaction between them is not possible since the AV mode exists in the domain $\omega\tau \ll 1$ (here τ is electron-impurity scattering time), whereas the plasmon mode exists at $\omega\tau \gg 1$, and these two conditions do not overlap. However, their interaction can become possible if the normal electron-gas layer (having the plasmon mode) and the SC layer (hosting the AV mode) are separated in space and electrons in each layer have different scattering times. Thus, if we denote as τ_n the scattering time of electrons in the normal layer and τ_s the one for quasiparticles in the SC layer, the simultaneous existence of AV and plasmon modes is possible when $\tau_s^{-1} > \omega > \tau_n^{-1}$. This is a necessary criterion for the coexistence of both the modes in one sample. However, it is not a sufficient condition for their hybridization. What are the other conditions? This is the main question which we address in this paper.

II. LONGITUDINAL DIELECTRIC FUNCTION AND THE DISPERSION RELATION

Let us consider a 2D layer of a semiconductor containing normal electrons with the equilibrium density N_n in the vicinity of a 2D SC layer containing both normal single-electron excitations above the SC gap $\Delta(T)$, where T is temperature (we will use $\Delta/T \ll 1$), and SC Cooper pairs described by the order parameter (Fig. 1). The total electron density in the SC layer is N_s, whereas the density of SC electrons we denote as n_s.

Furthermore, we expose the system to an external EM field, which represents a plane wave with 3D wave vector $\mathbf{Q} = (\mathbf{k}, Q_z)$, where the 2D wave vector $\mathbf{k} = (Q_x, Q_y)$ is its in-plane projection. The field creates electric currents in both the layers:

$$\mathbf{j}_s = \sigma_s(\mathbf{E}_0 + \mathbf{E}_s^i), \quad \mathbf{j}_n = \sigma_n(\mathbf{E}_0 + \mathbf{E}_n^i), \tag{1}$$

where $\mathbf{E}_0$ is the in-plane component of external EM field, which we assume to be equal in both layers (valid as long as $Q_z d \ll 1$); $\mathbf{E}_{n,s}^i$ are the internal EM fields, induced in the corresponding layers due to the oscillations of the particle densities;

$$\sigma_n = \frac{e^2 N_n \tau_n / m_n}{1 - i\omega\tau_n} \tag{2}$$

is a conventional dynamical Drude conductivity of the degenerate electron gas in the semiconductor, with m_n being the electron effective mass; σ_s describes the current response of single-particle excitations in the SC layer. It should be noted that in Eq. (1) we omitted the factor $\exp(i\mathbf{kr} - i\omega t)$ since all the time and in-plane position-dependent quantities are proportional to this factor.

The induced fields depend on z, $\mathbf{E}_{n,s}^i \equiv \mathbf{E}^i(z = \pm d)$, where $\mathbf{E}^i$ can be expressed via the scalar potential in the framework of quasistatic approximation, $\mathbf{E}^i = -\nabla\varphi^i$. The scalar potential, in turn, satisfies the Poisson equation

$$\left(\frac{d^2}{dz^2} - k^2\right)\varphi^i(z) = -\frac{4\pi}{\epsilon}[\delta\rho_s\delta(z-d) + \delta\rho_n\delta(z+d)], \tag{3}$$

where $\delta\rho_{n,s}$ are deviations of charge densities from their equilibrium values, and ϵ is a dielectric function of the material. Combining (1) and (3) with the continuity equation, we find

$$\begin{pmatrix} 1 + \frac{2\pi ik}{\epsilon\omega}\sigma_s & \frac{2\pi ik}{\epsilon\omega}e^{-2kd}\sigma_s \\ \frac{2\pi ik}{\epsilon\omega}e^{-2kd}\sigma_n & 1 + \frac{2\pi ik}{\epsilon\omega}\sigma_n \end{pmatrix}\begin{pmatrix}\delta\rho_s \\ \delta\rho_n\end{pmatrix} = \frac{kE_0}{\omega}\begin{pmatrix}\sigma_s \\ \sigma_n\end{pmatrix} \tag{4}$$

and

$$\begin{pmatrix} j_s \\ j_n \end{pmatrix} = \begin{pmatrix} \left[1 + \frac{2\pi ik}{\epsilon\omega}(1 - e^{-2kd})\sigma_n\right]\sigma_s \\ \left[1 + \frac{2\pi ik}{\epsilon\omega}(1 - e^{-2kd})\sigma_s\right]\sigma_n \end{pmatrix}\frac{E_0}{\varepsilon(k,\omega)}, \tag{5}$$

where we have introduced the longitudinal dielectric function

$$\varepsilon(k,\omega) = \left(1 + \frac{2\pi ik}{\epsilon\omega}\sigma_s\right)\left(1 + \frac{2\pi ik}{\epsilon\omega}\sigma_n\right) + \left(\frac{2\pi k e^{-2kd}}{\epsilon\omega}\right)^2\sigma_s\sigma_n. \tag{6}$$

Here and in what follows we assume that $\mathbf{k}$ and $\mathbf{E}_0$ only have x in-plane components.

Equation (6) is similar in form to the standard one, describing generic two-component systems. Equation $\varepsilon(k,\omega) = 0$ determines the dispersion and damping of collective modes. The key quantity here is σ_s, which has to be found accounting for the interaction of single-particle excitations in the SC layer with the Cooper pair condensate.

Let us, first, switch off the interaction between the layers and analyze the collective modes in each layer separately. Formally, the noninteracting case corresponds to putting $d \to \infty$ in Eq. (6). Then $\varepsilon(k,\omega) = 0$ splits into two independent conditions:

$$1+\frac{2\pi ik}{\epsilon\omega}\sigma_n=0,\quad 1+\frac{2\pi ik}{\epsilon\omega}\sigma_s=0. \tag{7}$$

Let us consider these cases separately.

A. Plasmons in the layer containing two-dimensional electron gas

Substituting Eq. (2) in (7), we find the plasmon dispersion and its damping due to the electron-impurity scattering:

$$\omega=\omega_n\sqrt{1-\frac{1}{(2\omega_n\tau_n)^2}}-\frac{i}{2\tau_n},\quad \omega_n=\sqrt{\frac{2\pi e^2kN_n}{\epsilon m_n}}. \tag{8}$$

The plasmon exists if $2\omega_n\tau_n>1$. In the case of weak scattering, when $\omega_n\tau_n\gg 1$, the plasmon damping is much smaller in comparison with its frequency, and in the EM power absorption spectrum the plasmons are seen as well-resolved resonances. The spatial dispersion of Drude conductivity (2) does not play a role in (8) and can be neglected if $\omega\gg kv_n$, where v_n is the Fermi velocity of electrons in the semiconducting layer.

B. Artemenko-Volkov modes in the SC layer

To find the dispersion law of the AV mode, we have to, first, find the conductivity σ_s of single-particle excitations above the SC gap. In their original work [10], Artemenko and Volkov used the quasiclassical approach based on the kinetic equations. Later, the validity of their results was confirmed by the quantum field theory methods [18]. We will thus follow the simpler original Boltzmann equation approach, where the AV modes are found by analyzing the longitudinal dielectric function (7). The calculation of σ_s for a 3D superconductor can be found elsewhere [19], and we adopt it for the 2D SC layer just presenting here the result (see the Appendix for the details of the derivation):

$$\sigma_s=\sigma_{0s}\frac{\omega^2-u^2k^2+i\omega\eta_s+ik^2v_s^2\omega\tau_sJ_\omega}{\omega^2-u^2k^2+ik^2v_s^2\omega\tau_sJ_\omega}, \tag{9}$$

where

$$\sigma_{0s}=\frac{e^2N_s\tau_s}{m_s},\quad u=v_s\sqrt{\frac{7\zeta(3)}{2\pi^3}\frac{\Delta}{T}},$$
$$\eta_s=\frac{n_s}{N_s}\frac{1}{\tau_s},\quad J_\omega=\frac{\ln(1/\omega\tau_s)}{\pi}. \tag{10}$$

Here we introduced a static Drude conductivity of normal electrons in the SC layer, σ_{0s}; v_s is the Fermi velocity in the SC layer; u is the phase velocity of the AV mode (see the description below); $\zeta(3)$ is the Riemann zeta function. We note also that $u\ll v_s$.

Formula (9) is valid under the conditions ω, kv_s, $\tau_s^{-1}\ll\Delta\ll T$ and $(\tau_skv_s)^2\ll\omega\tau_s\ll 1$ [19]. Using these assumptions and combining Eqs. (9) and (7), we find

$$1+i\frac{\omega_s^2\tau_s}{\omega}\left(1+i\frac{\omega\eta_s}{\omega^2-u^2k^2+ik^2v_s^2\omega\tau_sJ_\omega}\right)=0,$$

where $\omega_s=\sqrt{\dfrac{2\pi e^2kN_s}{\epsilon m_s}}$. (11)

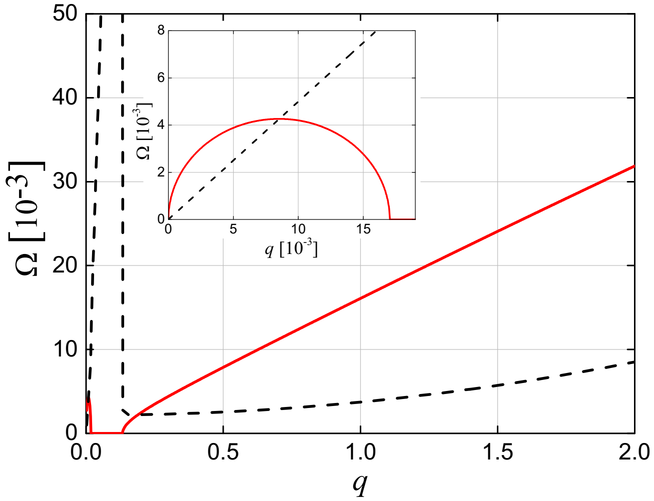

FIG. 2. Dimensionless complex-valued eigenfrequency $\Omega=\omega\tau_s$ as a function of dimensionless wave vector $q=(\omega_s\tau_s)^2$ for an isolated SC layer. We use $a_B/(v_s\tau_s)=1/10$, $\Delta/T=1/10$. The red curve stands for the dispersion Re Ω whereas the dotted black curve shows the absolute value of the damping $|\mathrm{Im}\,\Omega|$. Inset: Zoom-in of the low-wave-vector domain, where the qAB mode is visible and not suppressed by the damping.

In the limiting case $\omega_s^2\tau_s\gg\omega$ [19], there exists an analytical solution:

$$\omega^2=u^2k^2-ik^2v_s^2\omega\tau_sJ_\omega-i\omega\eta_s. \tag{12}$$

We note again that this solution describes a weakly damped soundlike AV mode in the frequency range

$$\left(\frac{\Delta}{T}\right)^2\ll\omega\tau_s\ll\frac{\Delta}{T}\ll 1. \tag{13}$$

The numerical solution of Eq. (11) gives two collective branches in the isolated 2D SC layer. The first dispersion is presented in Fig. 2, where we use dimensionless frequency $\Omega=\omega\tau_s$ and wave vector $q=(\omega_s\tau_s)^2$. Also, we use $m_s=m_n$, $a_B=\epsilon/e^2m_s$ in Fig. 2 and all the figures which follow. We see that the AV mode characterized by the linear dispersion uk lies above the almost horizontal part of the red-dotted line, which stands for the lower bound $(\Delta/T)^2$ in Eq. (13).

The second mode is located in the low-q domain, as shown in the inset in Fig. 2. At $q\ll 1$, this mode is weakly damped due to the smallness of its imaginary part ($\approx q$) in comparison with its eigenfrequency $\approx\sqrt{q}$. The dispersion of this mode can be found analytically from Eq. (11). Indeed, in the limit $k\to 0$, Eq. (11) reduces to

$$1+i\frac{\omega_s^2\tau_s}{\omega}\left(1+i\frac{\eta_s}{\omega}\right)=0, \tag{14}$$

which gives the dispersion of the 2D plasmon,

$$\omega=\omega_s\sqrt{\frac{n_s}{N_s}}\sqrt{1-\frac{N_s}{n_s}\left(\frac{\omega_s\tau_s}{2}\right)^2}-i\frac{\omega_s^2\tau_s}{2}, \tag{15}$$

existing when $n_s/N_s>(\omega_s\tau_s)^2/4$. On condition $n_s/N_s\gg(\omega_s\tau_s)^2/4$, formula (15) turns into nondamped 2D plasmon

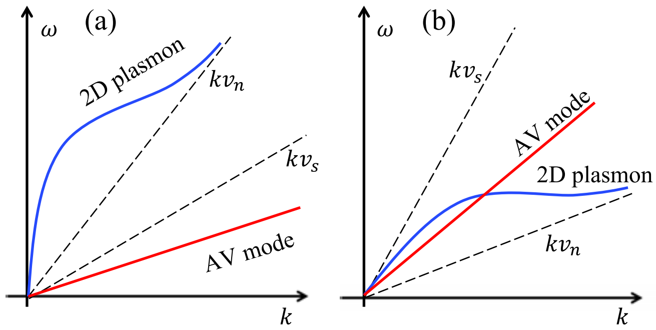

FIG. 3. Qualitative behavior of a noninteracting plasmon and AV modes in the case of (a) $v_n > v_s$ and (b) $v_s > v_n$. (The qAB mode is not shown.)

oscillations of SC electrons:

$$\omega \approx \omega_s \sqrt{\frac{n_s}{N_s}} = \sqrt{\frac{2\pi e^2 k n_s}{\epsilon m_s}}. \tag{16}$$

It should be noted that in contrast with the normal-state systems, where the plasmon exists if $\omega_n \tau_n \gg 1$, the mode Eq. (16) exists at $\omega_s \tau_s \ll 1$. This mode recalls the Anderson-Bogoliubov mode [2,3] studied at $T = 0$ in 3D superconductors. It represents plasmon oscillations of SC electrons, the density of which at $T = 0$ equals to the total electron density in the superconductor. The difference is that in our 2D case at $T \neq 0$ this mode is determined by the density of SC electrons $n_s \neq N_s$. We will call it the quasi-Anderson-Bogoliubov (qAB) mode. The qAB oscillations of the SC condensate at $T \neq 0$ are accompanied by the appearance of induced charges in the system which, obviously, interact with normal electrons. They exist in a superconductor at nonzero temperatures and play the role of a friction force influencing the oscillations of the condensate. At $T_c - T \ll T_c$ (the case which we consider in this paper), the density of normal electrons in the SC layer is of the order of total electron density N_s. It explains why the imaginary part in Eq. (15) is proportional to $\omega_s^2 \propto N_s$.

III. INTERACTION BETWEEN THE MODES

We have already found that the plasmon mode of the SC layer lies above the boundary of the particle-hole continuum, $\omega_n \gg kv_n$, whereas the AV mode has the phase velocity $u \ll v_s$. Hence there exist two limiting cases, depending on the ratio between the Fermi velocities in SC, v_s, and normal two-dimensional electron gas (2DEG), v_n, layers (Fig. 3). Let us consider both of these scenarios independently.

A. Modes interaction if $v_n > v_s$

In this case, the dispersions of the modes do not intersect. Thus at a given value of the wave vector k, these modes are excited independently in the system at different frequencies of external EM field. Nevertheless, the presence of the other layer results in the interlayer Coulomb interaction of the particles and produces an additional damping of the collective mode which is excited. Let us analyze this damping.

In the frequency range when the plasmon mode exists, $\omega \sim \omega_n$, $\omega_n \tau_n \gg 1$, we can simplify formulas for the conductivities (2) and (9),

$$\sigma_n \approx \frac{e^2 N_n}{-i\omega m_n}, \quad \sigma_s \approx \sigma_{s0} = \frac{e^2 N_s \tau_s}{m_s}, \tag{17}$$

and the dispersion equation $\varepsilon(k, \omega) = 0$ reduces to

$$\left(1 - \frac{\omega_n^2}{\omega^2}\right)\left(1 + i\frac{\omega_s^2 \tau_s}{\omega}\right) = -i\omega_s \tau_s \frac{\omega_n \omega_s}{\omega^2} e^{-4kd}. \tag{18}$$

Its iterative solution gives the renormalization of the plasmon dispersion and an additional damping:

$$\omega = \omega_n \left(1 - \omega_s \tau_s \frac{\omega_s \tau_s (1 + e^{-4kd}) + 2i\sqrt{N_n/N_s}}{4N_n/N_s + \omega_s^2 \tau_s^2 (1 + e^{-4kd})^2} e^{-4kd}\right). \tag{19}$$

We see that if kd is not very large and $N_n/N_s \ll \omega_s^2 \tau_s^2$ the presence of the SC layer can drastically change the plasmon dispersion, resulting in a dramatic *redshift* of the plasmon frequency.

If, instead, the frequency lies in the range of the AV mode, $\omega \approx uk$, the Drude conductivity of electrons in the semiconducting layer can be replaced by its static limit $\sigma_n \approx e_n^2 \tau_n / m_n$, and the dispersion equation reads

$$\left(1 + i\frac{\omega_n^2 \tau_n}{\omega}\right)\left[1 + i\frac{\omega_s^2 \tau_s}{\omega}\left(1 + i\frac{\omega \eta_s}{\omega^2 - u^2 k^2 + ik^2 v_s^2 \omega \tau_s J_\omega}\right)\right]$$
$$= \left(\frac{\omega_n^2 \tau_n}{\omega}\right)\left(\frac{\omega_s^2 \tau_s}{\omega}\right)\left(1 + i\frac{\omega \eta_s}{\omega^2 - u^2 k^2 + ik^2 v_s^2 \omega \tau_s J_\omega}\right) e^{-4kd}. \tag{20}$$

The bare AV mode can exist under the condition $\omega_s^2 \tau_s \gg \omega$, as we have discussed above. Due to the relation $\omega_n \tau_n \gg \omega_s \tau_s$, we can assume $\omega \ll \omega_s^2 \tau_s \ll \omega_n^2 \tau_n$ in Eq. (20), and we find that the AV mode is not renormalized in the presence of the semiconducting layer (in the first-order perturbation theory with respect to the parameter $\omega_s^2 \tau_s / \omega$). One can find the next-order corrections with respect to $1 \ll \omega_s^2 \tau_s / \omega \ll \omega_n^2 \tau_n / \omega$; however, they are small and we will not discuss them further.

B. Modes interaction if $v_n < v_s$

In this case, the density of electrons in the semiconducting layer is smaller than the total density of electrons in the SC layer, $N_n < N_s$. The intersection of the plasmon mode (of the semiconducting layer) with both qAB and AV modes (of the SC layer) is possible, depending on the system parameters, as it is shown by dashed curves in Figs. 4 and 5.

We analyze the interaction between the modes numerically (see solid curves presented in Figs. 4 and 5). The main conclusion is that the plasmon mode of the semiconducting layer interacts with the qAB mode stronger than with the AV mode since in the latter case the modes repel each other, shifting their positions in the q axis. This situation can be explained if we recall that the qAB mode corresponds to the oscillation of SC elections and it is accompanied by the oscillations of induced electric fields. In contrast, the AV mode corresponds to the antiphase oscillations of SC and normal electrons in

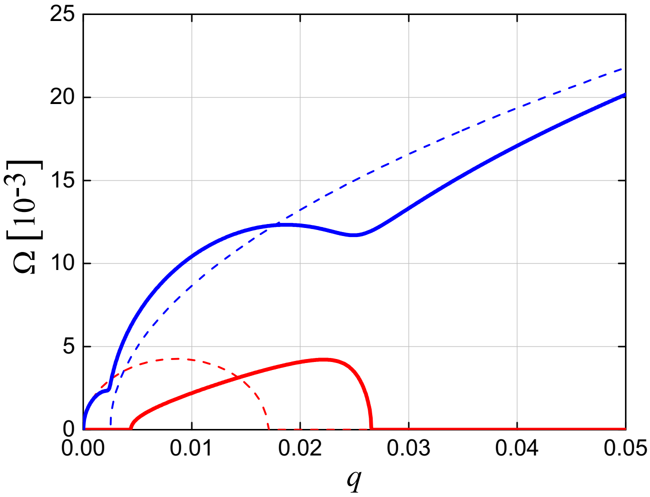

FIG. 4. Dispersions of hybrid modes in the system (solid curves). The dispersions of bare qAB and plasmon modes are shown in dashed red and blue curves, respectively. We use $a_B/v_s\tau_s = 1/10$, $\Delta/T = 1/10$, $\tau_n/\tau_s = 100$, $N_s/N_n = 100$; for solid curves, $d/v_s\tau_s = 100$.

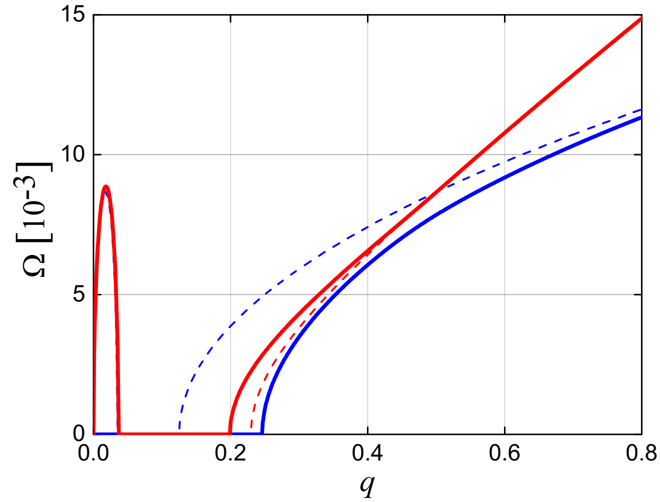

FIG. 5. Dispersions of hybrid modes in the system (solid curves). The dispersions of bare AV and plasmon modes are shown in dashed red and blue curves, respectively. We use $a_B/v_s\tau_s = 1/10$, $\Delta/T = 1/7$, $\tau_n/\tau_s = 100$, $N_s/N_n = 5000$; for solid curves, $d/v_s\tau_s = 10$.

the SC layer resulting in the quasineutrality. Therefore, since the interaction between the layers has Coulomb nature, it is not surprising that the plasmon mode interacts with the qAB mode much stronger.

IV. ELECTROMAGNETIC FIELD POWER ABSORPTION

Let us expose the system to an external EM field and study light-matter interaction. We will do a numerical analysis of EM power absorption $\mathrm{Re}\,(j_nE_0^*)$ [where j_n is defined in Eq. (5)] in the case when $v_s > v_n$ and, thus, $N_n < N_s$. In the opposite regime, the interaction of the modes is weak.

First, we switch off the interaction between the layers. The EM power absorption of an isolated semiconducting layer is characterized by a standard Lorenz-shaped resonance at the plasmon frequency. A more interesting situation arises in the isolated SC layer due to the presence of both the qAB and AV modes. The AV mode is quasineutral, and hence it interacts with the external EM field weakly, thus the EM power absorption is negligibly small. Indeed, the approximate analytical solution Eq. (12) if substituted into Eq. (9) gives $\sigma_s = 0$ and $j_n = 0$ reflecting the quasineutral nature of the AV mode and nearly zero EM power absorption. The exact numerical calculation of the power absorption in the absence of interaction between SC and semiconducting layers ($d \to \infty$), presented in Fig. 6(a), supports this conclusion. At large frequencies $\omega \gg ku$, the absorption in Fig. 6(a) reaches a plateau with the value equal to the static Drude absorption of EM radiation of normal electrons in the SC layer, as it is seen from Eq. (9). Indeed, at $\omega \gg ku$ the conductivity of the SC layer resembles the static Drude conductivity $\sigma_s \approx \sigma_{s0}$. The arrow in Fig. 6(a) indicates EM power absorption at the position of the AV mode which is negligibly small in comparison with the power absorption corresponding to the plateau value.

In contrast, the qAB mode strongly interacts with external EM field and demonstrates a pronounced peak of absorption with the amplitude much greater than the plateau value, as it is shown in Fig. 6(b).

Furthermore, let us switch the interlayer interaction on and consider the EM power absorption by the hybrid system when the collective modes of the SC layer are hybridized with the plasmon mode in the semiconducting layer (Fig. 7). Figure 7(a) shows the EM power absorption by the hybrid AV-plasmon modes. Weak hybridization of the modes results in two new eigenmodes. One of them almost resembles the original plasmon mode, and the other mostly possesses the properties of the bare AV mode. From our previous discussion of the light absorption by the bare modes, we expect that the quasiplasmon mode (the hybrid mode which mostly inherited the properties of the plasmon mode) will give a pronounced resonance in the EM absorption spectrum, whereas the other, quasi-AV mode (the hybrid mode which mostly inherited the properties of the AV mode) will not be seen. This reasoning is supported by the exact numerical calculations, shown in Fig. 7(a). We see a single resonance at the frequency of the quasiplasmon mode with a shifted position with respect to bare plasmon dispersion and a pronounced Lorenz shape. The quasi-AV mode is not seen in the figure.

At large Ω, the EM power absorption spectrum again reaches the plateau with the value equal to the Drude conductivity of the SC layer. The amplitude of the Lorenz-shaped resonance is smaller than the value at the plateau since the total density of electrons in the semiconducting layer is smaller than the density of electrons in the SC layer, $N_n < N_s$.

Finally, EM power absorption by the hybrid qAB-plasmon mode demonstrates a double-resonance structure, indicating that both the new modes formed from the original qAB and plasmon modes (before the coupling) are extremely sensitive to external EM radiation. We want to point out that such shape of the spectrum can be attributed to the Fano resonance, which is expected in hybrid Bose-Fermi systems [15]. And again, at

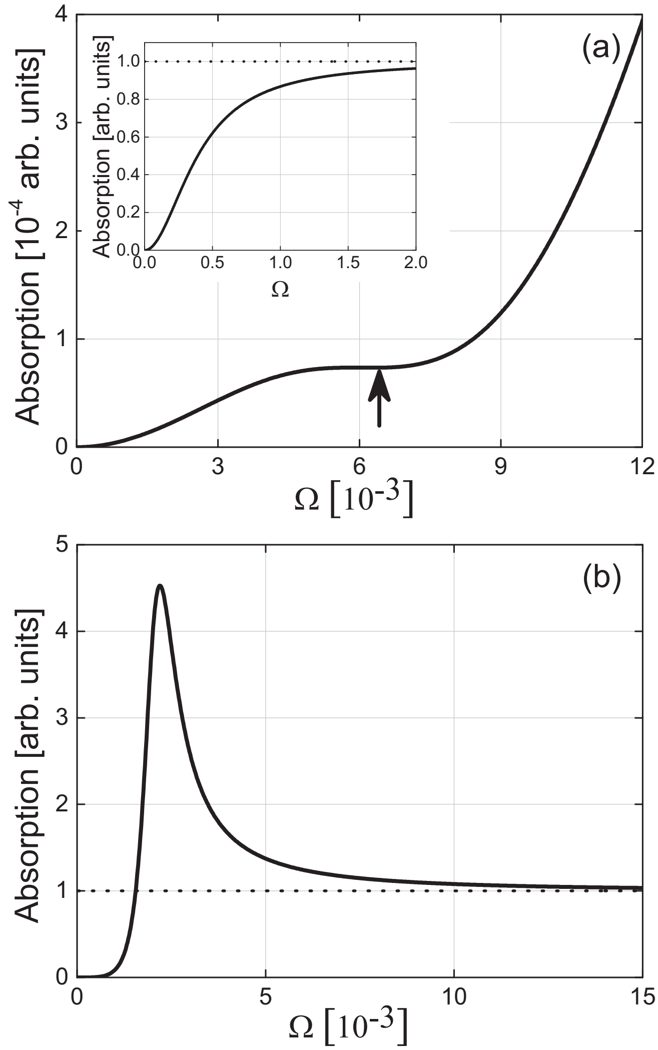

FIG. 6. EM power absorption by the isolated SC layer as a function of dimensionless frequency Ω given in arbitrary units. (a) EM power absorption by the AV mode. We use $q = 0.4$, $a_B/v_s\tau_s = 1/10$, $\Delta/T = 1/7$, $\tau_n/\tau_s = 100$, $N_s/N_n = 5000$. Inset: Absorption spectrum at large values of Ω; (b) EM power absorption by the qAB mode. We use $q = 0.001$, $a_B/v_s\tau_s = 1/10$, $\Delta/T = 1/10$, $\tau_n/\tau_s = 100$, $N_s/N_n = 100$.

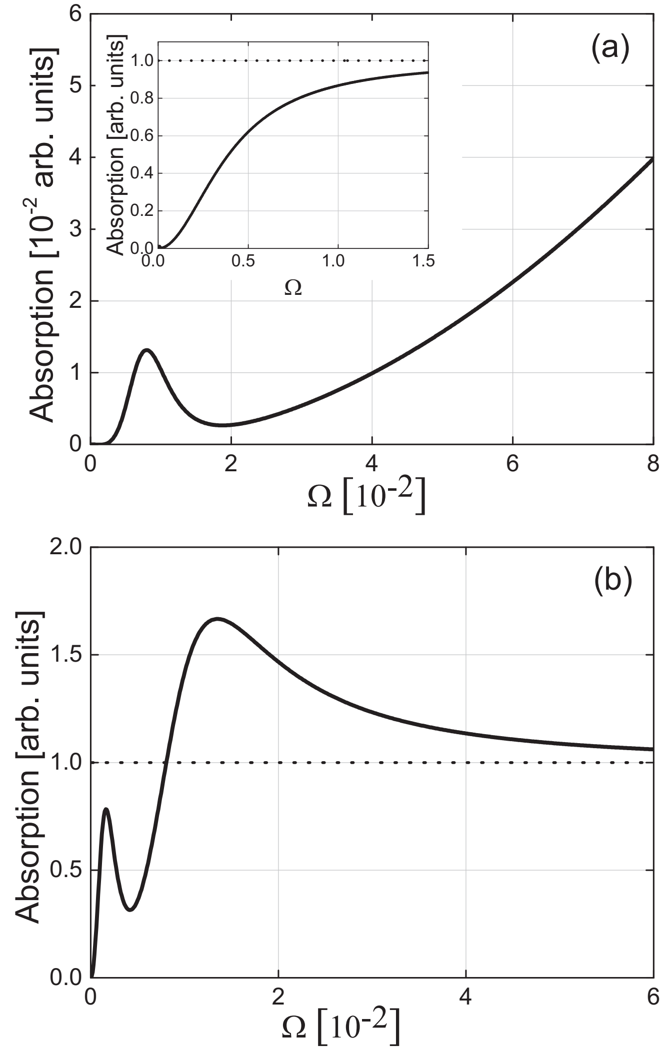

FIG. 7. EM power absorption by the hybrid system as a function of dimensionless frequency Ω given in arbitrary units. (a) EM power absorption by the AV-plasmon mode. We use $q = 0.4$, $a_B/v_s\tau_s = 1/10$, $\Delta/T = 1/7$, $\tau_n/\tau_s = 100$, $N_s/N_n = 5000$, $d/v_s\tau_s = 10$. Inset: Absorption spectrum at large values of Ω; (b) EM power absorption by the qAB-plasmon mode. We use $q = 0.006$, $a_B/v_s\tau_s = 1/10$, $\Delta/T = 1/10$, $\tau_n/\tau_s = 100$, $N_s/N_n = 100$, $d/v_s\tau_s = 100$.

large frequencies, the energy absorption reaches the plateau with the value equal to the Drude conductivity absorption of the SC layer.

V. ACTUAL PARAMETERS TO CONDUCT THE EXPERIMENT AND LIMITATIONS

Let us discuss the actual parameters of the sample to be used in the experiments. We will consider the qAB mode first. The Drude model is applicable if $k_F l \gg 1$, where l is the mean free path of normal electrons. We choose $N_s = 10^{14}$ cm^{-2} and $m_s = 0.5\ m_0$, which are typical for superconductors based on transition-metal dichalcogenides [20]. Then $k_F l \approx 1.38 \times 10^{15}\,\tau_s$, and thus we can take $\tau_s = 10^{-14}$ s. Note, with these parameters the resistivity amounts to kiloohm (which corresponds to experimentally measured values), while the Bohr radius becomes $a_B \approx \epsilon \times 10^{-10}$ m. For $\epsilon = 6$, the dimensionless ratio $a_B/v_F\tau_s$ used in all the plots is equal to $1/10$, that corresponds to the interlayer distance $d = 100 v_F\tau_s = 1000 a_B = 600$ nm. In the normal layer, we use typical parameters $N_n = 10^{12}$ cm^{-2} and $\tau_n = 10^{-12}$ s. They give for the ratio of static conductivities $\sigma_{s0}/\sigma_n(\omega = 0) = 1$.

Second, let us consider the AV mode. To bring the plasmon and AV mode dispersions together, we need a change of parameters. In our calculations, we make the ratio N_s/N_n 50 times bigger. If we keep $N_n = 10^{12}$ cm^{-2} (the same as before), then the scattering times τ_s and τ_n should become $\sqrt{50}$ times smaller to keep the dimensionless ratios τ_s/τ_n and $a_B/v_F\tau_s$ unchanged. Note, unlike the qAB mode, the AV mode starts feeling the presence of plasmons in the normal layer at order-of-magnitude smaller interlayer distances, $d = 10 v_F\tau_s = 100 a_B = 60$ nm.

In the calculations and plots, we used dimensionless wave vector and frequency, $\Omega = \omega\tau_s \sim 0.01$. Then $\omega/2\pi \sim$ 0.1–1 THz, which corresponds to the wavelength of the order of 0.1–1 mm. For $q = 0.4$ and 0.006 we find $2\pi/k \sim 1\,\mu$m and $\approx 70\,\mu$m, respectively, which corresponds to the period of typical diffraction gratings (which are required in the experiments to excite the plasmons).

Let us now discuss the limitations of our theoretical approach. When a normal 2DEG is in the vicinity of a 2D superconductor, there might arise several phenomena, one of which is the proximity effect [21,22], which might result in an induced superconductivity in the semiconducting layer. In our calculations, we used a relatively large 2DEG-superconductor separation. Evidently, the smaller the separation, the more pronounced the coupling between the qAB, AV, and plasmon modes. However, at very small separations, there will arise the proximity effect, which might quench the formation of collective modes and should be accounted for in engineering samples. It limits the separation distance to the coherence length of the particular superconductor. For instance, the coherence length amounts to several nanometers in the case of high-T_c superconductors.

An electron in 2DEG with energy below Δ can tunnel into the superconductor due to the Andreev scattering [23]. However, if the distance between the layers is large and most of the electrons are paired, the probability of such tunneling is small. If the superconductor is exposed to external light with the frequency above Δ, the tunneling can be enhanced. Indeed, external EM fields excite electron and hole quasiparticles (which coexist with SC fluctuations in the superconductor). In this case, in addition to the Andreev scattering, there occurs common tunneling of quasiparticles into the superconductor. Usually, this effect can also be disregarded when studying collective modes. In particular, the frequency of the external EM field which we consider in this paper (to excite the modes) is smaller than Δ.

VI. CONCLUSION

We have studied analytically and numerically the coupling of collective modes in a hybrid two-dimensional electron-gas–superconductor structure in the vicinity of the critical temperature of superconducting transition of the superconductor. The layer containing the electron gas in the normal state can be represented by a semiconductor or a metallic layer, used in hybrid normal metal-superconductor contacts [12]. We have shown that the superconducting layer lodges two collective modes with drastically different physical properties, which results in their different sensitivity to external electromagnetic fields. As a result, these modes couple differently with the plasmon mode of the normal electron gas spatially separated from the superconducting layer. It manifests itself in different spectra of the electromagnetic power absorption of the hybrid system.

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APPENDIX: DERIVATION OF THE CONDUCTIVITY IN 2D SYSTEMS

We present here the main steps of the derivation of conductivity (9) succeeding [19]. To find the form of SC conductivity (which describes the linear response of the system to external EM field, $\mathbf{j}_s = \sigma_s\mathbf{E}$), we can write the supercurrent within the Bogoliubov–de Gennes approach:

$$\mathbf{j}_s = \frac{en_s\mathbf{p}_s}{m_s} + \frac{2e}{m_s}\sum_{\mathbf{p}} \mathbf{p} f_{\mathbf{p}}(\mathbf{p}_s, \phi), \tag{A1}$$

where $\mathbf{p}_s = (\nabla\chi - 2e\mathbf{A})/2 = m_s\mathbf{V}_s$, $\mathbf{V}_s$ is the velocity of SC flow, χ is the phase of the order parameter, $\phi = e\varphi + \partial_t\chi/2$, $\mathbf{A}$ and φ are the vector and scalar potentials, respectively, and $f_{\mathbf{p}}(\mathbf{p}_s, \phi) \equiv f_{\mathbf{p}} = n_{\mathbf{p}}(\mathbf{p}_s, \phi) - n_{\mathbf{p}}^{(0)}$ is the nonequilibrium contribution to the distribution function of excitations in the superconductor. These excitations possess the following dispersion:

$$\tilde{\zeta}_{\mathbf{p}} = \sqrt{\tilde{\xi}_{\mathbf{p}}^2 + \Delta^2} + \mathbf{p}\mathbf{V}_s, \quad \text{with} \tag{A2}$$

$$\tilde{\xi}_{\mathbf{p}} = \frac{\mathbf{p}^2}{2m} - \mu + \phi + \frac{\mathbf{p}_s^2}{2m}, \tag{A3}$$

and their distribution function $n_{\mathbf{p}}(\mathbf{p}_s, \phi) \equiv n_{\mathbf{p}}$ obeys the conventional kinetic equation

$$\frac{\partial n_{\mathbf{p}}}{\partial t} + \frac{\partial\tilde{\zeta}_{\mathbf{p}}}{\partial\mathbf{p}}\frac{\partial n_{\mathbf{p}}}{\partial\mathbf{r}} - \frac{\partial\tilde{\zeta}_{\mathbf{p}}}{\partial\mathbf{r}}\frac{\partial n_{\mathbf{p}}}{\partial\mathbf{p}} + I\{n_{\mathbf{p}}\} = 0, \tag{A4}$$

where $I\{n_{\mathbf{p}}\}$ is the collision integral. Furthermore, we should linearize (A4) over $\mathbf{p}_s$ and ϕ, and then find the deviation $f_{\mathbf{p}}$ and substitute it in (A1). Then we can express $\mathbf{p}_s$ and ϕ via electric field strength $\mathbf{E}$. For that we need three equations (accounting for the fact that we deal with the vector $\mathbf{p}_s$, which has two components). Two of them result from the definitions of $\mathbf{p}_s$ and ϕ:

$$\frac{\partial\mathbf{p}_s}{\partial t} - \nabla\phi = e\mathbf{E}, \tag{A5}$$

while the third one is the continuity equation

$$e\frac{\partial\delta n}{\partial t} + \nabla\mathbf{j}_s = 0. \tag{A6}$$

Here δn is the deviation of electron density from equilibrium and it takes the following form by linearization:

$$\delta n = 2\sum_{\mathbf{p}}\left\{f_{\mathbf{p}}\frac{\xi_{\mathbf{p}}}{\zeta_{\mathbf{p}}} + \phi\left[\left(\frac{\xi_{\mathbf{p}}}{\zeta_{\mathbf{p}}}\right)^2\frac{\partial n_{\mathbf{p}}^{(0)}}{\partial\zeta_{\mathbf{p}}} - \frac{\Delta^2}{2\zeta_{\mathbf{p}}^3}\tanh\frac{\zeta_{\mathbf{p}}}{2T}\right]\right\}, \tag{A7}$$

where $\xi_{\mathbf{p}} = \tilde{\xi}_{\mathbf{p}}(\mathbf{p}_s = 0, \varphi = 0)$ and $\zeta_{\mathbf{p}} = \tilde{\zeta}_{\mathbf{p}}(\mathbf{p}_s = 0, \varphi = 0)$. Solving the set of equations (A1), (A5), (A6), (A7), and linearized (A4) we come up with formula (9) in the main text.

Note, [19] treats 3D superconductors. In the current 2D problem (which we deal with in this paper), we suppose that the density of states of quasiparticles $G(\xi)$ does not include any singularities in the vicinity of the Fermi energy, like in [19]. It means that performing usual transformations $\sum_{\mathbf{p}} \rightarrow G(\mu) \int d\xi$ we arrive at the same integrals over ξ as in the 3D case. Thus, the results are almost identical to ones reported in [19]. They coincide if we replace $3 \rightarrow 2$ and take the electron density per unit area (instead of the electron density per unit volume).

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