Transmission of crystallization waves across the edge between the rough and faceted crystalline surfaces at the superfluid-solid ^{4}He interface

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The wavelike processes of crystallization and melting or crystallization waves are well known to exist at the crystal ^{4}He surface in its rough state. Below the roughening transition temperature the crystalline surface experiences the transition to the smooth faceted state and the crystallization waves represent the propagation of a train of crystalline steps at the velocity depending on the crystal step height. Here we analyze the transmission and reflection of crystallization waves which propagate across the crystal edge separating the crystalline surfaces in the rough and faceted states.

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I. INTRODUCTION

Helium crystals as a model system can provide us with very general and unusual properties of liquid-solid interfaces [1,2]. On one hand, helium crystals demonstrate faceting as classical crystals. The so-called roughening transition is the transition from the atomically rough and fluctuating state of the crystalline surface at high temperatures to the smooth faceted surfaces at sufficiently low temperatures. The experimental observations have displayed several roughening transitions in hcp ^{4}He crystals, as follows: $T_{R1} = 1.3$ K for the c facet in the [0001] direction, $T_{R2} = 1.07$ K for the a facet in the $[10\bar{1}0]$ direction perpendicular to the c axis, and $T_{R3} = 0.36$ K for the s facet in the $[10\bar{1}1]$ direction. The $[10\bar{1}1]$ direction is tilted by 58.5° with respect to the [0001] direction.

On the other hand, as compared with classical crystals, the ^{4}He crystals under ultralow energy dissipation can demonstrate the growth dynamics when quantum mechanics plays a major role [1,2]. In particular, at sufficiently low temperatures, the ^{4}He crystal in contact with its superfluid phase can support oscillations of the superfluid-solid interface due to weakly damped processes of melting and crystallization [3]. From the dynamical point of view such weakly damped crystallization waves at the rough crystal surface are an immediate counterpart of the familiar gravitational-capillary waves at the interface between two normal liquids and have the similar dispersion as a function of wave vector.

On the contrary, no basic study of the melting-crystallization dynamics has been made at the well-faceted and atomically smooth ^{4}He crystal surfaces which, unlike the atomically rough crystal surfaces, have an infinitely large surface stiffness. Accordingly, the crystal surface curvature vanishes and the crystal facet takes the flat shape. The most striking distinction of smooth-faceted crystal surfaces from the rough ones is the existence of nonanalytical cusp-like behavior in the angle dependence of the surface tension; see, e.g., Ref. [4]. The crystal step energy becomes nonzero and positive below the roughening transition temperature T_R, vanishing at the higher temperatures. The origin of the singularity is directly connected with nonzero magnitude of the facet step energy below the roughening transition temperature.

As compared with the melting-crystallization wave-like processes at the rough crystal surface, the analogous processes at the faceted crystal surface demonstrate a more complicated picture than that at the rough crystal surface [5]. The frequency spectrum of crystallization waves at the faceted crystal surface has a sound-like dispersion with the velocity depending significantly on the wave perturbation amplitude and on the number of the facet steps distributed over the wavelength [5,6]. In essence, such crystallization waves represent a propagation of a train of crystal-facet steps along the crystal surface at the velocity governed with the crystal step height. Here we mention the formation of crystallization waves under heavy shake of an experimental cell [7] or in the process of anomalously fast growth of a ^{4}He crystal under high overpressures [8,9]. The progressive facet waves are observed at the crystal (001) facet in ^{3}He [10].

The presence of a singularity in the behavior of surface tension or nonzero crystal step energy results also in a number of interesting phenomena at the faceted ^{4}He crystal surface, e.g., amplitude-dependent velocity of traveling waves [5,6], quantum fingering of the inverted liquid-crystal interface in the field of gravity [11], Rayleigh-Taylor instability with generating the crystallization waves [12], and electrohydrodynamical instability [13] with breaking the faceted state down.

So far the crystallization waves have been studied only for the spatially homogeneous crystalline surfaces. Since the adjacent crystal surfaces have the different roughening transition temperatures, we can raise a question about the propagation of melting-crystallization waves across the edge between the crystalline surfaces in the rough and smooth states. For the first time, in the present paper we attempt the transmission and reflection of crystallization waves across the edge between the rough crystal surface and the smooth faceted surface at the superfluid-solid ^{4}He interface.

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II. LAGRANGIAN

The atomically rough surface and the atomically smooth surface of a ^{4}He crystal correspond to various crystallographic directions and the surfaces contact each other at the crystal edge. The transition from one direction to the other or from one surface to the other surface can be described with the polar angle which varies gradually from one value to another in order to parametrize two adjacent crystal surfaces.

To treat the transmission and reflection of crystallization waves in the most simple and obvious way, we consider the following model situation: For simplicity, we assume that both crystal surfaces, rough and smooth, are parallel to the xy plane with the vertical position at $z = 0$. In addition, we imply that one-half of the crystal surface, e.g., $x < 0$, is in the rough state and the other half $x > 0$ is in the smooth-faceted state.[1] First, we call $\zeta = \zeta(\boldsymbol{r})$ as a displacement of the crystal surface from its horizontal position $z = 0$ with $\boldsymbol{r} = (x, y)$ as a two-dimensional vector. We neglect any anisotropy of the crystal surface in the xy plane as well. We suppose a sufficiently low temperature range in order to neglect any possible energy dissipation and the damping of crystallization waves in both states of the surfaces. This implies temperatures lower than about 0.4 K. Neglecting the dissipation aspects simplifies mathematics as well.

As a result, in the lack of energy dissipation the surface oscillations of a ^{4}He crystal can be described with the following Lagrangian:

$$L[\zeta(t,\boldsymbol{r}),\dot\zeta(t,\boldsymbol{r})] = \frac{\rho_{\rm eff}}{2}\iint d^2r d^2r' \frac{\dot\zeta(t,\boldsymbol{r})\dot\zeta(t,\boldsymbol{r}')}{2\pi|\boldsymbol{r}-\boldsymbol{r}'|} - \int d^2r\left(\alpha(\boldsymbol{\nu})\sqrt{1+(\nabla\zeta)^2}+\frac{1}{2}\Delta\rho g\zeta^2\right). \quad (1)$$

Here we ignore the compressibility of both the liquid and solid phases and g is the acceleration of gravity. Because of low-temperature considerations we also neglect the normal component density in the superfluid phase or, equivalently, the difference between the superfluid density ρ_s and the density of the liquid phase ρ, i.e., $\rho_s = \rho$. Then the effective interface density $\rho_{\rm eff}$ is given by

$$\rho_{\rm eff} = \frac{(\rho'-\rho)^2}{\rho} \approx 1.9\ {\rm mg/cm^3}$$

and depends on the difference $\Delta\rho = \rho' - \rho$ between the solid density ρ' and the liquid density ρ.

Unlike the fluid-fluid interface, the surface tension coefficient $\alpha(\boldsymbol{\nu})$ for the crystal depends essentially on the direction of the normal $\boldsymbol{\nu}$ to the interface against crystallographic axes. In our simplest description this is a function of the angle ϑ alone between the normal and, say, crystallographic [0001] or c axis of the crystal hcp structure with the geometric relation $|\tan\vartheta| = |\nabla\zeta|$.

For the crystal facet tilted by small angle ϑ from the basal plane, the expansion of the surface tension $\alpha(\vartheta)$, usually written (see Refs. [1,4,14]) as

$$\alpha(\vartheta) = (\alpha_0 + \alpha_1\tan|\vartheta| + \cdots)\cos\vartheta, \quad |\tan\vartheta| = |\nabla\zeta|,$$

can be expanded for small angles into a series

$$\alpha(\vartheta) = \alpha_0 + \alpha_1|\vartheta| + \cdots, \quad |\vartheta| \ll 1.$$

We intentionally do not write the next terms of expansion, e.g., cubic ones due to step-step interactions, since we are studying only a small bending of the crystal surface. The rough or faceted state of the crystal surface is closely connected with the magnitude of $\alpha_1 = \beta(T)/a$ representing a ratio of the linear facet step energy $\beta(T)$ to the crystallographic interplane spacing a. Below the roughening transition temperature T_R, the linear facet step energy $\beta(T)$ is positive and vanishes for temperatures $T > T_R$. Obviously, the dynamics of the rough and the faceted surfaces differs drastically in kind.

To determine the spectrum of crystal surface oscillations, we minimize the action $S = \int L dt$ against the perturbation $\zeta(t,\boldsymbol{r})$ in order to derive the equation of motion. Within the framework of our approximation $|\nabla\zeta| \ll 1$, the excess Lagrangian $\Delta L[\zeta,\dot\zeta] = L[\zeta,\dot\zeta] - L[0,0]$ is given by the following expression:

$$\begin{aligned}\Delta L[\zeta(t,\boldsymbol{r}),\dot\zeta(t,\boldsymbol{r})] &= L[\zeta(t,\boldsymbol{r}),\dot\zeta(t,\boldsymbol{r})] - L[0,0] \\ &= \frac{\rho_{\rm eff}}{2}\iint d^2r d^2r' \frac{\dot\zeta(t,\boldsymbol{r})\dot\zeta(t,\boldsymbol{r}')}{2\pi|\boldsymbol{r}-\boldsymbol{r}'|} \\ &\quad - \int d^2r\left(\alpha_1|\nabla\zeta| + \frac{\alpha_0}{2}(\nabla\zeta)^2 + \frac{1}{2}\Delta\rho g\zeta^2\right). \quad (2)\end{aligned}$$

As for the step energy α_1, we assume that its low temperature magnitude [1] is approximately $\alpha_1 \approx 0.014$ erg/cm^2. This magnitude amounts to one-tenth of the surface tension [1] $\alpha_0 \approx 0.2$ erg/cm^2 and in the following we always keep the inequality $\alpha_1/\alpha_0 \ll 1$ in mind. Moreover, this small parameter justifies the approximations that will be made below.

Note here that the faceted crystal plane represents in essence a region of the crystal surface in the rough-like state if it is tilted with respect to the crystallographic axis by an angle exceeding about $\arctan(\alpha_1/\alpha_0) \sim 4^\circ$ in sense that $\alpha_1|\nabla\zeta| \ll \alpha_0(\nabla\zeta)^2$. In fact, from the physical point of view the angle of slope $\vartheta = \arctan(\alpha_1/\alpha_0) \sim 4^\circ$ is determined by the competition of two contributions to the total surface energy. One originates from the regular surface term $\alpha_0(\nabla\zeta)^2$ and the second comes from the irregular step tension $\alpha_1|\nabla\zeta|$. Provided that $\alpha_0(\nabla\zeta)^2 \gg \alpha_1|\nabla\zeta|$, the latter contribution becomes negligible and thus the dynamical interface properties should resemble those in the rough surface state. One can say that the crystal surface has too many crystal steps. On the contrary, if $\alpha_0(\nabla\zeta)^2 \ll \alpha_1|\nabla\zeta|$, the dominant term linear in $|\nabla\zeta|$ is responsible for faceting.

Here we mean no phase transition from the atomically smooth to the rough state at about $\theta \sim 4^\circ$. We expect only that the dynamical response of the atomically smooth surface to its perturbation at sufficiently large tilted angles should resemble and become similar to the dynamical response of the surface in the rough state. The dependence of the dynamical response on the slope of the surface will represent the smooth crossover from one type of behavior to another. Such a picture

[1]The variable x plays the role of polar angle.

can be supported with the experimental evidence [15]. The crossover from the smooth to the rough-like state is observed as a function of the tilt angle at the same magnitude between 3° and 4°.

III. TRANSMISSION AND REFLECTION

To consider the transmission and reflection of melting-crystallization waves across the edge separating the rough crystal surface and the crystal facet, we suppose a simple model to describe such phenomena. So, the step energy is approximated by the function

$$\alpha_1(x) = \begin{cases} 0, & x < 0 \\ \alpha_1, & x > 0. \end{cases}$$

In other words, the left-hand side of the crystal surface is in the rough state and the right-hand side of the crystal surface represents the smooth faceted state.

The approximation for step energy $\alpha_1(x)$ with the step-like function implies implicitly that the width of transition W from the rough to smooth faceted state is much smaller as compared with the inverse wave vector $1/k$ or wavelength. The smooth boundary when $W \sim 1/k$ or larger should change the transmission and reflection coefficients. The smooth transition region usually reduces the reflection and enhances the transmission of the wave.

As the crystallization wave propagates across the boundary between two crystalline surfaces, the wave transmits and reflects. The wave on the left-hand side of the boundary is a superposition of the incident and reflected waves. On the right-hand side from the boundary the transmitted wave alone propagates. The relation between all three waves is determined with the boundary conditions at the interface $x = 0$. We consider the case of the normal incidence.

Let us write the perturbations of the crystal surface due to the incident, reflected, and transmitted waves, respectively:

$$\zeta_0(x,t) = \zeta_0 e^{ikx - i\omega t}, \quad x < 0,$$
$$\zeta_1(x,t) = \zeta_1 e^{-ikx - i\omega t}, \quad x < 0,$$
$$\zeta_2(x,t) = \zeta_2 e^{iqx - i\omega t}, \quad x > 0.$$

Here ω and $k = k(\omega)$ are the frequency and wave vector of the incident and reflected waves. The transmitted wave has the same frequency but its wave vector $q = q(\omega, |\zeta_2|)$, unlike the case of the rough state of the surface, depends on the amplitude of the wave as well.

At the interface $x = 0$ we should provide a continuity of both the crystal surface distortion $\zeta(x)$ and the derivative of $\partial\zeta/\partial x$. Eventually, we choose the following boundary conditions in order to match the propagation of waves across the crystal edge:

$$\zeta(x = -0, t) = \zeta(x = +0, t),$$
$$\frac{\partial\zeta(x = -0, t)}{\partial x} = \frac{\partial\zeta(x = +0, t)}{\partial x}.$$

As it concerns the crystal surfaces apart, we have the well-established equations describing the wave-like melting-crystallization processes in full agreement with experiment. These differential equations are of second order. Strictly speaking, the equations are derived within the macroscopic approach and it is implicitly implied that the perturbation of the interface ζ is sufficiently large as compared with the interatomic distances. In general, the boundary conditions are not a direct consequence of the bulk equations but represent an additional condition to be introduced independently.

However, in the absence of any experimental observation the speculation is conventional and referred to as natural. To have the finite magnitudes for the derivatives of the highest order at the boundary region, we must put the continuity for the derivatives of lowest orders across the boundary, i.e., the matching of the interface perturbations and its first derivations. In addition, such a boundary condition satisfies the requirement of zero reflection and full transmission in the case of identical left- and right-hand surfaces.

Employing the above conditions, we obtain readily a pair of equations determining the reflection and transmission of the incident wave:

$$\zeta_0 + \zeta_1 = \zeta_2,$$
$$ik\zeta_0 - ik\zeta_1 = iq\zeta_2.$$

Introducing the reflection and transmission coefficients as a ratio of the reflected amplitude ζ_1 and transmitted amplitude ζ_2 to the incident one ζ_0, we arrive at the following magnitudes:

$$r = \frac{\zeta_1}{\zeta_0} = \frac{k - q}{k + q} \quad \text{and} \quad t = \frac{\zeta_2}{\zeta_0} = \frac{2k}{k + q}.$$

The striking distinction from the usual case of acoustic wave is associated with the dependence of wave vector q for the transmitted crystallization wave on its amplitude ζ_2. The equations which determine the amplitudes ζ_2 and ζ_1 of the transmitted and reflected waves are given by

$$\zeta_2 = \frac{2k(\omega)}{k(\omega) + q(\omega, \zeta_2)}\zeta_0 \quad \text{and} \quad \zeta_1 = \zeta_2(\zeta_0) - \zeta_0.$$

To understand the main features of the phenomenon, we first neglect the gravitational term proportional to the density difference $\Delta\rho$ in the Lagrangian, assuming that the wave vector is larger than the inverse magnitude of capillary length, $k > k_0 \sim \sqrt{\Delta\rho g/\alpha_0}$. Then, for the rough state of the crystalline surface, one has an ordinary capillary dispersion [3]

$$\rho_{\text{eff}}\frac{\omega^2}{k} = \alpha_0 k^2 \quad \text{and} \quad k(\omega) = \left(\frac{\omega^2\rho_{\text{eff}}}{\alpha_0}\right)^{1/3}.$$

For the faceted state of the crystal surface [6], the dispersion is more complicated and depends on the wave amplitude ζ according to

$$\rho_{\text{eff}}\frac{\omega^2}{q} = \begin{cases} \gamma\alpha_1 q/|\zeta|, & q|\zeta| \ll \alpha_1/\alpha_0 \\ \alpha_0 q^2, & q|\zeta| \gg \alpha_1/\alpha_0, \end{cases}$$

where $\gamma = \pi\zeta(3)/7 = 0.539\ldots$ is a numerical coefficient. Accordingly,

$$q(\omega, \zeta) = \begin{cases} \omega\left(\frac{\rho_{\text{eff}}|\zeta|}{\gamma\alpha_1}\right)^{1/2}, & \omega^2|\zeta|^3 \ll \frac{\alpha_1^3}{\alpha_0^2\rho_{\text{eff}}} \\ \left(\frac{\omega^2\rho_{\text{eff}}}{\alpha_0}\right)^{1/3}, & \omega^2|\zeta|^3 \gg \frac{\alpha_1^3}{\alpha_0^2\rho_{\text{eff}}}. \end{cases}$$

The most interesting case is that of sufficiently small amplitudes $|\zeta_0|$ for the incident crystallization wave satisfying the inequality $2k|\zeta_0| \ll (\alpha_1/\alpha_0)^{2/3} \lesssim 1$. The latter implies $q \ll k$.

As a final result, we arrive at

$$\zeta_2 \approx 2\zeta_0\,, \quad \zeta_1 = \left(1 - \frac{2q}{k}\right)\zeta_0, \quad \text{and} \quad \frac{q}{k} \approx \sqrt{\frac{2\alpha_0}{\alpha_1} k|\zeta_0|}.$$

Thus, we have approximately the following reflection and transmission coefficients: $r \approx 1$ and $t \approx 2$.

Let us discuss the result obtained. We see that the reflected crystallization wave has approximately the same amplitude and is similar to the incident wave but propagating in the opposite direction. At the same time the incident wave onto the boundary edge induces the transmitted crystallization wave with the double amplitude representing the flat kink at the smooth crystal surface. Such a solitonlike perturbation, whose size is about the wavelength $2\pi/k$, propagates away from the boundary at a velocity of about $V \approx (\gamma\alpha_1/2\rho_{\rm eff}|\zeta_0|)^{1/2}$. Briefly speaking, the incident wave produces the reflected wave and excites the transmitted wave in the shape of a soliton with the larger wavelength.

In the opposite case of sufficiently large amplitudes, if $2k|\zeta_0| \gg (\alpha_1/\alpha_0)^{2/3}$, the reflected crystallization wave is weak and practically vanishes since the facet step energy α_1 plays a negligible role. The reflection can appear only due to differences in the surface tension coefficients for the adjoint crystalline facets:

$$r \approx \frac{\alpha_{0r}^{1/3} - \alpha_{0l}^{1/3}}{\alpha_{0r}^{1/3} + \alpha_{0l}^{1/3}},$$

where coefficients α_{0l} and α_{0r} refer to the left- and right-hand sides of the crystalline surface. In its turn, the transmitted crystallization wave has an almost full similarity with the incident crystallization wave. Thus, we expect $r \approx 0$ and $t \approx 1$ if $\alpha_{0l} = \alpha_{0r}$.

IV. INCIDENCE FROM THE CRYSTAL FACET ONTO THE ROUGH CRYSTAL SURFACE

Let us consider the opposite situation when the crystallization wave or crystal step arrives at the boundary from the smooth-faceted surface to the rough crystal surface. So, we represent the incident, reflected, and transmitted waves as follows:

$$\zeta_0(x,t) = \zeta_0 e^{-iqx - i\omega t}, \quad x > 0,$$
$$\zeta_1(x,t) = \zeta_1 e^{iqx - i\omega t}, \quad x > 0,$$
$$\zeta_2(x,t) = \zeta_2 e^{-ikx - i\omega t}, \quad x < 0.$$

Then, we have the following conditions at the boundary $x = 0$:

$$\zeta_0 + \zeta_1 = \zeta_2 \quad \text{and} \quad -iq\zeta_0 + iq\zeta_1 = -ik\zeta_2.$$

Hence we arrive at

$$\zeta_2 = \frac{2q(\omega,\zeta_0)}{q(\omega,\zeta_0) + k(\omega)}\zeta_0 \quad \text{and} \quad \zeta_1 = \zeta_0 - \zeta_2(\zeta_0).$$

Again the most interesting case is when the amplitude of the incident wave is sufficiently small $q|\zeta_0| \lesssim \alpha_1/\alpha_0 \ll 1$. This means that, either the crystal step height is small, or the length of the protrusive crystal layer is rather extended. Thus, we find that the wave vector of the transmitted wave is given by

$$k = q(\omega)\left(\frac{\gamma\alpha_1}{\alpha_0}\frac{1}{q(\omega)|\zeta_0|}\right)^{1/3} \gg q(\omega)$$

and the reflection and transmission coefficients read

$$r = \frac{\zeta_1}{\zeta_0} \approx 1 \quad \text{and} \quad t = \frac{\zeta_2}{\zeta_0} \approx 2\left(\frac{\alpha_0}{\alpha_1} q\zeta_0\right)^{1/3} \ll 1.$$

Thus, we see that the crystallization wave or the crystal step, on the whole, reflects from the rough crystal surface. As it concerns the transmitted wave, its amplitude is much smaller than the amplitude of the incident wave, and the excitation of the crystallization wave at the rough surface with the incident crystal step is ineffective. Here we should underline some asymmetry between the incidence of crystallization waves from the rough and from the faceted crystal surface sides.

In the opposite limit when $q|\zeta_0| \gg \alpha_1/\alpha_0$, the difference between the faceted state and the rough state is not large. We expect practically no reflection from the boundary and the full transmission of the wave to the other side of the crystalline surface. In fact,

$$k \approx q(\omega) + \frac{\gamma\alpha_1}{3\alpha_0|\zeta_0|},$$

and the reflection and transmission coefficients are approximately given by

$$r = \frac{\zeta_1}{\zeta_0} \approx \frac{1}{6}\frac{\gamma\alpha_1}{\alpha_0 q|\zeta_0|} \ll 1 \quad \text{and} \quad t = \frac{\zeta_2}{\zeta_0} \approx 1.$$

The latter means that the noticeable reflection can again appear only due to large distinction in the surface tension coefficients of the left- and right-hand sides of a crystal.

V. SUMMARY

To conclude, we have attempted the transmission and reflection of crystallization waves propagating along the superfluid-crystal ^{4}He interface across the boundary edge between the crystalline surfaces in the rough and smooth states. The crystallization wave at the rough ^{4}He crystal surface resembles the usual gravitational-capillary waves at the fluid-fluid interface. In contrast, the crystallization wave at the smooth-faceted surface represents the propagation of crystal steps at a velocity depending on the crystal step height. To match two types of waves at the crystal edge, we use the natural boundary conditions.

Since the dispersion of crystallization waves at the smooth-faceted surface is essentially governed by the wave amplitude, the transmission and reflection coefficients depend on the amplitude of the incident wave. The incidence of the crystallization wave from the rough crystal surface onto the smooth-faceted one results in the practically mirror reflection of the incident wave and in inducing the crystal step or soliton of about double amplitude in the region of the crystal facet behind the crystal edge.

In the opposite situation of the incidence of the crystallization wave from the faceted crystal surface onto the

rough crystal surface we should observe practically the full reflection and the corresponding small transmission to the rough crystal surface. Note that we have no symmetry with respect to rearrangement between the crystal surfaces in the rough and flat states.

The experimental study on the dynamics of crystallization waves at the atomically smooth crystal facet requires an effective mechanism for their excitation. Apparently this is a tough challenge. To confirm, we can mention an unsuccessful attempt to produce a soliton-like crystallization wave with the aid of a Π-shaped crossbar oscillating in the vicinity of the crystal ^{4}He facet [16]. The oscillations of the crossbar are shown to be very effective for inducing the crystallization waves at the rough ^{4}He crystal surface but no effect is observed for the faceted ^{4}He crystal surface.

The present work proposes a mechanism for exciting the wave or soliton at the smooth flat facet with the help of a crystallization wave propagating along the nonfaceted rough surface across the crystal edge in the direction to the atomically smooth facet. One more possibility is to prepare the electron-charged crystal facet in order to induce the instability of the atomically smooth surface at the critical electron density. However, this will require a critical electron density [13] larger by a factor of about 50–100 as compared with that of about 10^9 cm^{-2} observed for the rough interface.

A special interest represents the experiment on the transmission and reflection of crystallization waves propagating across the crystal edge between the rough crystal surface and the vicinal surface whose orientation is tilted by the small angle ϑ with respect to the well-faceted surface. Provided the tilt angle ϑ is sufficiently small, the crystal steps are well separated. On the other hand, an existence of ready-made crystal steps should noticeably affect the transmission and reflection coefficients as a function of tilt angle of the vicinal surface.

Here we have described the transmission and reflection of the crystallization wave propagating across the facet boundary within the framework of phenomenological approach. The boundary condition is chosen in the form of continuity of total wave amplitude and its derivative. In addition to the transmission and reflection, the incidence of the crystallization wave onto the facet boundary can be accompanied by other possible effects. One can suppose excitation of sound modes propagating away from the boundary to the liquid and/or crystal bulk. For the oblique incidence at the facet boundary, one should expect excitation of specific crystallization wave which will propagate along the crystal edge. Provided that the wavelength of the incident wave is comparable with the width of the facet boundary, the interference effects may appear. In this case the transmission and reflection coefficients will display an oscillating behavior as a function of incident wavelength. Involving such aspects into consideration fails our simple description and requires the introduction of additional parameters and possible discontinuities to the formulation of the boundary conditions. Such a plentiful picture cannot be described within our phenomenological approach and will require a microscopic treatment.

Checking for the physical aspects of crystallization wave transmission across the facet boundaries in ^{4}He crystals is the main reason for experiments.

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