# **Breakdown of Arrhenius law of temperature-dependent vacancy concentration in fcc lanthanum**

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We measured the temperature-dependent equilibrium vacancy concentration using *in situ* positron annihilation spectroscopy in order to determine the enthalpy  $H_f$  and entropy  $S_f$  of vacancy formation in elementary fcc La. The Arrhenius law applied for the data analysis, however, is shown to fail in explaining the unexpected high values for both  $S_f$  and  $H_f$ : in particular  $S_f = 16(2)$   $k_B$  is one order of magnitude larger compared to other elemental metals, and the experimental value of  $H_f$  is found to be more than three standard deviations off the theoretical one  $H_f = 1.46$  eV (our DFT calculation for La at  $T = 0$  K). A consistent explanation is given beyond the classical Arrhenius approach in terms of a temperature dependence of the vacancy formation entropy with  $S_f' = 0.01119(13) k_B/K$  accounting for the anharmonic potential introduced by vacancies.

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#### **I. INTRODUCTION**

The dominant species of lattice defects, which are thermally created in metal samples, are monovacancies. The enthalpy  $H_f$  and entropy  $S_f$  for vacancy formation in thermal equilibrium are key features for the fundamental understanding of physical processes in crystals such as creation of lattice defects and diffusion properties.

There are several experimental techniques where the measured quantity depends on the concentration of (point) defects. Conventionally, measurements of the residual electrical resistivity, which is proportional to the total concentration of all species of lattice defects, are performed to examine the crystal quality or to provide detailed information of defect annealing [\[1\]](#page-3-0). Differential dilatometry is sensitive to the volume change associated with the formation of lattice defects in the sample and hence allows the estimation of the vacancy concentration in thermal equilibrium but is limited to temperatures close to the melting point [\[2\]](#page-3-0). In contrast, positron annihilation spectroscopy (PAS) is applied as true probing technique to study open-volume crystal defects on an atomic scale (see the pioneering work by MacKenzie *et al.* [\[3\]](#page-3-0) and reviews, e.g., [\[4\]](#page-3-0)). Due to the efficient trapping in the attractive potential formed by vacancies  $[2,5]$ , positrons exhibit an outstanding sensitivity for the detection of vacancy concentrations as low as  $c_v \sim 10^{-7}$  [\[6\]](#page-3-0).

PAS has been widely applied as a nondestructive technique to study the annealing behavior  $[7,8]$  and the thermal production of vacancies [\[9\]](#page-3-0). The measurement of the equilibrium vacancy concentrations as a function of temperature in turn allows the determination of the vacancy formation enthalpy *H*f. For a large number of elemental metals and alloys the Arrhenius law has been applied in order to obtain values for  $H_f$  and, to lesser extent, for  $S_f$  (see, e.g., [\[9–](#page-3-0)[12\]](#page-4-0)). Besides specific heat measurements on La providing an estimation of  $H_f = 1$  eV [\[13\]](#page-4-0), no further data or detailed studies of the vacancy formation in La is reported to the best of our knowledge.

In this article we present measurements of the vacancy concentration in thermal equilibrium in fcc La up to 1020 K using *in situ* PAS. For comparison of the experimental findings with theory we calculated  $H_f$  for La at  $T = 0$  K based on density-functional theory (DFT). For the first data analysis an Arrhenius-like behavior for thermal production of vacancies was assumed, which lead to unexpectedly high values for both the enthalpy  $H_f$  and entropy  $S_f$  for vacancy formation. This discrepancy was attributed to the fact that the formation entropy is mainly affected by the change of the phonon spectrum of the crystal due to the presence of vacancies. In order to obtain a consistent physical explanation we followed a theoretical study by Glensk *et al.* [\[14\]](#page-4-0) and introduced a temperature-dependent vacancy formation entropy.

### **II. EXPERIMENT**

Lanthanum exhibits phase transitions from dhcp to fcc at  $\sim$ 560 K and from fcc to bcc at  $\sim$ 1120 K [\[15\]](#page-4-0). The melting point of La amounts to 1193 K and its density at room temperature is  $6.145$  g cm<sup>-3</sup> [\[16\]](#page-4-0). The purity of the sample investigated in the present study is >99.9%. Since La is highly reactive and would, e.g., oxidize rapidly when exposed to air, it is kept in ethanol during and after sample preparation. A disk of 4 mm was cut and polished first with SiC grinding paper and subsequently with a  $H_2O$ -free diamond suspension with a final grain size of 1  $\mu$ m. Possible lattice defects have been annealed by heating the sample up to 1020 K with a heating rate of 13 K min<sup>-1</sup> and subsequent adiabatic cooling in the coincident Doppler broadening (CDB) spectrometer prior the temperature-dependent measurements. Thus the initial *S* parameter of the as-prepared sample at room temperature was reduced by 4%.

Positron-electron annihilation leads predominantly to the emission of two 511 keV  $\gamma$  quanta. These photons, which

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<span id="page-1-0"></span>experience a Doppler shift due to the momentum of the annihilating electrons (the momentum of the thermalized positrons is negligible), are examined by Doppler broadening spectroscopy (DBS) of the positron annihilation line. This broadening strongly depends on the vacancy concentration since the lower annihilation probability of positrons trapped in vacancies with high-momentum core electrons leads to a smaller Doppler shift compared to annihilation in the pure lattice. For the characterization of the Doppler broadening, the so-called *S* parameter is conventionally defined as the fraction of counts in a fixed central region of the annihilation photo peak. Hence, compared to the defect-free state the *S* parameter is usually enhanced for positrons trapped in a vacancy. For further details of DBS we refer to [\[17\]](#page-4-0).

For the present study we used the CDB spectrometer [\[18\]](#page-4-0) with a monoenergetic positron beam provided by the neutron induced positron source Munich (NEPOMUC) [\[19\]](#page-4-0) at the research neutron source Heinz-Maier Leibnitz (FRM II). DBS at this spectrometer with an accessible temperature range of 40–1100 K was shown to be particularly suited for the determination of the vacancy concentration, e.g., in Heusler alloys [\[20\]](#page-4-0) or the *in situ* observation of fast defect annealing after severe plastic deformation [\[21\]](#page-4-0). In addition, compared to conventional PAS with  $\beta^+$  emitters the usage of a positron beam has the advantage of simple sample heating under ultrahigh vacuum conditions and that the recorded signal exclusively stems from annihilation events inside the sample (absence of the so-called source component). The kinetic energy of the positron beam can be varied between 0.1 and 30 keV and the spot size of the beam at the sample position is typically 250  $\mu$ m. In this study a maximum implantation energy of  $E = 28$  keV is used corresponding to a mean positron implantation depth of  $\bar{z} = 1.2 \mu m$  in La. This value was calculated by using  $\bar{z} = A/\rho \cdot E^n$  of the so-called Makhovian implantation profile [\[22\]](#page-4-0) with the mass density ρ of La and the element specific parameters *A* and *n*. Since *A* and *n* for La are not known the corresponding values for Ag are used ( $A = 3.98 \ \mu g \text{ cm}^{-2} \text{ keV}^{-n}$  and  $n = 1.57$ ) [\[23\]](#page-4-0). The *S* parameter is calculated as the fraction of annihilation events of the photo peak in the energy interval  $(511 \pm 1.7)$ keV. For the determination of the bulk equilibrium vacancy concentration in La at elevated temperature we performed *in situ* DBS between 493 and 1023 K in steps of 10 K using the 28 keV positron beam. At each temperature step starting at 1023 K data were recorded for 5 min resulting in about 800 000 counts in the 511 keV annihilation photo peak.

## **III. RESULTS AND DISCUSSION**

The measured *S* parameter as a function of temperature is shown in Fig. 1. The total increase of *S* from 493 to 1023 K amounts to 7%. Up to temperatures of 700 K a linear rise of the *S* parameter is observed with a slope of  $\alpha \approx 2.4(3) \times$  $10^{-5}$  K<sup>-1</sup>. This linear increase is attributed to the lattice expansion without significant positron trapping in defects and can be very well explained by the thermal volume expansion coefficient of  $\alpha_{th,v} = 3\alpha_{th,l}$ , with  $\alpha_{th,l} \approx 8 \times 10^{-6} \text{ K}^{-1}$  being the thermal linear expansion coefficient of La [\[24\]](#page-4-0). This effect correlates to the decreasing overlap of the positron wave function with those of core electrons being proportional to



FIG. 1. Measured *S* parameter as a function of temperature for La. The experimental data (symbols) are fit by a two-state model for positrons annihilating in the bulk or trapped in vacancies according to Eq. (1) (lines). The linear increase of *S* at lower temperature is well described by the thermal lattice expansion (dashed line).

the volume expansion of the lattice as observed in studies on, e.g., Al, In, and Pb; the effect of a small contraction of the Fermi surface with higher temperature is negligible [\[25\]](#page-4-0). It is noteworthy that the phase transition for La at ∼580 K from dhcp to fcc does not affect the linear slope of the *S* parameter or any other of our fit parameters (within the errors) when starting the fit above 600 K. This behavior of  $S(T)$ , however, was expected since the crystal structures fcc and dhcp differ only in the stacking order. The thermal production of vacancies, which act as efficient positron trapping sites with significantly reduced core annihilation probability, leads to a steeper increase of *S* above 700 K according to the equilibrium vacancy concentration at the respective temperature. At about 950 K the *S* parameter starts to converge due to so-called saturation trapping, since the high vacancy concentration results in trapping of all positrons.

As first proposed by McKee *et al.* [\[10\]](#page-3-0), the behavior of the *S* parameter can be represented by a superposition of two positron states: positrons annihilate either from a delocalized state in the bulk or from the trapped state in a vacancy with characteristic values  $S_b$  and  $S_v$ , respectively. Hence the *S* parameter measured at a given temperature *S*(*T* ) can be described by

$$
S(T) = \frac{1}{1+Q} S_b(T) + \frac{Q}{1+Q} S_v(T). \tag{1}
$$

The temperature dependencies of  $S_b$  and  $S_v$  are considered to be linear in *T* with  $(1 + \alpha T)$  and  $(1 + \beta T)$ , respectively, and well explain the effect of the lattice expansion as discussed. The weighting factors in Eq. (1) are expressed in terms of *Q* containing properties of the positron and thermodynamical information

$$
Q(T) = \frac{S(T) - S_b(T)}{S_v(T) - S(T)} = \mu \tau_b c_v(T),
$$
 (2)

with the specific trapping coefficient of a monovacancy  $\mu$ , the bulk lifetime of a positron  $\tau_b$ , and the thermal equilibrium

<span id="page-2-0"></span>

FIG. 2. Arrhenius plot of the measured data. The equilibrium vacancy concentration  $c<sub>v</sub>$  (right axis) is deduced from the measured  $Q(T)$  using the approximation  $\mu \tau_{\rm b} \approx 8.3 \times 10^4$  [see Eq. [\(2\)](#page-1-0)]. For the different fits (lines) see text.

vacancy concentration  $c_v(T)$  at the temperature *T*. If not explicitly given,  $\mu \tau_b$  has to be estimated to provide a value for  $c_v$  from the measurements. Since  $\mu \tau_b$  is actually not known within about one order of magnitude we use the approximation  $\mu \tau_b \approx 8.3 \times 10^4$  in the following. A reasonable value for the positron trapping coefficient can be assumed by  $\mu = 4 \times 10^{14}$  s<sup>-1</sup> which corresponds to the value applied for Al [\[26,27\]](#page-4-0) and being in the range found for Cu, Au, and Pt [\[28\]](#page-4-0). The positron bulk lifetime in La has been calculated to  $\tau_b \approx 208$  ps [\[29\]](#page-4-0). Since the positron binding energy to a monovacancy in La is calculated to be at least 1.9 eV [\[30\]](#page-4-0) detrapping of positrons in the temperature regime relevant for the present study can be neglected.

Assuming monovacancies being the dominant species of lattice defects and in the limit of noninteracting vacancies their concentration is determined by the Gibbs free enthalpy of vacancy formation *G*f,

$$
c_{\rm v}(T) = \exp[-G_{\rm f}(T)/k_{\rm B}T],\tag{3}
$$

wherein  $k<sub>B</sub>$  is the Boltzmann constant. The temperature dependence of  $G_f$  is conventionally given by

$$
G_{\rm f}(T) = H_{\rm f} - TS_{\rm f},\tag{4}
$$

with enthalpy  $H_f$  and entropy  $S_f$  of vacancy formation, both assumed to be temperature independent. It has to be noted that the respective influence of  $H_f$  and  $S_f$  cannot be separated by any experiment measuring the vacancy concentration. The data shown in Fig. [1](#page-1-0) were fitted (Arrhenius, blue line) by applying Eq. [\(1\)](#page-1-0) using four parameters  $(S_b^0, S_v^0, \alpha, \text{ and } \beta)$ . Tests with different values revealed that any temperature dependence of  $\tau_b$  [see Eq. [\(2\)](#page-1-0)] can be well neglected. By applying Eqs.  $(3)$  and  $(4)$  this classical model yields a value for the vacancy formation enthalpy of  $H_f = (1.97 \pm 0.15)$  eV. Figure 2 shows the recorded data  $S(T)$  in the common Arrhenius representation where  $H_f$  is given by the linear slope of the data. A linear behavior is observed in the significant region of the covered temperature range. Note that due to the high sensitivity of defect spectroscopy with positrons low



FIG. 3. Gibbs free enthalpy as a function of temperature. The different curves correspond to Arrhenius fit (blue line) as well as fits using the Taylor expansion up to the second order with  $1<sup>st</sup>$  &  $2<sup>nd</sup>$  order (orange line) and without linear term  $2<sup>nd</sup>$  order (green line) with the calculated value of  $G_f^{\text{th}}(0) = 1.46 \text{ eV}$  (red symbol) as a constraint.

concentrations in the order of  $10^{-6}$  vacancies per atom are clearly visible and saturation trapping starts above  $c_v = 10^{-3}$ .

We computed the vacancy formation energy by DFT with the PBE-generalized gradient approximation [\[31\]](#page-4-0) using the ABINIT code in the projector-augmented wave framework [\[32\]](#page-4-0). We used a plane-wave cutoff of 680 eV and a  $12 \times 12 \times 12$ *k*-point grid with respect to the conventional cubic unit cell of the fcc lattice. We obtained a lattice constant of 5.29 Å for the ground state, in perfect agreement with the experimental fcc lattice constant extrapolated to zero temperature [\[33\]](#page-4-0). The cubic  $32 - 1$ -atom supercell with relaxed internal positions but fixed cell dimensions gave a vacancy formation energy of 1.46 eV, thus perfectly reproducing the previously reported values of 1.44 and 1.46 eV  $[34,35]$ . To test for a variation of the formation energy with thermal lattice expansion, we performed additional calculations at a lattice constant of 5.32 Å corresponding to the experimental value around 780 K representative of the temperatures of measurement, which however resulted in only a minute increase of the formation energy to 1.50 eV. In order to compare experiment with theory we calculated  $G(T)$  at each temperature from the measured data by combining Eqs. [\(2\)](#page-1-0) and (3) and the  $S(T)$  fit result (see Fig. [1\)](#page-1-0). Figure 3 displays the Gibbs free enthalpy as function of temperature with extrapolation of the Arrhenius fit to  $T = 0$  K. It becomes obvious that the experimental value  $G_f(0) \equiv H_f$  is significantly, i.e., more than three standard deviations, off the calculated one. For the physical interpretation of the data we formally describe the Gibbs free enthalpy in a more general way by applying the Taylor expansion up to the second order

$$
G_{\rm f}(T) \approx G(T_0) + G'(T - T_0) + G''(T - T_0)^2/2 \qquad (5)
$$

centered at  $T_0 = 850$  K, i.e., at the center of the  $S(T)$  data set; G' and G'' are first and second partial derivatives, respectively, of the Gibbs free enthalpy with respect to temperature. Two

<span id="page-3-0"></span>additional fits with fixed  $G_f^{\text{th}}(0) = 1.46 \text{ eV}$  from DFT calculation are performed with and without the linear term (indicated as  $1<sup>st</sup>$  &  $2<sup>nd</sup>$  order and  $2<sup>nd</sup>$  order) and plotted alongside with the Arrhenius fit in all Figs. [1](#page-1-0) to [3.](#page-2-0)

The classical Arrhenius law applied to our data for La would yield unrealistic values for both vacancy formation entropy  $S_f$  and Gibbs free enthalpy at  $T = 0$  K. The experimental value of  $G_f(0)$  was found to be 0.51 eV higher than the calculated one. Hence, the failure of the Arrhenius law becomes apparent in Fig. [3:](#page-2-0) consequently, the large difference between experimental and theoretical value of  $G_f(0)$  cannot be explained by the conventionally defined Gibbs free enthalpy being linear in temperature. Even more importantly, for the vacancy formation entropy we obtain a lower limit of  $S_f$  =  $-\partial G/\partial T = (16 \pm 2) k_B$ ; such high values of *S*<sub>f</sub> in metals have never been observed to the best of our knowledge. Typical *S*<sup>f</sup> for elemental metals, however, are in the range of 0.5– 2  $k_B$  [\[12\]](#page-4-0), i.e.,  $S_f \approx 1$   $k_B$  for fcc and  $S_f \approx 2$   $k_B$  for bcc crystal lattices [\[36\]](#page-4-0). Assuming such values also for fcc La would necessitate to postulate a trapping coefficient that is seven orders of magnitude lower than in comparable systems, which can safely be ruled out.

The influence of possible divacancies has been proven to be negligible in early experiments [\[37\]](#page-4-0) and divacancies, e.g., in Al, were shown to be unstable [\[38\]](#page-4-0). Theoretical studies yield that the anharmonicity of lattice vibrations are much more significant than the small effect of possible divacancies [\[39\]](#page-4-0). This was confirmed in more recent computations of the thermodynamics of divacancies in Al and Cu by Glensk *et al.* [\[14\]](#page-4-0), who obtained divacancy concentrations  $\leq 4 \times 10^{-3}c_v$ even at the melting point. The effect of positron detrapping from vacancies was found to somewhat influence the measurements at very high temperature as observed for refractory metals such as Ta [\[11\]](#page-4-0) but is assumed to be negligible in the temperature range of the present study. According to an empiric description based on a temperature-dependent vacancy formation enthalpy Schaefer *et al.* [\[40\]](#page-4-0) and assuming a "real" formation entropy of  $S_f = 1$   $k_B$  for La we would obtain a temperature dependence of  $H_f$  of about  $-1.3$  meV K<sup>-1</sup>. Besides being purely phenomenological, this approach relies on a Gibbs free enthalpy depending linearly on temperature, that in turn is unable to explain the theoretically calculated value of  $G_f^{th}(0)$ .

In order to describe the exceptionally high  $S_f$  our data can be fitted by using Eq. [\(5\)](#page-2-0) and the calculated value for  $G_f^{th}(0)$ . The resulting best fit (with  $G' = 1.48 \times 10^{-3}$  eV/K and  $G'' =$  $-1.6 \times 10^{-6}$  eV/K<sup>2</sup>) is displayed as 1<sup>st</sup> & 2<sup>nd</sup> order (orange line) in the figures. It has to be emphasized that this rather formal procedure, i.e., the mathematical description of the Taylor expansion of *G*(*T* ) around 850 K, is intrinsically not able to disentangle information of the temperature dependence of  $H_f$  and  $S_f$ . Therefore, we follow the theoretical approach proposed by Glensk *et al.* [\[14\]](#page-4-0) who performed demanding finite temperature DFT computations of the Gibbs free energy of vacancy formation by explicitly including anharmonicity due to phonon-phonon interactions, which is of particular importance at high temperatures. Compared to the classical Arrhenius behavior deviations for  $H<sub>f</sub>$  of 0.15 and 0.22 eV were found for Al and Cu, respectively [\[14\]](#page-4-0). The formation entropy of vacancies was described to be linear in temperature. We now apply this physically justified model (local Grüneisen theory) to our experimental results for La and expand  $S_f$  up to the first order in temperature (whereby its constant fraction is neglected as proven to be valid for Al and Cu [\[14\]](#page-4-0))

$$
S_{\rm f}(T) \approx S_{\rm f}'T,\tag{6}
$$

where  $S'_f$  is the partial temperature derivative of  $S_f$ . Using Eq. [\(4\)](#page-2-0) we obtain for the Gibbs free enthalpy

$$
G_{\rm f}(T) = H_{\rm f}^{0\,{\rm K}} - T^2 S_{\rm f}'.
$$
 (7)

Fitting the data hence requires only two free parameters  $H_f^{0 K}$ Fitting the data hence requires only two rice parameters  $n_f$  and  $S_f$ . According to our DFT calculation the first one is found to be  $H_f^{0K} = 1.46$  eV and the second one is  $S_f' =$ 0.01119(13)  $k_B/K$ . The according fit depicted as  $2<sup>nd</sup>$  order is shown in Figs. [1](#page-1-0) to [3.](#page-2-0) Using this model the linear increase with  $\alpha \approx 2.5(2) \times 10^{-5} \text{ K}^{-1}$  in the temperature range 480 to 700 K is indistinguishable from the other fits as shown in Fig. [1](#page-1-0) but clearly deviates from the Arrhenius law at lower temperature (see Fig. [3\)](#page-2-0).

#### **IV. CONCLUSION**

In summary we found unexpectedly high discrepancies for both  $H_f$  and  $S_f$  by applying the classical Arrhenius interpretation to our data obtained by *in situ* PAS at high temperatures: the Gibbs free enthalpy at  $T = 0$  K was more than three standard deviations higher than that resulting from our DFT calculation. Even more surprising, however, is the exceptional high value for the entropy  $S_f$ , which was found to be about one order of magnitude higher than typical ones for elemental metal crystals. In this article a consistent explanation is given in terms of a temperature-dependent vacancy formation entropy taking into account the anharmonicity of phonons introduced by the presence of monovacancies in the crystal lattice.

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