


**Dual Majorana universality in thermally induced nonequilibrium**Sergey Smirnov <sup>\*</sup>*P. N. Lebedev Physical Institute of the Russian Academy of Sciences, 119991 Moscow, Russia* (Received 17 December 2019; revised manuscript received 24 February 2020; accepted 26 February 2020; published 18 March 2020)

We demonstrate that nonequilibrium nanoscopic systems with Majorana zero modes admit a special kind of universality which cannot be classified as strictly transport or strictly thermodynamic in nature. To reveal such a type of Majorana universality we explore purely thermal nonequilibrium states of a quantum dot whose low-energy degrees of freedom are governed by Majorana zero modes. Specifically, the quantum dot is coupled to a topological superconductor, supporting Majorana zero modes, as well as to two normal metallic contacts with the same chemical potentials but different temperatures. It is shown that the Majorana universality in this setup is dual: it is stored inside both the response of the electric current, excited exclusively by the temperature difference, and the quantum dot compressibility. The latter is defined as the derivative of the quantum dot particle number with respect to the chemical potential and forms a universal Majorana ratio with a proper derivative of the electric current that flows in nonequilibrium states of a purely thermal nature.

DOI: [10.1103/PhysRevB.101.125417](https://doi.org/10.1103/PhysRevB.101.125417)**I. INTRODUCTION**

Nanoscope systems based on topological superconductors provide novel low-energy degrees of freedom as key players determining their universal physical properties. In particular, as suggested by experimental measurements [1–3] of the differential conductance, universality of nonequilibrium quantum transport through such systems is to a large extent governed by low-energy quasiparticles known as Majorana zero modes. These non-Abelian modes are often termed “Majorana fermions” since they represent their own antiparticles, as happens for Abelian Majorana fermions [4] in quantum field theory [5]. In the context of condensed-matter physics, Majorana zero modes are predicted to arise in the topological phase of the Kitaev tight-binding chain model [6]. The latter has become of extreme practical importance due to its various appealing mappings [7–11] onto experimentally feasible models such as setups combining superconductors with topological insulators [12,13] and semiconductors whose low-energy spectra result from an interplay between spin-orbit interactions and an induced superconducting order parameter [14,15].

An alternative to transport experiments measuring mean quantities is to access Majorana universality via fluctuations of transport quantities. For example, noise of the electric current offers unique Majorana behavior both in the static limit [16–19] and at finite frequencies [20,21]. In particular, fluctuation universality of Majorana zero modes is revealed in zero-frequency noise via universal effective charges [22] and in finite-frequency quantum noise via universal plateaus, resonances, and antiresonances located at specific frequencies

[23]. Moreover, current shot noise allows one to reveal also the nonlocality of Majorana zero modes [24].

Advanced experiments [25] on nanoscopic systems have set the stage for a very original access to Majorana universality via thermodynamic measurements, as proposed in Ref. [26]. This approach is fundamentally different from quantum transport experiments and allows one to describe the thermodynamics of Kitaev chains, in particular, its Majorana universal fractional entropy [27].

Setups where nonequilibrium is simultaneously induced by a bias voltage  $V$  and a temperature difference  $\Delta T$  uncover even more unique physics of Majorana zero modes which manifests, e.g., in a violation of the Wiedemann-Franz law [28] and in thermoelectric fluctuations having a high degree of universality. Indeed, in nanoscopic Majorana setups not only linear but also nonlinear zero-frequency thermoelectric noise turns out to be universal [29] while finite-frequency thermoelectric quantum noise reveals universal symmetry and dynamic resonances with universal maxima [30]. Recently, it has been proposed that one obtains the entropy of a nanoscopic system from thermoelectric transport experiments [31] which incorporate measurements of the differential conductance of the nanoscopic system and measurements of its thermopower. This shows that thermoelectric transport may also become an effective experimental tool able to detect the Majorana universal fractional entropy [27] of a Kitaev chain.

In this paper we focus on nonequilibrium states of purely thermal nature induced solely by a temperature difference  $\Delta T$  assuming  $V = 0$ . Remarkably, this type of nonequilibrium reveals the existence of Majorana universality, which is classified as neither strictly transport nor strictly thermodynamic, i.e., neither of the only two known in Majorana experiments. We show that combining transport measurements of the electric current in a quantum dot with measurements of the quantum dot compressibility allows one to access such a type of Majorana universality. Although compressibility itself

<sup>\*</sup>sergej.physik@gmail.com; sergej.smirnov@physik.uni-regensburg.de; ssmirnov@sci.lebedev.ru

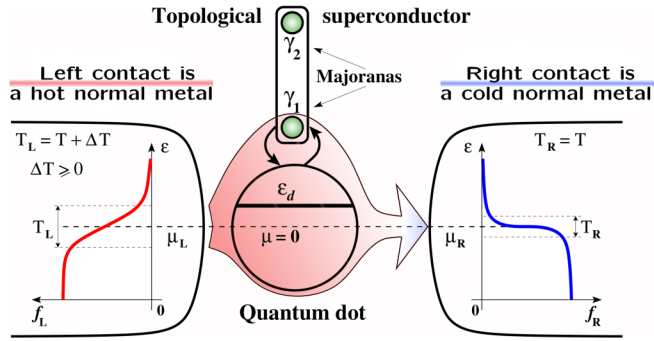


FIG. 1. A basic outline of a nanoscopic physical system where nonequilibrium quantum transport may be implemented experimentally by purely thermal means. The system is composed of a quantum dot interacting via tunneling mechanisms with left ( $L$ ) and right ( $R$ ) contacts as well as with a grounded one-dimensional topological superconductor hosting two Majorana zero modes at its ends, specified as  $\gamma_1$  and  $\gamma_2$  (green circles), of which the first one interacts with the quantum dot (arced arrows). In particular, there is no bias voltage in this system, i.e., the chemical potentials  $\mu_L$  and  $\mu_R$  of the left and right contacts, respectively, coincide,  $\mu_L = \mu_R = \mu = 0$ . The temperatures of the left and right contacts are, respectively,  $T_L = T + \Delta T$  and  $T_R = T$  with the temperature difference  $\Delta T \geq 0$  and the temperature  $T \geq 0$ .

may be used to probe quantum phase transitions leading to the formation of Majorana zero modes [32], it has never been employed in conjunction with low-energy quantum transport governed by non-Abelian Majorana quasiparticles.

The paper is organized as follows: In Sec. II we present a theoretical model of a nanoscopic system where Majorana zero modes are involved in nonequilibrium states of a purely thermal nature. Section III shows that the current induced in such nonequilibrium states does not allow one to access Majorana universal behavior. We demonstrate in Sec. IV that, in order to reveal Majorana universality in purely thermal nonequilibrium, it is necessary to combine the transport response stored in the induced current with the thermodynamic response stored in the quantum dot compressibility. Finally, in Sec. V we conclude the paper.

## II. MAJORANA NANOSCOPIC SETUP IN THERMAL NONEQUILIBRIUM

To reveal a special type of Majorana universality which cannot be accessed via pure transport or pure thermodynamic experiments it is enough to resort to the simple model schematically shown in Fig. 1. It includes a quantum dot with a nondegenerate single-particle energy level  $\epsilon_d$  as measured with reference to the chemical potential  $\mu$ . The left and right normal metals represent the contacts which are linked to the quantum dot. These links provide a quasiparticle exchange between the quantum dot and contacts via quantum-mechanical tunneling. The one-dimensional topological superconductor is grounded and supports a pair of Majorana zero modes at its ends. One of these ends is linked to the quantum dot. This link establishes a special channel for Majorana tunneling between the quantum dot and topological superconductor. We assume that the left and right contacts are described by equilibrium

Fermi-Dirac distributions with the chemical potentials  $\mu_L$ ,  $\mu_R$  and temperatures  $T_L$ ,  $T_R$ ,

$$f_L(\epsilon) = \left[ \exp\left(\frac{\epsilon - \mu_L}{k_B T_L}\right) + 1 \right]^{-1},$$

$$f_R(\epsilon) = \left[ \exp\left(\frac{\epsilon - \mu_R}{k_B T_R}\right) + 1 \right]^{-1}. \quad (1)$$

Since no bias voltage is applied,  $V = 0$ , we have  $\mu_L = \mu_R = \mu = 0$ . Nonequilibrium states arise only due to finite  $\Delta T$  or thermal voltage  $eV_T \equiv k_B \Delta T$ .

For quantitative analysis of the quantum transport in the above system we represent its Hamiltonian as the sum  $\hat{H} = \hat{H}_d + \hat{H}_c + \hat{H}_{ts} + \hat{H}_{d-c} + \hat{H}_{d-ts}$ . The Hamiltonians of the quantum dot, contacts, and topological superconductor are  $\hat{H}_d = \epsilon_d d^\dagger d$ ,  $\hat{H}_c = \sum_{l=L,R} \sum_k \epsilon_k c_{lk}^\dagger c_{lk}$ , and  $\hat{H}_{ts} = i\xi \gamma_2 \gamma_1 / 2$ . The tunneling Hamiltonians,  $\hat{H}_{d-c} = \sum_{l=L,R} \sum_k T_{lk} c_{lk}^\dagger d + \text{H.c.}$  and  $\hat{H}_{d-ts} = \eta^* d^\dagger \gamma_1 + \text{H.c.}$ , describe, respectively, the interactions of the quantum dot with the contacts and with the topological superconductor. The contacts are massive normal metals with a continuous energy spectrum  $\epsilon_k$ . For simplicity, we use the traditional approximation assuming that the contacts density of states  $\nu(\epsilon)$  varies sufficiently weakly over the energies involved in the quantum transport that is in fact energy independent,  $\nu(\epsilon) \approx \nu_c / 2$ . The Majorana operators  $\gamma_{1,2}$  are self-adjoint,  $\gamma_{1,2}^\dagger = \gamma_{1,2}$ , and their anticommutator is  $\{\gamma_i, \gamma_j\} = 2\delta_{ij}$ . The energy  $\xi$  characterizes the overlap of the Majorana modes so that perfectly separated Majoranas correspond to  $\xi = 0$ . Another conventional assumption is that the tunneling between the quantum dot and contacts is independent of the quantum numbers  $l, k$ , that is  $T_{lk} = \mathcal{T}$ . Then the coupling of the quantum dot and the left or right contact is expressed in terms of the quantity  $\Gamma = 2\pi \nu_c |\mathcal{T}|^2$ . The coupling between the quantum dot and topological superconductor is specified by the quantity  $|\eta|$ .

The quasiparticle current, induced by  $\Delta T$ , may be derived by means of the Keldysh field integral [33] written in terms of the Keldysh action  $S_K$  with sources  $J_l(t)$ ,

$$Z[J_l(t)] = \int \mathcal{D}[\theta(t)] e^{i S_K[\theta(t); J_l(t)]},$$

$$S_K = S_{\text{sys}} + S_{\text{scr}}, \quad (2)$$

where  $\{\theta(t)\} = \{\psi(t), \phi_{lk}(t), \zeta(t)\}$  are the Grassmann fields of the quantum dot,  $\psi(t)$ , contacts,  $\phi_{lk}(t)$ , and topological superconductor,  $\zeta(t)$ , defined on the forward ( $q = +$ ) and backward ( $q = -$ ) branches of the Keldysh contour,  $S_{\text{sys}}$  is the system action having the conventional form [23] in the retarded-advanced space, and  $S_{\text{scr}}$  is the source action involving the current operator,

$$S_{\text{scr}}[J_l(t)] = - \int_{-\infty}^{\infty} dt \sum_{l=L,R} \sum_{q=+,-} J_{lq}(t) \hat{I}_{lq}(t),$$

$$\hat{I}_{lq}(t) = \frac{ie}{\hbar} \sum_k [\mathcal{T} \bar{\phi}_{lkq}(t) \psi_q(t) - \mathcal{T}^* \bar{\psi}_q(t) \phi_{lkq}(t)]. \quad (3)$$

The current in contact  $l$  is obtained via the functional derivative over the source field taken at  $J_{lq}(t) = 0$  and

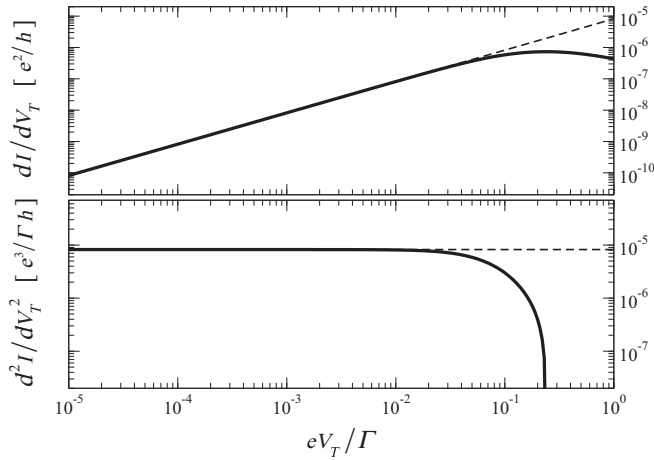


FIG. 2. The first and second derivatives of the electric current with respect to the thermal voltage,  $\partial I(V_T, \epsilon_d, |\eta|)/\partial V_T$ ,  $\partial^2 I(V_T, \epsilon_d, |\eta|)/\partial V_T^2$ , are shown as functions of the thermal voltage  $V_T$ . Here  $\epsilon_d/\Gamma = 10$ ,  $k_B T/\Gamma = 10^{-12}$ ,  $|\eta|/\Gamma = 10^3$ ,  $\xi/\Gamma = 10^{-8}$ .

arbitrary  $q$ ,

$$I_l = \langle \hat{I}_{lq}(t) \rangle = i\hbar \left. \frac{\delta Z[J_l(t)]}{\delta J_{lq}(t)} \right|_{J_{lq}(t)=0}. \quad (4)$$

Below we focus on the quasiparticle current  $I$  in the hot, i.e., left, contact,  $I \equiv I_L$ , as a function of  $V_T$ ,  $\epsilon_d$ , and  $|\eta|$ . From Eq. (4) one finds the following expression [34]:

$$I(V_T, \epsilon_d, |\eta|) = -\frac{e\Gamma}{\hbar^2} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \left\{ \text{Im}[G^R(\epsilon)] f_L(\epsilon) - \frac{i}{2} G^<(\epsilon) \right\}, \quad (5)$$

where the quantum dot retarded (lesser) Green's function  $G^R$  ( $G^<$ ) follows from the inverse kernel of  $S_{\text{sys}}$ .

### III. THERMALLY INDUCED MAJORANA CURRENT

In Fig. 2 we show the results obtained for the first and the second derivatives of the electric current with respect to the thermal voltage. At small values of the thermal voltage  $eV_T$  the first derivative (upper panel) has a linear dependence on  $eV_T$ . As a consequence, at small values of  $eV_T$  the second derivative (lower panel) is independent of the thermal voltage. As one can see, it saturates at a certain value, the coefficient in the linear dependence of the first derivative  $\partial I(V_T, \epsilon_d, |\eta|)/\partial V_T$  on  $eV_T$ . This coefficient is a function of the quantum dot energy level  $\epsilon_d$  (or, equivalently, the chemical potential  $\mu$ ) controlled by a gate voltage as well as a function of the energy  $|\eta|$  characterizing the strength of the Majorana tunneling between the quantum dot and topological superconductor.

Note that, although for the present setup it is not crucial whether  $\epsilon_d > 0$  or  $\epsilon_d < 0$ , we nevertheless prefer to use positive values of  $\epsilon_d$  to disentangle Majorana universality from Kondo universality in more general interacting setups [35,36]. As it is known [37], Kondo universality may arise when  $\epsilon_d < 0$ .

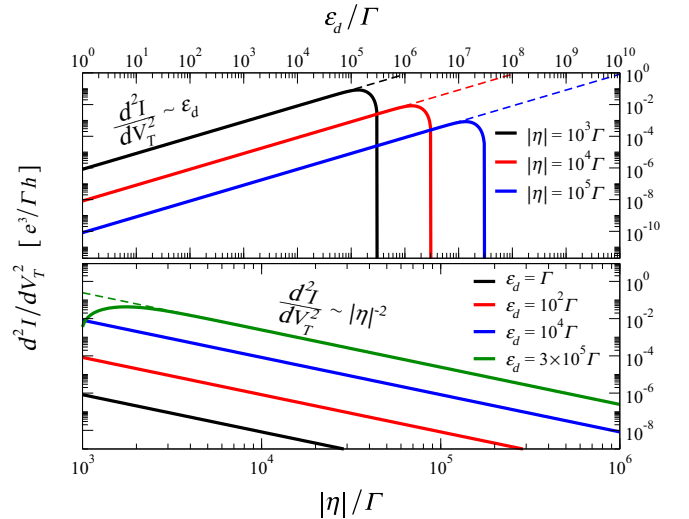


FIG. 3. The second derivative  $\partial^2 I(V_T, \epsilon_d, |\eta|)/\partial V_T^2$  is shown for a small value of the thermal voltage, specifically, for  $eV_T/\Gamma = 10^{-5}$ . The other parameters have the same values as in Fig. 2.

The behavior of the second derivative  $\partial^2 I(V_T, \epsilon_d, |\eta|)/\partial V_T^2$  as a function of  $\epsilon_d$  and  $|\eta|$  is analyzed in Fig. 3 at small  $eV_T$ . In the upper panel,  $\partial^2 I(V_T, \epsilon_d, |\eta|)/\partial V_T^2$  is shown for various values of the energy  $|\eta|$  as a function of the quantum dot energy level  $\epsilon_d$  (or, equivalently, the chemical potential  $\mu$ ) which may be controlled by a gate voltage. The curves exhibit a linear dependence of the second derivative  $\partial^2 I(V_T, \epsilon_d, |\eta|)/\partial V_T^2$  on the energy level  $\epsilon_d$  up to  $\epsilon_d \approx 10^2 |\eta|$ . In the lower panel  $\partial^2 I(V_T, \epsilon_d, |\eta|)/\partial V_T^2$  is shown for various values of  $\epsilon_d$  as a function of the Majorana tunneling strength  $|\eta|$ . The curves exhibit an inverse quadratic dependence of the second derivative  $\partial^2 I(V_T, \epsilon_d, |\eta|)/\partial V_T^2$  on the energy  $|\eta|$  down to  $|\eta| \approx 10^{-2} \epsilon_d$ .

Using the numerical approach of Ref. [29] for obtaining asymptotics, we find for  $\xi \ll eV_T \ll \Gamma$  the following asymptotic limit:

$$\frac{\partial I(V_T, \epsilon_d, |\eta|)}{\partial V_T} = \frac{e^2 \pi^2 \epsilon_d (eV_T)}{h 12 |\eta|^2}. \quad (6)$$

As in Ref. [29], the analytical expression in Eq. (6) is obtained by inspection of numerical results which reproduce Eq. (6) with any desired numerical precision when the corresponding inequalities are satisfied as strong as necessary for that precision.

The parameters which can be varied in an experiment are  $V_T$  and  $\epsilon_d$ . Independence of these parameters is achieved via measurements of the derivative  $\partial^3 I(V_T, \epsilon_d, |\eta|)/\partial \epsilon_d \partial V_T^2 = e^3 \pi^2 / 12 h |\eta|^2$ . The first derivative of the current with respect to the thermal voltage may be interpreted as a special kind of conductance which measures the current sensitivity to the temperature inhomogeneity  $\Delta T$ . One can call it thermoelectric conductance. The second derivative of the current with respect to the thermal voltage is therefore the first derivative of the thermoelectric conductance. It shows how the thermoelectric conductance of the quantum dot is enhanced or suppressed by temperature inhomogeneities of the external environment whose role is played here by the massive metallic contacts.

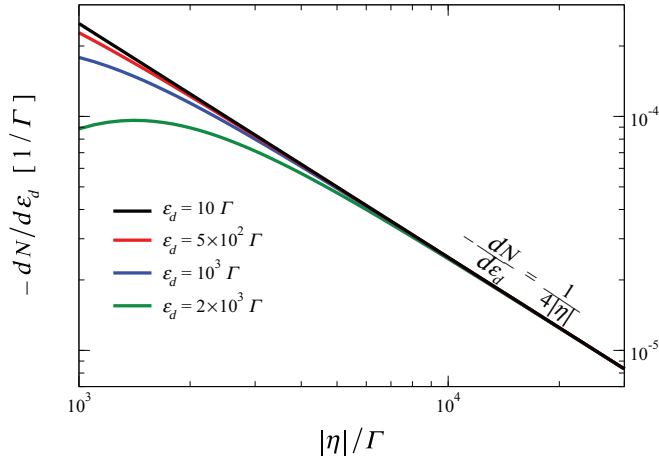


FIG. 4. The quantum dot compressibility, which is the first derivative of the quantum dot particle number with respect to the chemical potential (or, equivalently, the quantum dot energy level),  $\partial N(V_T, \epsilon_d, |\eta|)/\partial \mu = -\partial N(V_T, \epsilon_d, |\eta|)/\partial \epsilon_d$ . For all the curves the thermal voltage is small,  $eV_T/\Gamma = 10^{-5}$ . The other parameters have the same values as in Fig. 2.

In contrast, the third derivative of the current with respect to the chemical potential probes internal sensitivity of the tunneling density of states of the quantum dot when the energy level  $\epsilon_d$  is varied by a gate voltage. Therefore, the derivative  $\partial^3 I(V_T, \epsilon_d, |\eta|)/\partial \epsilon_d \partial V_T^2$  is a very comprehensive physical quantity: it measures the transport response to the temperature inhomogeneity of the external environment and at the same time it provides the internal spectral response of the system.

However, the dependence of this derivative on the parameter  $|\eta|$  still remains. This parameter is not directly controlled in an experiment and will have in general different values for different experimental setups. As a result, the strictly transport response does not allow one to obtain universal properties of the Majorana zero modes in purely thermal nonequilibrium states induced by  $\Delta T$  at  $V = 0$ .

#### IV. MAJORANA UNIVERSALITY VIA QUANTUM DOT COMPRESSIBILITY

Nevertheless, it is possible to uncover the Majorana universality encoded in this purely thermal nonequilibrium if in addition one turns attention to thermodynamic properties, namely, to the quantum dot compressibility defined as  $\partial N/\partial \mu$  or, equivalently,  $-\partial N/\partial \epsilon_d$ , where  $N$  is the quantum dot particle number,

$$N(V_T, \epsilon_d, |\eta|) = - \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi i\hbar} G^<(\epsilon). \quad (7)$$

In Fig. 4 the quantum dot compressibility is shown as a function of the energy  $|\eta|$  characterizing the strength of the Majorana tunneling between the quantum dot and topological superconductor. Various curves correspond to various values of the quantum dot energy level  $\epsilon_d$  (or, equivalently, the chemical potential  $\mu$ ) controlled by a gate voltage. The curves demonstrate that, down to  $|\eta| \approx 3\epsilon_d$ , the compressibility has an inverse dependence on the energy  $|\eta|$ . More exactly, similar to Eq. (6), we find for  $\xi \ll eV_T \ll \Gamma$ , the following asymp-

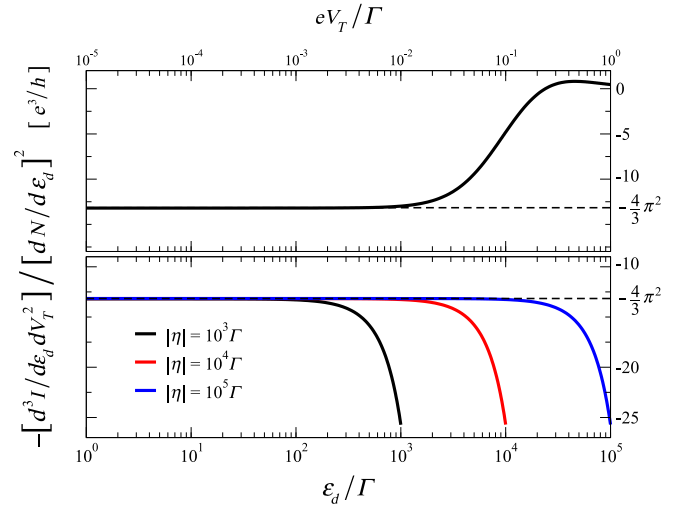


FIG. 5. The ratio between the derivative of the electric current,  $\partial^3 I/\partial \mu \partial V_T^2 = -\partial^3 I/\partial \epsilon_d \partial V_T^2$ , and the square of the quantum dot compressibility,  $[\partial N/\partial \mu]^2 = [\partial N/\partial \epsilon_d]^2$ . Upper panel shows the ratio as a function of the thermal voltage  $V_T$ . Lower panel shows the ratio as a function of the quantum dot energy level  $\epsilon_d$  for various values of the Majorana tunneling strength  $|\eta|$ .

otic limit:

$$-\frac{\partial N(V_T, \epsilon_d, |\eta|)}{\partial \epsilon_d} = \frac{1}{4|\eta|}. \quad (8)$$

From Eqs. (6) and (8) there follows the dual (that is, having both transport and thermodynamic nature) universal ratio:

$$-\frac{\partial^3 I(V_T, \epsilon_d, |\eta|)/\partial \epsilon_d \partial V_T^2}{[\partial N(V_T, \epsilon_d, |\eta|)/\partial \epsilon_d]^2} = -\frac{4\pi^2 e^3}{3 h}. \quad (9)$$

In the upper panel of Fig. 5 this ratio is shown as a function of the thermal voltage  $eV_T$  with all the other parameters having the same values as in Fig. 2. The curve demonstrates that the ratio does not depend on the thermal voltage when  $eV_T/\Gamma \lesssim 10^{-2}$ . In this regime the ratio reaches the universal Majorana value  $-4\pi^2 e^3/3h$ . In the lower panel of Fig. 5 this ratio is shown for a small value of the thermal voltage, specifically, for  $eV_T/\Gamma = 10^{-5}$ , as a function of the quantum dot energy level  $\epsilon_d$  which may be controlled by a gate voltage. The other parameters have the same values as in Fig. 2. The curves demonstrate that, as soon as  $\epsilon_d \lesssim 10^{-1}|\eta|$ , the ratio does not depend on both the quantum dot energy level  $\epsilon_d$  and the energy  $|\eta|$  which characterizes the strength of the Majorana tunneling. In this regime the ratio reaches the universal Majorana value  $-4\pi^2 e^3/3h$ .

When the two Majorana modes start to significantly overlap, they form a partially separated Andreev bound state. In many respects this situation may be adequately analyzed by using the energy  $\xi$  while, for a more complete analysis, a measure for spatial separation [38] of the two Majorana modes could be introduced. Here for simplicity we focus on purely energetic arguments. Thus, when  $\xi$  is large enough, our model describes a quantum dot coupled to one end of a topological superconductor supporting a partially separated Andreev bound state localized at that end [39]. Below we assume weak overlaps of the Majorana zero modes. This means that,

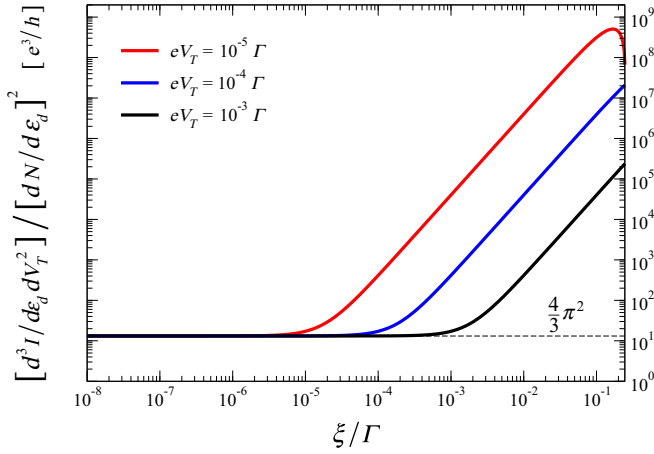


FIG. 6. The ratio between the derivative of the electric current  $-\partial^3 I / \partial \mu \partial V_T^2 = \partial^3 I / \partial \epsilon_d \partial V_T^2$  and the square of the quantum dot compressibility  $[\partial N / \partial \mu]^2 = [\partial N / \partial \epsilon_d]^2$ . Because of the logarithmic scale we invert the ratio's sign as compared with Fig. 5. The ratio is shown as a function of the Majorana overlap energy  $\xi$  for various values of the thermal voltage  $V_T$ . The other parameters have the same values as in Fig. 2.

in the partially separated Andreev bound state, composed of  $\gamma_1$  and  $\gamma_2$ , the second Majorana mode  $\gamma_2$  is still far enough so that the quantum dot does not directly couple to  $\gamma_2$ . A more complicated analysis of a model with additional direct coupling between the quantum dot and  $\gamma_2$  will be performed in another work. In pure electric transport ( $\Delta T = 0$ ) the differential conductance ( $G = \partial I / \partial V$ ) may behave similarly [40] for both Majorana zero modes and partially separated Andreev bound states. It is thus reasonable to explore what happens with the ratio  $[\partial^3 I / \partial \mu \partial V_T^2] / [\partial N / \partial \mu]^2$  when  $\xi$  is large enough. In the present context it is clear physically that large  $\xi$  means  $\xi > eV_T$ . In this case the coupling between the two Majorana zero modes is so strong that the second Majorana mode  $\gamma_2$  quickly adjusts to the dynamics of the first Majorana mode  $\gamma_1$  directly reacting to variations of  $V_T$ . So that in response to the thermal voltage  $V_T$  the two Majoranas behave together as a single state which represents a partially separated Andreev bound state. In contrast, if  $\xi < eV_T$ , the second Majorana mode  $\gamma_2$  does not follow  $\gamma_1$  involved in the dynamics induced by variations of the thermal voltage  $V_T$  and the nonequilibrium states emerging in this case are of unique Majorana nature. This is, indeed, what we see in Fig. 6, which shows the ratio  $[\partial^3 I / \partial \mu \partial V_T^2] / [\partial N / \partial \mu]^2$  as a function of the Majorana overlap energy  $\xi$  for three different values of the thermal voltage  $eV_T$ . As one can see, when  $\xi < eV_T$  and nonequilibrium states result from fully unpaired Majorana zero modes, the ratio is independent of  $\xi$  and is equal to the universal Majorana value,  $[\partial^3 I / \partial \mu \partial V_T^2] / [\partial N / \partial \mu]^2 = -4\pi^2 e^3 / 3h$ . However, when  $\xi \approx eV_T$ , the ratio starts to deviate from the universal Majorana plateau  $-4\pi^2 e^3 / 3h$ . At this point both Majorana modes  $\gamma_1$  and  $\gamma_2$  start to feel the variations of the thermal voltage  $V_T$  and respond to these variations together as a partially separated Andreev bound state. As a result,  $[\partial^3 I / \partial \mu \partial V_T^2] / [\partial N / \partial \mu]^2$  significantly deviates from the universal Majorana value  $-4\pi^2 e^3 / 3h$  when  $\xi$  is further increased, as can be seen in Fig. 6 for  $\xi > eV_T$ .

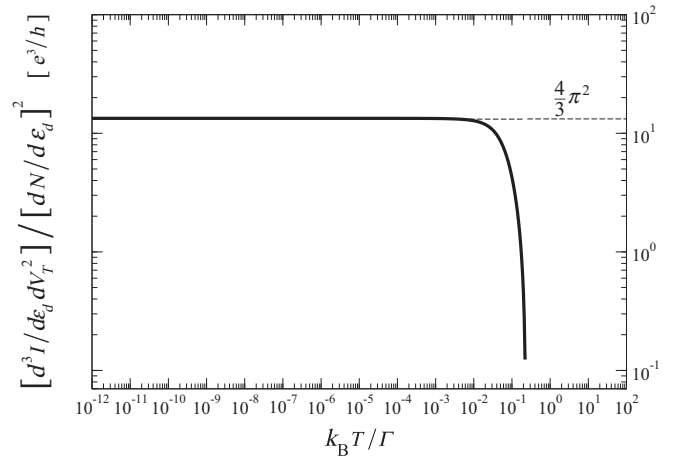


FIG. 7. The ratio between the derivative of the electric current  $-\partial^3 I / \partial \mu \partial V_T^2 = \partial^3 I / \partial \epsilon_d \partial V_T^2$  and the square of the quantum dot compressibility  $[\partial N / \partial \mu]^2 = [\partial N / \partial \epsilon_d]^2$ . As in Fig. 6, because of the logarithmic scale we invert the ratio's sign as compared with Fig. 5. The ratio is shown as a function of the temperature  $T$ . The thermal voltage is relatively large,  $eV_T / \Gamma = 10^{-2}$ . Here  $\epsilon_d / \Gamma = 0.5$ ,  $|\eta| / \Gamma = 60$ , and  $\xi = 10^{-8}$ .

The above analysis has been done for almost zero temperature,  $k_B T / \Gamma = 10^{-12}$ . Nevertheless, below we demonstrate that the dual Majorana universality analyzed above is robust against high temperatures and high thermal voltages. In Fig. 7 the ratio  $[\partial^3 I / \partial \mu \partial V_T^2] / [\partial N / \partial \mu]^2$  is shown as a function of the temperature  $k_B T$  for the thermal voltage  $eV_T / \Gamma = 10^{-2}$ . The Majorana overlap energy here is small,  $\xi / \Gamma = 10^{-8}$ , so that  $\xi \ll eV_T$  and, as discussed above, nonequilibrium states clearly reveal unique Majorana physics. From the curve shown in Fig. 7 one can see that the ratio has the universal Majorana value  $-4\pi^2 e^3 / 3h$  for temperatures  $k_B T \lesssim eV_T$ . For higher temperatures,  $k_B T > eV_T$ , the ratio strongly deviates from the universal Majorana plateau. This implies that, for very high temperatures, the Majorana zero modes are no longer effective in formation of nonequilibrium states of purely thermal nature. However, the temperature  $k_B T / \Gamma = 10^{-2}$  should be already high enough to be reached in modern experiments. Indeed, the largest energy scale in Fig. 7 is given by the energy  $|\eta|$ . This means that this energy should not exceed the induced superconducting gap  $\Delta$  and, for example, one can take  $|\eta| \sim \Delta$ . Experiments in Ref. [41] demonstrate that sufficiently high values, such as  $\Delta \approx 15$  meV, have already been achieved and may soon become regular for generating Majorana zero modes. Since in Fig. 7 for  $k_B T / \Gamma = 10^{-2}$  we have  $k_B T = 1.7 \times 10^{-4} |\eta| = 1.7 \times 10^{-4} \Delta$ , the temperature at which one observes the universal Majorana value  $[\partial^3 I / \partial \mu \partial V_T^2] / [\partial N / \partial \mu]^2 = -4\pi^2 e^3 / 3h$  is estimated to be  $T \approx 30$  mK in SI units. Such temperatures are already high enough to be reached in modern experiments.

## V. CONCLUSION

In conclusion, we have revealed Majorana universality originating simultaneously from both transport and thermodynamic properties emerging in nonequilibrium states of purely thermal nature. Combining two essentially different physical

properties of quantum dots, the current response and compressibility, one may observe their universal correlation as highly unique Majorana physics inaccessible within strictly transport or thermodynamic experiments. Although this special kind of Majorana universality requires measurements of both the current and compressibility, it is exactly this combination which makes it much more unique than measurements of only the current aiming to obtain the differential conductance. Importantly, since such measurements involve the mean current, they are much simpler than measurements of the current noise. Another possible benefit is that compressibility measurements [42] involving single-electron transistors indicate that this physical quantity may soon be accessed also in

quantum dot experiments and one expects that the complexity of such experiments will not be higher than the one related to the thermodynamic measurements [26] of the universal Majorana entropy [27]. The above advantages and practical accessibility of the universal range of the model parameters demonstrate that the previously unknown Majorana universality predicted here is a feasible goal for state-of-the-art experiments on nanoscopic Majorana setups.

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- [1] V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. A. M. Bakkers, and L. P. Kouwenhoven, Signatures of Majorana fermions in hybrid superconductor-semiconductor nanowire devices, *Science* **336**, 1003 (2012).
- [2] S. M. Albrecht, A. P. Higginbotham, M. Madsen, F. Kuemmeth, T. S. Jespersen, J. Nygård, P. Krogstrup, and C. M. Marcus, Exponential protection of zero modes in Majorana islands, *Nature (London)* **531**, 206 (2016).
- [3] H. Zhang, C.-X. Liu, S. Gazibegovic, D. Xu, J. A. Logan, G. Wang, N. van Loo, J. D. S. Bommer, M. W. A. de Moor, D. Car, R. L. M. O. het Veld, P. J. van Veldhoven, S. Koelling, M. A. Verheijen, M. Pendharkar, D. J. Pennachio, B. Shojaei, J. S. Lee, C. J. Palmstrøm, E. P. A. M. Bakkers, S. D. Sarma, and L. P. Kouwenhoven, Quantized Majorana conductance, *Nature (London)* **556**, 74 (2018).
- [4] E. Majorana, Teoria simmetrica dell'elettrone e del positrone, *Nuovo Cimento* **14**, 171 (1937).
- [5] C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, St. Louis, San Francisco, 1980).
- [6] A. Yu. Kitaev, Unpaired Majorana fermions in quantum wires, *Phys. Usp.* **44**, 131 (2001).
- [7] J. Alicea, New directions in the pursuit of Majorana fermions in solid state systems, *Rep. Prog. Phys.* **75**, 076501 (2012).
- [8] M. Leijnse and K. Flensberg, Introduction to topological superconductivity and Majorana fermions, *Semicond. Sci. Technol.* **27**, 124003 (2012).
- [9] M. Sato and S. Fujimoto, Majorana fermions and topology in superconductors, *J. Phys. Soc. Jpn.* **85**, 072001 (2016).
- [10] R. Aguado, Majorana quasiparticles in condensed matter, *Nuovo Cimento* **40**, 523 (2017).
- [11] R. M. Lutchyn, E. P. A. M. Bakkers, L. P. Kouwenhoven, P. Krogstrup, C. M. Marcus, and Y. Oreg, Majorana zero modes in superconductor-semiconductor heterostructures, *Nat. Rev. Mater.* **3**, 52 (2018).
- [12] L. Fu and C. L. Kane, Superconducting Proximity Effect and Majorana Fermions at the Surface of a Topological Insulator, *Phys. Rev. Lett.* **100**, 096407 (2008).
- [13] L. Fu and C. L. Kane, Josephson current and noise at a superconductor/quantum-spin-Hall-insulator/superconductor junction, *Phys. Rev. B* **79**, 161408(R) (2009).
- [14] R. M. Lutchyn, J. D. Sau, and S. Das Sarma, Majorana Fermions and a Topological Phase Transition in Semiconductor-Superconductor Heterostructures, *Phys. Rev. Lett.* **105**, 077001 (2010).
- [15] Y. Oreg, G. Refael, and F. von Oppen, Helical Liquids and Majorana Bound States in Quantum Wires, *Phys. Rev. Lett.* **105**, 177002 (2010).
- [16] D. E. Liu, M. Cheng, and R. M. Lutchyn, Probing Majorana physics in quantum-dot shot-noise experiments, *Phys. Rev. B* **91**, 081405(R) (2015).
- [17] D. E. Liu, A. Levchenko, and R. M. Lutchyn, Majorana zero modes choose Euler numbers as revealed by full counting statistics, *Phys. Rev. B* **92**, 205422 (2015).
- [18] C. W. J. Beenakker, Random-matrix theory of Majorana fermions and topological superconductors, *Rev. Mod. Phys.* **87**, 1037 (2015).
- [19] A. Haim, E. Berg, F. von Oppen, and Y. Oreg, Current correlations in a Majorana beam splitter, *Phys. Rev. B* **92**, 245112 (2015).
- [20] S. Valentini, M. Governale, R. Fazio, and F. Taddei, Finite-frequency noise in a topological superconducting wire, *Phys. E (Amsterdam, Neth.)* **75**, 15 (2016).
- [21] D. Bathellier, L. Raymond, T. Jonckheere, J. Rech, A. Zazunov, and T. Martin, Finite frequency noise in a normal metal-topological superconductor junction, *Phys. Rev. B* **99**, 104502 (2019).
- [22] S. Smirnov, Non-equilibrium Majorana fluctuations, *New J. Phys.* **19**, 063020 (2017).
- [23] S. Smirnov, Majorana finite-frequency nonequilibrium quantum noise, *Phys. Rev. B* **99**, 165427 (2019).
- [24] J. Manousakis, C. Wille, A. Altland, R. Egger, K. Flensberg, and F. Hassler, Weak Measurement Protocols for Majorana Bound State Identification, *Phys. Rev. Lett.* **124**, 096801 (2020).
- [25] N. Hartman, C. Olsen, S. Lüscher, M. Samani, S. Fallahi, G. C. Gardner, M. Manfra, and J. Folk, Direct entropy measurement in a mesoscopic quantum system, *Nat. Phys.* **14**, 1083 (2018).
- [26] E. Sela, Y. Oreg, S. Plugge, N. Hartman, S. Lüscher, and J. Folk, Detecting the Universal Fractional Entropy of Majorana Zero Modes, *Phys. Rev. Lett.* **123**, 147702 (2019).
- [27] S. Smirnov, Majorana tunneling entropy, *Phys. Rev. B* **92**, 195312 (2015).
- [28] J. P. Ramos-Andrade, O. Ávalos-Ovando, P. A. Orellana, and S. E. Ulloa, Thermoelectric transport through Majorana bound

- states and violation of Wiedemann-Franz law, *Phys. Rev. B* **94**, 155436 (2016).
- [29] S. Smirnov, Universal Majorana thermoelectric noise, *Phys. Rev. B* **97**, 165434 (2018).
- [30] S. Smirnov, Dynamic Majorana resonances and universal symmetry of nonequilibrium thermoelectric quantum noise, *Phys. Rev. B* **100**, 245410 (2019).
- [31] Y. Kleeorin, H. Thierschmann, H. Buhmann, A. Georges, L. W. Molenkamp, and Y. Meir, How to measure the entropy of a mesoscopic system via thermoelectric transport, *Nat. Commun.* **10**, 5801 (2019).
- [32] D. Nozadze and N. Trivedi, Compressibility as a probe of quantum phase transitions in topological superconductors, *Phys. Rev. B* **93**, 064512 (2016).
- [33] A. Altland and B. Simons, *Condensed Matter Field Theory*, 2nd ed. (Cambridge University Press, Cambridge, 2010).
- [34] Y. Meir and N. S. Wingreen, Landauer Formula for the Current Through an Interacting Electron Region, *Phys. Rev. Lett.* **68**, 2512 (1992).
- [35] J. F. Silva, L. G. G. V. D. da Silva, and E. Vernek, Robustness of the Kondo effect in a quantum dot coupled to Majorana zero modes, *Phys. Rev. B* **101**, 075428 (2020).
- [36] I. Weymann, K. P. Wójcik, and P. Majek, Majorana-Kondo interplay in T-shaped double quantum dots, [arXiv:2001.11280v1](https://arxiv.org/abs/2001.11280v1).
- [37] A. C. Hewson, *The Kondo Problem to Heavy Fermions* (Cambridge University Press, Cambridge, 1997).
- [38] M.-T. Deng, S. Vaitiekėnas, E. Prada, P. San-Jose, J. Nygård, P. Krogstrup, R. Aguado, and C. M. Marcus, Nonlocality of Majorana modes in hybrid nanowires, *Phys. Rev. B* **98**, 085125 (2018).
- [39] M. Hell, K. Flensberg, and M. Leijnse, Distinguishing Majorana bound states from localized Andreev bound states by interferometry, *Phys. Rev. B* **97**, 161401(R) (2018).
- [40] C. Moore, T. D. Stanescu, and S. Tewari, Two-terminal charge tunneling: Disentangling Majorana zero modes from partially separated Andreev bound states in semiconductor-superconductor heterostructures, *Phys. Rev. B* **97**, 165302 (2018).
- [41] E. Wang, H. Ding, A. V. Fedorov, W. Yao, Z. Li, Y.-F. Lv, K. Zhao, L.-G. Zhang, Z. Xu, J. Schneeloch, R. Zhong, S.-H. Ji, L. Wang, K. He, X. Ma, G. Gu, H. Yao, Q.-K. Xue, X. Chen, and S. Zhou, Fully gapped topological surface states in Bi<sub>2</sub>Se<sub>3</sub> films induced by a *d*-wave high-temperature superconductor, *Nat. Phys.* **9**, 621 (2013).
- [42] J. Martin, N. Akerman, G. Ulbricht, T. Lohmann, J. H. Smet, K. von Klitzing, and A. Yacoby, Observation of electron-hole puddles in graphene using a scanning single-electron transistor, *Nat. Phys.* **4**, 144 (2008).