Rapid Communications

Collective spinon spin wave in a magnetized U(1) spin liquid

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We study the transverse dynamical spin susceptibility of the two-dimensional U(1) spinon Fermi-surface spin liquid in a small applied Zeeman field. We show that both short-range interactions, present in a generic Fermi liquid, as well as gauge fluctuations, characteristic of the U(1) spin liquid, qualitatively change the result based on the frequently assumed noninteracting spinon approximation. The short-range part of the interaction lead to a collective "spinon spin wave" mode, which splits off from the two-spinon continuum at a small momentum and disperses downward. Gauge fluctuations renormalize the susceptibility, providing nonzero power-law weight in the region outside the spinon continuum and giving the spin wave a finite lifetime, which scales as momentum squared. We also study the effect of Dzyaloshinskii-Moriya anisotropy on the zero momentum susceptibility, which is measured in electron spin resonance (ESR), and obtain a resonance linewidth linear in temperature and varying as $B^{2/3}$ with magnetic field B at low temperatures. Our results form the basis for a theory of inelastic neutron scattering, ESR, and resonant inelastic x-ray scattering studies of this quantum spin-liquid state.

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The search for the enigmatic spin-liquid state has switched into high gear in recent years. Dramatic theoretical (the Kitaev model [1,2] and a spin liquid in a triangular lattice antiferromagnet [3]) and experimental ($YbMgGaO_4$ [4,5] and α-$RuCl_3$ [6]) developments leave no doubt as to the eventual success of this enterprise. To push this to the next stage, it is incumbent upon the community to identify specific experimental signatures that evince the unique aspects of these states. In this Rapid Communication, we focus on the two-dimensional U(1) quantum spin liquid (QSL) with a spinon Fermi surface. This is *a priori* the most exotic two-dimensional QSL state, and yet one which has repeatedly been advocated for in both theory [7–11] and experiment [5,12–14]. Specifically, we study the dynamical susceptibility of the $\mathbf{q}$ component of the spin operator $S^a_{\mathbf{q}}$ $(a = x, y, z)$,

$$X_{\pm}(\boldsymbol{q},\omega) = -i\int_0^{\infty} dt\,\langle[S^+_{\mathbf{q}}(t), S^-_{-\mathbf{q}}(0)]\rangle e^{i\omega t}, \tag{1}$$

which is an extremely information-rich quantity, and is accessible through inelastic neutron scattering [15], electron spin resonance (ESR) [16,17], and resonant inelastic x-ray scattering (RIXS) [18]. The fractionalization of triplet excitations into pairs of spinons is a fundamental aspect of a QSL, and is expected to manifest in $X_{\pm}$ as a two-particle continuum spectral weight [2,19,20], a surprising feature which appears more characteristic of a weakly correlated metal than a strongly correlated Mott insulator. In a mean-field treatment in which the spinons are approximated as noninteracting fermions, this continuum has a characteristic shape at small frequency and wave vector in the presence of an applied Zeeman magnetic field, as discussed in Ref. [21]. In particular, there is nonzero spectral weight in a wedge-shaped region which terminates at a single point along the energy axis at zero momentum. Our analysis reveals the full structure in this regime beyond the mean-field approximation. Notably, we find that interactions between spinons *qualitatively* modify the result from the mean-field form, introducing another collective mode—a *spinon spin wave*—and modifying the spectral weight significantly.

We recapitulate the derivation of the theory of the spinon Fermi-surface phase [22,23]. One introduces Abrikosov fermions by rewriting the spin operator $\boldsymbol{S}_i = \frac{1}{2}c^{\dagger}_{i\alpha}\boldsymbol{\sigma}_{\alpha\beta}c_{i\beta}$, where $c_{i\alpha}, c^{\dagger}_{i\alpha}$ are canonical fermionic spinors on site i with a spin-1/2 index α (repeated spin indices are summed). This is a faithful representation provided the constraint $c^{\dagger}_{i\alpha}c_{i\alpha} = 1$ is imposed—this constraint induces a gauge symmetry. In a path integral representation, the constraint is enforced by a Lagrange multiplier A_{i0}, which takes the role of the time component of a gauge field, i.e., scalar potential. Microscopic exchange interactions, which are quadratic in spins, and are therefore quartic in fermions, are decoupled to introduce other link fields whose phases act as the spatial components of the corresponding gauge fields $\mathbf{A}$, i.e., the vector potential.

To describe the universal low-energy physics, it is appropriate to consider "coarse-grained" fields $\psi_\alpha, \psi^{\dagger}_\alpha$ descending from the microscopic ones, and include the symmetry-allowed Maxwell terms for the U(1) gauge field. Furthermore, due to the finite density of states at the spinon Fermi surface, the longitudinal scalar potential is screened and the time component A_0 can then be integrated out to mediate a short-range repulsive interaction u between like charges. Therefore we consider the Euclidean action $S = S_\psi + S_A + S_u$, where [22–24]

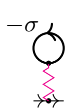
$$\begin{aligned} S_\psi &= \int d^3x\,\psi^{\dagger}\left(\partial_\tau - \mu - \frac{1}{2m}(\nabla_{\mathbf{r}} - i\mathbf{A})^2 - \omega_B\sigma^3\right)\psi,\\ S_A &= \int \frac{d^3q}{(2\pi)^3}\frac{1}{2}(\gamma|\omega_n|/q + \chi q^2)|A(q)|^2,\\ S_u &= \int d^3x\,u\,\psi^{\dagger}_{\uparrow}\psi_{\uparrow}\psi^{\dagger}_{\downarrow}\psi_{\downarrow}. \end{aligned} \tag{2}$$

Here, $x = (\tau, \boldsymbol{x})$ is the space-time coordinate, $q = (\omega_n, \boldsymbol{q})$ is the three-momentum, ψ_α is a two-component spinor, with spin indices $\alpha, \beta = \uparrow, \downarrow$ that are suppressed when possible, and ω_B describes static magnetic field $\mathbf{B} = B\hat{z}$ and includes the g-factor as well as the Bohr magneton. The gauge dynamics is derived in the Coulomb gauge $\nabla \cdot \boldsymbol{A} = 0$ with $\boldsymbol{A}(q) = i\hat{z} \times \hat{\boldsymbol{q}} A(q)$. The gauge action S_A is generated by spinons and $\gamma = 2\bar{n}/k_F$ and $\chi = 1/(24\pi m)$ represent Landau damping and the diamagnetic susceptibility of noninteracting spinon gas, correspondingly (m is the spinon mass, $\bar{n}$ is the spinon density, and k_F is the Fermi momentum of a nonmagnetized system).

We proceed with the assumption of SU(2) symmetry, a good first approximation for many spin-liquid materials, and address the effect of its violations in the latter part of this Rapid Communication. Previous investigations focused on the transverse vector potential $\mathbf{A}$, which is not screened but Landau damped, and hence induces exotic non-Fermi-liquid physics. For example, one finds a self-energy varying with frequency as $\omega^{2/3}$, and a singular contribution to the specific heat $c_v \sim T^{2/3}$ [22,23]. However, notably, the transverse gauge field has negligible effects on the hydrodynamic long-wavelength collective response [24]. Here, we instead focus on the short-range repulsion u, which produces an exchange field that dramatically alters the behavior in the presence of an external Zeeman magnetic field giving rise to finite magnetization. Gauge fluctuations play a subsidiary role which we also include.

An important constraint follows purely from symmetry. Provided the Hamiltonian in zero magnetic field has SU(2) symmetry, a Zeeman magnetic field leads to a fully determined structure factor at zero momentum. Specifically, the Larmor/Kohn theorem [25] dictates that the only response at $\mathbf{q} = 0$, $X''_{\pm} = -2M\delta(\omega - 2\omega_B)$, where $M = (\bar{n}_\uparrow - \bar{n}_\downarrow)/2$ is the magnetization and ω_B is the spinon Zeeman energy. For free fermions, the delta function is precisely at the corner of the *spinon* particle-hole continuum (also known as the two-spinon continuum). However, the contact exchange interaction shifts up the particle-hole continuum, at small momentum $\mathbf{q}$, away from the Zeeman energy $2\omega_B$ to $2\omega_B + 2uM$. This is seen by the trivial Hartree self-energy

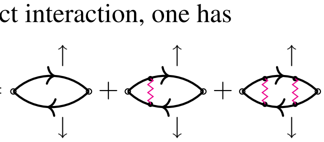
$$\Sigma_\sigma = \quad = u\bar{n}_{-\sigma} = -uM\sigma + u\bar{n}/2, \tag{3}$$



where we use a zigzag line to diagrammatically represent the local u interaction, $\sigma = \uparrow = 1$ and $\sigma = \downarrow = -1$, and $\bar{n}_\sigma$ is the expectation value of spin-σ spinon density in the presence of a magnetic field. Consequently, for the Larmor theorem to be obeyed, there *must be a collective transverse spin mode* at small momenta.

This collective spin mode is most conveniently described by the random phase approximation (RPA), which corresponds to a standard resummation of particle-hole ladder diagrams [26]. For the particular case of a momentum-independent contact interaction, one has

$$X_\pm(\boldsymbol{q}, i\omega_n) = \quad + \quad + \quad + \cdots$$



$$= \frac{\chi_\pm(\boldsymbol{q}, i\omega_n)}{1 + u\chi_\pm(\boldsymbol{q}, i\omega_n)}, \tag{4}$$

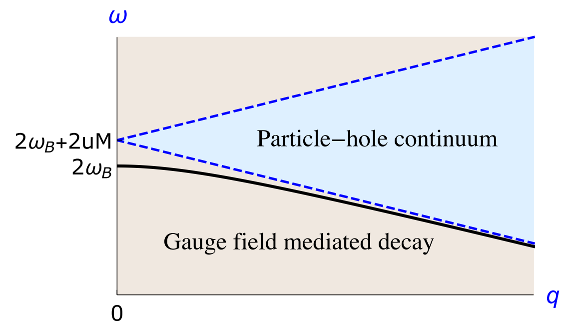


FIG. 1. Magnetic excitation spectrum of an interacting U(1) spin liquid with a spinon Fermi surface. The wedge-shaped (blue) region, bounded by dashed lines, denotes a spinon particle-hole continuum inside which $\mathrm{Im}\, \chi^0_\pm(\boldsymbol{q}, \omega) \neq 0$. The collective spinon spin wave, which is promoted by a short-ranged repulsive interaction u, is shown by the bold black line. The linewidth of this transverse spin wave is determined by the gauge fluctuations, which produce finite $\mathrm{Im}\, \chi^1_\pm(\boldsymbol{q}, \omega) \neq 0$ outside the noninteracting spinon continuum [see (11)].

where the fermion lines correspond to the spinon Green's functions *including the Hartree shift* (3), and in this approximation $\chi_\pm(\boldsymbol{q}, i\omega_n) = \chi^0_\pm(\boldsymbol{q}, i\omega_n)$ is the bare susceptibility bubble, calculated using these functions. We will, however, use the second line in Eq. (4) to later define the RPA approximation even when gauge field corrections (but not the local interaction u) are included in $\chi_\pm$. For the moment, we simply evaluate the bare susceptibility,

$$\chi^0_\pm(\boldsymbol{q}, i\omega_n) = \frac{1}{\beta V} \sum_{k_n, \boldsymbol{k}} \frac{1}{ik_n - \epsilon_{\boldsymbol{k}} + \omega_B - u\bar{n}_\downarrow} \times \frac{1}{ik_n + i\omega_n - \epsilon_{\boldsymbol{k}+\boldsymbol{q}} - \omega_B - u\bar{n}_\uparrow}. \tag{5}$$

Here, ω_n, k_n are bosonic and fermionic Matsubara frequencies, respectively. A simple calculation, followed by analytical continuation $i\omega_n \to \omega + i0$, gives

$$\mathrm{Re}\, \chi^0_\pm(\boldsymbol{q}, \omega) = \frac{2M\, \mathrm{sgn}(\omega - 2\omega_B - 2uM)}{\sqrt{(\omega - 2\omega_B - 2uM)^2 - v_F^2 q^2}},$$

$$\mathrm{Im}\, \chi^0_\pm(\boldsymbol{q}, \omega) = \frac{-2M}{\sqrt{v_F^2 q^2 - (\omega - 2\omega_B - 2uM)^2}}, \tag{6}$$

where square roots are defined when their arguments are positive. The real/imaginary spin susceptibility describes domains outside/inside two-spinon continuum in the (q, ω) plane (Fig. 1), correspondingly. At $\boldsymbol{q} = 0$,

$$\chi^0_\pm(\boldsymbol{q} = 0, \omega) = \frac{2M}{\omega - 2\omega_B - 2uM + i0}, \tag{7}$$

and therefore $\mathrm{Im}\ \chi^0_\pm(\boldsymbol{q} = 0, \omega) \sim \delta(\omega - 2\omega_B - 2uM)$: The position of the two-spinon continuum is renormalized by the interaction shift. However, inserting (7) in the RPA formula (4), one finds that the RPA successfully recovers the Larmor theorem at zero momentum for the interacting SU(2)-invariant system,

$$X_\pm(\boldsymbol{q} = 0, \omega) = \frac{2M}{\omega - 2\omega_B + i0}. \tag{8}$$

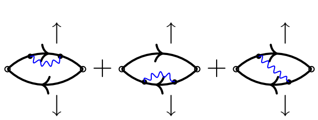
Therefore the contribution at $\boldsymbol{q} = 0$ is *solely* from the collective mode, with no spectral weight from the continuum at $2\omega_B + 2uM$. Dispersion of the collective spin mode is obtained with the help of (6) and $\mathrm{Im}\, X_\pm = \mathrm{Im}\, \chi^0_\pm/[(1 + u\, \mathrm{Re}\, \chi^0_\pm)^2 + (u\, \mathrm{Im}\, \chi^0_\pm)^2]$,

$$\omega_{\mathrm{swave}}(\mathbf{q}) = 2\omega_B + 2uM - \sqrt{4u^2M^2 + v_F^2 q^2}. \tag{9}$$

For small $q \ll uM/v_F$ the collective mode is dispersing downward quadratically $\omega \approx 2\omega_B - (v_F q)^2/(4uM)$, while in the opposite limit $q \gg uM/v_F$ it approaches the low boundary of the spinon continuum, $\omega \approx 2\omega_B + 2uM - v_F q$. Retaining quadratic in q terms in (5) will lead to the termination of the collective mode at some $q_{\max}$ at which the spin wave enters the two-spinon continuum.

This physics is not unique to spin liquids but applies to paramagnetic metals. Historically, this spin-wave mode was predicted by Silin in 1958 for nonferromagnetic metals within Landau Fermi-liquid theory [27–30], and observed via conduction electron spin resonance (CESR) in 1967 [31]. At the time, this observation was considered to be one of the first proofs of the validity of the Landau theory of Fermi liquids [32]. Unlike the more well-known zero sound [33], an external magnetic field is required in order to shift the particle-hole continuum up along the energy axis to allow for the undamped collective spin wave to appear outside the particle-hole continuum, in the triangular-shaped window below it. Second order in the interaction u corrections (beyond the ladder series) do cause damping of this spin mode [34–36].

However, in the U(1) spin liquid, there is an additional branch of low-energy excitations due to the gauge field $\boldsymbol{A}$, dispersing as $\omega \sim q^3$. The very flat dispersion of the gauge excitations suggests it may act as a momentum sink, so that, for example, an excitation consisting of a particle-hole pair plus a gauge quantum may exist in the "forbidden" region where the bare particle-hole continuum vanishes and the collective spin mode lives. It is therefore critical to understand the effect of the gauge interactions upon the dynamical susceptibility. To this end, we consider the dressing of the particle-hole bubble χ^0 by gauge propagators. Guided by the above thinking, we expect that it is sufficient to consider all diagrams with a single gauge propagator (denoted by the wavy line),

$$\chi^1_\pm(\boldsymbol{q}, i\omega_n) =$$

(10)

Calculations described in Ref. [37] lead to

$$\mathrm{Im}\, \chi^1(\boldsymbol{q}, \omega) = -\frac{\sqrt{3}\gamma^{1/3}k_F}{56\pi^2\chi^{4/3}} \frac{q^2\omega^{7/3}}{(\omega - 2\tilde{\omega}_B)^4}. \tag{11}$$

That is, the dressed susceptibility has a nonzero imaginary part in the previously kinematically forbidden region outside the spinon particle-hole continuum (see Fig. 1). This is another continuum weight. However, the weight in this continuum contribution vanishes quadratically in momentum as $\boldsymbol{q} = 0$ is approached. This is an important check on the calculations, since the Larmor theorem still applies to the full theory (2) with the gauge field, which implies that precisely at zero momentum, there can be no different contributions. Similar to Kim *et al.* [24], who considered diagrams for the density correlations and optical conductivity, this result relies on important cancellations between self-energy (first two diagrams) and vertex corrections (last diagram), which are needed to obtain this proper behavior of χ^1.

Within the RPA approximation of Eq. (4), but now with $\chi_\pm = \chi^0_\pm + \chi^1_\pm$, we see that the q^2 dependence of $\mathrm{Im}\, \chi^1_\pm$ is sufficient to ensure that the width (in energy) of the collective spin mode becomes narrow compared to its frequency at small momentum: This is the standard criteria for sharpness and observability of a collective excitation. The real part of χ^1, derived in Eq. (S53) in the Supplemental Material, modifies the dispersion of the collective mode, too, but preserves its downward $\mathbf{q}^2$ character within the $1/N$ approximation [37]. The final result for the dynamical susceptibility is summarized in Fig. 1. Away from the zero momentum axis $\mathrm{Im}\, X_\pm(\boldsymbol{q}, \omega)$ is always nonzero, and is the sum of several distinct contributions. Inside this spinon-gauge continuum, the spinon spin wave appears as a resonance which is asymptotically sharp at small momentum. We note that, while our calculations are done in two dimensions, a spinon spin wave with the same qualitative features is also present in the three-dimensional U(1) QSL.

For observation of the spinon spin wave via an inelastic neutron or RIXS experiment, the mode should be present over a range of momenta which is not too narrow. Because the extent of the "decay-free" triangular-shaped region in Fig. 1 is determined by $2\omega_B/v_F \sim \sqrt{m\omega_B(\omega_B/E_F)}$, this requires that the Zeeman energy should be a substantial fraction of the exchange integral (effective Fermi energy). This makes spin-liquid materials with J of order 10 K ideal candidates for observation, in contrast with the usual metals for which ω_B/E_F is vanishingly small.

The above results apply to the case in which SU(2) spin rotation symmetry is broken only by the applied Zeeman field. Breaking of the SU(2) invariance by anisotropies invalidates the Larmor theorem and causes a shift and, more importantly, a broadening of the spin collective mode even at zero momentum [38]. This is of particular importance for electron spin resonance, which has a high-energy resolution but measures directly at zero momentum only [25]. The way in which the resonance is broadened depends in detail on the nature of the anisotropy, the orientation of the applied magnetic field, etc., so it is not possible to give a single general result. Instead, we provide one example of this physics and consider the influence of a Dzyaloshinskii-Moriya (DM) interaction, which is typically the dominant form of anisotropy for weakly spin-orbit coupled systems, provided it is symmetry allowed by the lattice. See, for example, Refs. [39,40].

Guided by the arguments of symmetry and simplicity we next suppose that when projected to the spin-liquid ground state manifold of the two-dimensional spin model, the DM interaction appears as a familiar spin-orbit interaction of the Rashba kind. A momentum-dependent spin splitting, of which this is the simplest example, is expected to appear in a model without spatial inversion symmetry because in the spinon Fermi surface state the spinons transform under lattice point group symmetries in the same way as the usual electrons [41]. Then the term in the Hamiltonian breaking SU(2) spin invariance H' reads

$$H' = \int d^2\boldsymbol{x}\, \alpha_{\rm so} \psi^\dagger[(\hat{p}_x + A_x)\sigma^y - (\hat{p}_y + A_y)\sigma^x]\psi. \quad (12)$$

Here, $\hat{p}_\mu$ is the ith component of the momentum operator, and $\alpha_{\rm so}$ is the strength of the Rashba coupling. The dependence on the minimal combination $\hat{\mathbf{p}} + \mathbf{A}$ is required by the gauge invariance of the action in Eq. (2). Note that the magnetic field continues to couple to σ^z.

In the fixed Coulomb gauge, the momentum and gauge terms within the Rashba anisotropy of Eq. (12) have distinct effects. The former, momentum term, may be considered at the mean-field level, as an intrinsic spin splitting in the spinon dispersion. Taking this into account, the ESR signal arises from vertical interband transitions [42]. The variation of these transitions with momentum leads to an intrinsic line shape, from which useful information about van Hove and other special points of the spinon bands may be extracted by a detailed analysis [43]. In Fermi liquids, this physics is responsible for chiral spin resonance [44,45]. In one-dimensional spin chains with a *uniform* DM interaction, the same basic physics leads to a splitting of the ESR line into a *doublet* [46].

The gauge field part of Eq. (12) consists mathematically of coupling of $\mathbf{A}$ to the spin-non-conserving bilinear operators of spinons,

$$H'_A = \frac{i\alpha_{\rm so}}{2}\sum_{p,q}[\psi^\dagger_{p+q,\downarrow}\psi_{p,\uparrow}A^+(q) - \psi^\dagger_{p+q,\uparrow}\psi_{p,\downarrow}A^-(q)], \quad (13)$$

where $A^\pm = A_x \pm iA_y$. This term has no simple mean-field description, and is responsible for the magnetic field and temperature dependence of the dynamical spin susceptibility, and in particular the ESR linewidth η.

Instead of a technically involved diagrammatic calculation (which is also possible and confirms the results otherwise obtained) we employ an elegant shortcut which is based on the modern reincarnation [25,47] of the classic ESR formulation by Mori and Kawasaki [48]. We are interested in the retarded Green's function of the transverse spin fluctuations $G^R_{S^+S^-}(\boldsymbol{q} = 0, \omega) = 2M/[\omega - 2\omega_B - \Sigma(\omega)]$, which defines the zero momentum self-energy $\Sigma(\omega)$. The ESR theory shows (see Ref. [37]) that this self-energy is related to the correlations of the perturbation operator $\mathcal{R} = [H'_A, S^+]$, according to

$$\eta = \mathrm{Im}\,\Sigma(\omega) = -\frac{1}{2M}\mathrm{Im}\,G^R_{\mathcal{R}\mathcal{R}^\dagger}(\omega). \quad (14)$$

Equation (14) directly expresses the ESR linewidth in terms of the retarded Green's function of the perturbing operator $\mathcal{R}$.

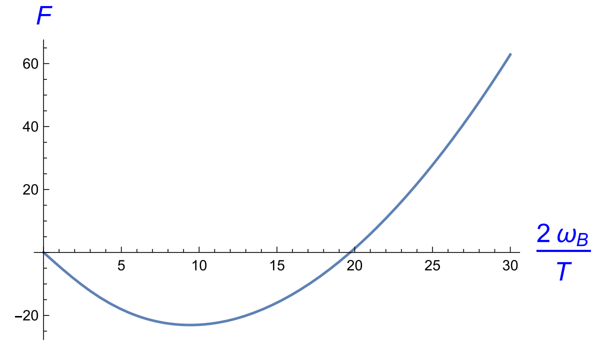


FIG. 2. Scaling function $F(x)$.

Observe that $\mathcal{R} \propto \alpha_{\rm so}$ and hence to the second order in the spin-orbit coupling, Eq. (14) may be calculated with respect to the isotropic Hamiltonian of the ideal spin liquid subject to the Zeeman magnetic field ω_B.

For the Rashba coupling, one obtains $\mathcal{R} = -i\frac{\alpha_{\rm so}}{2}\sum_{p,q}\psi^\dagger_{p+q}\sigma^z\psi_p A^-(q)$, so that the calculation of η reduces to a convolution-type integral over energy and momentum of the spectral functions of the spinon magnetization density $S^z(\mathbf{q})$ and the gauge field $\boldsymbol{A}(\boldsymbol{q})$. This instructive calculation is described in Ref. [37] and results in the full scaling function prediction for the ESR linewidth,

$$\eta(\omega_B, T) = \frac{\alpha^2_{\rm so}}{2M}\left[\frac{mT}{8\pi\chi} + \mathrm{const}\, T^{5/3}F\left(\frac{2\omega_B}{T}\right)\right]. \quad (15)$$

The scaling function $F(x)$ is plotted in Fig. 2 and is characterized by these limits: $F(x \ll 1) = -4.4x$ and $F(x \gg 1) = 0.75x^{5/3}$. Consequently, in the low-temperature limit the linewidth follows a "fractional" scaling with the magnetic field, $\eta \to \alpha^2_{\rm so}(2\omega_B)^{5/3}/M \sim \alpha^2_{\rm so}B^{2/3}$. Also notable is the nonmonotonic dependence of the scaling function F on its argument. The full scaling function represents a nontrivial quantitative prediction for the the present model of magnetic anisotropy.

However, while all isotropic magnets are alike, all anisotropic magnets are anisotropic in their own way. We leave an exhaustive study of different mechanisms of anisotropy on ESR in spin liquids for future work.

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