Kondo effect in a Aharonov-Casher interferometer

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We consider a model describing a spin field-effect transistor based on a quantum nanowire with a tunable spin-orbit interaction embedded between two ferromagnetic leads with anticollinear magnetization. We investigate a regime of a strong interplay between resonance Kondo scattering and interference associated with the Aharonov-Casher effect. Using the Keldysh technique at a weak-coupling regime we calculate perturbatively the charge current. It is predicted that the effects of the spin-orbit interaction result in a nonvanishing current for any spin polarization of the leads including the case of fully polarized anticollinear contacts. We analyze the influence of the Aharonov-Casher phase and degree of spin polarization in the leads onto a Kondo temperature.

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I. INTRODUCTION

The Kondo effect is known to play a very important role for charge transport through nanostructures, facilitating the maximal conductance of a nanodevice at zero bias [1]. Having a spin nature, the Kondo effect is associated with a resonance scattering accompanied by a spin flip through the multiple cotunneling processes in Coulomb blockaded nanodevices [2]. The Kondo effect in GaAs-based semiconductor nanostructures (quantum dots, quantum point contacts, quantum wires, etc.) attracted enormous attention in both the experimental and theoretical communities during the last two decades [2–5]. Recently, semiconductor quantum wires fabricated on InAs and InSb heterostructures started to be widely used in new quantum technological devices [6,7]. One of the most important properties of these materials is related to the effects of a strong spin-orbit interaction (SOI) which does not conserve spin in the resonance scattering processes (see, e.g., Ref. [8]). The high tunability of the interplay between a SOI and the resonance Kondo effect and its influence on the charge and spin transport through nanostructures paves a way for practical applications of these materials in spintronics devices. It is known that, in contrast to the effects of an external magnetic field, the effect of SOI on electron scattering is in preserving time-reversal symmetry. While the magnetic field is destructive for the Kondo effect due to the suppression of spin-flip processes, the influence of SOI on resonance scattering is more delicate.

One of the most remarkable manifestations of SOI in quantum devices (e.g., a Datta-Das spin field-effect transistor) is associated with an accumulation of a spin-dependent phase difference in the electron (spinor) wave function [9,10]. This phase accumulation being controlled by an external electric field applied to a nanodevice is known as the Aharonov-Casher effect [9]. The electric field manipulation of Aharonov-Casher interference provides a big advantage compared to an external magnetic field control. In particular, no magnetization currents are generated both in the nanodevice and in the leads and there is no extra decoherence associated with extra heating. An additional degree of control associated with the use of ferromagnetic leads allows one to open (enhance) and close (suppress) a charge current through the nanostructure [10,11], similar to the effects of a spin valve [12–14].

In this paper we present an example when a strong Coulomb blockade and an established quantum coherence of electrons are simultaneously present and controlled in a quantum nanowire. We consider the Kondo tunneling of electrons through the nanowire in the presence of a strong SOI. We show that quantum interference originating from the Aharonov-Casher phase accumulated during the tunneling process affects qualitatively the many-body Kondo transmission and results in a strong renormalization of the Kondo temperature and a significant enhancement of the charge current.

The paper is organized as follows: In Sec. II we introduce a model Hamiltonian of a spin-orbit active one-dimensional nanowire placed between spin-polarized electrodes and derive an effective Kondo model. In Sec. III we analyze the charge current through the nanowire calculated in the lowest order of perturbation theory. In Sec. IV we obtain the contribution to the charge current in the second order of perturbation theory and evaluate a Kondo temperature as a function of the Aharonov-Casher phase and degree of spin polarization in the leads. In Sec. V we analyze the Kondo temperature in a particular limit of fully polarized anticollinear contacts.

II. MODEL

We investigate a Datta-Das spin field-effect transistor [10] with a spin-orbit active weak link in the Kondo regime.

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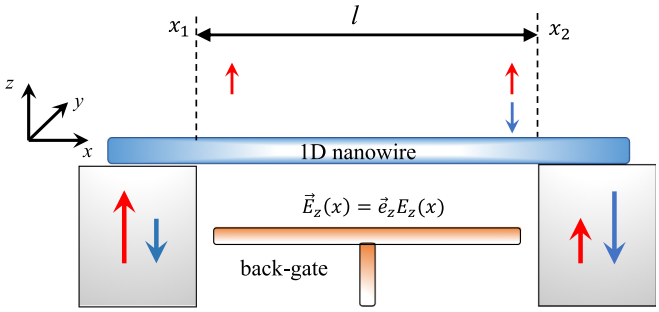

FIG. 1. Scheme of a nanodevice. A 1D nanowire of length L is placed between two massive magnetically polarized electrodes. Polarization is chosen to be collinear antiparallel (AP). Spin-dependent density of states in the leads is defined through $\nu_{L\uparrow} = \nu_{R\downarrow} = \nu(1+p)$ and $\nu_{L\downarrow} = \nu_{R\uparrow} = \nu(1-p)$. A back-gate electrode situated near the nanowire creates an electric field in the z direction, inducing a spin-orbit interaction (SOI) in the nanowire. The short nanowire is treated as a quantum dot (QD) (see the main text for a discussion). We assume that the QD is in a strong Coulomb blockade regime. Odd Coulomb valleys provide access to the Kondo physics. SOI leads to an accumulation of an Aharonov-Casher phase in the electron wave function which is equivalent semiclassically to an electron spin precession.

We consider a one-dimensional (1D) nanowire embedded between two magnetically polarized electrodes in an antiparallel configuration (AP). The back gate controls a spin-orbit interaction (SOI) in the nanowire (see Fig. 1). The one-dimensional nanowire can be treated as a quantum dot (QD) in a regime when the temperature and bias voltage are smaller compared to a mean level spacing in the QD, $\delta\varepsilon \sim \hbar v_F L^{-1}$. We assume an odd number of electrons in the QD to access the Kondo regime. The model is described by the Hamiltonian $H = H_0 + H_{\rm tun}$, where

$$H_0 = \sum_{k,\alpha,\sigma} \left(\varepsilon_k - \mu_\alpha^\sigma\right) c_{k\alpha\sigma}^\dagger c_{k\alpha\sigma} + \sum_\lambda (\varepsilon_0 d_\lambda^\dagger d_\lambda + U_C \hat{n}_\lambda \hat{n}_{\bar{\lambda}}) \tag{1}$$

characterizes a 1D nanowire and the magnetically polarized left (right) electrodes with chemical potentials $\mu_{L(R)}^\sigma$. Here, ε_0 stands for the energy of the first half-filled level of the dot counted from the Fermi level of the leads, and U_C is the charging energy in the nanowire. The annihilation (creation) operators of the conduction electrons are denoted by $c_{k\alpha\sigma}$ ($c_{k\alpha\sigma}^\dagger$), where $\alpha = L, R$. The electron states in the leads are characterized by a spin quantum number $\sigma = (\uparrow, \downarrow)$. The twofold degenerate quantum level in the dot represented by a linear superposition of states with $\sigma = \uparrow$ and $\sigma = \downarrow$ is described by the pseudospin quantum number with two eigenvalues $\lambda, \bar{\lambda}$. We use notations d_λ ($d_\lambda^\dagger$) for the electrons in the QD, $\hat{n}_\lambda = d_\lambda^\dagger d_\lambda$ (see Appendix A). To describe the partial spin polarization of the electrodes we introduce a spin-dependent density of states at Fermi energies $\nu_{L\uparrow} = \nu_{R\downarrow} = \nu(1+p)$ and $\nu_{L\downarrow} = \nu_{R\uparrow} = \nu(1-p)$, where parameter p defines a degree of polarization (see Fig. 1). If magnetization in the leads is collinear and oriented antiparallel, the net magnetic field produced by the leads at the position of the nanowire is zero. Therefore, the net magnetic field does not lift a twofold degeneracy of the pseudospin state in the QD. If the orientation of the magnetization is parallel, the net magnetic field at the position of the nanowire is nonzero, resulting in time-reversal symmetry-breaking effects.

While the spin is a good quantum number in the leads, it cannot be used for the characterization of the state in the nanowire due to the presence of SOI. Thus, the tunnel matrix element computed using wave functions of electrons in the leads and in QD (see, e.g., Ref. [15]) is characterized by two indices σ (spin) and λ (pseudospin). The most general form of the tunneling Hamiltonian is given by

$$H_{\rm tun} = \sum_{k,\alpha;\sigma\lambda} \left(V_{k\alpha}^{\sigma\lambda} c_{k\alpha\sigma}^\dagger d_\lambda + \text{H.c.}\right). \tag{2}$$

We solve the 1D Schrödinger equation for an electron in the nanowire in the presence of SOI (see details in Appendix A), and express the tunnel matrix amplitudes in terms of the SOI parameters. The two-component electron wave functions $\vec{\psi}_\lambda$ at different points are connected through the operator $\hat{U}$ in such a way, $\vec{\psi}_\lambda(x_2) = \hat{U}(x_2, x_1)\vec{\psi}_\lambda(x_1)$, where

$$\hat{U} = \exp\left[\frac{i\hat{\sigma}^y \vartheta(x_1, x_2)}{2}\right], \quad \vartheta = \frac{2\alpha p_F l}{\hbar v_F} \tag{3}$$

characterizes the accumulation of the Aharonov-Casher phase, $l = |x_2 - x_1|$ (see Fig. 1). In Eq. (3), $\alpha \propto E_z$ is the SOI coupling constant, E_z is the electric field in the z direction produced by the back-gate electrode, and $\hat{\sigma}^y$ is the y-Pauli matrix. Using Eq. (3) and assuming that the tunneling occurs at the points x_1, x_2, we parametrize the tunnel matrix elements as follows,

$$V_{k\alpha}^{\tau\tau'} = V_{k\alpha}\left(\delta_{\tau\tau'}\cos(\vartheta/4) \mp i\hat{\sigma}_{\tau\tau'}^y \sin(\vartheta/4)\right). \tag{4}$$

Here, $-/+$ stands for the L/R lead correspondingly. The effect of the SOI on the tunneling processes is characterized by the parameter ϑ. The SOI vanishes for $\vartheta = 0$ and reaches its maximal value at $\vartheta = \pi$, when both tunneling processes, diagonal and off diagonal in spin (pseudospin) indices, contribute equally [see Eq. (4)]. We assume full symmetry in the tunneling junction $V_{kL} = V_{kR} = V_{\rm tun}$.

The mapping of the Anderson impurity model Eq. (1) onto a Kondo-like model is done using the standard Schrieffer-Wolff transformation [16] (see Appendix B). We assume a single occupied twofold degenerate level in the QD and consider the energy level width to be smaller compared to the charging energy $\Gamma = 2\pi\nu|V_{\rm tun}|^2 \ll U_C$.

The effective Hamiltonian $H_{\rm eff} = H_{\rm dir} + H_{\rm ex}$ contains $H_{\rm dir}$ describing a direct (potential) electron scattering between the leads and

$$\begin{aligned} H_{\rm ex} = &\sum_\alpha J\left[\left(s_{\alpha\alpha}^z S^z + s_{\alpha\alpha}^x S^x\right)\cos(\vartheta/2) + s_{\alpha\alpha}^y S^y\right] \\ &+ \sum_{\alpha\neq\alpha'} J\left[s_{\alpha\alpha'}^z S^z + s_{\alpha\alpha'}^x S^x + s_{\alpha\alpha'}^y S^y \cos(\vartheta/2)\right] \\ &- J\sin(\vartheta/2)\left[\left(s_{LL}^z - s_{RR}^z\right)S^x - \left(s_{LL}^x - s_{RR}^x\right)S^z\right] \\ &- \frac{J}{2}\sin(\vartheta/2) j_{LR} S^y \end{aligned} \tag{5}$$

constituting the effective exchange interaction between pseudospin-1/2 in the QD, $\vec{S}$, and spin $\vec{s}_{\alpha\alpha'} = \sum_{kk'} (1/2) c^{\dagger}_{k\alpha\sigma} \hat{\sigma}_{\sigma\sigma'} c_{k'\alpha'\sigma'}$ of the conduction electrons in the L/R leads and charge transfer $j_{LR} = i \sum_{kk'} (c^{\dagger}_{kL\uparrow} c_{k'R\uparrow} + c^{\dagger}_{kL\downarrow} c_{k'R\downarrow} - \text{H.c.})$. We used the following notations for the exchange interaction constant in Eq. (5),

$$J = 2U_C \frac{|V_{\text{tun}}|^2}{|\varepsilon_0|(U_C - |\varepsilon_0|)}. \tag{6}$$

We concentrate below on the case of electron-hole symmetry, $\varepsilon_0 \to (-U_C/2)$, and ignore the irrelevant processes of potential scattering [17].

The influence of SOI effects onto the Kondo scattering has several facets. First, SOI is responsible for the different types of spin anisotropies in the terms diagonal and off diagonal in the lead indices [first and second lines in Eq. (5)]. Second, SOI produces an additional coupling between the pseudospin in QD and spin density of the conduction electrons, also known as the Dzyaloshinskii-Moriya (DM) interaction ($\propto \vec{e}_y \cdot [\vec{s}_{\alpha\alpha} \times \vec{S}]$) [18]. Third, the SOI mediates the interaction between the pseudospin in the QD and the charge transfer.

III. COTUNNELING CURRENT

Assuming a high-temperature (compared to some emerging energy scale to be defined below) regime we calculate the current through the nanowire perturbatively in $\nu J \ll 1$. The first nonvanishing contribution to the charge current is $\propto (\nu J)^2$. The cotunneling current given by Eq. (7) can be straightforwardly derived either through an equation of motion method or using the nonequilibrium Keldysh Green's function technique (we adopt below the units $k_B = 1$),

$$I^{(2)} = \frac{e\pi^2}{4\pi\hbar} (\nu J)^2 eV \Big\{ (1-p^2)(2-q^2) + (1+q^2)(1+p^2) - 8pq \coth\left(\frac{eV}{2T}\right) \langle S^z \rangle \Big\}, \tag{7}$$

where we use shorthand notations $q = \cos(\vartheta/2)$ for the parametrization of the accumulated Aharonov-Casher phase ($q = 1$ for the case of the absence of SOI, $\vartheta = 0$, and $q = 0$ is when the SOI is maximal, $\vartheta = \pi$). In Eq. (7), $\langle S^z \rangle$ denotes an out-of-equilibrium QD (nanowire) magnetization [19,20]. The QD magnetization that appears because of an applied bias voltage in the presence of a finite polarization p is nonvanishing even without an external magnetic field (see Ref. [21]). The temperature T in Eq. (7) stands for the temperature in the contacts which are assumed to be in equilibrium. The expression for QD magnetization $\langle S^z \rangle$ is obtained from the steady-state solution of the quantum Langevin equation of motion [21,22] for the QD spin-1/2 in the lowest order of perturbation theory in νJ,

$$\langle S^z \rangle = \frac{pq(eV/T)}{2(1-q^2p^2) + \varphi\left(\frac{eV}{T}\right)(p^2+q^2)}, \tag{8}$$

where $\varphi(x) = x \coth(x/2)$. Nonequilibrium QD magnetization described by Eq. (8) is limited by $\langle S^z \rangle = \pm pq/(p^2+q^2)$ achieved at a large bias voltage $eV \gg T$.

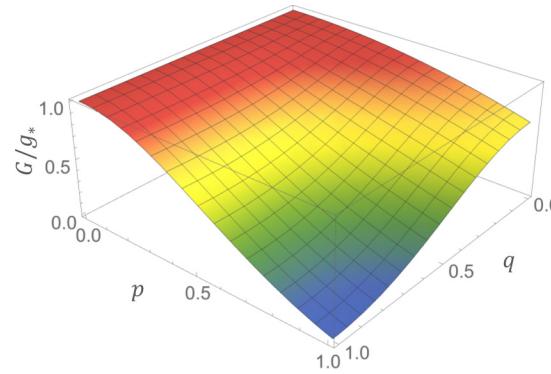

FIG. 2. Differential conductance in units of $g_* = 3e^2\pi^2(\nu J)^2/(4\pi\hbar)$ as a function of both polarization in the leads p and effects associated with the accumulation of the Aharonov-Casher phase ϑ parametrized by $q = \cos(\vartheta/2)$for the case of the bias voltage $eV/T \gg 1$.

The appearance of nonequilibrium QD magnetization $\langle S^z \rangle$ in Eq. (7) influences the shape of the peak in the differential conductance, $G^{(2)} = dI^{(2)}/dV|_{V\to 0}$ (see Refs. [21,23]). At large bias voltages $eV \gtrsim T$ the effects of saturation of the QD magnetization result in a suppression of the charge current. At low bias voltages $eV \ll T$ the contribution to the current proportional to the QD magnetization is vanishing and the current reaches its maximum value. The height of the conductance peak depends on the QD magnetization slope $\partial\langle S^z \rangle/\partial(eV) = pq/[2T(1+p^2+q^2-p^2q^2)]$.

The p and q dependence of the differential conductance given by Eq. (7) is illustrated in Fig. 2 at $eV \gg T$. The leading contribution to the differential conductance calculated in the lowest nonvanishing order of the perturbation theory for the case of nonmagnetic leads ($p = 0$) in the absence of the SOI ($q = 1$) saturates at $G^{(2)} = g_*$, where $g_* = (e^2/\pi\hbar)(3/4)(\pi\nu J)^2$. The conductance peak at zero-bias voltage for the case of partial polarization of the leads ($p \neq 1$) is $G^{(2)} = g_*(1-p^2)$. The linear response (voltage-independent) part of the differential conductance at a large bias voltage ($eV \gg T$) is asymptotically given by $G^{(2)} = g_*(3-4p^2+p^4)/[3(1+p^2)]$ (see Ref. [23]). Spin-dependent tunneling induced by SOI at $q < 1$ enhances the charge transport through QD at any collinear AP of the reservoirs ($p \neq 0$). The zero-bias conductance in the case of fully polarized leads $p = 1$ is given by $G^{(2)}|_{V\to 0} = g_*(2/3)(1-q^2)$, while the linear response conductance at large bias voltages is $G^{(2)}|_{V\to\infty} = g_*(2/3)(1-q^2)^2/(1+q^2)$. The effect of SOI on the charge current is maximal at $q = 0$. The polarization dependence of the differential conductance in this case is given by $G^{(2)} = g_*(1-p^2/3)$.

IV. KONDO CONTRIBUTION TO THE CHARGE CURRENT

The next nonvanishing contribution to the charge current $\propto (\nu J)^3$ depends on the spin-flip processes and is therefore described by the Kondo physics. We apply the nonequilibrium Keldysh Green's function technique and Abrikosov's *pseudofermion* representation [24] (see details in Ref. [20] to proceed with the calculations). The current $I^{(3)} = I^{(3)}_K + I^{(3)}_{\text{an}}$

consists of two parts: (i) $I_K^{(3)}$ is originating from the anisotropic Kondo model [first two lines in Eq. (5)],

$$\frac{I_K^{(3)}}{g_* 4\nu J} = \frac{1}{3}\{(1-p^2)[(1+q^2)V - 2pq\mathcal{S}^z] + q^2(1+3p^2)V - pq(1+q^2)(3+p^2)\mathcal{S}^z\} \times \log\left(\frac{D}{T^*}\right), \tag{9}$$

and (ii) $I_{\rm an}^{(3)}$ is accounting for both the Dzyaloshinskii-Moriya and charge transfer processes in Eq. (5),

$$\frac{I_{\rm an}^{(3)}}{g_* 4\nu J} = \frac{(1-q^2)}{3}(1-p^2)\{2V - 3pq\mathcal{S}^z\}\log\left(\frac{D}{T^*}\right). \tag{10}$$

Here, D is the bandwidth of the leads. We use the shorthand notations $\mathcal{S}^z = V\coth(eV/2T)\langle S^z\rangle$ and $T^* = \max[|eV|, T]$. The third order in the $(\nu J)^3$ correction to the charge current logarithmically grows with a decrease of both the temperature and the applied bias voltage, revealing *a Kondo anomaly*. The validity of the perturbation theory approximation (weak-coupling regime) of Eqs. (9) and (10) determines the energy scale T_K, the Kondo temperature. Perturbation theory breaks down at $T \lesssim T_K$. The effective coupling constants in this (strong-coupling) regime flow towards the strong-coupling fixed point.

The dependence of the Kondo temperature on the parameters p and q is in general determined by the solution of a system of coupled renormalization group (RG) equations [25,26]. Without losing generality, we parametrize T_K by the function $f(p,q)$ [27,28],

$$T_K = D\exp\left(-\frac{f(p,q)}{2\nu J}\right). \tag{11}$$

The form of $f(p,q)$ is known for several limiting cases [29–31]. In particular, $f(p,1) = 1$ for all $|p| \leqslant 1$ [29,30]. Besides, in the case of nonmagnetic leads ($p = 0$) we get $f(0,q) = 1$. The form of $f(p,q)$ on a line $p = 1$ for $r = 1 - q \ll 1$ is found perturbatively from the condition of breaking down perturbation theory for the differential conductance, $f(1,r) = 1 + r$ (here, $r = \vartheta^2/4$). With the same logic we found $f(p \ll 1, 0) = 1 + 2p^2/3$. A perturbative analysis leads to a conclusion that $f(p,q) \geqslant 1$. Moreover, the absolute minimum of $f(p,q)$ is reached at the symmetry lines (points), where original Hamiltonian (5) is mapped onto an isotropic Kondo model. We conclude that the Kondo temperature in the Datta-Das transistor with AP polarized electrodes becomes a function of the Aharonov-Casher phase and degree of spin polarization in the leads.

The most striking effect of the influence of the Aharonov-Casher interference onto Kondo scattering is manifested in the case of full AP polarization, $p = 1$. In particular, the nonvanishing charge current is controlled by the Aharonov-Casher phase.

The expression for the differential conductance $G = G^{(2)} + G^{(3)}$ derived in the limit of small bias voltage $eV \ll T$ is given by

$$G = \frac{e^2}{2\pi\hbar}(\pi\nu J)^2(1-q^2)\left(1 + q^2 4\nu J\log\frac{D}{T}\right). \tag{12}$$

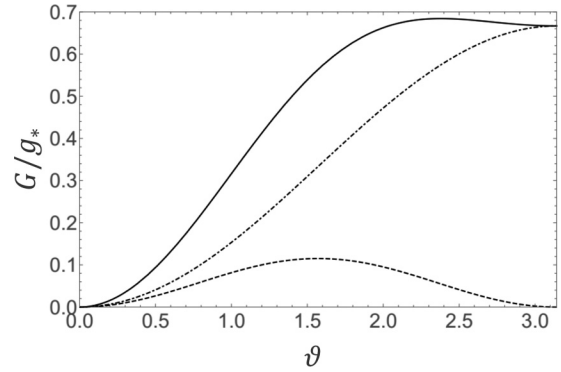

FIG. 3. Dependence of zero-bias differential conductance [in units of $g_* = (3/4)(e^2/\pi\hbar)(\pi\nu J)^2$] on the Aharonov-Casher phase for the case of full AP polarization of the leads $p = 1$. The dotted-dashed line denotes differential conductance $G^{(2)}$ obtained from Eq. (7), and the dashed line is the third-order $\propto(\nu J)^3$ perturbative correction to the conductance $G^{(3)}$ determined by Eqs. (9) and (10). The solid line represents the sum of $G^{(2)}$ and $G^{(3)}$. The figure is plotted for the following values of the model parameters: $D = 1$ eV, $T = 1$ K, and $\nu J = 0.1$.

The conductance dependence on the Aharonov-Casher phase ϑ is shown in Fig. 3. Charge current at $p = 1$ and $q = 1$ is blocked by the Pauli principle. However, the absence of charge current is not in contradiction to the presence of resonant Kondo scattering. The Kondo effects (and T_K) are fully determined by multiple pseudospin-flip processes on a QD and a single (L or R) contact [29]. As it is seen from Eq. (12), the logarithmic corrections to the conductance are positive for $J > 0$ when $q \neq 1$ and increase with decreasing T. While $G^{(2)}$ is monotonously increased with ϑ, the behavior of $G^{(3)}$ is nonmonotonous due to the interplay between the Kondo effect and the Aharonov-Casher interferometer. As a result, the maximal current is reached at some critical value $q_{\rm cr} = \sqrt{[1-(4\nu J)^{-1}\log(T/D)]/2}$ which depends on T and the initial parameters of the model.

V. SOI INFLUENCE ON KONDO TEMPERATURE

The Hamiltonian Eq. (5) casts a simple form for $p = 1$,

$$\tilde{H}_{\rm ex} = J\{(s^zS^z + s^yS^y)\cos(\vartheta/2) + s^xS^x\} - \frac{1}{2}J\sin(\vartheta/2)\sum_{kk'}c_{k\gamma}^\dagger c_{k'\gamma}S^x. \tag{13}$$

The Hamiltonian (13) is derived from Eq. (5) by retaining the operators $c_{k\gamma}$ ($c_{k\gamma}^\dagger$) with $\gamma = 1$ for $(L,\uparrow)$, $\gamma = -1$ for $(R,\downarrow)$, and $\vec{s} = \sum_{kk'}(1/2)c_{k\gamma}^\dagger\hat{\sigma}_{\gamma\gamma'}c_{k'\gamma'}$. We omit all other terms in Eq. (5) with zero expectation values. Equation (13) describes an *anisotropic Kondo-like model* with an additional term, which couples the spin in the QD with the charge density in the leads. The last term in (13) can be viewed as an extra potential scattering and is therefore disregarded for the particle-hole symmetric limit.

As it is known both from the RG and exact Bethe anzatz solution of the Kondo model, the maximal Kondo temperature is achieved in the isotropic case [27]. The Kondo temperature in the anisotropic case is defined through the Bethe anzatz

solution [27] by an equation equivalent to (11) with the dimensionless function $f(1,q)$ dependent on the anisotropy parameter $q=\cos(\vartheta/2)$,

$$f(1,q)=-\frac{1}{\sqrt{1-q^2}}\log\left(\frac{1-\sqrt{1-q^2}}{q}\right). \tag{14}$$

The anisotropy controlled by SOI suppresses the Kondo temperature. The asymptotic behavior of Eq. (14) in the limit of weak ($\vartheta\to 0$) SOI is given by $f(1,\vartheta)-1\approx\vartheta^2/12$. Similar behavior is also obtained from the renormalization group treatment under condition $\nu J\ll 1$. While the cases of small anisotropy can be accessed perturbatively [see the discussion after Eq. (11)], or by the Bethe anzatz solution ($p=1$) [see Eq. (14)], the solution for the function $f(p,q)$ for arbitrary values of its arguments $-1<(p,q)<1$ remains an interesting and unsolved problem [32].

VI. SUMMARY AND OUTLOOK

The interplay between resonance Kondo scattering in the quantum wire, effects of SOI in the tunnel barriers, and partial spin polarization in the leads provides a high level of controllability for charge transport through a nanodevice. In particular, the fine tuning of the Kondo temperature is achieved by control of three independent tunable parameters of the system. First, the Aharonov-Casher phase is tuned by the electric field applied to the area of the nanowire. Second, the degree of spin polarization in the leads is manipulated by the spin valve [12–14]. Third, the local out-of-equilibrium QD magnetization of the nanodevice is controlled by the source-drain voltage. The central result of the paper is a prediction of a finite charge current through the nanowire even at full AP polarization of the leads in the presence of a nonzero spin-orbit interaction. Besides, perturbative (weak-coupling) calculations demonstrate pronounced (logarithmic) effects of enhancement of the current by SOI at any given partial polarization of the leads. Competition between the resonance scattering resulting in a maximal Kondo temperature in the absence of SOI at $\vartheta=0$ and quantum interference due to the Aharonov-Casher effect that is maximal at $\vartheta=\pi$ allows one to find an optimal strength of SOI at $\vartheta_{\rm cr}$ ($q_{\rm cr}$) under the condition of maximizing the electric current.

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APPENDIX A: SPIN-ORBIT INTERACTION AND SPIN-DEPENDENT TUNNEL MATRIX ELEMENTS

In this Appendix we calculate the Aharonov-Casher phase ϑ [see Eq. (3)]. We start from a Schrödinger equation for the electron in a 1D nanowire in the presence of an external homogeneous electric field E_z produced by the back gate and directed perpendicular to nanowire,

$$-\frac{\hbar^2}{2m}\frac{\partial^2\vec{\psi}}{\partial x^2}-\alpha\hat{\sigma}^y i\hbar\frac{\partial\vec{\psi}}{\partial x}=(E_F-\varepsilon)\vec{\psi}, \tag{A1}$$

where $\vec{\psi}$ is a two-component electron wave function (spinor), and $\alpha\propto E_z$ is a spin-orbit interaction coupling constant. Since $\varepsilon\leqslant\hbar v_F L^{-1}\ll E_F$, we can present the electron's wave function in terms of right- and left-moving parts,

$$\vec{\psi}=e^{ip_F x/\hbar}\vec{\psi}_+(x)+e^{-ip_F x/\hbar}\vec{\psi}_-(x), \tag{A2}$$

where $p_F=\sqrt{2m(E_F-\varepsilon)}$ is a Fermi momentum. Substituting the wave function Eq. (A2) into Eq. (A1) and neglecting the second derivative, we get

$$-iv_F\hbar\frac{\partial\vec{\psi}_\pm}{\partial x}=\alpha p_F\hat{\sigma}^y\vec{\psi}_\pm. \tag{A3}$$

From this equation one can see that the spinor function $\vec{\psi}_\pm$ satisfies the relation

$$\vec{\psi}_\pm(x)=\hat{U}(x)\vec{\psi}_\pm(0),\quad \hat{U}(x)=e^{i\hat{\sigma}^y\vartheta(x)/2}, \tag{A4}$$

where $\vartheta(x)=2\alpha x p_F/(\hbar v_F)$ is an Aharonov-Casher phase.

Despite the fact that in the presence of SOI the electronic spin is an unsuitable quantum number for the classification of the electronic states, the energy levels continue to be doubly degenerate. If $\vec{\psi}_\lambda$ is the eigenstate with energy ε, then $\vec{\psi}_{\bar\lambda}=i\hat{\sigma}^y\vec{\psi}^*_\lambda$ is also an eigenstate with the same energy. Here, we used notations $\vec{\psi}_{\lambda(\bar\lambda)}$ for the normalized electron wave function in the nanowire. As this takes place one can classify the states by the spin structure of the wave functions at a fixed point, for example, at $x=0$ the middle point of the nanowire between the left and right electrodes. Assuming that eigenstates $\vec{\psi}_{\lambda(\bar\lambda)}(0)$ in the middle of the nanowire correspond to a state with spin up and spin down, we define the value of the wave function in the point x_1, x_2 where tunneling into the leads occurs ($x_{1(2)}=\pm l/2$),

$$\begin{aligned}
\vec{\psi}_\lambda(x_1)&=\hat{U}(x_1,0)\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}\cos(\vartheta/4)\\ \sin(\vartheta/4)\end{pmatrix},\\
\vec{\psi}_\lambda(x_2)&=\hat{U}(x_2,0)\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}\cos(\vartheta/4)\\ -\sin(\vartheta/4)\end{pmatrix},\\
\vec{\psi}_{\bar\lambda}(x_1)&=\hat{U}(x_1,0)\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}-\sin(\vartheta/4)\\ \cos(\vartheta/4)\end{pmatrix},
\end{aligned} \tag{A5}$$

$$\vec{\psi}_{\bar\lambda}(x_2)=\hat{U}(x_2,0)\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}\sin(\vartheta/4)\\ \cos(\vartheta/4)\end{pmatrix}. \tag{A6}$$

The amplitude of electron tunneling from the left lead to the nanowire in state $\lambda,\bar\lambda$ can be found as follows,

$$\begin{aligned}
V_{kL}^{\uparrow\lambda}&=\sum_k V_{kL}\langle(\uparrow,0)|\vec{\psi}_\lambda(x_1)\rangle,\\
V_{kL}^{\downarrow\lambda}&=\sum_k V_{kL}\langle(0,\downarrow)|\vec{\psi}_\lambda(x_1)\rangle,\\
V_{kL}^{\uparrow\bar\lambda}&=\sum_k V_{kL}\langle(\uparrow,0)|\vec{\psi}_{\bar\lambda}(x_1)\rangle,
\end{aligned} \tag{A7}$$

$$V_{kL}^{\downarrow\bar\lambda}=\sum_k V_{kL}\langle(0,\downarrow)|\vec{\psi}_{\bar\lambda}(x_1)\rangle, \tag{A8}$$

where V_{kL} is the transition amplitude. The tunnel matrix element for tunneling processes from the right lead can be defined in a similar way by using wave functions $\vec{\psi}_{\lambda(\bar{\lambda})}(x_2)$.

APPENDIX B: SCHRIEFFER-WOLFF TRANSFORMATION

In this Appendix we derive the effective Kondo Hamiltonian for the general case of spin-dependent tunnel matrix elements. The mapping of the Anderson-like impurity model Eqs. (1) and (2) onto a Kondo-like model is done using a standard Schrieffer-Wolff transformation,

$$H_{\rm eff} = e^{S} H e^{-S} \equiv H + [S, H] + \tfrac{1}{2}[S, [S, H]] + \cdots . \tag{B1}$$

The first step is to eliminate the first order in the tunneling amplitude terms using the following condition,

$$[S, H_0] = -H_{\rm tun}. \tag{B2}$$

As a result, the effective Hamiltonian is transformed to

$$H_{\rm eff} = H_0 + \tfrac{1}{2}[S, H_{\rm tun}] + \cdots . \tag{B3}$$

We choose operator S in the following form,

$$S = \left[\sum (A_{\alpha\sigma\lambda} + B_{\alpha\sigma\lambda}\hat{n}_{\bar{\lambda}}) c^{\dagger}_{k\alpha\sigma} d_{\lambda} - \text{H.c.} \right]. \tag{B4}$$

Using the condition given by Eq. (B2) one can determine the constants $A_{\alpha\sigma\lambda}, B_{\alpha\sigma\lambda}$. After straightforward calculations using Eqs. (B3) and (B4), we obtain the effective Hamiltonian $H_{\rm eff} = H_{\rm dir} + H_{\rm ex}$. The first term responsible for the electron potential scattering between leads is given by

$$\begin{aligned} H_{\rm dir} = \sum \frac{K_{\alpha\alpha'}}{4} \Big[& \frac{n_{\alpha\alpha'}}{2} [V^{\uparrow\uparrow}_{k\alpha} (V^{\uparrow\uparrow}_{k'\alpha'})^* + V^{\downarrow\downarrow}_{k\alpha} (V^{\downarrow\downarrow}_{k'\alpha'})^* + V^{\downarrow\uparrow}_{k\alpha} (V^{\uparrow\downarrow}_{k'\alpha'})^* + V^{\uparrow\downarrow}_{k\alpha} (V^{\downarrow\uparrow}_{k'\alpha'})^*] \\ & + s^{z}_{\alpha\alpha'} [V^{\uparrow\uparrow}_{k\alpha} (V^{\uparrow\uparrow}_{k'\alpha'})^* - V^{\downarrow\downarrow}_{k\alpha} (V^{\downarrow\downarrow}_{k'\alpha'})^* - V^{\downarrow\uparrow}_{k\alpha} (V^{\uparrow\downarrow}_{k'\alpha'})^* + V^{\uparrow\downarrow}_{k\alpha} (V^{\downarrow\uparrow}_{k'\alpha'})^*] \\ & + s^{x}_{\alpha\alpha'} [V^{\uparrow\uparrow}_{k\alpha} (V^{\uparrow\downarrow}_{k'\alpha'})^* + V^{\uparrow\downarrow}_{k\alpha} (V^{\downarrow\downarrow}_{k'\alpha'})^* + V^{\downarrow\downarrow}_{k\alpha} (V^{\downarrow\uparrow}_{k'\alpha'})^* + V^{\downarrow\uparrow}_{k\alpha} (V^{\uparrow\uparrow}_{k'\alpha'})^*] \\ & + i s^{y}_{\alpha\alpha'} [V^{\uparrow\uparrow}_{k\alpha} (V^{\uparrow\downarrow}_{k'\alpha'})^* + V^{\uparrow\downarrow}_{k\alpha} (V^{\downarrow\downarrow}_{k'\alpha'})^* - V^{\downarrow\downarrow}_{k\alpha} (V^{\downarrow\uparrow}_{k'\alpha'})^* - V^{\downarrow\uparrow}_{k\alpha} (V^{\uparrow\uparrow}_{k'\alpha'})^*] \Big]. \end{aligned} \tag{B5}$$

The matrix elements used in Eq. (B5) are given by

$$K_{\alpha\alpha'} = \frac{1}{\varepsilon_{k\alpha} - \varepsilon_0} + \frac{1}{\varepsilon_{k'\alpha'} - \varepsilon_0} - \frac{1}{U_C + \varepsilon_0 - \varepsilon_{k\alpha}} - \frac{1}{U_C + \varepsilon_0 - \varepsilon_{k'\alpha'}}. \tag{B6}$$

The next step is to find the Hamiltonian responsible for the exchange processes,

$$H_{\rm ex} = H_1 + H_2 + H_3, \tag{B7}$$

where the Hamiltonian H_1,

$$\begin{aligned} H_1 = \sum \frac{J_{\alpha\alpha'}}{2} \Big[& s^{z}_{\alpha\alpha'} S^{z} [V^{\uparrow\uparrow}_{k\alpha} (V^{\uparrow\uparrow}_{k'\alpha'})^* + V^{\downarrow\downarrow}_{k\alpha} (V^{\downarrow\downarrow}_{k'\alpha'})^* - V^{\downarrow\uparrow}_{k\alpha} (V^{\uparrow\downarrow}_{k'\alpha'})^* - V^{\uparrow\downarrow}_{k\alpha} (V^{\downarrow\uparrow}_{k'\alpha'})^*] \\ & + s^{x}_{\alpha\alpha'} S^{x} [V^{\uparrow\uparrow}_{k\alpha} (V^{\downarrow\downarrow}_{k'\alpha'})^* + V^{\downarrow\downarrow}_{k\alpha} (V^{\uparrow\uparrow}_{k'\alpha'})^* + V^{\downarrow\uparrow}_{k\alpha} (V^{\downarrow\uparrow}_{k'\alpha'})^* + V^{\uparrow\downarrow}_{k\alpha} (V^{\uparrow\downarrow}_{k'\alpha'})^*] \\ & + s^{y}_{\alpha\alpha'} S^{y} [V^{\uparrow\uparrow}_{k\alpha} (V^{\downarrow\downarrow}_{k'\alpha'})^* + V^{\downarrow\downarrow}_{k\alpha} (V^{\uparrow\uparrow}_{k'\alpha'})^* - V^{\downarrow\uparrow}_{k\alpha} (V^{\downarrow\uparrow}_{k'\alpha'})^* - V^{\uparrow\downarrow}_{k\alpha} (V^{\uparrow\downarrow}_{k'\alpha'})^*] \Big], \end{aligned} \tag{B8}$$

is equivalent to an anisotropic Kondo model. The Hamiltonian H_2,

$$\begin{aligned} H_2 = \sum \frac{J_{\alpha\alpha'}}{4} \Big[& n_{\alpha\alpha'} S^{z} [V^{\uparrow\uparrow}_{k\alpha} (V^{\uparrow\uparrow}_{k'\alpha'})^* - V^{\downarrow\downarrow}_{k\alpha} (V^{\downarrow\downarrow}_{k'\alpha'})^* + V^{\downarrow\uparrow}_{k\alpha} (V^{\uparrow\downarrow}_{k'\alpha'})^* - V^{\uparrow\downarrow}_{k\alpha} (V^{\downarrow\uparrow}_{k'\alpha'})^*] \\ & + n_{\alpha\alpha'} S^{x} [V^{\uparrow\uparrow}_{k\alpha} (V^{\downarrow\uparrow}_{k'\alpha'})^* + V^{\uparrow\downarrow}_{k\alpha} (V^{\uparrow\uparrow}_{k'\alpha'})^* + V^{\downarrow\downarrow}_{k\alpha} (V^{\uparrow\downarrow}_{k'\alpha'})^* + V^{\downarrow\uparrow}_{k\alpha} (V^{\downarrow\downarrow}_{k'\alpha'})^*] \\ & + i n_{\alpha\alpha'} S^{y} [-V^{\uparrow\uparrow}_{k\alpha} (V^{\downarrow\uparrow}_{k'\alpha'})^* + V^{\uparrow\downarrow}_{k\alpha} (V^{\uparrow\uparrow}_{k'\alpha'})^* + V^{\downarrow\downarrow}_{k\alpha} (V^{\uparrow\downarrow}_{k'\alpha'})^* - V^{\downarrow\uparrow}_{k\alpha} (V^{\downarrow\downarrow}_{k'\alpha'})^*] \Big], \end{aligned} \tag{B9}$$

describes the coupling between the spin-1/2 on the dot and the "charge" density in the leads $n_{\alpha\alpha'}$.

The Hamiltonian H_3,

$$\begin{aligned} H_3 = \sum \frac{J_{\alpha\alpha'}}{2} \Big[& i s^{y}_{\alpha\alpha'} S^{x} [V^{\uparrow\uparrow}_{k\alpha} (V^{\downarrow\downarrow}_{k'\alpha'})^* - V^{\downarrow\downarrow}_{k\alpha} (V^{\uparrow\uparrow}_{k'\alpha'})^* - V^{\downarrow\uparrow}_{k\alpha} (V^{\downarrow\uparrow}_{k'\alpha'})^* + V^{\uparrow\downarrow}_{k\alpha} (V^{\uparrow\downarrow}_{k'\alpha'})^*] \\ & + i s^{x}_{\alpha\alpha'} S^{y} [-V^{\uparrow\uparrow}_{k\alpha} (V^{\downarrow\downarrow}_{k'\alpha'})^* + V^{\downarrow\downarrow}_{k\alpha} (V^{\uparrow\uparrow}_{k'\alpha'})^* - V^{\downarrow\uparrow}_{k\alpha} (V^{\downarrow\uparrow}_{k'\alpha'})^* + V^{\uparrow\downarrow}_{k\alpha} (V^{\uparrow\downarrow}_{k'\alpha'})^*] \\ & + s^{x}_{\alpha\alpha'} S^{z} [V^{\uparrow\uparrow}_{k\alpha} (V^{\uparrow\downarrow}_{k'\alpha'})^* - V^{\uparrow\downarrow}_{k\alpha} (V^{\downarrow\downarrow}_{k'\alpha'})^* - V^{\downarrow\downarrow}_{k\alpha} (V^{\downarrow\uparrow}_{k'\alpha'})^* + V^{\downarrow\uparrow}_{k\alpha} (V^{\uparrow\uparrow}_{k'\alpha'})^*] \\ & + s^{z}_{\alpha\alpha'} S^{x} [V^{\uparrow\uparrow}_{k\alpha} (V^{\downarrow\uparrow}_{k'\alpha'})^* + V^{\uparrow\downarrow}_{k\alpha} (V^{\uparrow\uparrow}_{k'\alpha'})^* - V^{\downarrow\downarrow}_{k\alpha} (V^{\uparrow\downarrow}_{k'\alpha'})^* - V^{\downarrow\uparrow}_{k\alpha} (V^{\downarrow\downarrow}_{k'\alpha'})^*] \end{aligned}$$

$$+ is^{y}_{\alpha\alpha'} S^{z} [V^{\uparrow\uparrow}_{k\alpha} (V^{\uparrow\downarrow}_{k'\alpha'})^{*} - V^{\uparrow\downarrow}_{k\alpha} (V^{\downarrow\downarrow}_{k'\alpha'})^{*} + V^{\downarrow\downarrow}_{k\alpha} (V^{\downarrow\uparrow}_{k'\alpha'})^{*} - V^{\downarrow\uparrow}_{k\alpha} (V^{\uparrow\uparrow}_{k'\alpha'})^{*}]$$

$$+ is^{z}_{\alpha\alpha'} S^{y} [-V^{\uparrow\uparrow}_{k\alpha} (V^{\downarrow\uparrow}_{k'\alpha'})^{*} + V^{\uparrow\downarrow}_{k\alpha} (V^{\uparrow\uparrow}_{k'\alpha'})^{*} - V^{\downarrow\downarrow}_{k\alpha} (V^{\uparrow\downarrow}_{k'\alpha'})^{*} + V^{\downarrow\uparrow}_{k\alpha} (V^{\downarrow\downarrow}_{k'\alpha'})^{*}] \Big], \tag{B10}$$

accounts for the Dzyaloshinskii-Moriya-like interaction.

The exchange coupling constant reads as follows,

$$J_{\alpha\alpha'} = \frac{1}{\varepsilon_{k\alpha} - \varepsilon_0} + \frac{1}{U_C + \varepsilon_0 - \varepsilon_{k\alpha}} + \frac{1}{\varepsilon_{k'\alpha'} - \varepsilon_0} + \frac{1}{U_C + \varepsilon_0 - \varepsilon_{k'\alpha'}}. \tag{B11}$$

Substituting parametrization Eq. (4) into Eqs. (B6) and (B7) and considering exchange coupling constants of conduction electrons at the Fermi energy, $\varepsilon_{k\alpha} = \varepsilon_k - \mu^{\sigma}_{\alpha} \approx 0$, we can obtain Hamiltonian Eq. (5).

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