Nonreciprocal transport of a super-Ohmic quantum ratchet

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Nonreciprocal transport, which refers to directional transport, is known to occur in noncentrosymmetric systems with broken time-reversal symmetry. In this paper, we study the nonreciprocal motion of a super-Ohmic dissipative particle—with a quadratic spectral density function of the environment ($s = 2$)—in an asymmetric periodic potential. Using the Keldysh formalism, we derive general expressions for the perturbative corrections to finite-frequency mobility up to third order in potential strength. We find from numerical integration that as a function of temperature, second-order mobility in the low-frequency limit scales exponentially in the low-temperature region, $\mu_2^{(3)}(T;\omega_1,\omega_2) \sim \exp(-b/T)$, $(b > 0)$ and scales as a power law in the high-temperature region, $\mu_2^{(3)}(T;\omega_1,\omega_2) \sim T^{-3}$. As a function of frequency, we show that second-order mobility scales according to $\mu_2^{(3)}(\omega_1,\omega_2;T) \sim i(\omega \log(\omega/\omega_c))^{-3}$ for the $s = 2$ case, but scales as a power law, $\mu_2^{(3)}(\omega_1,\omega_2;T) \sim i\omega^{-3}$, for $s \geqslant 3$, $(s \in \mathbb{Z}^+)$.

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I. INTRODUCTION

Nonreciprocal transport refers to directional transport. For example, in a one-dimensional system, charge transport is nonreciprocal if transport properties change when current is reversed or, to be precise, when second-order conductivity is finite. The two necessary conditions for nonreciprocal transport are noncentrosymmetry and broken time-reversal symmetry. Time-reversal symmetry can be broken in two ways: microscopically using an external magnetic field or by the irreversibility of dissipative effects [1]. When it is broken by a magnetic field, nonreciprocal transport known as magnetochiral anisotropy is known to occur in noncentrosymmetric systems [2]. When it is broken by both a magnetic field and dissipative effects, nonreciprocal spin currents can be generated in noncentrosymmetric systems with spin-orbit coupling [3–6]. However, in this paper, we focus solely on the latter case when time-reversal symmetry is broken only by dissipation, and we study nonreciprocal transport in an open system.

To study nonreciprocal transport induced by dissipative effects, we consider a dissipative particle in an asymmetric periodic potential. This problem has been extensively studied in the past for the case of Ohmic dissipation when the potential is symmetric [7–12], but when the potential is asymmetric, the motion of the dissipative particle becomes nonreciprocal under the influence of an external force. We call this model the ratchet model. The classical dynamics of the ratchet model has been researched extensively [13–17], but when studying the ratchet model at low temperatures, it becomes necessary to include quantum effects—namely quantum fluctuations. The quantum ratchet model that incorporates quantum effects into the ratchet model has been studied theoretically in the strong and weak potential limit for the case of Ohmic dissipation [18–21]. Experimentally, the quantum ratchet effect has been observed in artificially fabricated nanostructures [22–24] and possibly in MoS_2, a chiral two-dimensional (2D) superconductor [20,25].

All previous theoretical works on the quantum ratchet model, including our recent work [20], have been limited to the case of Ohmic dissipation, but the super-Ohmic case is also of general interest. For example, acoustic polarons which can be found in clean metals with few impurities are super-Ohmic [26]. In the area of quantum information, super-Ohmic baths are also important when studying the coherence trapping of qubits [27,28].

In general, the spectral density of the environment has the form

$$J(\omega) \propto \omega^s, \qquad \omega \ll \omega_c, \tag{1}$$

where s is the exponent of the frequency dependence. In this paper, we study the quantum ratchet model for the $s = 2$ super-Ohmic case:

$$J(\omega) = \eta \frac{\omega^2}{\omega_{\text{ph}}} \Theta(\omega_c - \omega), \tag{2}$$

where $\Theta(\cdot)$ is the Heaviside step function, ω_c is a cutoff frequency, and ω_{ph} is an arbitrary constant introduced to fix the dimension of the dissipation strength η. We use the sharp cutoff function since it is the most convenient function when calculating the Green's functions that will be introduced later. Like our previous work [20], we focus on the weak-potential limit and calculate velocity perturbatively in powers of potential strength. When dissipation is super-Ohmic, there is no quantum phase transition and the periodic potential is an irrelevant operator for all values of dissipation strength. Hence, the perturbation is valid even at low temperatures for all values of dissipation strength, but for the same reason, DC velocity diverges. Therefore, in this paper, we study finite-frequency mobility.

II. KELDYSH APPROACH

The velocity of a dissipative particle in a periodic potential, $V(x) = V(x+a)$, under a general external force, $F(t)$, can be computed using the Keldysh formalism. We begin by determining the Keldysh action and then derive an expression for finite-frequency mobility in the low-frequency limit.

A. Keldysh action

The Keldysh action has three parts,

$$S = S_0 + S_V + S_F, \tag{3}$$

where S_V and S_F are contributions from the periodic potential and external force respectively. Their exact forms are

$$S_V = -\oint_{\mathcal{C}} dt\, V(x(t)) = -\int_{-\infty}^{\infty} dt\, [V(x_+(t)) - V(x_-(t))], \tag{4}$$

$$S_F = \oint_{\mathcal{C}} dt\, F(t)x(t) = \int_{-\infty}^{\infty} dt\, F(t)[x_+(t) - x_-(t)], \tag{5}$$

where $\mathcal{C}$ is the Keldysh contour from $t = -\infty$ to $t = +\infty$ and back. $x_+(t), x_-(t)$ denote the position variables for the forward and backward contours, respectively.

The first term in Eq. (3), S_0, is the action of the free dissipative particle and it contains the kinetic energy and dissipative effects of the particle. We use the Caldeira-Leggett model [26,27,29,30] to include the effects of dissipation and assume the system-reservoir coupling is linear in both system and reservoir coordinates. This model has the advantage of being quadratic even after tracing out the reservoir. If we rotate the coordinates,

$$\begin{bmatrix} x_+ \\ x_- \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_c \\ x_q \end{bmatrix}, \tag{6}$$

the action S_0 is

$$S_0 = \int_{-\infty}^{\infty} dt\, dt' [x_+(t) \quad x_-(t)] D_0^{-1}(t-t') \begin{bmatrix} x_+(t') \\ x_-(t') \end{bmatrix}, \tag{7}$$

where $D_0^{-1}(t)$ is a 2×2 matrix of Green's functions,

$$D_0^{-1}(t-t') = \begin{bmatrix} 0 & D_0^A(t-t')^{-1} \\ D_0^R(t-t')^{-1} & D_0^K(t-t')^{-1} \end{bmatrix}, \tag{8}$$

where entries

$$D_0^R(t-t') = -i\theta(t-t')\langle [x(t)x(t')] \rangle_0, \tag{9a}$$

$$D_0^A(t-t') = i\theta(t'-t)\langle [x(t)x(t')] \rangle_0, \tag{9b}$$

$$D_0^K(t-t') = -i\langle \{x(t)x(t')\} \rangle_0, \tag{9c}$$

are the retarded, advanced, and Keldysh Green's functions, respectively. $\langle \cdot \rangle_0$ indicates the average is taken with respect to S_0 [31]. These Green's functions are not independent and they satisfy

$$\tilde{D}_0^K(\omega) = \left[\tilde{D}_0^R(\omega) - \tilde{D}_0^A(\omega)\right] \coth(\beta\omega/2), \tag{10a}$$

$$\tilde{D}_0^R(\omega) = \tilde{D}_0^A(\omega)^*, \tag{10b}$$

where $\beta = 1/T$ is inverse temperature. Notice that the temperature dependence of the Keldysh action comes from the Keldysh Green's function.

The Heisenberg equation of motion of $x(t)$—which is the quantum mechanical Langevin equation—is linear, so using the Ehrenfest theorem it is clear the quantum mechanical response function is identical to the classical response function. Hence, the retarded Green's function is

$$\tilde{D}_0^R(\omega) = \frac{1}{M\omega(\omega + i\tilde{\gamma}(\omega))}. \tag{11}$$

The friction kernel is given by [26]

$$\gamma(t) = \Theta(t) \frac{2}{M\pi} \int_0^{\infty} d\omega \frac{J(\omega)}{\omega} \cos(\omega t). \tag{12}$$

Hence, using Eqs. (10a), (11), and (12), we can calculate the free Green's functions from the spectral density function given in Eq. (2) and obtain the explicit form of the Keldysh action.

B. Perturbative correction to finite-frequency mobility

Given the Keldysh action, we can perturbatively compute velocity,

$$v(t) = \frac{1}{2} \frac{\partial}{\partial t} \langle x_+(t) + x_-(t) \rangle, \tag{13}$$

in powers of V and compute finite-frequency mobility. Superscripts are used to indicate the order of the correction. To be precise, nth-order mobility is

$$\mu_n(\vec{\omega}^{(n)}) = \sum_{m=0}^{\infty} \mu_n^{(m)}(\vec{\omega}^{(n)}), \quad \left(\mu_n^{(m)} \sim \mathcal{O}(V^m)\right), \tag{14}$$

where $\vec{\omega}^{(n)} \equiv (\omega_1, \ldots, \omega_n)$. At zeroth order, mobility is linear, so

$$\mu_1^{(0)}(\omega) = i\omega D_0^R(\omega). \tag{15}$$

First-order corrections vanish, so we begin computing corrections from second order. In the low-frequency limit, the second-order mobility is

$$\begin{aligned} &\mu_{n:\mathrm{odd}}^{(2)}(\vec{\omega}^{(n)}) \\ &\approx (-1)^{(n-1)/2} \left(i \sum_{m=1}^{n} \omega_m \right) D_0^R \left(\sum_{m=1}^{n} \omega_m \right) \prod_{m=1}^{n} i\omega_m D_0^R(\omega_m) \\ &\quad \times 2 \sum_k k^{n+1} |V_k{}^2| \int_0^{\infty} t^n \exp\left[\frac{k^2}{2} i\delta D_0^K(t)\right] \sin\left[\frac{k^2}{2} D_0^R(t)\right] dt, \end{aligned} \tag{16}$$

where $\delta D_0^K(t) \equiv D_0^K(t) - D_0^K(0)$ and the summation is over the reciprocal lattice vectors, $k = 2\pi n/a$ $(n \in \mathbb{Z})$. Note that the low-frequency limit is convenient because the temperature-dependent and frequency-dependent factors are decoupled. The exact expression for second-order correction is given by Eq. (A16). Second-order correction is finite only for odd-order responses, so we continue to third-order corrections to find the leading-order contribution to nonreciprocal response.

In the low-frequency limit, third-order mobility is

$$\mu^{(3)}_{n:\text{odd}}(\vec{\omega}^{(n)}) \approx (-1)^{(n-1)/2} 4\left(i\sum_{m=1}^{n}\omega_m\right) D_0^R\left(\sum_{m=1}^{n}\omega_m\right)\prod_{j=1}^{n} i\omega_j D_0^R(\omega_j)\sum_{k_1+k_2+k_3=0}\text{Re}\big[V_{k_1}V_{k_2}V_{k_3}\big]$$
$$\times \int_0^\infty dt_1\int_0^\infty dt_2\, k_1(k_1t_1-k_2t_2)^n \sin\left[\frac{k_1k_2}{2}D_0^R(t_1+t_2)+\frac{k_2k_3}{2}D_0^R(t_2)\right]\sin\left[\frac{k_1k_3}{2}D_0^R(t_1)\right]$$
$$\times \exp\left\{-\frac{i}{2}\big[k_1k_2\delta D_0^K(t_1+t_2)+k_2k_3\delta D_0^K(t_2)+k_1k_3\delta D_0^K(t_1)\big]\right\}, \tag{17}$$

$$\mu^{(3)}_{n:\text{even}}(\vec{\omega}^{(n)}) \approx (-1)^{n/2} 4\left(i\sum_{m=1}^{n}\omega_m\right) D_0^R\left(\sum_{m=1}^{n}\omega_m\right)\prod_{j=1}^{n} i\omega_j D_0^R(\omega_j)\sum_{k_1+k_2+k_3=0}\text{Im}\big[V_{k_1}V_{k_2}V_{k_3}\big]$$
$$\times \int_0^\infty dt_1\int_0^\infty dt_2\, k_1(k_1t_1-k_2t_2)^n \sin\left[\frac{k_1k_2}{2}D_0^R(t_1+t_2)+\frac{k_2k_3}{2}D_0^R(t_2)\right]\sin\left[\frac{k_1k_3}{2}D_0^R(t_1)\right]$$
$$\times \exp\left\{-\frac{i}{2}\big[k_1k_2\delta D_0^K(t_1+t_2)+k_2k_3\delta D_0^K(t_2)+k_1k_3\delta D_0^K(t_1)\big]\right\}. \tag{18}$$

Third-order corrections to even-order mobility is finite if $\text{Im}[V_{k_1}V_{k_2}V_{k_3}] \neq 0$, which is true if the potential is asymmetric [19,20]. Like second-order correction, third-order corrections have a decoupled frequency- and temperature-dependent part in the low-frequency limit. The exact expressions of the third-order corrections of odd- and even-order responses are given by Eqs. (A18) and (A19).

If we assume the periodic potential has the asymmetric form,

$$V(x) = V_1\cos\left(\frac{2\pi x}{a}\right) + V_2\sin\left(\frac{4\pi x}{a}\right), \tag{19}$$

the summation over reciprocal lattice vectors in the third-order corrections are over permutations of $(k_1, k_2, k_3) = \pm 2\pi/a(1, 1, -2)$, and we can define the functions

$$O_n^{(2)}(T) = \int_0^\infty \exp\left[\frac{k^2}{2}i\delta D_0^K(t;T)\right]\sin\left[\frac{k^2}{2}D_0^R(t)\right]t^n\, dt, \tag{20}$$

$$O_n^{(3)}(T) = \sum_{k_1+k_2+k_3=0}\int_0^\infty dt_1\int_0^\infty dt_2\, k_1(k_1t_1-k_2t_2)^n$$
$$\times \sin\left[\frac{k_1k_2}{2}D_0^R(t_1+t_2)+\frac{k_2k_3}{2}D_0^R(t_2)\right]$$
$$\times \sin\left[\frac{k_1k_3}{2}D_0^R(t_1)\right]\exp\left\{-\frac{i}{2}\big[k_1k_2\delta D_0^K(t_1+t_2;T)\right.$$
$$\left.+\,k_2k_3\delta D_0^K(t_2;T)+k_1k_3\delta D_0^K(t_1;T)\big]\right\}, \tag{21}$$

that give us the temperature dependence of the perturbative corrections in the low-frequency limit. In this paper, we focus on examining the behavior of Eqs. (20) and (21).

III. RESULTS

To calculate the second-order and third-order corrections to mobility, which were derived in the previous section for the $s = 2$ super-Ohmic case, we first compute the retarded and Keldysh Green's functions.

A. Green's functions

From the definition of the friction kernel, Eq. (12), and the specified spectral density function, Eq. (2), we have

$$\tilde{\gamma}(\omega) = -i\frac{\gamma_2}{\pi\omega_{\text{ph}}}\omega\log\left(1-\frac{\omega_c^2}{\omega^2}\right). \tag{22}$$

Using Eq. (11), the Fourier transform of the retarded Green's function can be trivially obtained. If we take the branch cut of the complex log to lie along the negative real axis, it is easy to determine that the Fourier transform of the retarded Green's function has two simple poles at $\pm\omega_0$, $(\omega_0 > \omega_c)$ and a branch cut connecting branch points located at $\omega = \pm\omega_c$. When computing the inverse Fourier transform, we push the simple poles and branch cut below the real axis to guarantee the causality of the retarded Green's function. Then, by deforming the contour integral to go around the simple poles and branch cut, we have

$$D_0^R(t) = -\frac{1}{M\omega_c\alpha}\frac{\exp(-1/\alpha)}{\sqrt{1-e^{-1/\alpha}}}\sin(\omega_0 t)$$
$$-\frac{2\alpha}{M}\int_0^{\omega_c}\frac{d\omega}{\omega^2}\frac{\sin(\omega t)}{\left[1+\alpha\log\left(\frac{\omega_c^2}{\omega^2}-1\right)\right]^2+\pi^2\alpha^2}. \tag{23}$$

We defined the dimensionless dissipation strength, $\alpha = \eta/(\pi M\omega_{\text{ph}}) = \gamma/(\pi\omega_{\text{ph}})$, where $\gamma = \eta/M$ is the characteristic frequency of dissipation. The Keldysh Green's function can be determined from Eq. (10a), and we have

$$\delta D_0^K(t) = i\frac{2\alpha}{M}\int_0^{\omega_c}\frac{d\omega}{\omega^2}\frac{\coth\left(\frac{\omega\beta}{2}\right)[1-\cos(\omega t)]}{\left[1+\alpha\log\left(\frac{\omega_c^2}{\omega^2}-1\right)\right]^2+\pi^2\alpha^2}. \tag{24}$$

The retarded Green's function, Eq. (23), has a term that oscillates with constant amplitude. This contribution, which comes from the residue of the simple poles, makes the numerical integration of Eqs. (20) and (21) computationally time consuming. Luckily, it can be discarded when studying the temperature dependence of the perturbative corrections. The justification for this is as follows. Numerically, Eqs. (20)

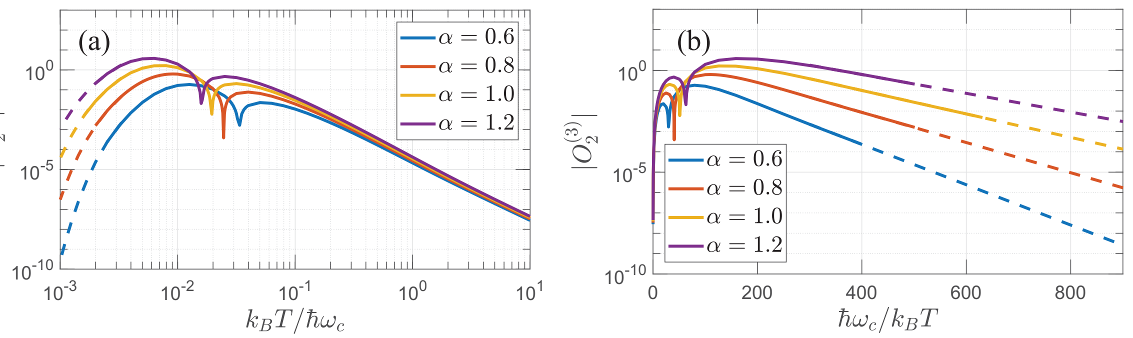

FIG. 1. Temperature dependence of second-order finite-frequency mobility in the low-frequency limit given by Eq. (21) evaluated for the dimensionless dissipation strengths, $\alpha = 0.6, 0.8, 1, 1.2$. As temperature decreases, the convergence of the oscillating integrand becomes slower. At very low temperatures, the round-off error becomes too large. Hence, the data were extrapolated to present a clear picture of the low-temperature behavior; the extrapolated data are represented by the dotted lines. (a) Log-log plot of $O_2^{(3)}$ plotted against temperature. (b) Lin-log plot of $O_2^{(3)}$ plotted against inverse temperature.

and (21) approximately evaluate to the same value and exhibit the same qualitative temperature dependence when the residue contribution is included and excluded as confirmed for the $\alpha = 1$ case. Analytically, the dominant contribution to the integrals come from the long-time region ($t_i > \omega_c^{-1}$) of the integration region and from low-frequency variations ($\omega \ll \omega_c^{-1}$) of the integrand, but the residue term oscillates at the frequency $\omega_0 > \omega_c$, which implies the residue term is essentially canceled out. At high temperatures, the dominant contribution to the integrals come from the short-time region, $t < \omega_c^{-1}$, where both the residue contribution and branch-cut contribution are linear in time, so there is no qualitative difference when the residue contribution is discarded. Physically, the residue contribution from the simple poles is an artifact of the artificial sharp frequency cutoff. It does not represent an essential feature of super-Ohmic dissipation, and we expect it to disappear for a different choice of a cutoff function. Therefore, it is safe to ignore the contribution from the simple poles and simply evaluate the retarded Green's function using the branch-cut integral.

B. Numerical calculation of $O_2^{(3)}$

The numerical calculation of $O_2^{(3)}$, the temperature dependence of second-order finite frequency, is shown in Fig. 1. At high temperatures, $O_2^{(3)}$ is linear with the same slope for different α in the log-log plot, so it clearly decays according to a power law with an exponent that is independent of α. In fact, numerical analysis of our results and an analytic derivation gives us the result, $O_2^{(3)} \sim T^{-3}$. Details of the analytic derivation of the exponent is presented in Appendix B. On the other hand, at low temperatures, $O_2^{(3)}$ is linear in the linear-log plot, so it clearly decays exponentially as a function of inverse temperature, i.e., $O_2^{(3)} \sim \exp(-b/T)$, where $b > 0$ is a constant that is dependent on α. It is clear from Fig. 1(b) that the constant $b > 0$ is a monotonically decreasing function of α. In other words, second-order mobility vanishes more rapidly as temperature decreases for weaker dissipation.

Note that there is a crossover temperature at $T \sim \omega_c$ that divides the low-temperature and high-temperature behavior of $O_2^{(3)}$. The power-law behavior of $O_2^{(3)}$ occurs for $T \gtrsim \omega_c$; on the other hand, for $T \lesssim \omega_c$, the temperature dependence of second-order mobility exhibits a sign change and decays exponentially as temperature decreases.

Hence, to summarize, the temperature dependence of second-order finite frequency is

$$\left|O_2^{(3)}\right| \sim \begin{cases} \exp(-b/T), & T \ll \omega_c, \\ T^{-3}, & T \gg \omega_c. \end{cases} \quad (25)$$

For the case of Ohmic dissipation, the second-order mobility vanishes as a power law of T instead of an exponential decay [20]. To understand this difference, note that super-Ohmic dissipation is, in a sense, weak dissipation. This can be readily understood by considering the following example. Consider a free dissipative particle and assume we apply a constant force F starting at $t = 0$. Then the velocity of this particle is $v = -FD_0^R(t)$. The limit of the Green's function as $t \to \infty$ is

$$D_0^R(t) \to \begin{cases} -\eta, & s = 1 \text{ (Ohmic)} \\ -\infty, & s > 1 \text{ (super-Ohmic)} \end{cases} \quad (t \to \infty). \quad (26)$$

Hence, it is clear that friction for the super-Ohmic case is weaker than the Ohmic case, and as temperature decreases and quantum fluctuations become dominant, we expect second-order mobility to vanish more quickly for the super-Ohmic case than the Ohmic case. This argument gives a rough explanation for why finite-frequency second-order mobility decays exponentially as $T \to 0$ for the $s = 2$ super-Ohmic dissipation case.

On the other hand, second-order mobility vanishes according to a power law as $T \to +\infty$. In this regime, thermal fluctuations are dominant and the mechanism for particle transport comes from thermal hopping instead of quantum tunneling, which is why mobility does not decay exponentially like in the low-temperature limit. Regardless, the high-temperature behavior of mobility is not universal like the low-temperature limit. As discussed in our previous work [20], the exponent of the power law is dependent on the choice of the cutoff function. In our case, we chose a sharp cutoff function, while in our previous work we chose the exponential cutoff function function, so we cannot compare the exponents of the power-law decay of the two cases.

C. General super-Ohmic case ($s > 1$)

To understand the general super-Ohmic case, we also consider the $s = 3$ super-Ohmic case. If we define the spectral density function

$$J_3(\omega) = \eta_3 \frac{\omega^3}{\omega_{\rm ph}^2} \Theta(\omega_c - \omega), \tag{27}$$

where the subscripted numeral 3 serves to differentiate the constants from the $s = 2$ case, the corresponding retarded and Keldysh Green's functions are

$$\begin{aligned} D_0^R(t) = &-\frac{t}{M(1+2\alpha)} + \frac{2\left(\omega_{0,3}^2 - \omega_c^2\right)}{M\omega_{0,3}\left[\omega_{0,3}^2 - \omega_c^2\left(1+2\alpha_3\right)\right]} \\ &\times \sin(\omega_{0,3} t) - \frac{2\alpha_3}{M\omega_c}\int_0^{\omega_c} \frac{d\omega}{\omega} \\ &\times \frac{\sin(\omega t)}{\left\{1+\alpha_3\left[2 - \frac{\omega}{\omega_c}\ln\left(\frac{\omega_c+\omega}{\omega_c-\omega}\right)\right]\right\}^2 + \left(\frac{\pi\alpha_3\omega}{\omega_c}\right)^2}, \end{aligned} \tag{28}$$

and

$$\begin{aligned} \delta D_0^K(t) = &\, i\frac{2\alpha_3}{M\omega_c}\int_0^{\omega_c} \frac{d\omega}{\omega} \\ &\times \frac{\coth\left(\frac{\omega\beta}{2}\right)[1-\cos(\omega t)]}{\left\{1+\alpha_3\left[2 - \frac{\omega}{\omega_c}\ln\left(\frac{\omega_c+\omega}{\omega_c-\omega}\right)\right]\right\}^2 + \left(\frac{\pi\alpha_3\omega}{\omega_c}\right)^2}, \end{aligned} \tag{29}$$

where $\alpha_3 = \eta_3\omega_c/M\pi\omega_{\rm ph}^2$ and $\omega_{0,3} > 0$ is the positive solution to

$$\omega_{0,3} = \frac{\omega_c}{2\alpha_3}\frac{1+2\alpha_3}{\tanh^{-1}\left(\frac{\omega_c}{\omega_{0,3}}\right)}. \tag{30}$$

The numerical integration of $O_2^{(3)}(T)$ for the $s = 3$ case cannot be performed because the integrand oscillates rapidly and round-off error becomes significant. However, computation of $O_1^{(2)}(T)$ is possible, and the numerical results indicate that it also decays exponentially as function of inverse temperature as $T \to 0$.

Therefore, given this result for the $s = 3$ case and our previous generalized argument on how super-Ohmic dissipation corresponds to weak dissipation, we speculate that the exponential decay, $O_2^{(3)} \sim \exp(-b/T)$, as $T \to 0$ is a qualitative feature that occurs for all super-Ohmic cases.

D. Frequency dependence of second-order mobility

Lastly, we comment on the frequency dependence of mobility. An important thing to keep in mind is that the results in Sec. III and the discussion presented in this section are limited to the low-frequency case where $\omega_i \ll \min(\omega_c, \beta^{-1})$. Notice that the frequency dependence of the perturbative correction to mobility comes from products of the factors, $i\omega\tilde{D}_0^R(\omega) = i/M(\omega + i\tilde{\gamma}(\omega))$. The leading-order behavior of $\tilde{\gamma}(\omega)$ is already known for different exponents of the spectral density function [26]. The leading-order behavior is

$$i\omega\tilde{D}_0^R(\omega) \approx \begin{cases} 1/M\tilde{\gamma}(\omega), & s \leqslant 2 \\ i/\omega(M+\Delta M_s), & s > 2 \end{cases} \quad (\omega \to 0^+), \tag{31}$$

where $\Delta M_s = (2/\pi)\Gamma(s-2)(\omega_c/\omega_{\rm ph})^{s-1}(\eta_s/\omega_c)$ is the mass renormalization term. For the $s = 2$ super-Ohmic case, we have

$$i\omega\tilde{D}_0^R(\omega) \approx \frac{-i}{2\alpha\omega\log(\omega/\omega_c)} \quad (\omega \to 0+). \tag{32}$$

Second-harmonic mobility, $\mu_2(\omega, \omega)$, should scale as a function of frequency according to

$$\mu_2(\omega, \omega) \sim \frac{i}{\omega^3\log(\omega/\omega_c)^3}. \tag{33}$$

For other super-Ohmic cases, $s > 2$, second-harmonic mobility scales as

$$\mu_2(\omega, \omega) \sim \frac{i}{\omega^3}. \tag{34}$$

Hence, second-harmonic mobility typically scales as an inverse cube of frequency but has a logarithmic multiplicative correction for the $s = 2$ case.

IV. SUMMARY

In this paper, we studied the nonreciprocal motion of a super-Ohmic dissipative particle in an asymmetric periodic potential. By perturbative expansions in powers of the potential strength, we derived general expressions for second-order and third-order corrections to finite-frequency mobility and numerically calculated their temperature dependence in the low-frequency limit for the case when the spectral density function of the environment has a quadratic frequency dependence ($s = 2$). In the low-temperature limit, the corrections vanished exponentially, and in the high-temperature limit they decayed as a power law. We argued that this is the qualitative behavior for the general super-Ohmic case ($s \geqslant 2$, $s \in \mathbb{Z}^+$). We also derived the frequency dependence of first-order and second-order mobility in the low-frequency limit, which diverge in the DC limit. The main results are summarized in Table I and compared with the results for the case of Ohmic dissipation [20].

It has been shown that the classical-quantum crossover as the temperature is lowered reflects itself as the nonmonotonous temperature dependence of the nonreciprocal nonlinear mobility μ_2. Namely, the quantum mechanical wave does not show nonreciprocal response as the temperature, T, goes to 0. This conclusion is similar to the case of Ohmic dissipation, where the main difference is that T dependence is exponentially activated type at low temperature even though the spectrum of the environment is gapless. This prediction can be tested in a polaron system in noncentrosymmetric crystals, where the phonon heat bath corresponds to $s = d$.

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TABLE I. Temperature dependence and frequency dependence of linear mobility, $\mu_1(T,\omega)$, and second-harmonic mobility, $\mu_2(T,\omega,\omega)$, in the low-frequency ($\omega \to 0$) and low-temperature ($T \to 0$) limit to leading order for the Ohmic [20] and super-Ohmic dissipative cases.

	Ohmic ($s=1$)	super-Ohmic ($s>1,\ s\in\mathbb{Z}^+$)
Quantum phase transition	$\begin{cases}\text{delocalized}, & \alpha<1\\ \text{localized}, & \alpha>1\end{cases}$	None
Linear mobility ($T\to 0,\ \omega\to 0$)	$\mu_1(T)\approx\begin{cases}\frac{1}{\eta}-CT^{2/\alpha-2}+\mathcal{O}(V^3), & \alpha<1\\ C'T^{2\alpha-2}+\mathcal{O}(V^3), & \alpha>1\end{cases}$	$\mu_1(T,\omega)\approx\begin{cases}\frac{1}{i\omega\log\frac{\omega}{\omega_c}}\left(1-\frac{C_{1,s}(\alpha)}{i\omega\log\frac{\omega}{\omega_c}}e^{-C_{2,s}(\alpha)/T}\right)\\ \quad+\mathcal{O}(V^3), & s=2\\ \frac{1}{i\omega}\left(1-\frac{C_{1,s}(\alpha)}{i\omega}e^{-C_{2,s}(\alpha)/T}\right)+\mathcal{O}(V^3), & s\geqslant 3\end{cases}$
Second-order mobility ($T\to 0,\ \omega\to 0$)	$\mu_2(T)\sim\begin{cases}T^{6/\alpha-3}+\mathcal{O}(V^4), & \alpha<1\\ T^{2\alpha-2}+\mathcal{O}(V^4), & \alpha>1\end{cases}$	$\mu_2(T,\omega,\omega)\approx\begin{cases}\frac{C'_{1,s}(\alpha)}{\left(i\omega\log\frac{\omega}{\omega_c}\right)^3}e^{-C'_{2,s}(\alpha)/T}+\mathcal{O}(V^4), & s=2\\ \frac{C'_{1,s}(\alpha)}{(i\omega)^3}e^{-C'_{2,s}(\alpha)/T}+\mathcal{O}(V^4), & s\geqslant 3\end{cases}$

APPENDIX A: DERIVATION OF FINITE-FREQUENCY MOBILITY

In this Appendix, we provide an outline for the derivation of the expressions for second-order and third-order perturbative corrections to finite-frequency mobility.

Before we dive in, we express the action, S_V, S_F, defined in Eqs. (5) and (4) using the rotated coordinates $x_c(t), x_q(t)$ defined by Eq. (6):

$$S_V = -\sum_{\delta=\pm 1}\sum_k V_k\delta\int_{-\infty}^{\infty} dt\,\exp\left\{i\frac{k}{\sqrt{2}}[x_c(t)+\delta x_q(t)]\right\}, \tag{A1}$$

$$S_F = \sqrt{2}\int_{-\infty}^{\infty} dt F(t)x_q(t). \tag{A2}$$

Here, we used the Fourier series representation of the periodic potential: $V(x+a)=V(x)\Rightarrow V(x)=\sum_k V_k e^{ikx}$, where $k=2\pi n/a$ and $n\in\mathbb{Z}$.

1. Velocity

The velocity of the particle is

$$v(t) = \frac{\partial}{\partial t}\langle x(t)\rangle = \frac{1}{2}\frac{\partial}{\partial t}\langle x_+(t)+x_-(t)\rangle$$
$$= \frac{1}{\sqrt{2}}\frac{\partial}{\partial t}\langle x_c(t)\rangle. \tag{A3}$$

If we define the following generating functional,

$$Z[\eta] = \int \mathcal{D}(x_c,x_q)e^{iS[x_c,x_q]+i\int dt'\eta(t')x_c(t')}, \tag{A4}$$

velocity is

$$v(t) = -i\frac{\partial}{\partial t}\left(\frac{\delta Z[\eta]}{\delta\eta(t)}\bigg|_{\eta=0}\right). \tag{A5}$$

We then calculate this functional derivative using a procedure similar to the one given in Ref. [9]. It is straightforward to find

$$v(t) = -\partial_t\int_{-\infty}^{\infty} dt' D_0^R(t-t')\left[F(t')+\frac{1}{\sqrt{2}}\,\mathrm{Tr}\{\rho_q(t')G[F]\}\right], \tag{A6}$$

where $\mathrm{Tr}\{\cdot\}$ is defined as

$$\mathrm{Tr}\{\cdot\} \equiv \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\sum_{\delta_1\cdots\delta_n=\pm 1}\sum_{k_1\cdots k_n}\prod_{i=1}^{n}V_{k_i}\delta_i\int dt_i\{\cdot\}. \tag{A7}$$

Notice that the nth term of the overall summation is the nth-order perturbative correction to velocity. To simplify calculations, we introduced the charge densities, $\rho_c^{(n)}$ and $\rho_q^{(n)}$, where the superscript denotes the order of perturbation they belong to. For nth-order perturbation, there are n charges and the charge densities are defined as

$$\rho_c^{(n)}(t) = \frac{1}{\sqrt{2}}\sum_{j=1}^{n}k_j\delta(t-t_j), \tag{A8a}$$

$$\rho_q^{(n)}(t) = \frac{1}{\sqrt{2}}\sum_{j=1}^{n}k_j\delta_j\delta(t-t_j). \tag{A8b}$$

The trace simply sums over all possible configurations of these charge densities. The functional, $G[F]$, is defined,

$$G^{(n)}[F] = \exp\left\{-\frac{i}{4}\sum_{i,j}k_ik_j\left[D_0^K(t_i-t_j)+2\delta_iD_0^R(t_j-t_i)\right]\right.$$
$$\left.-i\sum_i k_i\int dt'F(t')D_0^R(t_i-t')\right\}. \tag{A9}$$

Now that we have an expression for velocity, $v(t)$, we define finite-frequency mobility in the following way:

$$v(t)[F] = \int \mu_1(t-t_1)F(t_1)dt_1$$
$$+\frac{1}{2!}\int \mu_2(t-t_1,t-t_2)F(t_1)F(t_2)dt_1\,dt_2$$
$$+\cdots \tag{A10}$$
$$= \sum_{n=1}^{\infty}\frac{1}{n!}\int dt_1\ldots dt_n\mu_n(t-t_1,\ldots,t-t_n)$$
$$\times F(t_1)\ldots F(t_n) \tag{A11}$$

$$= \sum_{n=1}^{\infty} \frac{1}{n!} \int \frac{d\omega_1}{2\pi} \dots \frac{d\omega_n}{2\pi} e^{-i(\omega_1+\dots+\omega_n)t} \times \mu(\omega_1, \dots, \omega_n) F(\omega_1) \dots F(\omega_n). \quad \text{(A12)}$$

The kernels of the integrals are response functions which we define as the finite-frequency mobilities. The nth-order mobility can be obtained from velocity by a functional derivative with respect to the external force,

$$\mu_n(\vec{\omega}^{(n)}) = (2\pi)^n \frac{\delta^n}{\delta F(\omega_1) \dots \delta F(\omega_n)} v(t=0)|_{F=0}, \quad \text{(A13)}$$

where $\vec{\omega}^{(n)} \equiv (\omega_1, \dots, \omega_n)$.

2. Momentum conservation

Before we perturbatively expand the expression for velocity given in Eq. (A6), we prove that only $\sum_{j=1}^{n} k_j = 0$ configurations contribute to the trace defined in Eq. (A7).

First, note that the Keldysh Green's function diverges as a result of a nonintegrable divergence in the integrand at zero frequency and goes to $D_0^K(t) \to -i\infty$. However, $\delta D_0^K(t) \equiv D_0^K(t) - D_0^K(0)$ converges nicely. From this result, we can derive the condition of momentum conservation, $\sum_{i=1}^{n} k_i = 0$. Note that $G^{(n)}[F]$ contains the factor $\exp[-\frac{i}{4}\sum_{i,j} k_i k_j D_0^K(t_i - t_j)]$. Rearranging the terms of the argument gives us

$$\begin{aligned} &\sum_{l,j} k_l k_j i D_0^K(t_l - t_j) \\ &= \sum_{l \neq j} k_l k_j i\delta D_0^K(t_l - t_j) + \left(\sum_j k_j\right)^2 i D_0^K(0) \\ &= \begin{cases} +\infty, & \sum_j k_j \neq 0 \\ 0, & \sum_j k_j = 0 \end{cases}. \end{aligned} \quad \text{(A14)}$$

Therefore, $G^{(n)}[F]$ vanishes if the sum of the momentum is nonzero, i.e., $\sum_j k_j \neq 0$, and so the only contribution to Eq. (A6) comes from configurations that conserve momentum. We refer to this condition as momentum conservation and heavily use it to derive the exact expression of finite-frequency mobility.

3. Second-order perturbative correction

In this subsection, we compute the second-order perturbative correction to velocity and mobility. We begin with computing the trace in Eq. (A6). From the definitions, Eqs. (A7) and (A9),

$$\begin{aligned} &\frac{1}{\sqrt{2}} \mathrm{Tr}\{\rho_q(t') G[F]\}^{(2)} \\ &= \frac{(-i)^2}{2!\sqrt{2}} \sum_{\delta_1,\delta_2=\pm1} \sum_{k_1+k_2=0} V_{k_1} V_{k_2} \delta_1 \delta_2 \\ &\quad \times \iint dt_1\, dt_2\, \rho_q(t') G^{(2)}[F] \\ &= 2 \sum_k k |V_k{}^2| \int_0^\infty ds \exp\left[\frac{k^2}{2} i\delta D_0^K(s)\right] \sin\left[\frac{k^2}{2} D_0^R(s)\right] \\ &\quad \times \sin\left[k \int \frac{d\omega'}{2\pi} D_0^R(\omega') F(\omega') e^{-i\omega' t'} (e^{i\omega' s} - 1)\right], \end{aligned} \quad \text{(A15)}$$

where the last equality was obtained after substituting the definitions of ρ_q and $G^{(2)}[F]$ and rearranging some terms. Next, after substituting this into the definition of velocity, Eq. (A6), and using the definition of mobility, Eq. (A13), we have

$$\begin{aligned} \mu_{n:\mathrm{odd}}^{(2)}(\vec{\omega}^{(n)}) &= (-1)^{(n-1)/2} 2 \left(i \sum_{m=1}^{n} \omega_m\right) D_0^R\left(\sum_{m=1}^{n} \omega_m\right) \\ &\quad \times \prod_{m=1}^{n} D_0^R(\omega_m) \sum_k k^{n+1} |V_k{}^2| \\ &\quad \times \int_0^\infty dt \exp\left[\frac{k^2}{2} i\delta D_0^K(t)\right] \\ &\quad \times \sin\left[\frac{k^2}{2} D_0^R(t)\right] \prod_{m=1}^{n} (e^{i\omega_m t} - 1), \end{aligned} \quad \text{(A16)}$$

where $\vec{\omega}^{(n)} \equiv (\omega_1, \dots, \omega_n)$. The low-frequency limit of this equation is given by Eq. (16) in the main text of this paper.

4. Third-order perturbative correction

Next, we compute the third-order perturbative correction to velocity and mobility. The steps are the same for the second-order correction but each step is more involved. The trace in the definition of velocity, Eq. (A6), is

$$\begin{aligned} &\frac{1}{\sqrt{2}} \mathrm{Tr}\{\rho_q(t') G[F]\}^{(3)} \\ &= \frac{(-i)^3}{3!\sqrt{2}} \sum_{\delta_1,\delta_2,\delta_3=\pm1} \sum_{k_1+k_2+k_3=0} V_{k_1} V_{k_2} V_{k_3} \delta_1 \delta_2 \delta_3 \iiint dt_1\, dt_2\, dt_1\, \rho_q(t') G^{(3)}[F] \\ &= -4 \sum_{k_1+k_2+k_3=0} k_1 \mathrm{Re}[V_{k_1} V_{k_2} V_{k_3}] \int_0^\infty dt_1 \int_0^\infty dt_2 \sin\left[\int \frac{d\omega'}{2\pi} F(\omega') D_0^R(\omega') e^{-i\omega' t'} \left(k_1 + k_2 e^{i\omega'(t_2+t_1)} + k_3 e^{i\omega' t_1}\right)\right] \\ &\quad \times \exp\left\{-\frac{i}{2}\left[k_1 k_2 \delta D_0^K(t_2+t_1) + k_2 k_3 \delta D_0^K(t_2) + k_1 k_3 \delta D_0^K(t_1)\right]\right\} \sin\left[\frac{k_1 k_2}{2} D_0^R(t_2+t_1) + \frac{k_2 k_3}{2} D_0^R(t_2)\right] \sin\left[\frac{k_1 k_3}{2} D_0^R(t_1)\right] \end{aligned}$$

$$+ 4 \sum_{k_1+k_2+k_3=0} k_1 \mathrm{Im}\big[V_{k_1}V_{k_2}V_{k_3}\big] \int_0^\infty dt_1 \int_0^\infty dt_2 \cos\left[\int \frac{d\omega'}{2\pi} F(\omega') D_0^R(\omega') e^{-i\omega' t'} \big(k_1 + k_2 e^{i\omega'(t_2+t_1)} + k_3 e^{i\omega' t_1}\big)\right]$$
$$\times \exp\left\{-\frac{i}{2}\big[k_1k_2\delta D_0^K(t_2+t_1) + k_2k_3\delta D_0^K(t_2) + k_1k_3\delta D_0^K(t_1)\big]\right\} \sin\left[\frac{k_1k_2}{2}D_0^R(t_2+t_1) + \frac{k_2k_3}{2}D_0^R(t_2)\right] \sin\left[\frac{k_1k_3}{2}D_0^R(t_1)\right] \quad \text{(A17)}$$

Notice that there the trace is separated into two terms that are odd and even with respect to the external force, $F(\omega)$. To obtain the third equality, we took advantage of momentum conservation and the causality of the retarded Green's function. Next, we use the definition of mobility, Eq. (A13), and find two separate expressions for odd- and even-order mobilities:

$$\mu^{(3)}_{n:\mathrm{odd}}(\vec{\omega}^{(n)}) = (-1)^{(n-1)/2} 4 \left(i\sum_{m=1}^n \omega_m\right) D_0^R\left(\sum_{m=1}^n \omega_m\right) \sum_{k_1+k_2+k_3=0} k_1 \mathrm{Re}\big[V_{k_1}V_{k_2}V_{k_3}\big] \int_0^\infty dt_1 \int_0^\infty dt_2 \sin\left[\frac{k_1k_3}{2}D_0^R(t_1)\right]$$
$$\times \sin\left[\frac{k_1k_2}{2}D_0^R(t_1+t_2) + \frac{k_2k_3}{2}D_0^R(t_2)\right] \prod_{j=1}^n D_0^R(\omega_j)\big[k_2(1-e^{i\omega_j(t_1+t_2)}) + k_3(1-e^{i\omega_j t_1})\big]$$
$$\times \exp\left\{-\frac{i}{2}\big[k_1k_2\delta D_0^K(t_1+t_2) + k_2k_3\delta D_0^K(t_2) + k_1k_3\delta D_0^K(t_1)\big]\right\}, \quad \text{(A18)}$$

$$\mu^{(3)}_{n:\mathrm{even}}(\vec{\omega}^{(n)}) = (-1)^{n/2} 4 \left(i\sum_{m=1}^n \omega_m\right) D_0^R\left(\sum_{m=1}^n \omega_m\right) \sum_{k_1+k_2+k_3=0} k_1 \mathrm{Im}\big[V_{k_1}V_{k_2}V_{k_3}\big] \int_0^\infty dt_1 \int_0^\infty dt_2 \sin\left[\frac{k_1k_3}{2}D_0^R(t_1)\right]$$
$$\times \sin\left[\frac{k_1k_2}{2}D_0^R(t_1+t_2) + \frac{k_2k_3}{2}D_0^R(t_2)\right] \prod_{j=1}^n D_0^R(\omega_j)\big[k_2(1-e^{i\omega_j(t_1+t_2)}) + k_3(1-e^{i\omega_j t_1})\big]$$
$$\times \exp\left\{-\frac{i}{2}\big[k_1k_2\delta D_0^K(t_1+t_2) + k_2k_3\delta D_0^K(t_2) + k_1k_3\delta D_0^K(t_1)\big]\right\}. \quad \text{(A19)}$$

The low-frequency limit of these two equations are given by Eqs. (17) and (18) in the main text of this paper. They were obtained by naively expanding the integrands of the exact expression to first order in ω_i. Roughly speaking, this approximation is valid because the integrand has a rapidly decaying exponential term that cuts off the integration region at $t \sim \max(\omega_c^{-1}, \beta)$. Hence, the expressions for the corrections in the low-frequency limit are valid for $\omega \ll \min(\omega_c, \beta^{-1})$.

APPENDIX B: DERIVATION OF THE POWER-LAW DECAY OF $O_2^{(3)}$ IN THE HIGH-TEMPERATURE LIMIT

At high temperatures, $T \gg \omega_c$, the temperature-dependent part of second-order finite-frequency mobility in the low-frequency limit is proportional to $O_2^{(3)}(T) \sim T^{-3}$. This can be readily observed from our numerical results presented in Fig. 1. This result can also be derived analytically.

In the high-temperature limit, as $T \to \infty$, short-time or high-frequency dynamics becomes relevant. Hence, we assume that the largest contribution to the double integral of $O_n^{(3)}$ comes from the integration region, $\{(t_1, t_2)\,|\, t_1, t_2 < \omega_c^{-1}\}$. In this region, the retarded and Keldysh Green's function can clearly be approximated by expanding the integrand to leading-order in t. This gives us

$$i\delta D_0^K(t) \approx -\frac{t^2}{\beta}\int_0^{\omega_c} d\omega \frac{2\alpha/M\omega}{\big[1+\alpha\log\big(\frac{\omega_c^2}{\omega^2}-1\big)\big]^2 + \pi^2\alpha^2}$$
$$= -J\frac{t^2}{\beta}, \quad (\omega_c t \ll 1), \quad \text{(B1)}$$

and

$$D_0^R(t) \approx -t\int_0^{\omega_c} d\omega \frac{2\alpha/M\omega}{\big[1+\alpha\log\big(\frac{\omega_c^2}{\omega^2}-1\big)\big]^2 + \pi^2\alpha^2}$$
$$= -Jt, \quad (\omega_c t \ll 1), \quad \text{(B2)}$$

where $J > 0$ is a constant. We substitute these approximations into the integrand of $O_n^{(3)}$ given by Eq. (21). Assuming that the retarded Green's function is sufficiently small, the sine function is expanded to first order. Hence, we have

$$O_n^{(3)} \sim \int_0^{\omega_c^{-1}}\int_0^{\omega_c^{-1}} dt_1\, dt_2\, (k_1t_1 - k_2t_2)^n \big(k_2^2 t_2 - k_1k_2t_1\big)t_1$$
$$\times \exp\left\{-\frac{J}{2\beta}\big[k_1k_2(t_1+t_2)^2 + k_2k_3t_2^2 + k_1k_3t_1^2\big]\right\}. \quad \text{(B3)}$$

Next, we perform the change of coordinates, $u_i = t_i/\sqrt{\beta}$, which produces the factor, $\sqrt{\beta}$, for each t_i,

$$O_n^{(3)} \sim \beta^{(4+n)/2}\int_0^{\omega_c^{-1}/\sqrt{\beta}}\int_0^{\omega_c^{-1}/\sqrt{\beta}} du_1\, du_2$$
$$\times (k_1u_1 - k_2u_2)^n \big(k_2^2u_2 - k_1k_2u_1\big)t_1$$
$$\times \exp\left\{-\frac{J}{2}\big[k_1k_2(u_1+u_2)^2 + k_2k_3u_2^2 + k_1k_3u_1^2\big]\right\}. \quad \text{(B4)}$$

Because of the Gaussian cutoff in the integrand, we assume we can safely extend the upper limit of the integration region to infinity, i.e., $\omega_c^{-1}/\sqrt{\beta} \to +\infty$. This gives us

$$\therefore O_n^{(3)} \sim T^{-(4+n)/2} \quad \Rightarrow \quad O_2^{(3)} \sim T^{-3}. \quad \text{(B5)}$$

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