Rapid Communications

Alternating current-induced interfacial spin-transfer torque

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We investigate an interfacial spin-transfer torque and β-term torque with alternating current (AC) parallel to a magnetic interface. We find that both torques are resonantly enhanced as the AC frequency approaches the exchange splitting energy. We show that this resonance allows us to estimate directly the interfacial exchange interaction strength from the domain wall motion. We also find that the β term includes an unconventional contribution which is proportional to the time derivative of the current and exists even in the absence of any spin relaxation processes.

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Introduction. A variety of physical phenomena arises near interfaces, such as spin-dependent transports [1–6], interfacial magnetic phenomena [7–12], and chiral/topological phenomena [13–16], which have attracted attention for many years [17]. Among these, the spin-dependent transport has been closely related to aspects of not only fundamental physics but device application; especially, tunneling magnetoresistance [2–4] has impacted the invention of magnetoresistive random access memory [18].

Spin-dependent transport near the interfaces is important from the viewpoint of understanding recent developments in spintronics, such as the spin pumping effect (SPE) [19–25] and the spin Seebeck effect (SSE) [26,27], because the mutual dependence between the magnetization dynamics and the spin-dependent transport is the key mechanism in various spin-dependent phenomena. The two effects are ways of generating spin currents without electric currents, in a bilayer system consisting of a ferromagnet (FM) and a normal metal (NM); the spin precession due to the rf microwave in FM induces the spin current in NM in the case of SPE, and the temperature difference between FM and NM induces that for SSE. Both effects can be described by the tunnel Hamiltonian method [24,27,28], which also captures tunneling magnetoresistance.

The interfacial exchange interaction between conduction electrons in NM and magnetization in FM plays a crucial role in SPE and SSE, which are proportional to J_{sd}^2, where J_{sd} is the interfacial exchange interaction strength [24,27]. In general, the exchange interaction possibly gives rise to an essential contribution to spin-related phenomena near the interfaces, such as spin Hall magnetoresistance [29]. However, this physically essential parameter J_{sd} has not been directly measured, and a direct method of evaluating it lacking.

In this Rapid Communication, we present a direct method of evaluating the interfacial exchange interaction strength J_{sd} from the domain wall dynamics in FM adjoined by NM, by applying an alternating current (AC) parallel to the interface [Fig. 1(a)]. It is a well-known fact that in bulk ferromagnetic metals with noncollinear magnetic textures such as domain walls, the direct current (DC) accompanying spin polarization exerts spin torques on the magnetization, which leads to dynamics such as the domain wall motion [30–35]. We here extend the DC-induced spin torques into the region of a finite frequency of the current, based on the quantum field theoretical approach, and apply this to the interfacial exchange interacting system of a FM-NM bilayer. We consider a thin NM so that we focus only on the spin-polarized electronic states near the interface due to the interfacial exchange interaction. Combined with the magnetization in FM having a texture such as the domain wall, the interfacial exchange interaction may lead to types of spin-transfer and β-term torques by applying an electric field. Our setup could be realized, for example, in a bilayer of ultrathin copper (Cu) and permalloy (Py). We find that AC-induced spin torques consist of corresponding extensions of the spin-transfer torque [34,36–38] and the so-called β-term torque [34,39–42]. However, we also find that the results we obtain include physically another contribution to the β-term torque, which depends on the time derivative of the current density. Our important finding is that both spin torques are proportional to $(1-\omega^2\tau_{sd}^2)^{-1}$ for the case of no spin relaxation processes, where ω is the AC frequency and $\tau_{sd} = \hbar/2\Delta$ with 2Δ being the interfacial exchange splitting. The exchange splitting is related to J_{sd} by $\Delta = SJ_{sd}$, where S is the localized spin length constructing the magnetization. This dependence suggests that we can evaluate J_{sd} from the magnetization dynamics driven by the spin torques. From the viewpoint of application, the enhancement of the spin torques has an advantage in that less current density is needed to excite the magnetization dynamics.

We then solve the equation of motion of a rigid domain wall (DW) [35,38] driven by the obtained spin torques, in the presence of a spin relaxation process. The equation is expressed by the two collective coordinates, the position of the DW center X and the angle ϕ_0 [Fig. 1(b)], and the spin torques act as the forces to X and ϕ_0. We find that X and ϕ_0 oscillate

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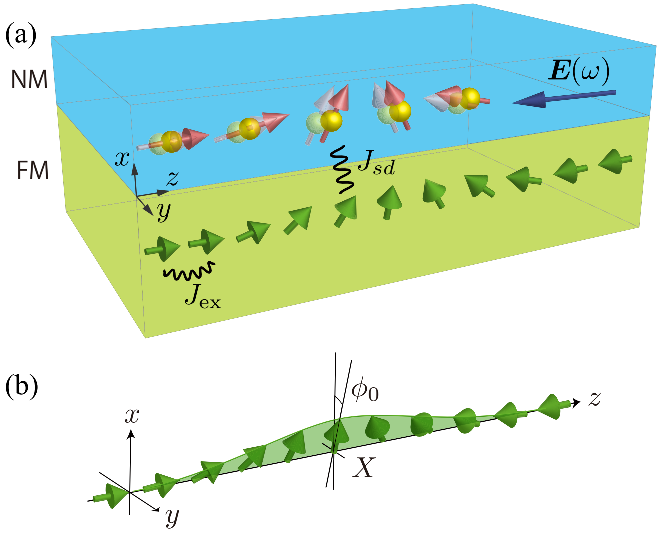

FIG. 1. (a) Schematic description of the configuration, where a ferromagnet (FM) having two magnetic domains with one domain wall (DW) is adjoined by a normal metal (NM) whose conduction electron couples with the magnetization in FM through an interfacial exchange interaction. Alternating current (AC) is applied parallel to the interface due to an electric field with finite frequency. AC-induced spin torques on the DW lead to the oscillation of the position of the DW center, which allows us to evaluate the interfacial exchange interaction strength. (b) DW configuration, which is described by the corrective coordinates of the DW center X and the angle ϕ_0.

along with the frequency ω in the region of the small electric current density, and the amplitude of the oscillation of X increases resonantly near $\omega\tau_{sd} \simeq 1$. Hence, we conclude that the dependence of X on the frequency allows us to estimate the interfacial exchange splitting.

This Rapid Communication is organized as follows. We first present the total Lagrangian of the magnetization in FM and the conduction electron in NM as well as their interfacial exchange interaction, and introduce the *rotated frame picture* sometimes used in the context of ferromagnetic spintronics. Then, the AC-induced spin torques are evaluated based on the linear response theory with the thermal Green's function method. As an application, we consider the DW dynamics driven by the obtained spin torques.

Theory. The total Lagrangian that we consider is given by $\mathcal{L} = \mathcal{L}_m + \mathcal{L}_e - \mathcal{H}_{sd}$, where $\mathcal{L}_m$ is the Lagrangian of the magnetization in the FM layer, $\mathcal{L}_e$ is that of the conduction electron in the NM layer, and $\mathcal{H}_{sd}$ is the sd-like interfacial exchange interaction between them.

Considering that the magnetization is constructed by localized spin ordering, we express the Lagrangian of the magnetization as that of a localized spin, $\boldsymbol{M} = -M_S\boldsymbol{m}$ with $\boldsymbol{m} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$, where M_S is the saturated magnetization, and $\theta = \theta(\boldsymbol{r}, t)$, $\phi = \phi(\boldsymbol{r}, t)$. Here, $\boldsymbol{m}$ does not represent the unit vector of the magnetization, but that of the localized spin, whose signs are opposite. The Lagrangian of the localized spin is defined as $\mathcal{L}_m = \int d\boldsymbol{r}(\hbar S/a^3)\dot{\phi}(\cos\theta - 1) - \mathcal{H}_m$ with

$$\mathcal{H}_m = \int \frac{d\boldsymbol{r}}{a^3}\left[\frac{J_{\rm ex}}{2}S^2(\nabla\boldsymbol{m})^2 - \frac{K}{2}S^2m_z^2 + \frac{K_\perp}{2}S^2m_y^2\right], \quad (1)$$

where a is the lattice constant of FM, $J_{\rm ex}$ is the exchange interaction between the localized spins, and K and $K_\perp$ are easy- and hard-axis magnetic anisotropies, respectively. Note that the saturated magnetization M_S is related to the localized spin length by $M_S = \gamma_e\hbar S/a^3$ with the gyromagnetic ratio γ_e, and $J_{\rm ex}$, K, and $K_\perp$ are all positive.

We show the rest of the Lagrangian, which is written as $\mathcal{L}_e - \mathcal{H}_{sd} = \int d\boldsymbol{r}\psi^\dagger(\boldsymbol{r}, t)(i\hbar\partial_t - H_e - H_{sd})\psi(\boldsymbol{r}, t)$, where $\psi^{(\dagger)}$ is the field operator of electrons, $H_e = \boldsymbol{p}^2/2m_e + V$ describes the kinetic energy with the electron mass m_e, and the nonmagnetic and magnetic impurity potentials given by $V = u_{\rm i}\sum_{i=1}^{N_{\rm i}}\delta(\boldsymbol{r} - \boldsymbol{R}_i) + u_{\rm s}\sum_{j=1}^{N_{\rm s}}(\boldsymbol{S}_{{\rm imp},j}\cdot\boldsymbol{\sigma})\delta(\boldsymbol{r} - \boldsymbol{R}'_j)$ with the impurity numbers $N_{\rm i}$ and $N_{\rm s}$ and with the strengths $u_{\rm i}$ and $u_{\rm s}$, and $H_{sd} = -\Delta\boldsymbol{m}(\boldsymbol{r}, t)\cdot\boldsymbol{\sigma}$ represents the interfacial exchange interaction with the coupling constant $\Delta > 0$ with $\boldsymbol{\sigma} = (\sigma^x, \sigma^y, \sigma^z)$ being the Pauli matrices. The magnetic impurity spin $\boldsymbol{S}_{{\rm imp},j}$ is assumed to be quenched.

Then, we transform the Hamiltonian into the "rotated frame" [35,43] by using the unitary transformation $U(\boldsymbol{r}, t)$ defined by $U^\dagger(\boldsymbol{r}, t)[\boldsymbol{m}(\boldsymbol{r}, t)\cdot\boldsymbol{\sigma}]U(\boldsymbol{r}, t) = \sigma^z$ with $\bar{\psi} = U^\dagger\psi$. The physical meaning of the unitary transformation is that the quantization axis of the electron spin is to be reoriented to $\boldsymbol{m}(\boldsymbol{r}, t)$ at each position and time. Hence, we call the frame after the transformation as the rotated frame and denote $\bar{A}$ as the quantity A in the rotated frame. The electron described by $\bar{\psi}^{(\dagger)}$ feels the uniform exchange interaction in the rotated frame. We also express the rotational unitary transformation by using the rotational matrix $\mathcal{R}(\boldsymbol{r}, t)$ for the three-dimensional vector defined by $U^\dagger(\boldsymbol{r}, t)\boldsymbol{\sigma}U(\boldsymbol{r}, t) = \mathcal{R}(\boldsymbol{r}, t)\boldsymbol{\sigma}$. This expression of unitary transformation is useful for the magnetic impurity potential and the spin torques. Note that the relation to the definition of U is $U^\dagger(\boldsymbol{m}\cdot\boldsymbol{\sigma})U = \boldsymbol{m}\cdot(\mathcal{R}\boldsymbol{\sigma}) = (\mathcal{R}^{-1}\boldsymbol{m})\cdot\boldsymbol{\sigma} = \sigma^z$, hence $\mathcal{R}^{-1}\boldsymbol{m} = \hat{z}$, where $\hat{z}$ is the unit vector along the z axis.

We now look into the equation of motion of the localized spin, which is obtained from the Euler-Lagrange equation with the relaxation function $\mathcal{W}$ [35],

$$\frac{d}{dt}\left(\frac{\delta\mathcal{L}}{\delta\dot{q}}\right) - \frac{\delta\mathcal{L}}{\delta q} = -\frac{\delta\mathcal{W}}{\delta\dot{q}}, \quad (2)$$

where $q \in \{\theta, \phi\}$, and $\mathcal{W} = \int d\boldsymbol{r}(\hbar S\alpha_{\rm G}/2a^3)\dot{\boldsymbol{m}}^2$ with the Gilbert damping constant $\alpha_{\rm G}$. Equation (2) leads to the Landau-Lifshitz-Gilbert equation, $\dot{\boldsymbol{m}} = \gamma_e\boldsymbol{H}_{\rm eff}\times\boldsymbol{m} + \alpha_{\rm G}\dot{\boldsymbol{m}}\times\boldsymbol{m} + \boldsymbol{\tau}_e$, where $\boldsymbol{H}_{\rm eff}$ is the effective magnetic field defined as $\gamma_e\boldsymbol{H}_{\rm eff} = (1/\hbar S)\,\delta\mathcal{H}_m/\delta\boldsymbol{m}$, and $\boldsymbol{\tau}_e$ is the spin torque through the interfacial exchange interaction,

$$\gamma_e\boldsymbol{H}_{sd} = \frac{1}{\hbar S}\left\langle\frac{\delta\mathcal{H}_{sd}}{\delta\boldsymbol{m}}\right\rangle_{\rm neq}, \quad (3)$$

$$\boldsymbol{\tau}_e = \gamma_e\boldsymbol{H}_{sd}\times\boldsymbol{m} = -\frac{\Delta}{\hbar S}\langle\boldsymbol{s}\rangle_{\rm neq}\times\boldsymbol{m}. \quad (4)$$

Here, $\boldsymbol{s} = \boldsymbol{s}(\boldsymbol{r}, t) = \psi^\dagger(\boldsymbol{r}, t)\boldsymbol{\sigma}\psi(\boldsymbol{r}, t)$ is the spin density operator divided by $\hbar/2$, and $\langle\cdots\rangle_{\rm neq}$ describes the statistical average in the nonequilibrium.

The spin torque is expressed in the rotated frame as $\bar{\boldsymbol{\tau}}_e = \mathcal{R}^{-1}\boldsymbol{\tau}_e = -(\Delta/\hbar S)\langle\bar{\boldsymbol{s}}\rangle_{\rm neq}\times\hat{z}$. We emphasize that, in the rotated frame, the perpendicular components of the nonequilibrium spin polarization $\langle\bar{s}\rangle_{\rm neq}$ to the $\hat{z}$ axis only act as torques.

In this Rapid Communication, we evaluate the nonequilibrium spin polarization $\langle\bar{s}\rangle_{\rm neq}$ in the linear response to the spatially uniform electric field $\boldsymbol{E}(t)$ with the frequency ω, $\boldsymbol{E}(t)=\boldsymbol{E}_0e^{-i\omega t}$, as

$$\langle\bar{s}^\alpha(\boldsymbol{r},t)\rangle_{\rm neq}=\int_{-\infty}^{\infty}dt'\,\bar{\chi}^{\alpha i}(\boldsymbol{r},t-t')E_i(t')$$

($\alpha=x,y$ and $i=y,z$). In the Fourier space, the form is expressed as

$$\langle\bar{s}^\alpha(\boldsymbol{q},t)\rangle_{\rm neq}=\bar{\chi}^{\alpha i}(\boldsymbol{q},\omega)E_{0,i}e^{-i\omega t}. \tag{5}$$

From the linear response theory, we can obtain the response coefficient $\bar{\chi}^{\alpha i}(\boldsymbol{q},\omega)$ from $\bar{\chi}^{\alpha i}(\boldsymbol{q};\omega)=[\bar{K}^{\alpha i}(\boldsymbol{q};\omega)-\bar{K}^{\alpha i}(\boldsymbol{q};0)]/i\omega$, where $\bar{K}^{\alpha i}(\boldsymbol{q};\omega)$ can be evaluated from the following spin-current correlation function in the Matsubara form

$$\bar{\mathcal{K}}^{\alpha i}(\boldsymbol{q};i\omega_\lambda)=\int_0^{1/k_{\rm B}T}d\tau\,e^{i\omega_\lambda\tau}\langle{\rm T}_\tau\bar{s}^\alpha(\boldsymbol{q},\tau)\bar{J}_i(0)\rangle_{\rm eq} \tag{6}$$

through the analytic continuation $i\omega_\lambda\to\hbar\omega+i0$, $\bar{K}^{\alpha i}(\boldsymbol{q};\omega)=\bar{\mathcal{K}}^{\alpha i}(\boldsymbol{q};\hbar\omega+i0)$. Here, $\omega_\lambda=2\pi\lambda k_{\rm B}T$ is the Matsubara frequency of bosons with the temperature T [44], and the spin and the electric current operator in the rotated frame are given by

$$\bar{s}^\alpha(\boldsymbol{q})=\frac{1}{V}\sum_{\boldsymbol{k}}\bar{c}^\dagger_{\boldsymbol{k}-\boldsymbol{q}}\sigma^\alpha\bar{c}_{\boldsymbol{k}}, \tag{7}$$

$$\bar{J}_i=-e\sum_{\boldsymbol{k}}\frac{\hbar k_i}{m_e}\bar{c}^\dagger_{\boldsymbol{k}}\bar{c}_{\boldsymbol{k}}-\frac{e\hbar}{2m_e}\sum_{\boldsymbol{q}'}A_i^\beta(\boldsymbol{q}')\sum_{\boldsymbol{k}}\bar{c}^\dagger_{\boldsymbol{k}}\sigma^\beta\bar{c}_{\boldsymbol{k}-\boldsymbol{q}'}, \tag{8}$$

where $\bar{c}^{(\dagger)}_{\boldsymbol{k}}$ is the Fourier transform of the field operator $\bar{\psi}^{(\dagger)}(\boldsymbol{r})$. The first term of Eq. (8) is the normal velocity term and the second is the anomalous velocity term due to the spin gauge field $A_i^\beta(\boldsymbol{r})=-i\,{\rm tr}\,[U^\dagger\partial_iU\sigma^\beta]$ with $\beta=x,y,z$.

As the detailed calculation will be shown elsewhere, here we sketch out the procedures of the calculation. By substituting Eqs. (7) and (8) into Eq. (6), we rewrite the correlation function by using the thermal Green's functions according to Wick's theorem. We expand the Green's function by the spin gauge field up to the first order, take the statistical average on the impurity positions, and then we obtain

$$\bar{\mathcal{K}}^{\alpha i}(\boldsymbol{q};i\omega_\lambda)=\tfrac{e\hbar}{4m_eV}A_j^\beta(\boldsymbol{q})k_{\rm B}T\sum_n\sum_{\boldsymbol{k}}\Phi_{ij}^{\alpha\beta}(\boldsymbol{k};i\epsilon_n^+,i\epsilon_n), \tag{9}$$

where $\epsilon_n^+=\epsilon_n+\omega_\lambda$, $\epsilon_n=(2n+1)k_{\rm B}T$ is the Matsubara frequency of fermions, and

$$\begin{aligned}\Phi_{ij}^{\alpha\beta}(\boldsymbol{k};i\epsilon_n^+,i\epsilon_n)&=2\delta_{ij}\,{\rm tr}\,[\Lambda^\alpha g_{\boldsymbol{k}}(i\epsilon_n^+)\sigma^\beta g_{\boldsymbol{k}}(i\epsilon_n)]\\&\quad-\frac{\hbar^2k_ik_j}{m_e}{\rm tr}\,[\Lambda^\alpha g_{\boldsymbol{k}}(i\epsilon_n^+)\sigma^\beta g_{\boldsymbol{k}}(i\epsilon_n^+)g_{\boldsymbol{k}}(i\epsilon_n)]\\&\quad-\frac{\hbar^2k_ik_j}{m_e}{\rm tr}\,[\Lambda^\alpha g_{\boldsymbol{k}}(i\epsilon_n^+)g_{\boldsymbol{k}}(i\epsilon_n)\sigma^\beta g_{\boldsymbol{k}}(i\epsilon_n)].\end{aligned} \tag{10}$$

Here, $g_{\boldsymbol{k}}(i\epsilon_n^{(+)})=[i\epsilon_n^{(+)}+\mu-\hbar^2k^2/2m_e-\Delta\sigma^z-\Sigma(i\epsilon_n^{(+)})]^{-1}$ is the thermal Green's function with the self-energy within the self-consistent Born approximation $\Sigma(i\epsilon_n)=n_{\rm i}u_{\rm i}^2\sum_{\boldsymbol{k}}g_{\boldsymbol{k}}(i\epsilon_n)+n_{\rm s}u_s^2S_{\rm imp}^2\sum_{\boldsymbol{k}}\sigma^\gamma g_{\boldsymbol{k}}(i\epsilon_n)\sigma^\gamma$, where $n_{\rm i}$ and $n_{\rm s}$ are the impurity concentrations of nonmagnetic and magnetic impurities, respectively, and we have taken the statistical average on the impurity spins and assume the spherical spins, $\overline{S^\alpha_{{\rm imp},i}S^\beta_{{\rm imp},j}}=(S^2_{\rm imp}/3)\delta_{ij}\delta^{\alpha\beta}$. In Eq. (10), we have evaluated $\Phi_{ij}^{\alpha\beta}$ by assuming $\boldsymbol{q}=\boldsymbol{0}$ since $\bar{\mathcal{K}}^{\alpha i}(\boldsymbol{q};i\omega_\lambda)$ is already in the q-linear order because of $A_i^\beta(\boldsymbol{q})$. The full vertex of spin $\Lambda^\sigma=\Lambda^\sigma(i\epsilon_n^+,i\epsilon_n)$ is given by $\Lambda^\alpha=\sigma^\alpha+n_{\rm i}u_{\rm i}^2\sum_{\boldsymbol{k}}g_{\boldsymbol{k}}(i\epsilon_n^+)\Lambda^\alpha g_{\boldsymbol{k}}(i\epsilon_n)+\frac{1}{3}n_{\rm s}u_{\rm s}^2S^2_{\rm imp}\sum_{\boldsymbol{k}}\sigma^\gamma g_{\boldsymbol{k}}(i\epsilon_n^+)\Lambda^\alpha g_{\boldsymbol{k}}(i\epsilon_n)\sigma^\gamma$. After some straightforward calculation and taking the analytic continuation with the assumption of $T=0$, we then obtain

$$\bar{\chi}^{\alpha i}(\boldsymbol{q},\omega)=\frac{\hbar\sigma_{\rm s}}{e}A_i^\beta(\boldsymbol{q})\sum_{\sigma=\pm}(\delta^{\alpha\beta}+i\sigma\epsilon^{\alpha\beta z})\frac{\sigma}{2\sigma\Delta-\hbar\omega+i\hbar/\tau_{\rm s}}, \tag{11}$$

where we neglected the higher-order contribution of $\hbar/\epsilon_{{\rm F}\sigma}\tau_\sigma$ with the spin-dependent Fermi energy $\epsilon_{{\rm F}\sigma}$ and the momentum lifetime τ_σ. Here, $\sigma_{\rm s}=\sigma_{\rm s}(\omega)$ is the spin conductivity, $\sigma_{\rm s}(\omega)=(e^2/m_e)\sum_{\sigma=\pm}\sigma n_\sigma\tau_\sigma/(1-i\omega\tau_\sigma)$, with the spin-dependent electron density n_σ and lifetime τ_σ with $\sigma=\pm$, and$\tau_{\rm s}$ is the relaxation time due to the magnetic impurity scattering defined as $\tau_{\rm s}^{-1}=(2\pi/3\hbar)n_{\rm s}u_{\rm s}^2S^2_{\rm imp}(\nu_++\nu_-)$, where $\nu_\pm$ is the density of states at the Fermi level.

Results. Here, we show the expression of the AC-induced spin-transfer torque and β-term torque obtained from Eq. (11) combined with Eqs. (5) and (4),

$$\boldsymbol{\tau}_e(\boldsymbol{r},t)=\frac{(\boldsymbol{j}_{\rm s}\cdot\nabla)\boldsymbol{m}+(i\omega\tau_{sd}+\zeta_{\rm s})\boldsymbol{m}\times(\boldsymbol{j}_{\rm s}\cdot\nabla)\boldsymbol{m}}{1+(i\omega\tau_{sd}+\zeta_{\rm s})^2} \tag{12}$$

in the laboratory frame, where $\tau_{sd}=\hbar/2\Delta$, $\zeta_{\rm s}=\tau_{sd}/\tau_{\rm s}$, and we used $\mathcal{R}\boldsymbol{A}_i^\perp=-\boldsymbol{m}\times\partial_i\boldsymbol{m}$ and $\mathcal{R}(\hat{z}\times\boldsymbol{A}_i^\perp)=\partial_i\boldsymbol{m}$. The frequency-dependent spin current is denoted by $\boldsymbol{j}_{\rm s}=\boldsymbol{j}_{\rm s}(t)=(\hbar/2eS)\sigma_{\rm s}(\omega)\boldsymbol{E}_0e^{-i\omega t}$. By taking the static field limit of $\omega\to0$, we find that the first term proportional to $(\boldsymbol{j}_{\rm s}\cdot\nabla)\boldsymbol{m}$ corresponds to the spin-transfer torque and the second term proportional to $\boldsymbol{m}\times(\boldsymbol{j}_{\rm s}\cdot\nabla)\boldsymbol{m}$ coincides with the β-term torque, and we confirm that our result agrees with that of Zhang and Li [34] and of Kohno and Shibata [42] for the model of the conduction electron in a ferromagnet, although they are not for the interfacial exchange interaction as in our situation. Our result (12) is an extension of the DC-induced spin-transfer and β-term torques into a case with finite frequency. Note that our theory can be adapted for the bulk conducting ferromagnet, where the exchange interaction is not an interfacial one but an s-d exchange interaction. In the bulk case, our theory keeps the same form, where we just interpret the interfacial exchange interaction here as the bulk s-d exchange interaction. Equation (12) is the main result of this Rapid Communication.

Considering the case of the dilute magnetic-impurity concentration, $\tau_{\rm s}\to\infty$, so that $\zeta_{\rm s}\to0$, we find

$$\boldsymbol{\tau}_e(\boldsymbol{r},t)=\frac{1}{1-\omega^2\tau_{sd}^2}[(\boldsymbol{j}_{\rm s}\cdot\nabla)\boldsymbol{m}+i\omega\tau_{sd}\boldsymbol{m}\times(\boldsymbol{j}_{\rm s}\cdot\nabla)\boldsymbol{m}],$$

which implies that the spin torques increase resonantly as the AC frequency approaches to the $1/\tau_{sd}$. As shown below, this frequency dependence allows us to determine the magnitude of the interfacial exchange interaction.

We also find that the β-term torque is present proportional to the frequency ω, without the magnetic impurity which results in a spin relaxation process. The β-term torques are known to arise from the spin relaxation process [34], such as the scattering due to the magnetic impurity potential [45] and spin-orbit impurity potential [46]. Actually, Eq. (12) shows that there is also a β term proportional to the magnetic impurity concentration, $\zeta_s \sim n_s$. The β-term torques also arise from *nonadiabaticity*, which stands for the higher order of the derivatives, such as the terms proportional to $\boldsymbol{m} \times \partial_t \partial_i \boldsymbol{m}$ [40,47]. From $\boldsymbol{j}_s(t) \propto e^{-i\omega t}$, we can write the obtained β-term torque as

$$(\beta\text{-term torque}) = -\boldsymbol{m} \times \left(\tau_{sd} \frac{d\boldsymbol{j}_s}{dt} \cdot \nabla\right)\boldsymbol{m}$$

for $\omega \ll \tau_{sd}^{-1}$, which is the first-order derivative for the magnetization, not higher orders. For these reasons, the β-term torque we obtain is different from the ones which are already known.

It should be discussed how the spin torques obtained here relate to the Rashba spin-orbit torques (SOTs) [48–51] and the spin Hall torques (SHTs) [52–54]. Both Rashba SOTs and SHTs originate from spin-orbit couplings (SOCs); the Rashba SOT comes from the interfacial SOC due to the inversion symmetry breaking and SHT arises from the bulk SOC in NM. We have assumed that these SOCs are weak so that these torques do not contribute much; for instance, that is the case for Cu as a NM and Py as a FM. For strong SOCs, we have to develop our theory which contains these strong SOCs, but that is out of the focus of this Rapid Communication.

Application. Now, we focus on the domain wall (DW) motion as an application of the obtained torques. Following Tatara *et al.* [35] and assuming $K_\perp \ll K$ and no pinning potentials, we rewrite the Lagrangian $\mathcal{L}_m$ into that of the DW, introducing the corrective coordinates of the DW center $X(t)$ and the angle $\phi_0(t)$ [Fig. 1(b)],

$$\boldsymbol{m} = \left(\frac{\cos\phi_0(t)}{\cosh\frac{z-X(t)}{\lambda}}, \quad \frac{\sin\phi_0(t)}{\cosh\frac{z-X(t)}{\lambda}}, \quad \tanh\frac{z-X(t)}{\lambda}\right), \tag{13}$$

where $\lambda = \sqrt{J_{ex}/K}$ is the DW width. By using $X(t)$ and $\phi_0(t)$, the DW Lagrangian and the dissipation function are written by $\mathcal{L}_w = N_w S(\hbar \dot{X}\phi_0/\lambda - (K_\perp S/2)\sin^2\phi_0)$ and $\mathcal{W}_w = \frac{\alpha N_w \hbar S}{2}(\frac{\dot{X}^2}{\lambda^2} + \dot{\phi}_0^2)$, where $N_w = 2\lambda A/a^3$ is the number of spins in the wall with A being the cross-sectional area. We have neglected the spin-wave excitations. From these, the equation of motion is written as

$$\dot{\phi}_0 + \alpha\frac{\dot{X}}{\lambda} = -\tau_{sd}\frac{d\mathcal{T}}{dt} + \zeta_s\mathcal{T}, \tag{14a}$$

$$\frac{\dot{X}}{\lambda} - \alpha\dot{\phi}_0 = \frac{v_c}{\lambda}\sin 2\phi_0 + \mathcal{T}, \tag{14b}$$

where

$$\mathcal{T} = \frac{a^3}{2eS\lambda}\frac{Pj_0 e^{-i\omega t}}{1 + (i\omega\tau_{sd} + \zeta_s)^2}, \quad v_c = \frac{K_\perp \lambda S}{2\hbar}, \tag{14c}$$

with the electric current density j_0 and its polarization P. Here, $\mathcal{T}$ is the spin torques that we obtain and act as the forces on $X(t)$ and $\phi_0(t)$. Solving Eqs. (14a) and (14b) numerically, we find that the DW position $X(t)$ and angle $\phi_0(t)$ oscillate with the period $2\pi/\omega$ for the low current density $(a^3/2eS)Pj_0 \lesssim v_c$. We also find that the amplitude of the oscillations becomes larger as $\omega\tau_{sd}$ approaches unitarity (Fig. 2). Figure 2 depicts the oscillation amplitude of the DW position for the case of $(a^3/2eS)Pj_0/v_c = 10^{-4}$ and $v_c\tau_{sd}/\lambda = 0.1$, which are equivalent to the case where, for $v_c \simeq 3$ m/s [33] and $a \sim 1.5$ Å, $j_0/S \sim 3 \times 10^8$ A/m^2 for $P = 0.1$ and $\tau_{sd} = 6.7 \times 10^{-10}$ s assuming $\lambda = 20$ nm. Hence, when observing the DW position as changing the AC frequency, we estimate the exchange interaction strength from the particular frequency in which the oscillation amplitude takes a maximum value. Note that the current density is four orders smaller than the common one [33].

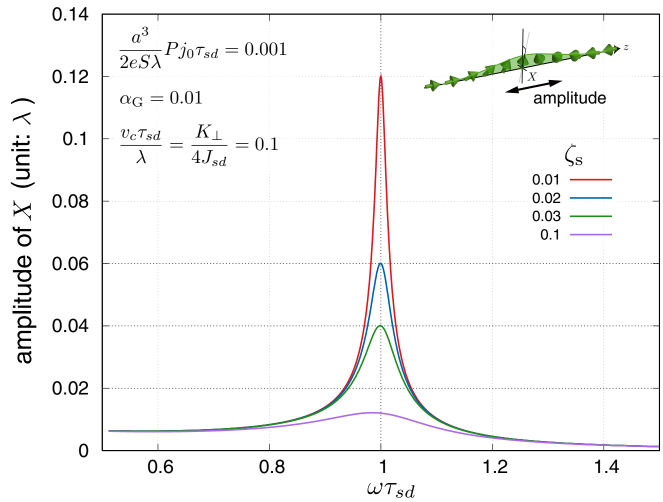

FIG. 2. Amplitude of the oscillation of the position X during the period $1/2\pi\omega$ for various ζ_s. It is clear that the amplitude enhances near the resonance point $\omega\tau_{sd} = 1$.

In conclusion, we have developed a theory of the interfacial spin-transfer torque and β-term torque, by consider a bilayer structure of a normal metal and ferromagnet with a spatially varying magnetic texture, applying alternating current parallel to the interface. We find that both torques are enhanced as the alternating current frequency ω approaches $1/\tau_{sd} = 2\Delta/\hbar$. We also find that the β-term torque we obtain here includes a contribution which is proportional to the time derivative of the current and exists even in the absence of spin relaxation processes. Evaluating the domain wall motion due to the spin torques, we directly estimate the interfacial exchange interaction strength. We have revealed an aspect of the spin-transfer torque with finite frequency, which is enhanced by the resonance of electronic states. By using this enhancement, less current density is needed for magnetization dynamics, which may lead to low-energy consuming magnetic devices.

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