Rapid Communications

Emergent mode and bound states in single-component one-dimensional lattice fermionic systems

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(Received 30 November 2018; revised manuscript received 9 October 2019; published 7 November 2019)

We study the formation of bound states in a one-dimensional, single-component Fermi chain with attractive interactions. The phase diagram, computed from DMRG (density matrix renormalization group), shows not only a superfluid of paired fermions (pair phase) and a liquid of three-fermion bound states (trion phase), but also a phase with two gapless modes. We show that the latter phase is described by a two-component Tomonaga-Luttinger liquid (TLL) theory, consisting of one charged and one neutral mode. We argue based on our numerical data, that the single, pair, and trion phases are descendants of the two-component TLL theory. We speculate on the nature of the phase transitions amongst these phases.

DOI: 10.1103/PhysRevB.100.201101

Tomonaga-Luttinger liquid (TLL) theory captures the physics of many one-dimensional (1D) quantum systems such as spin chains, spin ladders, nanotubes [1], nanowires [2], and cold atoms confined to 1D tubes [3–7]. In higher-dimensional systems, TLL is a tool that is often used, e.g., in edge theory [8] and coupled-wire constructions [9–11].

Recently, there has been significant interest in the study of 1D systems that cannot be described by the standard TLL theory [12–18]. In describing 1D interacting fermions, TLL theory naturally arises through bosonization that maps fermionic modes to bosonic modes. Nearby phases (i.e., descendants) such as charge density order appear as instabilities of the parent TLL theory [19–26]. This approach breaks down at the weak to strong pairing transition, i.e., the transition to the p-wave paired liquid. As recently pointed out in Ref. [27,28], the p-wave pairing phase cannot be described as a descendant phase of a single-mode TLL; instead the transition is described by an emergent mode theory, with the weak and strong pairing phases being descendants of this theory. Which raises the question: what other phases, beyond p-wave pairing, can appear in one-component interacting fermions and how are these phases connected to some emergent-mode description?

In this Rapid Communication, we investigate the formation of multifermion bound states in 1D single-component systems. We perform DMRG numerics on a lattice model with finite-range interactions, and find liquids of singles, pairs, trions, etc., in addition to an extended phase with two gapless modes [two-mode (2M) phase]. We unify these findings by constructing an effective theory with an emergent mode that characterizes the 2M phase; the descendants of this theory describe the liquid phases of single fermions as well as multifermion bound states (i.e., bound states of $2, 3, 4, \dots$ fermions). Our construction [Eq. (4)] is not equivalent to the band-bending construction in Ref. [28] (see Supplemenal Material [29]) but is similar to Ref. [27] (see discussion at the end of the Rapid Communication).

Microscopic model. We study the lattice Hamiltonian

$$H = \sum_i \left[-\frac{1}{2}(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) + \sum_{m=1}^{3} V_m n_i n_{i+m} \right], \qquad (1)$$

where c_i and $c_i^\dagger$ are the fermion annihilation and creation operators at lattice site i, $n_i = c_i^\dagger c_i$ is the number operator, and V_m defines the shape of the fermion-fermion interaction potential. We choose short-ranged attractive interactions ($V_1 < 0$ and $V_2 < 0$) to promote the formation of pairs and trions, but with $V_3 > 0$ to prevent phase separation [27]. To decrease the parameter space, we restrict our attention to the subspace $V_1 = V_2$. We expect that extending the range of attractive interactions will result in more liquid phases of multifermion bound states. For example, we demonstrate that extending the attractive interactions to three sites results in a quaternion liquid phase [29].

We use infinite-system DMRG (iDMRG) [30–32] to study the ground-state properties of the Hamiltonian (1) with a focus on the 1/5 filling. The accuracy of iDMRG is controlled by the bond dimension χ; the result becomes exact as $\chi \to \infty$ [29]. To identify the various phases, we use two types of diagnostics: central charge c and various two-point correlators.

We obtain c as follows. We study the bipartite entanglement entropy S, i.e., the von Neumann entropy of DMRG ground state traced over either half of the system. Both S and the correlation length ξ are infinite for the true ground state, but are cut off by finite χ. The manner in which these two variables diverge gives the central charge: $S = \frac{c}{6}\log(\xi) +$ const [33,34].

We also compute the single, pair, and trion two-point correlators

$$G_1(r) = \langle c_i^\dagger\, c_{i+r} \rangle, \qquad (2a)$$

$$G_2(r) = \langle (c_i c_{i+1})^\dagger\, c_{i+r} c_{i+r+1} \rangle, \qquad (2b)$$

$$G_3(r) = \langle (c_i c_{i+1} c_{i+2})^\dagger\, c_{i+r} c_{i+r+1} c_{i+r+2} \rangle. \qquad (2c)$$

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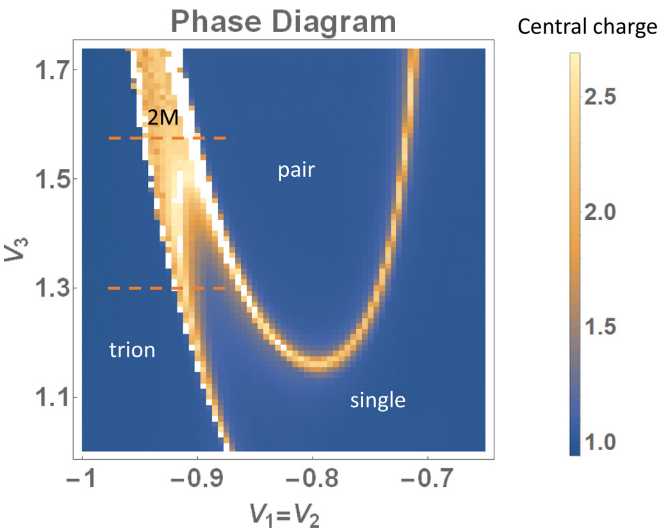

FIG. 1. Central charge as a function of interactions in the lattice model (1) computed at filling fraction 1/5. Four phases labeled are identified with further analysis of correlation functions. We have checked that all the reported phases exist for $V_1 = 2V_2$ and we expect the qualitative features of the phase diagram to hold for generic values of $V_1 \neq V_2$ in the vicinity of $V_1 = V_2$.

In the single phase, all correlators decay algebraically; in the pair phase, only G_2 decays algebraically while G_1 and G_3 decay exponentially; in the trion phase, G_3 is algebraic while G_1 and G_2 are exponential. This behavior implies that there is a bulk gap to adding a single fermion into the pair (trion) phase but no bulk gap to adding two (three) fermions.

Figure 1 shows c as a function of the interaction parameters $V_1 = V_2$ and V_3. The blue regions denote the single-mode phases with $c = 1$; we identify these as single, pair, and trion phases based on their two-point correlators [Eq. (1)]. While we observe a direct transition between the pair and single phases [27,28], we do not find a direct transition between the pair and trion phases; instead we find an intermediate phase with $c \approx 2$ which we call the 2M (two-mode) phase. The 2M phase neighbors all other phases and indicates a parent theory with an emergent mode, which enables a unified description of the multifermion bound-state phases and their transitions.

Our strategy for the remainder of the Rapid Communication is as follows. First, we introduce a field theory to describe the 2M phase, writing down its operators and free Hamiltonian. Next, we introduce the possible interaction terms, and examine the descendant phases that result. Finally, we show that these predictions are consistent with our numerics, justifying our theoretical model.

Theory of the emergent mode. Motivated by Refs. [27,28] and our data, we introduce a theory with two modes. In this theory, the charge-1 operators in the lowest harmonic are [35]

$$\psi_{0,\eta}^{(\pm)} = e^{\pm i\theta_1} e^{i\theta_0 + i\eta(\phi_0 + k_F x)},$$
$$\psi_{1,\eta}^{(\pm)} = e^{i\theta_0} e^{\pm i\theta_1 \pm i\eta(\phi_1 + k' x)}, \tag{3}$$

where $\eta = +1$ (-1) denotes a right (left) mover; θ_μ is the dual field of the compact bosonic field ϕ_μ and satisfies $[\partial_x\theta_\mu(x), \phi_\nu(x')] = i\pi\delta_{\mu\nu}\delta(x - x')$. The charge is carried by the θ_0 mode, while θ_1 is neutral; as a result, k_F is fixed by the density of electrons while k' is a free parameter.

The set of local physical operators can be generated via products of operators from Eq. (3), i.e., $(\psi_{0,1}^{+})^l(\psi_{0,-1}^{+})^m(\psi_{1,1}^{-})^n\cdots$. [Note that the generators Eq. (3) are overcomplete.] As a result, primary operators of charge q take the form

$$c(x)^q \sim \sum_{q_1, r_0, r_1} e^{i[q\theta_0 + q_1\theta_1 + r_0(\phi_0 + k_F x) + r_1(\phi_1 + k' x)]},$$
$$\text{where } q_1 \equiv r_0 + r_1 \equiv q \pmod 2). \tag{4}$$

Due to the restrictions on the coefficients q_1, r_0, and r_1 of physical operators, we cannot simply treat this theory as a product of decoupled θ_0/ϕ_0 and θ_1/ϕ_1 theories.

The theory must obey charge conservation, and be invariant under both parity ($\phi_{0,1} \to -\phi_{0,1}$ and $x \to -x$) and time reversal ($\theta_{0,1} \to -\theta_{0,1}$, $i \to -i$, and $t \to -t$). The kinetic part of the Hamiltonian takes the form

$$\mathcal{H}_{\text{KE}} = \sum_{\mu,\nu}[A_{\mu\nu}(\partial_x\theta_\mu)(\partial_x\theta_\nu) + B_{\mu\nu}(\partial_x\phi_\mu)(\partial_x\phi_\nu)]. \tag{5}$$

$\mathcal{H}_{\text{KE}}$ describes a two-mode TLL, which we later demonstrate to be consistent with the 2M phase found in the numerics.

Single-mode phases as descendants of the 2M theory. The single-mode phases (single, pair, trion,...) are constructed by introducing locking terms, shown in Table I, to the Hamiltonian (5). For a term to appear, it must be of the form of Eq. (4) with $q = 0$, and also respect parity and time reversal. At large interaction strength, some of these terms may "lock" [26], taking an expectation value and reducing the theory to a one-component TLL.

Our analysis for the locking terms follows [36]. For an interaction term to lock, it should have no oscillation (i.e., x dependence), which places constraints on the Fermi momenta. For each locking term of the form $\cos\Lambda$, we find linear combinations of the θ's and ϕ's that commute with Λ. Among this set we find a conjugate pair which we denote as θ_+ and ϕ_+. The set of gapless operators is then generated by $e^{i\Lambda}$, $e^{i\theta_+}$, and $e^{i\phi_+}$, and must be a subset of Eq. (4) [37]. We show that the minimal (unit) charges for these operators are indeed $q_{\min} = 1, 2,$ and 3 for the single, pair, and trion phases, respectively, from the given locking terms. We extend our analysis to arbitrary $q_{\min}$ in the Supplemental Material [29].

We first analyze the locking term $\cos(2\theta_1)$ which induces the single phase. The gapless mode is described by the dual fields $\theta_+ = \theta_0$ and $\phi_+ = \phi_0$. Thus the gapless operators take the form $c(x) \sim \sum e^{ia\theta_1} e^{i\theta_0} e^{i(2n+1)(\phi_0 + k_F x)}$ where n is an integer and a an odd integer. (The dual field ϕ_1 is disordered and cannot appear here.) As $e^{i\theta_1}$ is a constant, $c(x)$ reduces to the standard bosonization form of a fermion mode [26,38].

Next, we show that the locking term $\cos(2\phi_1 + 2k'x)$ induces the pair phase. Notably, for this term to lock we must enforce $k' = 0$. As θ_1 is disordered, it cannot appear in a gapless operator, i.e., $q_1 = 0$. From the parity relation (4), we see that q must be an even integer and thus the single and trion correlators decay exponentially. Letting $\theta_+ = 2\theta_0$ and $\phi_+ = \phi_0/2$, we recover the standard bosonization expansion of a boson mode [26,38] for the pair operator: $c(x)^2 \sim b(x) \approx \sum e^{i\theta_+} e^{i(2n)(\phi_+ + k_B x)}$ with $n \in \mathbb{Z}$ and $k_B = k_F/2$. We interpret

TABLE I. Locking terms and correlators of single-mode phases. The first line lists interaction terms and the second line shows the corresponding phases when interaction terms get locked. The remaining rows show the algebraic decay form of correlators $G_{1,2,3}$; the coefficient of each term is neglected for simplicity. Figure 2 shows the numeric data verifying the predicted dependence.

Locking term	$\cos(2\theta_1)$	$\cos(2k'x+2\phi_1)$	$\cos[(3k'-k_F)x+3\phi_1-\phi_0]$	…
Resulting phase	single	pair	trion	…
Single correlator $G_1(r)$	$\sum_n \frac{\sin[(2n+1)k_F\|r\|]}{\|r\|^{(1/K+(2n+1)^2K)/2}}$			
Pair correlator $G_2(r)$	$\sum_n \frac{\cos[(2n)k_F\|r\|]}{\|r\|^{2/K+2n^2K}}$	$\sum_n \frac{\cos[(2n)\frac{k_F}{2}\|r\|]}{\|r\|^{(1/K+(2n)^2K)/2}}$		
Trion correlator $G_3(r)$	$\sum_n \frac{\sin[(2n+1)k_F\|r\|]}{\|r\|^{(9/K+(2n+1)^2K)/2}}$		$\sum_n \frac{\sin[(2n+1)\frac{k_F}{3}\|r\|]}{\|r\|^{(1/K+(2n+1)^2K)/2}}$	

this descendant theory as a TLL of fermion pairs, with the density of pairs being half of the density of elementary fermions.

Finally, we address the locking term $\Lambda = 3(\phi_1 + k'x) - (\phi_0 + k_F x)$ which yields the trion phase while fixing $k' = k_F/3$. As Λ commutes with $\theta_+ = 3\theta_0 + \theta_1$ and $\phi_+ = \phi_1$, the gapless operators take the form $c(x)^q \sim \sum e^{i(q/3)\theta_+} e^{ia(\phi_+ + k'x)+ibL}$. Mapping the expression to Eq. (4), we get $q_1 = q/3$, $r_0 = -b$, and $r_1 = a + 3b$; we determine the consistency conditions $q/3, a, b \in \mathbb{Z}$ and $a \equiv q \pmod 2$. Hence for any gapless operator, q must be a multiple of 3, which implies exponential decay of G_1 and G_2. The trion operator expansion reduces to $c(x)^3 \sim \sum e^{i\theta_+} e^{i(2n+1)(\phi_+ + k'x)}$, where $k' = k_F/3$ is the Fermi wave vector of the trions and n is an integer.

Within the low-energy theory for each of the three single TLL mode phases, $c(x)^{q_{\min}}$ admits a standard bosonization expansion in terms of θ_+ and ϕ_+. The effective Hamiltonian is thus

$$\mathcal{H}_+ = \frac{v_+}{2\pi}\left[K(\partial_x\theta_+)^2 + \frac{1}{K}(\partial_x\phi_+)^2\right], \tag{6}$$

where K is the Luttinger parameter.

Fourier spectra of the correlators. The long-distance behavior of the correlation functions of gapless operators can be written as a sum of algebraically decaying terms of the form

$$\frac{\cos(k_{\mathrm{osc}}|r| + \varphi)}{|r|^\eta}. \tag{7}$$

Our theory puts a restriction on the allowed values of k_{osc} in the 2M phase and the single-mode phases. Table I summarizes the long-distance behavior of the correlation functions within the single-mode phases; observe that the (leading) decay exponents η of all harmonics k_{osc} depend only on the Luttinger parameter K; this is verified in Fig. 2.

To connect the effective theory to our microscopic model, we compare the k_{osc} in correlation functions obtained from field theory and DMRG. We perform Fourier transforms on the correlation functions $G_{1,2,3}(r)$ and take the nth derivative, such that terms of the form Eq. (7) with $\eta < n+1$ will show a divergent peak at k_{osc}. We then match the set of predicted oscillation wave vectors to peaks in the Fourier transforms. Figure 3 presents the correlation functions along cuts at constant V_3. Panels (a)–(c) show a cut through the trion, 2M, and pair phases; while panels (d)–(f) cut from trion to single phase (with a possible intervening 2M phase).

In the 2M phase, both modes are gapless and the allowed k_{osc}'s are given by the oscillatory part of $c(x)^q$ in Eq. (4):

$$k_{\mathrm{osc}} = r_0 k_F + r_1 k'. \tag{8}$$

For G_1 and G_3, $r_0 + r_1$ is odd, hence the first several k_{osc} are k', k_F, $k_F \pm 2k'$, and $2k_F \pm k'$. For G_2, $r_0 + r_1$ is even and so $k_{\mathrm{osc}} = 0$, $k_F \pm k'$, $2k'$, $2k_F$, etc. These wave vectors are fitted to numerical data and are marked by the dotted lines in Fig. 3. As k_F is fixed, k' is the only fitting parameter at each point of phase space. In the 2M region of panels (a)–(c), we observe unambiguous peaks at the predicted wave vectors. (In the numerics, we have not resolved peaks at some of the predicted k_{osc}, as these peaks are too weak or have large exponent η.)

A key feature of the DMRG data in the 2M phase is that k' varies continuously between the two limiting values: $k' = k_F/3$ on the trion side and $k' = 0$ on the pair side. The variation of this wave vector is a clear sign of a neutral emergent mode and confirms our effective two-mode TLL [39].

In the single-mode phases, some of the peaks found for the 2M phase persist while others are no longer divergent as modes become gapped out. The trion phase is characterized by the absence of singular behavior in panels (a),(b),(d),(e) as $G_{1,2}$ decay exponentially. We observe that G_3 decays algebraically with peaks in Figs. 3(c) and 3(f) at odd multiples of $k' = k_F/3$. In the pair phase, $k' = 0$ and only G_2 shows divergent peaks at multiples of k_F, as predicted. [The features at 0, k_F in panel (a) are not divergent and broadened out due to G_1 being gapped. Deep in the pair phase, they become invisible.] Finally, for the single phase all three correlators

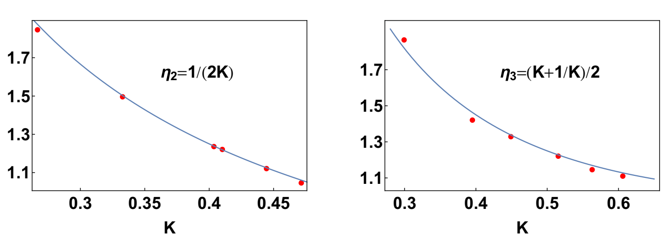

FIG. 2. Verification of predicted decay exponents. Left: leading decay exponent (η_2) of pair correlator in pair phase at $V_1 = V_2 = -0.8$, $V_3 = 1.4$. Right: leading decay exponent (η_3) of trion correlator in trion phase at $V_1 = V_2 = -1$, $V_3 = 1.4$. The two lines are predictions from TLL theory (cf. Table I), $\eta_2 = \frac{1}{2K}$ and $\eta_3 = \frac{1}{2}(K + \frac{1}{K})$. The values of Luttinger parameter K are extracted from the neutral sector [29]. In order to cover a larger range of K, we use DMRG data from fillings (left to right) $\frac{1}{5}, \frac{1}{6}, \ldots, \frac{1}{10}$.

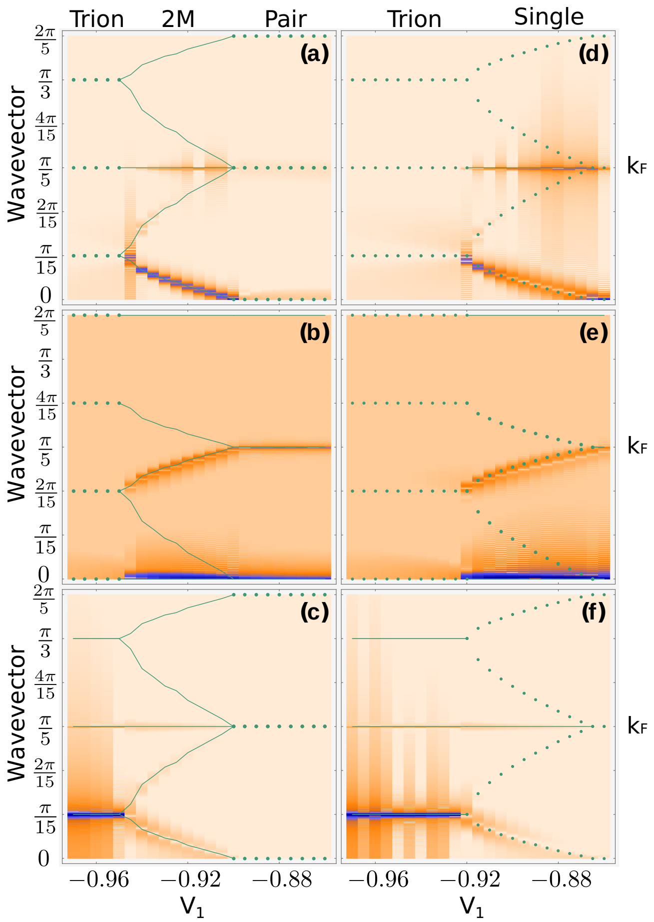

FIG. 3. Spectra G_1, G_2, and G_3 (from top to bottom) as a function of wave vector and interaction strength ($V_1 = V_2$), showing agreement of peak locations between DMRG and theory. The data is taken at cuts shown in Fig. 1. Plots (a)–(c) taken at $V_3 = 1.56$ show the trion, 2M, and pair phases; plots (d)–(f) taken at $V_3 = 1.3$ show the trion and single phases (with a possible 2M phase in between). Darker (blue) colors represent larger values of amplitudes. The peak in the data of G_1, which continuously varies between 0 and $k_F/3$ in the 2M/single phase, is identified as k'. The lines added to the color plot are theoretic predictions with the determined parameter k'. The solid lines denote several long-distance k_{osc} associated with algebraic decay; the dotted lines denote several exponential-decay "peaks," which are possibly visible if the decay length scale is large. The parameters for the plots are explained in the Supplemental Material [29].

are algebraically decaying [Figs. 3(d)–3(f)] with peaks at multiples of k_F. Remarkably, we also observe exponentially decaying features at the moving k' which are remnants of the 2M parent theory.

Phase transitions. There are five potential phase transitions in our phase diagram. The locking mechanisms give hints about the possible phase transitions, which we discuss in relation to our data.

(1) Single-pair transition. The transition is controlled by the competition between the terms $\cos(2\theta_1)$ and $\cos(2\phi_1)$, and results in a quantum Ising transition [27,28,40–42]. In the Supplemental Material [29], we provide definitive evidence that the single-pair transition is Ising via finite-χ scaling.

(2) 2M-single transition. This transition is driven by the term $\cos(2\theta_1)$, and is likely a Berezinskii-Kosterlitz-Thouless (BKT) transition.

(3) 2M-pair/trion transition. Both 2M-to-pair and 2M-to-trion transitions are accompanied by k' reaching a commensurate value. This suggests a commensurate-incommensurate transition.

(4) Single-trion transition. We are unable to determine if there is a direct transition between the trion and single phase, or whether there is an intervening 2M phase which extends down as V_3 is decreased. In both cases, our numerical analysis suggests a (at least one) first-order transition (see the Supplemental Material [29]).

Discussion. In summary we find conclusive evidence for an emergent mode in a one-dimensional attractive fermion chain. This emergent mode results in the formation of a stable 2M phase with two Fermi surfaces. We argue that the multifermion bound-state liquids are not descendants of the single-mode TLL phase but are rather descendants of this 2M phase. Here the 2M parent theory is written as a mixture of charged/neutral modes. Curiously, we can also rewrite the theory in terms of a mixture of charge-1/charge-2 modes [27], or more generally charge-n/charge-$(n+1)$ modes.

The two ingredients required to realize the proposed phenomenology are (1) confining the fermions to one dimension and (2) controlling the form of the interaction potential between the fermions. In the setting of solid-state systems the two ingredients could be realized in nanowires made of superconducting semiconductors [43–48]. In ultracold atoms, confinement could be provided by either optical lattices [3,6,49], or atom chips [5] and tunable long-range interaction by the use of dipolar interactions [50,51], or Rydberg-state-mediated interactions [52].

The 1D systems studied here can also be used to construct higher dimensional topological phases via the coupled-wire construction [9,27,53,54]. TLL enriched by emergent mode(s) may give a pathway to a wide range of new phases in condensed matter.

Acknowledgments. We acknowledge enlightening discussions with J. Levy. This work was supported by the Charles E. Kaufman foundation KA2017-91784 and NSF PIRE-1743717.

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