# Rashba splitting of Dirac points and symmetry breaking in strained artificial graphene

Vram Mughnetsyan<sup>®</sup>,<sup>1</sup> Aram Manaselyan,<sup>1</sup> Manuk Barseghyan<sup>®</sup>,<sup>1</sup> Albert Kirakosyan,<sup>1</sup> and David Laroze<sup>2,\*</sup>

<sup>1</sup>Department of Solid State Physics, Yerevan State University, Alex Manoogian 1, 0025 Yerevan, Armenia

<sup>2</sup>Instituto de Alta Investigación, CEDENNA, Universidad de Tarapacá, Casilla 7D, Arica, Chile

(Received 1 August 2019; revised manuscript received 30 October 2019; published 20 November 2019)

The effect of Rashba spin-orbit interaction and anisotropic elastic strain on the electronic, optical, and thermodynamic properties of an artificial graphenelike superlattice composed of InAs/GaAs quantum dots has been theoretically considered. The electronic energy dispersions have been obtained using Green's function formalism in combination with the Fourier transformation to the reciprocal space and an exact diagonalization technique. We have observed a splitting of Dirac points and the appearance of additional Dirac-like points due to the Rashba spin-orbit interaction. Furthermore, a breaking of the hexagonal symmetry of the dispersion surfaces caused by the strain anisotropy is observed as well. It is shown that both the spin-orbit interaction and strain anisotropy have a qualitative impact on the measurable characteristics of the considered structure and can be used as effective tools to control the performance of devices based on artificial graphene.

DOI: 10.1103/PhysRevB.100.195132

### I. INTRODUCTION

Dirac materials are foreseen to be of paramount importance because of their universal behavior and the robustness of their properties which are linked to symmetry [1,2]. Their band structure is similar to one of relativistic massless particles where the energy dependence on the momentum is linear in the vicinity of touching (Dirac) points of the electronic bands. Graphene is an innate example of a one-atom-thick 2D electron system composed by carbon atoms on a honeycomb lattice with two inequivalent sites in the unit cell. Due to its unique electronic spectrum, graphene makes possible the observation and test of table-top quantum relativistic phenomena in experiments, which are unobservable in high-energy physics [3]. The preference of graphene for spin logic applications instead of other metals and semiconductors has been recently experimentally tested [4,5]. The main features which make graphene an advantageous option for these applications are the large spin signal, the long spin diffusion length, and the relatively long spin lifetime [6,7].

In principle, Dirac-type singularities may exist in any 2D lattice with the similar underlying symmetry as graphene. Advanced methods such as atom-by-atom assembling [8], nanopatterning of 2D electron gas in semiconductors [9], and optical trapping of ultracold atoms in crystals of light [10] make it possible to design and fabricate artificial honeycomb lattices or artificial graphene (AG), which is a unique structure for the investigation and manipulation of several systems displaying massless Dirac quasiparticles, topological phases, and strong correlations. One of the reasons for pursuing the study of AG is the opportunity for regimes difficult to achieve in these systems, such as high magnetic fluxes, tunable lattice constants, and precise manipulation of defects, edges, and strain [11]. These studies enable tests of several predictions

for massless Dirac fermions. For future experiments based on AG, the availability of semiconductors and metals with large spin-orbit coupling opens new exciting potential features to investigate topological phases of artificial matter.

It has been shown that two-dimensional electron gas in a periodic potential of the honeycomb array of GaAs/GaAlAs quantum dots (QDs) can result in isolated massless Dirac points with controlable Fermi velocity (FV) [12]. The controllable FV, in turn, can lead to bound states of Dirac fermions [13], which is crucial for building practical digital devices with well-defined on/off logical states [14]. The realization of massless Dirac fermions in standard semiconductors opens interesting possibilities regarding the impact of the spin-orbit interaction (SOI), especially when using InAs-based materials such as honeycomb lattice of InAs/GaAs QDs [15].

Although the growth of homogeneous and spatially ordered arrays of InAs/GaAs QDs is a technological challenge [15,16], recent studies point to the possibility of controlling the size and shape, as well as the electron concentration in them, using strain engineering and selective area epitaxy. In this regard, there is a good prospect to achieve uniform, position-controlled InAs QDs in the near future [17–22].

It is known that the elastic strains at the InAs/GaAs heterojunction due to the lattice mismatch dramatically alters the electronic band structure [23–26]. It has been shown that the strain anisotropy in InAs/GaAs AG leads to the shift of the Dirac points from the K and the K' points of the first Brillouin zone (FBZ) resulting in anisotropy in the FV and qualitative changes in the density of states (DOS) [27].

The optical properties of transistors [28], optical switches [29–31], midinfrared photodetectors [32,33], photovoltaic devices [34], ultrafast lasers [35], etc., significantly rest on the light-matter interaction, limited in graphene (optical absorption is less than 2.5%). One of the advantages of AG is the possibility to overcome this limitation and tune the absorption coefficient (AC) by means of external factors such as Rashba SOI. The possibility to study the collective optical response

<sup>\*</sup>dlarozen@uta.cl

of modulated nearly 2D electrons [36,37] and holes [38] in semiconductors is another advantage of AGs based on QDs.

The heat capacity (HC) is a measurable thermodynamic quantity that can be considered as a sensitive tool to bring out the modifications in the electron energy spectrum in QD, as well as in graphene structures due to internal and external factors [36,39–41]. The study of HC in AG is of great interest due to the possibility to observe the combined effects originated by the quantum confinement in QDs and the underlying honeycomb symmetry.

In this regard, the consideration of the Rashba SOI and the elastic strain field in AG opens perspectives for the control of the optical and thermal properties of Dirac fermions.

In the present paper, the effect of Rashba SOI on the electronic band structure and DOS, as well as optical and thermodynamic properties of AG composed of highly strained InAs/GaAs QDs has been considered. The paper is organized as follows: In Sec. II, the model and the method are presented. In Sec. III, the results are displayed and the corresponding discussion is given. The conclusions are presented in Sec. IV.

#### **II. THEORETICAL MODEL**

Our theoretical model is based on the following assumptions. In the view of strong quantization in the direction perpendicular to the plane of the superlattice (SL), we will assume that electron makes a two-dimensional motion in the plane of the SL. Further, due to very weak dependence of the hydrostatic strain on the coordinate in the transverse direction, only the in-plane variations of the strain will influence the motion of the electron [42].

The method developed in Ref. [43] allows one to derive an analytic expression for the Fourier components of the strain tensor for a single QD of arbitrary shape in a material with a lattice of cubic symmetry (see the Appendix). In the framework of the mentioned approach, the hydrostatic strain in a two-dimensional SL of honeycomb symmetry, composed of cylindrical QDs of the height  $h_d$  and the radius  $r_d$  is as follows [27]:

$$\tilde{\varepsilon}_{h}(\xi_{1},\xi_{2}) = \sum_{i=1}^{3} \int d\xi_{3} \tilde{\varepsilon}_{ii}(\xi_{1},\xi_{2},\xi_{3}) = \varepsilon_{0} \tilde{\chi}_{\text{QD}}(n_{1},n_{2}) \\ \times \left(3 - \frac{C_{11} + 2C_{12}}{\pi} \int_{-\infty}^{\infty} \frac{\xi_{3}^{-1} \sin(\xi_{3}h_{d}/2)d\xi_{3}}{C_{12} + C_{44} + \Lambda_{\xi}^{-1}}\right),$$
(1)

where

$$\Lambda_{\xi} = \sum_{p=1}^{3} \frac{\xi_p^2}{C_{44}\xi^2 + C_{an}\xi_p^2}$$
(2)

and

$$\tilde{\chi}_{\text{QD}}(n_1, n_2) = \frac{2\pi r_d J_1(r_d |\vec{G}|)}{s_0 |\vec{G}|} A(n_1, n_2) (1 - A(n_1, n_2))$$
(3)

is the Fourier component of the SL's shape function [42,44],  $J_1(t)$  is the first kind of Bessel function of the first order,  $\vec{G} = n_1 \vec{g}_1 + n_2 \vec{g}_2$  is the 2D lattice vector in reciprocal space,  $\vec{g}_1 = (2\pi/3a)(1;\sqrt{3})$  and  $\vec{g}_2 = (2\pi/3a)(1;-\sqrt{3})$  are the elementary vectors of the reciprocal lattice, *a* is the smallest distance between the centers of QDs in the SL,  $A(n_1, n_2) = \exp(-i2\pi(n_1 + n_2)/3)$ ,  $n_{1,2}$  are integers,  $s_0$  is the area of the SL's unite cell,  $\tilde{\varepsilon}_{ii}(\vec{\xi})$  is the 3D Fourier transform of the diagonal element of the strain tensor in SL,  $\xi_1 = G_x$ ,  $\xi_2 = G_y$ ,  $\xi^2 = \sum_{i=1}^{3} \xi_i^2$ ,  $C_{11}$ ,  $C_{12}$ , and  $C_{44}$  are the elastic moduli of the matrix material (GaAs),  $C_{an} = C_{11} - C_{12} - 2C_{44}$  is the parameter of anisotropy,  $\varepsilon_0 = (a_1 - a_2)/a_2$  is the initial strain [45], and  $a_1$  and  $a_2$  are the lattice constants of the GaAs and InAs lattices, respectively.

It should be noted that when the condition  $h_d \ll r_d$  is satisfied, the dependence of hydrostatic strain on the *z* coordinate is weak [42] and Eq. (1) can be used for the calculation of the hydrostatic strain in 2D space:

$$\varepsilon_h(\vec{r}) = \sum_{\vec{G}} \tilde{\varepsilon}_h(\vec{G}) e^{i\vec{G}\vec{r}}.$$
(4)

The Hamiltonian of the considered system is

$$\mathcal{H} = \frac{1}{2}\hat{p}\frac{1}{m(\vec{r})}\hat{p} + \mathcal{H}_{\rm SO} + V(\vec{r}),\tag{5}$$

where

$$\mathcal{H}_{\rm SO} = \frac{\alpha}{\hbar} (\vec{\sigma} \times \vec{p})_z \tag{6}$$

is the Rashba SOI Hamiltonian which arises in 2D electron systems due to an inversion of asymmetry of the confinement potential perpendicular to the 2D plain direction. The SOI constant  $\alpha$  can be tuned by an external electric field in that direction [46,47]. In Eq. (5),  $V(\vec{r}) = v_0(\vec{r}) + a_c \varepsilon_h(\vec{r})$  is the periodic potential of QD SL,  $v_0(\vec{r}) = Q(E_{g,GaAs} - E_{g,InAs})$ is the potential of unstrained structure,  $E_{g,GaAs(InAs)}$  is the band gap of GaAs(InAs) material, Q is the conduction band offset,  $a_c$  is the hydrostatic potential constant, and  $m(\vec{r})$  is the electron effective mass. Due to the periodicity of the Hamiltonian Eq. (5), one can make a Fourier transformation to the momentum space [44,48]:

$$\psi_{\uparrow(\downarrow)}(\vec{r}) = \frac{1}{S} e^{i\vec{k}\vec{r}} u_{\vec{k}\uparrow(\downarrow)}(\vec{r}) = \frac{1}{S} \sum_{\vec{G}} u_{\vec{k},\vec{G}\uparrow(\downarrow)} e^{i(\vec{k}+\vec{G})\vec{r}}, \quad (7)$$

$$V(\vec{r}) = \sum_{\vec{G}} V_{\vec{G}} e^{i\vec{G}\vec{r}},\tag{8}$$

$$\frac{1}{m(\vec{r})} = \sum_{\vec{G}} m_{\vec{G}}^{-1} e^{i\vec{G}\vec{r}}.$$
(9)

Note that in Eq. (7),  $u_{\vec{k}\uparrow(\downarrow)}(\vec{r})$  and  $u_{\vec{k},\vec{G}\uparrow(\downarrow)}$  are the Bloch amplitude and its Fourier transform for the spin-up (spindown) component of the spinor  $\hat{\psi}$ , respectively. Also,  $V_{\vec{G}}$  and  $m_{\vec{G}}^{-1}$  are the Fourier transforms of the SL potential and inverse effective mass, respectively. Finally,  $\vec{k}$  is quasimomentum and *S* is the effective area of the AG. Substituting the expressions Eq. (7)–(9) to the Ben Daniel-Duke's equation  $\mathcal{H} \ \hat{\psi} = E \ \hat{\psi}$ , one can arrive at the following set of linear equations in reciprocal space:

$$\sum_{\vec{G}'} \left( \left[ \frac{\hbar^2}{2} m_{\vec{G}-\vec{G}'}^{-1} (\vec{k}+\vec{G})(\vec{k}+\vec{G}') + V_{\vec{G}-\vec{G}'} - E\delta_{\vec{G},\vec{G}'} \right] u_{\vec{k},\vec{G}'\uparrow} + \alpha \delta_{\vec{G}} \bar{\alpha} \left[ i(k_x + G') + (k_y + G') \right] u_{\vec{k},\vec{G}'\downarrow} \right) = 0, \quad (10)$$

$$\sum_{\vec{G}'} \left( \left[ \frac{\hbar^2}{2} m_{\vec{G}-\vec{G}'}^{-1}(\vec{k}+\vec{G})(\vec{k}+\vec{G}') + V_{\vec{G}-\vec{G}'} - E\delta_{\vec{G},\vec{G}'} \right] u_{\vec{k},\vec{G}'\downarrow} \right)$$

$$-\alpha \delta_{\vec{G},\vec{G}'}[i(k_x + G'_x) - (k_y + G'_y)]u_{\vec{k},\vec{G}'\uparrow} = 0, \qquad (11)$$

where  $m_{\vec{G}}^{-1} = \delta_{\vec{G},0} m_{\text{GaAs}}^{-1} + (m_{\text{InAs}}^{-1} - m_{\text{GaAs}}^{-1}) \tilde{\chi}_{\text{QD}}(\vec{G})$  and  $V_{\vec{G}} = (v_0/s_0) \tilde{\chi}_{\text{QD}}(\vec{G})$  are the Fourier transforms of the electron's inverse mass and the SL potential, respectively. The electronic dispersions are obtained by means of diagonalization of the set of Eqs. (10) and (11) for each value of the quasimomentum  $\vec{k}$ .

The DOS of the considered structure can be expressed as follows:

$$\rho(E) = \frac{1}{(2\pi)^2} \sum_{j} \int_{\text{FBZ}} \delta(E - E_j(\vec{k})) d^2k,$$
(12)

where the integration is carried out over the FBZ and j denotes the number of the miniband.

Assuming that the Fermi energy  $E_F$  is on the touching point between two couples of splitted minibands, the AC caused by the allowed direct transitions is

$$\alpha(\omega) = \alpha_0 \sum_{i=1}^2 \sum_{j=3}^4 \int_{\text{FBZ}} d^2 k |M_{i,j}(\vec{k})|^2$$
$$\times \delta(\hbar\omega - (E_j(\vec{k}) - E_i(\vec{k})), \qquad (13)$$

where

$$M_{i,j}(\vec{k}) = \hbar \sum_{\vec{G}} \left( u_{\vec{k},\vec{G}\uparrow}^{(i)} u_{\vec{k},\vec{G}\uparrow}^{(j)} + u_{\vec{k},\vec{G}\downarrow}^{(i)} u_{\vec{k},\vec{G}\downarrow}^{(j)} \right) (\vec{G}\vec{\eta})$$
(14)

is the dipole matrix element of the transitions from the *i*th to the *j*th miniband,  $\alpha_0 = e^2 (m_0^2 ch_d \omega \sqrt{\epsilon})^{-1}$ ,  $\omega$  and  $\vec{\eta}$  are the frequency and the polarization vector of the incident photon,  $\epsilon$  is the dielectric constant,  $m_0$  and e are the free mass and the electron charge, respectively, and c is speed of light.

We have also calculated the electronic HC of the system using the following expression [41,49]:

$$c_V = \int E\rho(E) \frac{\partial f(E,T)}{\partial T} dE, \qquad (15)$$

where integration is carried out over all the conduction bands,  $f(E, T) = (e^{\beta(E-\mu(T))} + 1)^{-1}$  is the Fermi-Dirac distribution function, and  $\beta = 1/k_BT$  and  $\mu(T)$  is the chemical potential. One can obtain the dependence of the chemical potential on the temperature by solving the following equation:

$$n = \int \rho(E) f(E, T) dE, \qquad (16)$$

where it is assumed that the electron 2D concentration *n* in the conduction band is constant and the Fermi energy  $E_F = \mu(T = 0)$  is on the touching point between two couples of split minibands.

### **III. DISCUSSION**

The numerical calculations are carried out for the following values of the parameters: a = 22 nm,  $r_d = 10$  nm,  $h_d = 2$  nm,  $m_{InAs} = 0.023m_0$ ,  $m_{GaAs} = 0.067m_0$ ,  $E_{g,GaAs} =$ 1518 meV,  $E_{g,InAs} = 413$  meV, and Q = 0.6 [50]. Taking into account that the electron is mostly localized in the QD regions, we use the value of the dielectric constant in InAs material ( $\epsilon = 12.3$ ) for the AC Eq. (13). The energy level broadening is taken into account, replacing the Dirac  $\delta$  function in Eq. (13) by the Lorentzian function with the value of the broadening parameter  $\Gamma = 0.2$  meV [51].

Figure 1 represents the electronic dispersion surfaces without (a) and with [(b) and (c)] Rashba SOI for isotropically strained AG. The vicinity of the K' point is mentioned by a dashed rectangle in Fig. 1(b), while the zoom of the corresponding region is shown in Fig. 1(c). It is obvious from the comparison of Figs. 1(a) and 1(b) that each surface splits in two due to SOI. Moreover, in the zoom of the vicinity of the K' point one can observe an obvious multiplication of Dirac points [see Fig. 1(c)]. Namely, around each Dirac point which is in the corner of the FBZ [the red line in Fig. 1(c)], three extra Dirac-like points appear where the minibands are attached. These points are shifted from the corner of the FBZ along the diagonals of the three hexagons with the same



FIG. 1. Dispersion surfaces for the splitted by SOI electronic minibands of isotropically strained AG. (a) The entire picture, (b) the first four minibands in the vicinity of the K' point, and (c) two touching minibands in the vicinity of K' point (the position of the K' point is indicated by a red line).



FIG. 2. Dispersion surfaces for the splitted by SOI electronic minibands of anisotropically strained AG. (a) The entire picture, (b) two touching minibands in the vicinity of K' point (the position of the K' point is indicated by a red line), and (c) the top view of the first miniband dispersion surface (the red dashed lines indicate the diagonals of three hexagons with the same corner in reciprocal space and their crossing point coincides with the K' point).

corner and compose an equilateral triangle  $K_1K_2K_3$  [Fig. 1(c)]. There are also three Dirac-like points of touching for each couple of the splitted surfaces which we refer as points  $S_1$ ,  $S_2$ , and  $S_3$ . These points compose two equilateral triangles which are rotated by 180° with respect to the triangle  $K_1K_2K_3$  around the energy axis passing through the K' point [the red line in Fig. 1(c)].

Figure 2 represents the electronic dispersion surfaces in the presence of Rashba SOI for anisotropically strained AG. Figure 2(b) shows the zoom of the region mentioned by the dashed rectangle in Fig. 2(a), while Fig. 2(c) represents the top view of the dispersion surface of the first miniband. From Fig. 2(b), the effect of the strain anisotropy on the symmetry of the dispersion surfaces is obvious. One can observe that the dispersion surfaces in the vicinity of K points coincide with those in the vicinity of K' points when rotated by 180° [Fig. 2(a)]. Importantly, both the Dirac points and the Diraclike points are shifted from the corners of the FBZ [Figs. 2(b) and 2(c)]. A more detailed examination shows that the shift of  $K_1$  is along one of the axes of the FBZ, while the shifts of  $K_2$  and  $K_3$  are no longer along corresponding diagonals. In addition, the shift of  $K_1$  is significantly larger than the shifts of the two other Dirac-like points [Fig. 2(c)]. As a result, the dispersion surfaces are neither of hexagonal nor of square



FIG. 3. Dependence of the FV on the angle between the *x* axis and the  $\vec{k} - \vec{k}_D$  in the vicinity of 1: *K'* point for isotropically strained AG, 2: *K*<sub>1</sub> point for isotropically strained AG, 3: Dirac point near the *K'* point for the anisotropically strained AG, 4: *K*<sub>1</sub> point for anisotropically strained AG, 5: *K*<sub>3</sub> point for anisotropically strained AG.

symmetry and they keep only the symmetry of reflection with respect to  $k_x$  and  $k_y$  axes. The accurate analysis of the solutions of the set of Eqs. (10) and (11) in the vicinity of Dirac and Dirac-like points indicate the linear dependence of the energy on  $|\vec{k} - \vec{k}_D|$ , where  $\vec{k}_D$  stands for the position of the Dirac or the Dirac-like point in the FBZ. Furthermore, the proportionality coefficient, which is the analog of the FV, depends on the orientation of the  $\vec{k} - \vec{k}_D$  in contrast to the case of the conventional honeycomb lattice. Based on the above-mentioned regularities, one can introduce an effective low-energy Hamiltonian in the vicinity of each Dirac and Dirac-like point in the following way:  $H_{\text{eff}} = \hbar v_F(\varphi) |\vec{k} - \vec{k}_D|$ , where  $v_F(\varphi)$  is the projection of the energy gradient at point  $\vec{k}_D$  on the direction of  $\vec{k} - \vec{k}_D$  and  $\varphi$  is the angle between the  $\vec{k} - \vec{k}_D$  and the *x* axis.



FIG. 4. Density of states for isotropically (a) and anisotropically (b) strained AG with (red lines) and without (black lines) Rashba SOI. Arrows indicate the touching points of the first and the second minibands.



FIG. 5. Absorption coefficient for isotropically (a) and anisotropically (b) strained AG with Rashba SOI for different directions of polarization vector of incident photon. The insets represent the corresponding graphs in the absence of SOI.

Figure 3 illustrates the dependence of the FV on the angle  $\varphi$ in the vicinity of the touching points (Dirac and Dirac-like) of the second and the third minibands. For isotropically strained AG, there is a third-order rotational symmetry around the K'point; that is why the results for only the K' and  $K_1$  points are illustrated. At the K' point, the FV is almost constant leading to a dispersion like in graphene. However, the FV at the Dirac-like point  $K_1$  has an oscillatory dependence on  $\varphi$ . For anisotropically strained AG,  $K_1$  and  $K_{2(3)}$  represent physically different points in the FBZ, which leads to different dependencies of FV on  $\varphi$ . One can observe that the curve, which corresponds to  $K_3$  point, is not symmetric regarding the line  $\varphi = \pi/2$  in contrast to all other curves. This fact is connected with the shift of the point  $K_3$  from the diagonal of corresponding hexagon and the breaking of the structure hexagonal symmetry due to the strain anisotropy.

The DOS in the presence (red lines) and the absence (black lines) of Rashba SOI is plotted in Fig. 4 for isotropically [Fig. 4(a)] and anisotropically [Fig. 4(b)] strained AG (in the figure,  $a_B \approx 3.57$  nm is the effective Bohr radius in InAs). The results for AG without SOI are taken from the Ref. [27] for comparison. An obvious multiplication of the maxima of DOS is observed. Namely, for an isotropically strained AG [Fig. 4(a)], two peaks are replaced by eight. Each of these peaks corresponds to the energy when the gradient of one of





FIG. 6. Heat capacity of isotropically (a) and anisotropically (b) strained AG with (red solid lines) and without (black dashed lines) Rashba SOI. The insets show the corresponding chemical potential dependence on the temperature.

the splitted surfaces is zero. The comparison of Figs. 4(a) and 4(b) shows that each peak of the DOS is duplicated because of the strain anisotropy. The splittings of the left and the right peaks are very weak (less then 0.05 meV) because they correspond to the zero gradient regions of dispersion surfaces which are very close to the center of FBZ.

The effect of the miniband splitting and the symmetry change of the dispersion surfaces on the AC of AG is presented in Fig. 5. The dependencies of the AC on incident photon energy for four different values of the angle  $\varphi$  between the light polarization vector and the x axis are shown for isotropically [Fig. 5(a)] and anisotropically [Fig. 5(b)] strained AG. The insets correspond to the absorption spectrum due to the transitions between the first and the second minibands in the absence of the Rashba SOI. It is noteworthy that when SOI is absent, the absorption curves corresponding to different light polarizations almost coincide for isotropically strained AG [see the inset of Fig. 5(a)]. This effect is connected with the hexagonal symmetry (symmetry of the SL) of the section of dispersion surfaces by the plane of constant energy corresponding to the allowed optical transitions in momentum space. However, the strain anisotropy removes the abovementioned symmetry, leading to the significant splitting of the curves which intersect at a fixed value of the incident photon energy [see the inset of Fig. 5(b)]. Furthermore, one can observe a pronounced maxima of AC in the presence of SOI [in both Figs. 5(a) and 5(b)], which is associated with the corresponding peaks of the DOS.

An obvious splitting of the curves corresponding to different polarizations of incident photon caused by SOI is also observed for both isotropically and anisotropically strained AG. Like the anisotropically strained SL without SOI [the inset of Fig. 5(b)], there is a certain value of the incident photon energy [indicated by arrows in Figs. 5(a) and 5(b)] at which the values of the AC for different light polarizations coincide.

In Fig. 6, the temperature dependence of the HC on the AG is presented with and without Rashba SOI for two considered different cases of strain. The insets of Fig. 6 show the temperature dependence of the chemical potential, assuming that the 2D concentration of electrons in the conduction band is constant and defined by the Fermi level, which is on the touching point between two couples of splitted minibands. An obvious increase can be observed for the chemical potential which has larger values when there is no SOI. As we can see from the figures, HC has a nonmonotonic behavior. It is zero at T = 0, because of the vanishing DOS at the Fermi energy. With the increase of the temperature, the HC initially increases as well due to the occupation of the states in the energy regions where the DOS is maxima. However, with further increase of the temperature, the states with higher density in the energy scale become saturated, leading to a smaller increase of the system's mean energy. As a result, the HC starts to decrease at some value of T. A comparison of Figs. 6(a) and 6(b) shows that the strain anisotropy results in the shift of the maximum of the HC to the region of lower temperatures. On the other hand, the Rashba SOI always increases the value of the HC because it removes the twofold spin degeneracy of minibands, leading to a necessity of extra energy for occupation of the splitted minibands.

### **IV. CONCLUSION**

In summary, we have considered the effect of Rashba SOI on the energy dispersion, DOS, AC, and the HC of the AG composed by InAs/GaAs QDs, taking into account the anisotropic elastic strain field due to the lattice mismatch between the materials of QDs and the matrix. Splitting of Dirac points due to the SOI have been observed. The Diraclike points  $K_1$ ,  $K_2$ , and  $K_3$  are shifted from the K point along the diagonals of adjacent hexagons in k-space when an isotropic strain is considered. However, in the case of anisotropic strain, only the  $K_1$  point is shifted along a diagonal of the Brillouin zone. The detailed analysis of the dispersion surfaces in the vicinity of touching points of minibands indicate the oscillatory behavior of the FV as a function of the rotation angle around the touching point. The oscillations are not only observed for the rotation around the K' point in the isotropically strained structure. The DOS in the presence of SOI has eight characteristic peaks. Moreover, each of these peaks is duplicated due to the strain anisotropy. Additionally, it is shown that both the Rashba SOI and the strain anisotropy have a qualitative effect on the measurable quantities of AG, like AC and HC. In particular, the splitting of the absorption spectra for different polarizations of incident photons, as well as the significant change in the HC make the Rashba coupling

an effective tool for controlling the optical and thermal characteristics of AG.

#### ACKNOWLEDGMENTS

This work was supported by the State Committee of Science of RA (Research Project No. 18T-1C223). V.M. acknowledges partial financial support from EU H2020 RISE Project CoExAN (Grant No. H2020-644076). D.L. acknowledges partial financial support from Centers of Excellence with BASAL/CONICYT financing, Grant No. FB0807, CE-DENNA.

## APPENDIX: DERIVATION OF THE FOURIER TRANSFORM OF HYDROSTATIC STRAIN IN TWO-DIMENSIONAL SUPERLATTICE

It is well known that in an elastic media, the displacement at position  $\vec{r}$  in the *l* direction due to the united point force applied at the origin of coordinates in the *n* direction can be expressed by Green's tensor  $G_{\ln}(\vec{r})$ , which satisfies the following equation [52]:

$$\sum_{klm} \lambda_{iklm} \frac{\partial G_{\ln}(\vec{r})}{\partial x_k \partial x_m} = -\delta(\vec{r})\delta_{i,n},\tag{A1}$$

where  $\lambda_{iklm}$  is the tensor of elastic moduli. Making the following Fourier transformations in Eq. (A1):

$$G_{\rm ln}(\vec{r}) = \int \tilde{G}_{\rm ln}(\vec{\xi}) \exp(i\vec{\xi}\vec{r})d^3\xi,$$
  
$$\delta(\vec{r}) = (2\pi)^{-3} \int \exp(i\vec{\xi}\vec{r})d^3\xi, \qquad (A2)$$

one arrives at the following equation for the Green's function Fourier transform:

$$\sum_{klm} \lambda_{iklm} \xi_k \xi_m \tilde{G}_{\rm ln}(\vec{\xi}) = (2\pi)^{-3} \delta_{\rm in}.$$
 (A3)

In particular, for materials with cubic crystal structure  $\lambda_{iklm} = C_{12}\delta_{ik}\delta_{lm} + C_{44}(\delta_{il}\delta_{mk} + \delta_{im}\delta_{kl}) + C_{an}\sum_{p=1}^{3}\delta_{ip}\delta_{kp}\delta_{lp}\delta_{mp}$ , where  $C_{an} = C_{11} - C_{12} - 2C_{44}$  is the parameter of anisotropy. For this case, after simple mathematical manipulations, one gets from Eq. (A3) the following expression:

$$(\vec{\xi}\tilde{G})_n \equiv \sum_{l=1}^{3} \xi_l \tilde{G}_{ln}(\vec{\xi}) = \frac{1}{(2\pi)^3} \frac{\xi_n}{C_{44}\xi^2 + C_{an}\xi_n^2} \times \left(1 + (C_{12} + C_{44}) \sum_{p=1}^{3} \frac{\xi_p^2}{C_{44}\xi^2 + C_{an}\xi_p^2}\right)^{-1}.$$
 (A4)

In the framework of the method of inclusions [53], the i component of the displacement caused by the existence of a single QD is as follows:

$$D_{i}^{s}(\vec{r}) = D_{i}^{s}\chi_{\text{QD}}(\vec{r}) + \sum_{n,k} \int G_{i,n}(\vec{r} - \vec{r'})\sigma_{nk}^{s} dS_{k}', \quad (A5)$$

where  $\sigma_{nk}^s = \sum_{pr} \lambda_{nkpr} \varepsilon_{pr}^s$  is the initial stress tensor,  $\varepsilon_{pr}^s$  and  $D_i^s$  are the initial strain tensor component and the initial displacement due to the lattice mismatch between the QD

$$\varepsilon_{ij}^{s} = \frac{1}{2} \left( \frac{\partial D_{i}^{s}(\vec{r})}{\partial x_{j}} + \frac{\partial D_{j}^{s}(\vec{r})}{\partial x_{i}} \right), \tag{A6}$$

and implying the Gauss's theorem, one obtains

$$\varepsilon_{ij}^{s}(\vec{r}) = \varepsilon_{ij}^{s} \chi_{\text{QD}}(\vec{r}) + \frac{1}{2} \sum_{nkpr} \int \left[ \frac{\partial^2 G_{\text{in}}(\vec{r} - \vec{r'})}{\partial x_j \partial x_k} + \frac{\partial^2 G_{jn}(\vec{r} - \vec{r'})}{\partial x_i \partial x_k} \right] \lambda_{nkpr} \varepsilon_{pr}^{s} \chi_{\text{QD}}(\vec{r'}) d^3r', \quad (A7)$$

where integration is carried out over the whole 3D space. Applying the operator  $\mathfrak{F}$  of the inverse Fourier transformation to both sides of Eq. (A7) and taking into account the convolution theorem according to which  $\mathfrak{F}(\int P(\vec{r} - \vec{r'})Q(\vec{r'})d\vec{r'}) =$  $(2\pi)^3 \mathfrak{F}(P(\vec{r}))\mathfrak{F}(Q(\vec{r}))$  for the functions  $P(\vec{r})$  and  $Q(\vec{r})$ , we arrive at the following expression of the strain tensor Fourier transform  $\varepsilon_{i}^s(\vec{\xi})$  for a single QD in an elastic media:

$$\tilde{\varepsilon}_{ij}^{s}(\vec{\xi}) = \varepsilon_{ij}^{s} \tilde{\chi}_{\text{QD}}(\vec{\xi}) - \frac{(2\pi)^{s}}{2} \times \sum_{nkpr} (\xi_{i}\tilde{G}_{jn}(\vec{\xi}) + \xi_{j}\tilde{G}_{in}(\vec{\xi}))\tilde{\chi}_{\text{QD}}(\vec{\xi})\lambda_{nkpr}\xi_{k}\varepsilon_{pr}^{s}, \text{ (A8)}$$

where  $\tilde{\chi}_{QD}(\vec{\xi})$  is the Fourier transform of the shape function and  $\vec{\xi}$  is the position vector in the inverse space. Taking into account that for cubic crystals, the initial strain tensor  $\varepsilon_{ij}^s = \varepsilon_0 \delta_{ij}$ , it is not hard to obtain from Eq. (A8) the following expression:

$$\tilde{\varepsilon}_{ij}^{s}(\vec{\xi}) = \varepsilon_{0} \tilde{\chi}_{\text{QD}}(\vec{\xi}) \bigg( \delta_{ij} - \frac{(2\pi)^{3}}{2} (C_{11} + 2C_{12}) \\ \times [\xi_{i}(\vec{\xi}\tilde{G})_{j} + \xi_{j}(\vec{\xi}\tilde{G})_{i}] \bigg).$$
(A9)

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in the dot products  $(\vec{\xi}\tilde{G})$  in Eq. (A9) by their cor-

Substituting the dot products  $(\bar{\xi}\tilde{G})$  in Eq. (A9) by their corresponding expressions presented in Eq. (A4), we arrive at an analytic expression for the Fourier transform of the strain tensor:

$$\tilde{\varepsilon}_{ij}^{s} = \varepsilon_{0} \tilde{\chi}_{\text{QD}}(\vec{\xi}) \Biggl( \delta_{ij} - \frac{1}{2} \frac{(C_{11} + 2C_{12})\xi_{i}\xi_{j}/\xi^{2}}{1 + (C_{12} + C_{44}) \sum_{p=1}^{3} \frac{\xi_{p}^{2}}{C_{44}\xi^{2} + C_{an}\xi_{p}^{2}}} \times \Biggl[ \frac{1}{C_{44} + C_{an}\xi_{i}^{2}/\xi^{2}} + \frac{1}{C_{44} + C_{an}\xi_{j}^{2}/\xi^{2}} \Biggr] \Biggr).$$
(A10)

Because of the linearity of the elasticity problem, the strain tensor component for a one-layer QD SL is as follows:

$$\varepsilon_{ij}(\vec{r}) = \sum_{\vec{R}} \varepsilon_{ij}^s(\vec{r} - \vec{R}) = \sum_{\vec{R}} \int \tilde{\varepsilon}_{ij}^s \exp{(\vec{\xi}(\vec{r} - \vec{R}))} d\vec{\xi},$$
(A11)

where  $\vec{R}$  runs over the in-plane cite vectors of QDs. On the other hand, the Fourier expansion of  $\varepsilon_{ij}(\vec{r})$  for the 2D QD lattice has the following form:

$$\varepsilon_{ij}(\vec{r}) = \sum_{\vec{G}} \exp(i\vec{G}\vec{\rho}) \int_{-\infty}^{\infty} \tilde{\varepsilon}(\vec{G},\xi_3) \exp(i\xi_3 z) d\xi_3, \quad (A12)$$

where  $\vec{G}$  runs over the vectors of the 2D reciprocal lattice and  $\vec{\rho}$  is the in-plane position vector. Comparison of Eqs. (A11) and (A12) leads to the expression for the strain tensor Fourier transform for a one-layer QD SL:

$$\tilde{\varepsilon}_{ij}(\vec{G},\xi_3) = \frac{(2\pi)^2}{s_0} \tilde{\varepsilon}_{ij}^s(\vec{G},\xi_3).$$
 (A13)

The hydrostatic strain is defined as the trace of the strain tensor:

$$\varepsilon_h = \sum_{i=1}^3 \varepsilon_{ii}.$$
 (A14)

Finally, acting on the both sides of Eq. (A14) by the operator  $\mathfrak{F}$  and applying Eqs. (A10) and (A13), we arrive at Eq. (1) of Sec. II.

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