

Dissipative features of the driven spin-fermion systemRuofan Chen¹ and Xiansong Xu²¹*Science and Technology on Surface Physics and Chemistry Laboratory, Mianyang 621908, China*²*Science and Math Cluster, Singapore University of Technology and Design, 8 Somapah Road, Singapore 487372*

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We study a generic spin-fermion model, where a two-level system (spin) is coupled to two metallic leads with different chemical potentials, in the presence of monochromatic driving fields. The real-time dynamics of the system is simulated beyond the Markovian limit by an iterative numerically exact influence functional path integral method. Our results show that although both system-bath coupling and chemical potential difference contribute to dissipation, their effects are distinct. In particular, under certain drivings the asymptotic Floquet states of the system exhibit robustness against a range of system-bath coupling strength: the asymptotic behaviors of the system are insensitive to different system-bath coupling strength, while they are highly tunable by the chemical potential difference of baths. Further simulations show that such robustness may be essentially a result of the interplay between driving, bath electronic structure, and system-bath coupling. Therefore, the robustness could break down depending on the characteristics of the interplay. In addition, under fast linearly polarized driving the quantum stochastic resonance is demonstrated that stronger system-bath coupling (stronger dissipation) enhances rather than suppresses the amplitude of coherent oscillations of the system.

DOI: [10.1103/PhysRevB.100.115437](https://doi.org/10.1103/PhysRevB.100.115437)**I. INTRODUCTION**

A wide range of physical and chemical systems can be effectively described by quantum two-level systems (TLSs). The most common example for a TLS is a particle of total spin $\frac{1}{2}$ under an external magnetic field which is shown in nuclear magnetic resonance experiments [1]. Another common situation is a particle moving in an effective double-well potential in which only the lowest energy doublet is occupied [2,3]. A well-known example is the ammonia molecule, NH_3 : quantum mechanically the hydrogens can tunnel back and forth between two potential minima [4,5]. The TLS is also the simplest nontrivial physical model used as a starting point to study time-dependent quantum systems. The explicitly time-dependent quantum problem generates a variety of novel phenomena that are not accessible within stationary quantum mechanics. A comprehensive review is given by Grifoni and Hänggi [6].

An isolated TLS is an ideal model and often fails to describe thermal and dynamical properties of real physical or chemical systems when the system is in contact with external environments. If environments can be effectively described as a collection of harmonic oscillators, then we obtain the so-called spin-boson model [2,7] which has been widely studied and exhibits rich phenomena [6,8]. The environments can also be fermionic and in this case we have the spin-fermion model [7,9,10]. The spin-boson and spin-fermion models represent the simplest nontrivial quantum dissipative models and are related to various physical and chemical problems. They are relevant for modeling charge transfer in photosynthesis [11], the Kondo problem for magnetic impurities [12,13], quantum stochastic resonance [14–16], and quantum decoherence in the context of a superconducting charge qubit [17–20].

The possibility of controlling the time evolution of the molecular system by time-dependent external field has long appealed to chemists and physicists in order to lead a chemical reaction toward the desired product, design better nanodevices, etc. For instance, it is shown that time-dependent control can boost the thermoelectric efficiency of nanodevices [21]. Manipulating the time evolution of a molecular system requires controlling dissipative mechanisms. Understanding such mechanisms is of both practical and fundamental theoretical interest. A driven spin-boson or spin-fermion model could serve as a simple but nontrivial model for theoretical investigations.

While the spin-boson model has been extensively studied both in the presence [6,8,22–31] and absence [2,8,32–46] of external driving, the spin-fermion model is less well understood. In particular, the interplay of external driving on the system and the system-environment coupling strength is less explored due to the limitations of both theoretical and numerical tools. In this article we employ an iterative numerically exact influence functional path integral method to study the driven spin-fermion model. This method was first developed by Makarov and Makri and applied to the spin-boson model without driving [33,34]; then it was successfully applied to investigate the driven spin-boson model [23–25]. It is a non-perturbative method beyond the Markovian limit and thus well suited for handling real-time dynamics problems. Later Segal *et al.* adopted a more flexible discretized scheme for tracing out the bath and generalized this method to investigate the spin-fermion and some other generic models in the absence of driving [47–51]. In this article we adopt such a discretized scheme and extend the method to the spin-fermion model in the presence of driving. Our implementation is compared with the Born-Markov master equation in the Floquet basis [52]. In

Sec. II we give a brief review of the method, and the details of the model are given in Sec. III.

In this article we investigate the spin-fermion model under monochromatic driving at zero temperature with two fermionic leads kept at different chemical potentials. Monochromatic circularly and linearly polarized driving fields are considered in Sec. V.

A noticeable phenomenon appears that under certain drivings the asymptotic Floquet states of the system exhibit robustness against different system-bath coupling strength: the asymptotic behaviors of the system tend to be almost the same even with different coupling strength. It seems that although both the system-bath coupling and the chemical potential difference of the baths affect the dissipative rate greatly, the asymptotic behavior of the system is insensitive to system-bath coupling strength but dominated by the chemical potential difference of baths. Such a feature may be useful in designing a nanodetector driven by an external field to detect the electronic structure of the environment. In this case the driving field must be also considered as a part of the detector and the time-dependent form of the driving field should be treated as a parameter of essential importance.

More simulations indicate that there may exist a complex interplay between driving, bath electronic structure, and system-bath coupling. Such robustness may be essentially a result of the interplay and can break down depending on the characteristics of the interplay. Moreover, under fast linearly polarized driving (in Sec. VB 2), quantum stochastic resonance has shown that stronger system-bath coupling (stronger dissipation) enhances rather than suppresses the amplitude of coherent oscillations of the system.

Moreover, a convergence test is given in Appendix A and a benchmark against the Born-Markov master equation in the Floquet basis is given in Appendix B.

II. GENERAL FORMULATION OF ITERATIVE PATH INTEGRAL METHOD

Here we give a brief review of the general formalism of the iterative path integral method (for more details refer to Refs. [33,34,47]). Let us consider a generic many-body system which is modeled by a finite system of interest coupled with two noninteracting baths. Let $H(t)$ denote the total Hamiltonian and $\rho(t)$ denote the total density matrix. Then the time evolution of $\rho(t)$ is given by

$$\rho(t) = U(t)\rho(0)U^\dagger(t), \quad (1)$$

where

$$U(t) = \mathsf{T} \exp \left[-i \int_0^t H(\tau) d\tau \right] = \prod_{t_i=0}^t e^{-iH(t_i)\delta t}. \quad (2)$$

Here T denotes the chronological ordering symbol and the product is understood in that we take the limit over all the infinitesimal intervals δt between zero and t . Basically, the evolution is split into N pieces for which $\delta t = t/N$ with $N \rightarrow \infty$. Now we introduce the reduced density matrix of the system, $\rho_S = \text{Tr}_B \rho$, which is obtained by tracing the total density matrix over the bath degrees of freedom. The time

evolution of $\rho_S(t)$ is then exactly given by

$$\rho_S(s'', s'; t) = \text{Tr}_B \langle s'' | U(t) \rho(0) U^\dagger(t) | s' \rangle. \quad (3)$$

Employing finite δt in Eq. (2) approximates the evolution operator $U(t)$ into a product of finite N exponentials where $U(t) \approx \prod_{t_i=0}^t [e^{-iH(t_i)\delta t}]$. Defining the discrete time evolution operator $T = e^{iH\delta t}$, then the reduced density matrix can be written as

$$\rho(s'', s'; t) = \text{Tr}_B \langle s'' | (T^\dagger)^N \rho(0) T^N | s' \rangle. \quad (4)$$

Inserting the identity operator $\int ds |s\rangle \langle s|$ between every two T and relabeling s'', s' as s_N^+, s_N^- yields

$$\begin{aligned} \rho(s_N^+, s_N^-; t) &= \int ds_0^+ \cdots ds_{N-1}^+ \int ds_0^- \cdots ds_{N-1}^- \\ &\times \text{Tr}_B [\langle s_N^+ | T^\dagger | s_{N-1}^+ \rangle \langle s_{N-1}^+ | T^\dagger | s_{N-2}^+ \rangle \\ &\times \cdots \langle s_0^+ | \rho(0) | s_0^- \rangle \cdots \\ &\times \langle s_{N-2}^- | T | s_{N-1}^- \rangle \langle s_{N-1}^- | T | s_N^- \rangle]. \end{aligned} \quad (5)$$

The integrand in Eq. (5) is referred to as the ‘‘influence functional’’ [47] (IF) and is denoted by $I(s_0^\pm, \dots, s_N^\pm)$. The IF contains the information of the system and bath degrees of freedom with system-bath interactions. The IF has an important property that allows us to greatly simplify the calculation: nonlocal correlations contained in the IF decay exponentially under certain conditions [33], which enables a (controlled) truncation of the IF. It means in practical calculation we need to only keep a finite memory length. Basically, for a system under a chemical potential bias $\Delta\mu$ at zero temperature the exponentially decaying of the correlations is guaranteed by finite $\Delta\mu$, while in a large-temperature situation ($T > \Delta\mu$) the temperature sets the scale of the memory length that needs to be kept [47,53]. Based on this feature, an iterative scheme for evaluating the path integral has been developed [33,34]. The original quasiadiabatic path integral algorithm was based on the analytical pairwise form of the IF specific to harmonic baths; later a more general approach was proposed which was based on the fact that memory effects generically vanish exponentially [36]. The idea was further developed to simulate the dynamics of a generic nonequilibrium bias driven system [53].

Since only a finite memory length needs to be considered, the IF can be truncated beyond a memory time $\tau_c = N_s \delta t$ (here N_s is a positive integer), which corresponds to the time beyond which bath correlations can be ignored controllably. Therefore, the total IF can be written approximately as [33,34,36,47,48]

$$\begin{aligned} I(s_0^\pm, \dots, s_N^\pm) &\approx I(s_0^\pm, \dots, s_{N_s}^\pm) I_s(s_1^\pm, \dots, s_{N_s+1}^\pm) \\ &\times \cdots I_s(s_{N-N_s}, \dots, s_N^\pm) \end{aligned} \quad (6)$$

with

$$I_s(s_k^\pm, \dots, s_{k+N_s}) = \frac{I(s_k^\pm, \dots, s_{k+N_s}^\pm)}{I(s_k^\pm, \dots, s_{k+N_s-1}^\pm)}. \quad (7)$$

The approach becomes exact when $\tau_c \rightarrow \infty$ and its physical content is discussed in Refs. [36,47].

To integrate Eq. (6) iteratively we define a multiple time reduced density matrix $\tilde{\rho}_S(s_k^\pm, \dots, s_{k+N_s-1})$ with an initial

value $\tilde{\rho}_S(s_0^\pm, \dots, s_{N_s-1}^\pm) = 1$; i.e., all of the initial components are identity. Its first evolution step is dictated by

$$\tilde{\rho}_S(s_1^\pm, \dots, s_{N_s}^\pm) = \int ds_0^\pm I(s_0^\pm, \dots, s_{N_s}^\pm), \quad (8)$$

and beyond the first step the evolution step is given by

$$\tilde{\rho}_S(s_{k+1}^\pm, \dots, s_{k+N_s}^\pm) = \int ds_k^\pm \tilde{\rho}_S(s_k^\pm, \dots, s_{k+N_s-1}^\pm) \times I_S(s_k^\pm, \dots, s_{k+N_s}^\pm). \quad (9)$$

Then the time-local ($t_k = k\delta t$) reduced density matrix is obtained by summing over all intermediate states:

$$\rho_S(t_k) = \int ds_{k-1}^\pm \cdots ds_{k-N_s+1}^\pm \tilde{\rho}_S(s_{k-N_s+1}^\pm, \dots, s_k^\pm). \quad (10)$$

In practical calculation, we need to keep track of $\tilde{\rho}_S(s_{k+1}^\pm, \dots, s_{k+N_s}^\pm)$, which is a $2N_s$ rank ‘‘tensor.’’ Suppose the size of Hilbert space of the system is M ; then a space with size proportional to M^{2N_s} is needed to store the tensor. Similarly, to store $I_S(s_k^\pm, \dots, s_{k+N_s}^\pm)$ one needs a space with size proportional to $M^{2(N_s+1)}$. The space size increases dramatically with increasing M and N_s , which means for practical calculations we need to ensure M and N_s are not too large. In other words, the size of the system and truncation time τ_c need to be small; otherwise the system may lose feasibility in the numerical evaluation.

However, there is no restriction or difficulty in the development of the method that the Hamiltonian must be time independent. On the contrary, since it is an iterative method which is truncated in time it is rather easy to deal with a time-dependent Hamiltonian. What we need to do is just calculate the IF with the Hamiltonian in corresponding time. Moreover, this method is a nonperturbative method beyond the Markovian limit and thus it is well suited for the investigation of long-time behaviors of driven systems.

III. THE MODEL

In this article we consider the spin-fermion model where a spin is coupled to two fermionic leads with chemical potential difference $\Delta\mu$ at zero temperature. Such a model serves as a simple but nontrivial model to study bias-driven nonequilibrium systems [9,47,54,55]. The Hamiltonian of the model is written as

$$H = H_0 + H_1, \quad (11)$$

where

$$H_0 = H_S, \quad H_1 = H_B + H_{SB}. \quad (12)$$

The bath Hamiltonian H_B is that of two independent free fermion baths ($\alpha = L, R$) whose statistics are determined by chemical potentials, i.e.,

$$H_B = \sum_{\alpha,k} \varepsilon_k c_{\alpha k}^\dagger c_{\alpha k}. \quad (13)$$

The operator $c_{\alpha k}^\dagger$ ($c_{\alpha k}$) creates (annihilates) an electron with state k in the α th bath. The system Hamiltonian H_S is that of

a driven TLS,

$$H_S = \frac{1}{2} \mathbf{B}(t) \cdot \boldsymbol{\sigma}, \quad (14)$$

where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are Pauli matrices and $\mathbf{B}(t)$ is the external field. The system-bath coupling is taken to be

$$H_{SB} = \sum_{\alpha\beta} V_{\alpha\beta} c_{\alpha k}^\dagger c_{\beta k'} \sigma_z, \quad (15)$$

where $\alpha, \beta = L, R$ are the bath indices. In this article we focus on the model [9,47,56,57] where the momentum dependence of the scattering potential is neglected. In particular, we consider only interbath system-bath couplings for which $gV_{\alpha\beta} = \lambda(1 - \delta_{\alpha\beta})$, where g is the density of states of each Fermi bath and λ is the control parameter.

For numerical evaluation we need to employ a second-order Trotter-Suzuki decomposition [58,59] on the discrete evolution operator $e^{iH\delta t}$ for which

$$e^{iH\delta t} \approx e^{iH_1\delta t/2} e^{iH_0\delta t} e^{iH_1\delta t/2}. \quad (16)$$

With this decomposition and assuming separable initial conditions $\rho(0) = \rho_S(0)\rho_B(0)$, the IF of the present model can be identified as

$$\begin{aligned} I(s_0^\pm, \dots, s_{N_s}^\pm) &= \langle s_0^\pm | \rho_S(0) | s_0^\pm \rangle \\ &\times K(s_{N_s}^\pm, s_{N_s-1}^\pm) \cdots K(s_1^\pm, s_0^\pm) \\ &\times \text{Tr}_B [e^{-iH_1(s_{N_s}^\pm)\delta t/2} e^{-iH_1(s_{N_s-1}^\pm)\delta t} \\ &\times \cdots e^{-iH_1(s_1^\pm)\delta t} e^{-iH_1(s_0^\pm)\delta t/2} \\ &\times \rho_B(0) e^{iH_1(s_0^-)\delta t/2} e^{iH_1(s_1^-)\delta t} \\ &\times \cdots e^{iH_1(s_{N_s-1}^-)\delta t} e^{iH_1(s_{N_s}^-)\delta t/2}], \end{aligned} \quad (17)$$

where

$$\begin{aligned} K(s_{k+1}^\pm, s_k^\pm) &= \langle s_{k+1}^\pm | e^{-iH_0(t_k)\delta t} | s_k^\pm \rangle \\ &\times \langle s_k^\mp | e^{iH_0(t_k)\delta t} | s_{k+1}^\mp \rangle \end{aligned} \quad (18)$$

is the propagator matrix for the isolated system.

It is more flexible to describe the bath as discrete levels and the infinite bath result can be easily reached even with a small number (about 40) of effective bath fermions [47]. The trace in Eq. (17) can be numerically eliminated via the Blankenbecler-Scalapino-Sugar (BSS) identity [60] and Levitov’s formula [61–63]; then the analytic structure of the trace is not required. This feature gives the method feasibility to investigate various system-bath coupling other than linear coupling including nonadditive system-bath coupling [64] used in our model. The generalization to finite temperature is also straightforward [47,61].

IV. FLOQUET FORMALISM

Alternatively, one could also use the Floquet master equation to study time-dependent systems. Comparing to the iterative path integral technique, the Floquet master equation approach is restricted to periodically driven systems and it is based on perturbative expansions. We employ a Floquet Born-Markov master equation with nonadditive system-bath interaction to calculate our model as a benchmark. We give a brief introduction to the Floquet formalism here and a

detailed derivation of the Floquet master equation is given in Appendix B.

Let us consider an isolated system with Hamiltonian $H_S(t)$. If the driving field $\mathbf{B}(t)$ is a periodic function with period \mathcal{T} for which $H_S(t) = H_S(t + \mathcal{T})$, then the Floquet theorem states that [65], for Schrödinger equation with system coordinate q ,

$$i \frac{\partial}{\partial t} \psi(q, t) = H_S(t) \psi(q, t), \quad (19)$$

there exist solutions in the form

$$\psi_i(q, t) = e^{-i\varepsilon_i t} \varphi_i(q, t), \quad (20)$$

where $\varphi_i(q, t)$ is periodic in time with period \mathcal{T} and ε_i is a real-valued function. The term φ_i is called the Floquet state and term ε_i is called the quasienergy. It is clear that ε_i is unique up to multiples of $\Omega = 2\pi/\mathcal{T}$ for which $\varepsilon_i + n\Omega$ with n being an integer corresponds to the same physical state. The Floquet states $\varphi_i(q, t)$ form a complete orthonormal basis for the system at given time t . The time evolution operator (for $t \geq t'$)

$$U_S(t, t') = \text{T exp} \left[-i \int_{t'}^t H_S(\tau) d\tau \right] \quad (21)$$

then can be expressed in the Floquet basis by

$$U_S(t, t') = \sum_n |\varphi_n(t)\rangle \langle \varphi_n(t')| e^{-i\varepsilon_n(t-t')}. \quad (22)$$

In the Floquet basis, the density matrix can be defined as

$$\varrho_{ij}(t) = \langle \varphi_i(t) | \rho_S(t) | \varphi_j(t) \rangle. \quad (23)$$

In this representation the time-dependent part of the Hamiltonian is absorbed in the Floquet states $\varphi_i(t)$; therefore, the time-dependent part of $\varrho_{ij}(t)$ is greatly simplified. In particular, $t \rightarrow \infty$ leads to the time-independent density matrix element ϱ_{ij} which represents the asymptotic Floquet states [52]. In Appendix B we give a comparison between asymptotic Floquet states calculated by the iterative path integral method used in this article and those calculated by the Born-Markov Redfield master equation in Floquet representation.

V. RESULTS

Back in 1927, Hund [4] pointed out the importance of the quantum tunneling effect in intramolecular rearrangements. Since then quantum tunneling between two levels in isolated TLSs under external driving has been widely studied. Rich tunneling phenomena are found in such problems. For instance, several physicists, including Landau, Zener, and Stückelberg, studied the transition between two levels in isolated TLSs under a time-dependent energy sweep external field [66–68]. Such a model is commonly known as a Landau-Zener model and it has a wide range of applications in physics and quantum chemistry. Another example is the coherence destruction of the tunneling phenomenon for which in isolated TLSs under monochromatic driving [69] the tunneling could be suppressed by the external driving.

According to our simulations, under monochromatic driving the spin-fermion model would exhibit rich tunneling

phenomena. Many of them could also be found in the spin-boson model [6,23,31]. These may be common phenomena of driven dissipative TLSs regardless of what kind of bath is present. To shorten the length and reduce the number of figures of this article, we do not represent them here and instead focus on showing the results which are relevant to the robustness of asymptotic behaviors of the system against different λ . Both circularly and linearly polarized driving fields are considered in this article. Although tunneling behaviors of the system differ a lot under different driving, under certain conditions all of them exhibit robustness behaviors.

A. Circularly polarized fields

Let us first consider the case of a spin-fermion model driven by a spatially homogeneous, circularly polarized field. We set the spin to the z direction at the initial time $t = 0$, i.e.,

$$\langle \sigma_z \rangle(0) = 1 \quad \text{and} \quad \rho_S(0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \quad (24)$$

A pioneering work on isolated driven TLSs in a circularly polarized field is given by Rabi [70] and it is shown that in this case analytic solutions can be found [6,70]. However, it is difficult to find analytic solutions in general cases and thus for consistency we simulated the isolated driven TLS numerically in this article.

1. Field in the x - y plane

Here we consider the case where the time-dependent field is orthogonal to $\langle \sigma_z \rangle(0)$, i.e., in the x - y plane. The system Hamiltonian can be written as

$$H_S(t) = \frac{B}{2} \sigma_z + \frac{\Delta}{2} (\sigma_x \cos \Omega t + \sigma_y \sin \Omega t). \quad (25)$$

Note that if we turn off the time-dependent part of the field, namely, set $\Delta = 0$ and only retain the static part $\frac{B}{2} \sigma_z$, then the system becomes localized; i.e., there would be no tunneling between two levels. In other words the tunneling is totally induced by the time-dependent field and in the dissipationless case $\langle \sigma_z \rangle(t)$ would stay as 1 if $\Delta = 0$ with the initial condition in Eq. (24).

With small local potential $B = 0.1$ and under slow and weak driving ($\Delta = 0.1$, $\Omega = 0.1$), the behavior of $\langle \sigma_z \rangle(t)$ is similar to that of a spin-fermion model without driving [47]: the chemical potential difference acts as a temperaturelike contributor to dephasing [9] for which $\langle \sigma_z \rangle(t)$ would be eventually dissipate to zero. The dissipation rate would be larger with larger λ and $\Delta\mu$.

If the driving field is both fast and strong ($\Delta = 1$ and $\Omega = 1$), the driving and dissipation eventually reach a balanced state and $\langle \sigma_z \rangle(t)$ would remain a finite value instead of decaying to zero [see Fig. 1(a)]. At first glance this behavior seems similar to the case in coherent destruction tunneling [71] (CDT) or driving-induced tunneling oscillations [26,27,30] (DITOs). However, the situation is different here. In CDT, without a time-dependent field there would be tunneling between two levels, and the field suppresses the tunneling. Here the time-dependent field induces, rather than

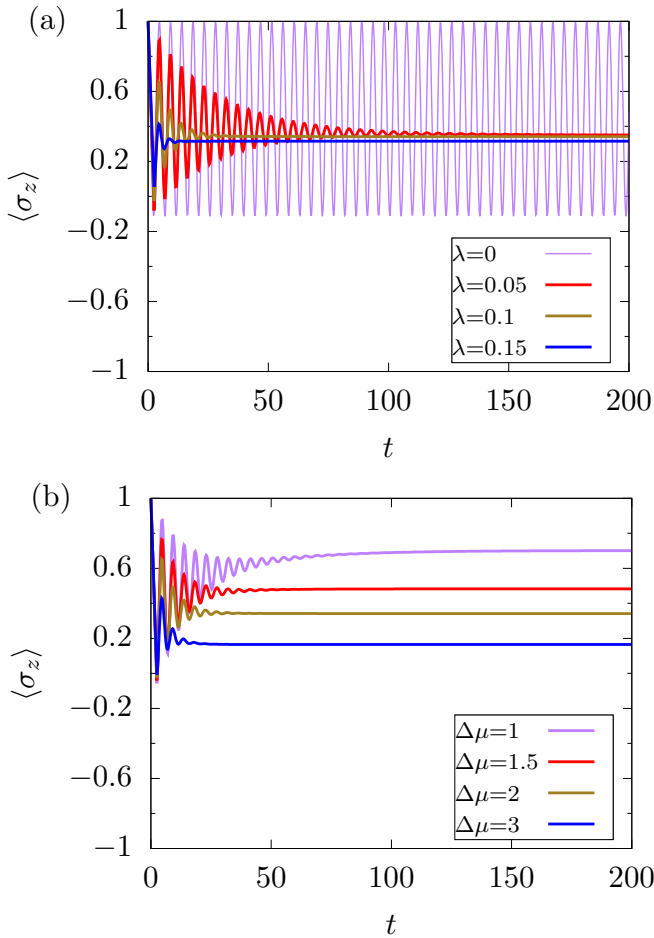


FIG. 1. Dynamics of $\langle \sigma_z \rangle(t)$ under a circularly driving field in the x - y plane for (a) varying system-bath coupling strength with $\Delta\mu = 2$ and (b) varying chemical potential difference with $\lambda = 0.1$. We have used $B = 0.1$, $\Delta = 1$, and $\Omega = 1$ for both (a) and (b).

suppresses, the tunneling between the two levels. In DITO, large amplitude oscillations are induced by a field with high static energy bias [30], which is not in our Hamiltonian. Without dissipation ($\lambda = 0$), $\langle \sigma_z \rangle(t)$ oscillates in the positive region, which means the system is tunneling between two levels but stays more time in the spin-up state. Such tunneling is suppressed by the dissipation and eventually the driving and dissipation reach a balanced state.

Here we can see an interesting phenomenon from Fig. 1(a). Lines have the same $\Delta\mu$ but different λ , and the dissipation rate is larger with larger λ . However, $\langle \sigma_z \rangle(t)$ with $\lambda = 0.05$, 0.1, and 0.15 reach almost the same value in the asymptotic state, while from Fig. 1(b) we could see that with the same λ but different $\Delta\mu$ the asymptotic $\langle \sigma_z \rangle(t)$ differ greatly. We could say that the system-bath coupling strength affects the dissipation rate of the fast oscillation greatly but has little effect on the asymptotic behavior, whereas the asymptotic behavior is dominated by $\Delta\mu$, or, in other words, by the electronic structure of the baths. We may say the asymptotic Floquet states of the system are *robust* against different system-bath coupling strengths λ . Such phenomena can be commonly seen in our simulations and we discuss it further.

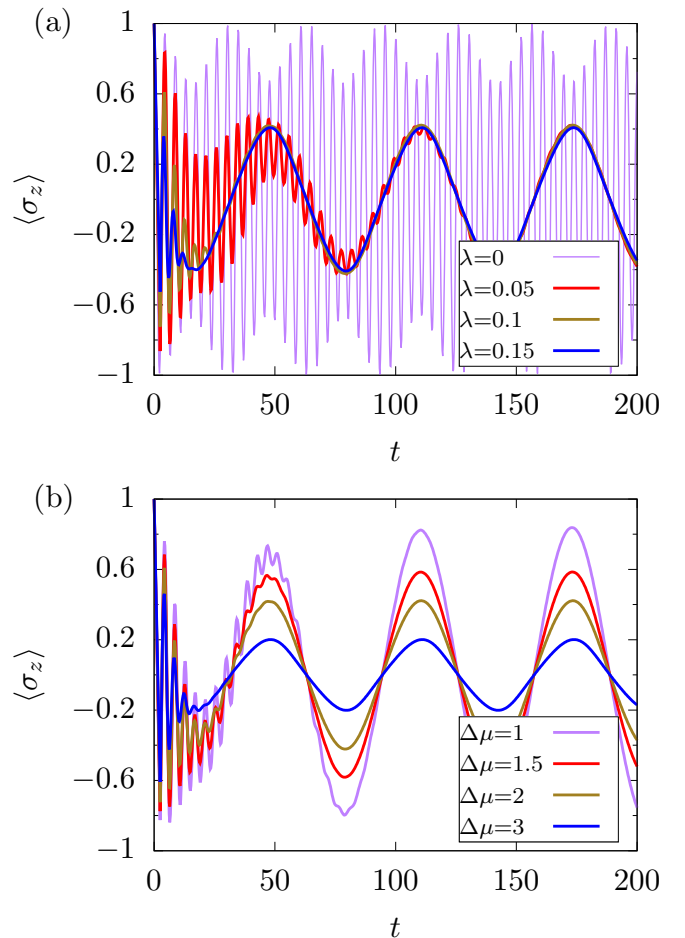


FIG. 2. Dynamics of $\langle \sigma_z \rangle(t)$ under a circularly driving field in the y - z plane for (a) varying system-bath coupling strength with $\Delta\mu = 2$ and (b) varying chemical potential difference with $\lambda = 0.1$. We have used $B = 1$, $\Delta = 1$, and $\Omega = 0.1$ for both (a) and (b).

2. Field in the y - z plane

Now consider the case where the time-dependent field is in the y - z plane. The system Hamiltonian is

$$H_S(t) = \frac{B}{2}(\sigma_y \cos \Omega t + \sigma_z \sin \Omega t) + \frac{\Delta}{2}\sigma_x. \quad (26)$$

Note that in this case even if the time-dependent field is turned off, namely, we set $B = 0$, the system is not localized since $\frac{\Delta}{2}\sigma_x$ remains in the Hamiltonian. The static field can be viewed as a rotation axis along the x direction and the spin would have uniform rotation around the axis. If we plot $\langle \sigma_z \rangle(t)$ in the dissipationless and static field case we would see $\langle \sigma_z \rangle(t)$ oscillates between 1 and -1 .

With large local potential $\Delta = 1$ and under strong but slow driving ($B = 1$, $\Omega = 0.1$), coherent oscillations are induced (see Fig. 2). The fast oscillation part due to system Hamiltonian dynamics is eventually dissipated out with a different rate, and the coherent oscillation due to driving remains. Similar behaviors have been reported in the spin-boson model [23,31]. Such oscillation behaviors present typical asymptotic Floquet states of a dissipative spin system under periodic driving [23,31,72].

Lines in Fig. 2(a) are simulated with the same $\Delta\mu$ but different λ . It is worth noting that although their dissipation rate is larger with larger λ , the asymptotic $\langle\sigma_z\rangle(t)$ with different λ eventually coincide.

Lines in Fig. 2(b) are simulated with the same λ but different $\Delta\mu$. It can be clearly seen that their final $\langle\sigma_z\rangle(t)$ differ greatly. Comparing Figs. 2(a) and 2(b), we may say that under this driving asymptotic Floquet states of $\langle\sigma_z\rangle(t)$ are dominated by the electronic structure of the baths and they are robust against different system-bath coupling strength λ .

B. Linearly polarized fields

Let us next consider the case where a spin-fermion model is driven by a spatially homogeneous, linearly polarized field. The initial condition is the same as in Eq. (24). It was pointed out long ago by Bloch and Siegert [73] that the driven TLS problem is no longer analytically solvable when the field is linearly rather than circularly polarized. To obtain an approximating solution for a dissipationless driven TLS under a linearly polarized field, the rotating wave approximation, which approximately transforms the linearly polarized field to the form of a circularly polarized field [6], is widely used. An iterative approach for strong-coupling periodically driven TLSs also exists [74]. However, in this article we directly use numerical results for the dissipationless case rather than analytical approximations.

1. Field in the x direction

Here we consider the case where the time-dependent field is orthogonal to $\langle\sigma_z\rangle(0)$, say, along the x direction, and write the system Hamiltonian as

$$H_S(t) = \frac{B}{2}\sigma_z + \frac{\Delta}{2}\sigma_x \cos \Omega t. \quad (27)$$

With this Hamiltonian the system would be localized if the time-dependent field is turned off. In other words, the tunneling between two levels is totally induced by the time-dependent field. This is a similar situation to that where the driving field is circularly polarized in the x - y plane (Sec. V A 1).

In Fig. 3, the simulations are done under strong but slow driving ($\Delta = 1$ and $\Omega = 0.1$) and with large local potential $B = 1$.

Figure 3(a) shows that $\langle\sigma_z\rangle(t)$ oscillates rapidly in the dissipationless ($\lambda = 0$) case. When dissipation is turned on, the system coherently oscillates around a nonzero value when dissipation and driving are balanced. Such kinds of asymptotic Floquet states are also reported in driven spin-boson models [31]. The robustness occurs again for which lines with the same $\Delta\mu$ but different nonzero λ almost coincide eventually, whereas in Fig. 3(b) the lines have the same λ but different $\Delta\mu$, and they coherently oscillate in different places.

2. Field in the z direction

Now let us consider the case where the time-dependent field is parallel to $\langle\sigma_z\rangle(0)$ with the system Hamiltonian

$$H_S = \frac{B}{2}\sigma_z \cos \Omega t + \frac{\Delta}{2}\sigma_x. \quad (28)$$

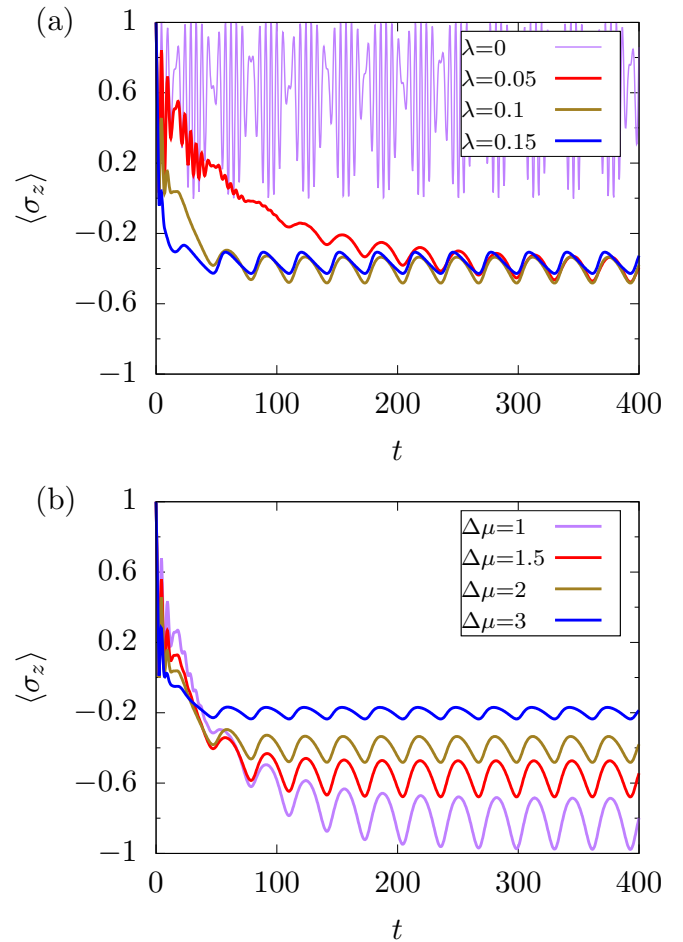


FIG. 3. Dynamics of $\langle\sigma_z(t)\rangle$ under a linearly driving field in the x direction for (a) varying system-bath coupling strength with $\Delta\mu = 2$ and (b) varying chemical potential difference with $\lambda = 0.1$. We have used $B = 1$, $\Delta = 1$, and $\Omega = 0.1$ for both (a) and (b).

There is always a static field along the x direction; thus, even if the time-dependent driving field is off there is still tunneling between the two levels. This is the usual case when studying the dissipative TLS [9,22,23,25–27,30,31,33,34,44,47,49]; thus, we present more simulations here.

In Fig. 4 the simulations are done under strong but slow driving ($B = 1$, $\Omega = 0.1$) with large local potential $\Delta = 1$. It can be seen under such driving that coherent oscillations are induced which are similar to the circularly driving field in the y - z plane case (Fig. 2). It can be seen from Fig. 4(a) that the asymptotic $\langle\sigma_z\rangle(t)$ with $\Delta\mu = 2$ and different λ almost coincide. Under this driving the asymptotic Floquet states exhibit robustness against different system-bath coupling, whereas the asymptotic $\langle\sigma_z\rangle(t)$ with $\lambda = 0.1$ and different $\Delta\mu$ differ a lot [see Fig. 4(b)].

The robustness *breaks down* when we enter the fast driving region by increasing Ω to 1 [see Fig. 5(a)]. It can be clearly seen that the asymptotic $\langle\sigma_z\rangle(t)$ with different λ differ a lot. Interestingly, by intuition we may think that the amplitude of oscillations with smaller λ would be larger since dissipation is smaller. However, conversely, the situation is that the amplitude of oscillations with the largest $\lambda = 0.15$ is

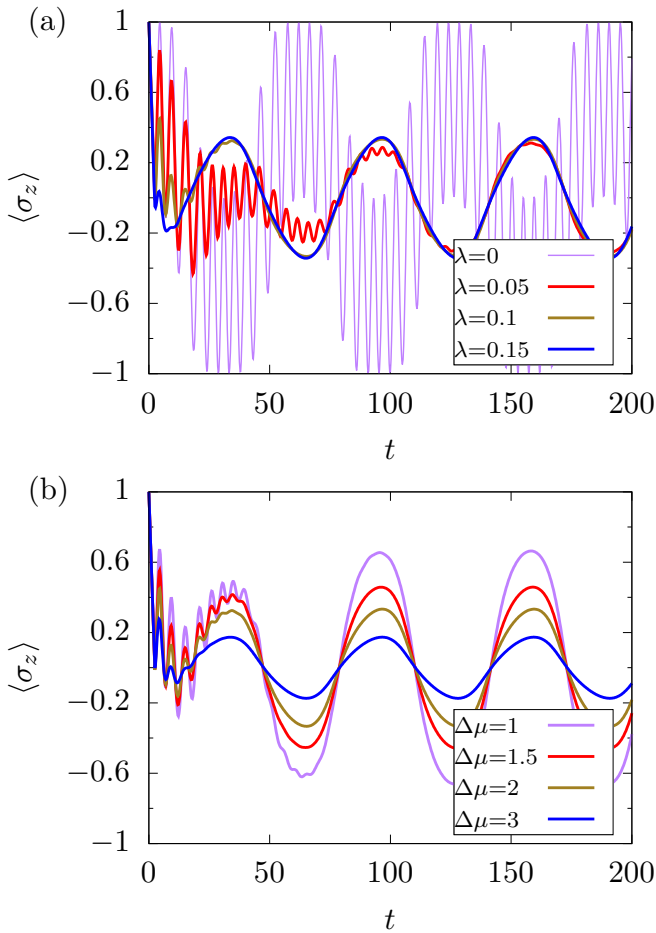


FIG. 4. Dynamics of $\langle \sigma_z \rangle(t)$ under a slow linearly driving field in the z direction for (a) varying system-bath strength with $\Delta\mu = 2$ and (b) varying chemical potential difference with $\lambda = 0.1$. We have used $B = 1$, $\Delta = 1$, and $\Omega = 0.1$ for (a) and (b).

the largest while the amplitude of the line with the smallest $\lambda = 0.05$ is the smallest. In other words, stronger dissipation enhances the amplitude of oscillations of the system rather than suppresses it. Such a phenomenon is a kind of quantum stochastic resonance in which noises amplify and optimize the response of a driven system [14–16].

Figure 5(b) shows fast driving simulations with $\lambda = 0.1$ and various $\Delta\mu$. We found the situation is, in another way around, that the asymptotic $\langle \sigma_z \rangle(t)$ coincide with different $\Delta\mu$.

Situations in Figs. 2 and 4 are similar. However, if Ω is increased to 1 in Fig. 2 (not shown in this article) we could not obtain similar results as in Fig. 5(b): the robustness against λ breaks down but the asymptotic $\langle \sigma_z \rangle(t)$ with different $\Delta\mu$ would not coincide either.

At first glance we may conclude that the robustness against different λ breaks down under fast driving. However, simulations in Fig. 1 are also under fast driving but they still show the robustness against λ . Right now we could only propose a hypothesis that the robustness would appear at least under slow driving.

The results above indicate that there exists a complex interplay between driving, bath electronic structure, and

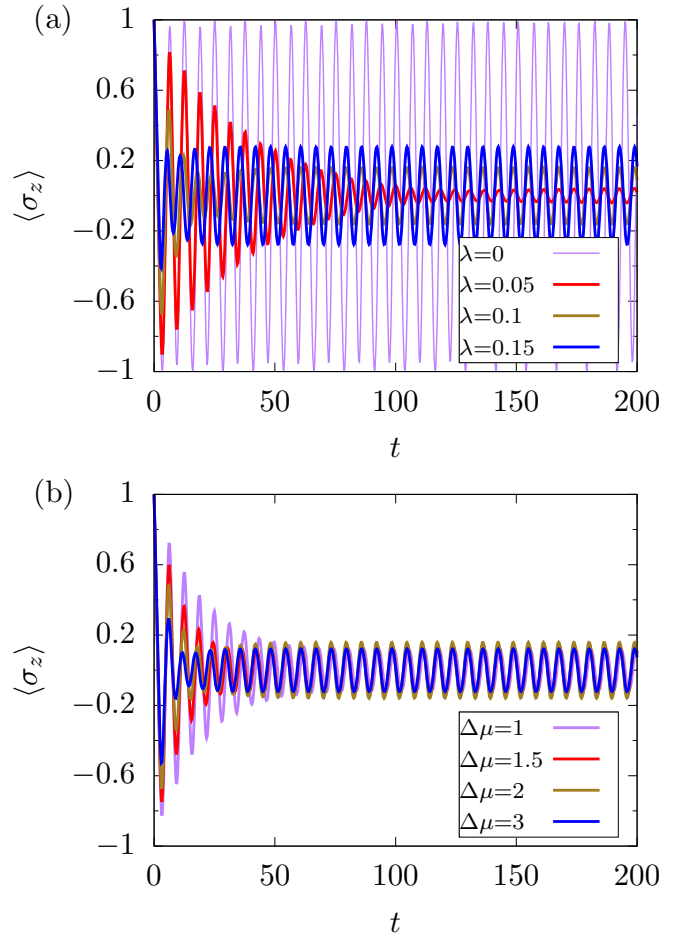


FIG. 5. Dynamics of $\langle \sigma_z \rangle(t)$ under a fast linearly driving field in the z direction for (a) varying system-bath coupling strength with $\Delta\mu = 2$ and (b) varying chemical potential difference with $\lambda = 0.1$. We have used $B = 1$, $\Delta = 1$, and $\Omega = 1$ for both (a) and (b).

bath-system coupling. Dissipation no longer simply acts as a contributor to dephasing under time-dependent driving. Due to the interplay the robustness of asymptotic Floquet states against λ is exhibited under certain drivings. The interplay can lead to quantum stochastic resonance as well.

VI. DISCUSSION AND CONCLUSIONS

We have numerically simulated the dynamics of the spin-fermion model under various external monochromatic driving fields via a numerically exact path integral method beyond the Markovian limit. Under time-dependent driving, the spin-fermion model exhibits rich phenomena which are not accessible in stationary situations.

We have also employed a Floquet master equation [52] with the nonadditive [64] system-bath interaction to check our results. The Floquet master equation is in qualitative agreement with the path integral method. However, the iterative path integral approach is nonperturbative and can be applied to the system with nonperiodic driving while the Floquet master equation, on the other hand, is perturbative and restricted to periodic driving. To further validate our approach, possible

future work is to apply chain-mapping approaches [75–77] which consider the evolution of both system and bath and are also nonperturbative.

It can be seen in our simulations that under a monochromatic driving field coherent oscillation can be induced in many circumstances (Figs. 2, 4, and 5). Such coherent oscillations are also reported in the spin-boson model under a monochromatic linearly polarized driving field in the z direction [23,31]. Such oscillations present typical asymptotic Floquet states of a dissipative spin system under periodic driving [23,31,72].

In addition, we also show that under a strong and fast circularly polarized driving field in the x - y plane (Fig. 1) the system exhibits behaviors similar to CDT [71] and DITO [26,27,30]. However, as we pointed out in Sec. V A 1 there is an essential difference between the behaviors in Fig. 1 and CDT or DITO. In CDT or DITO, the driving field suppresses rather than induces the tunneling between the two states. Here $\langle\sigma_z\rangle(t)$ finally stays in a positive value because driving and dissipation reach a balanced state.

A linearly polarized driving field in the x direction can induce coherent oscillations around a nonzero value (Fig. 3): without dissipation $\langle\sigma_z\rangle(t)$ oscillates around a nonzero value with wave packets, while the dissipation suppresses fast oscillations and $\langle\sigma_z\rangle(t)$ eventually coherently oscillates around a nonzero value.

Based on the observations of simulations in Sec. V we find an interesting phenomenon that under certain drivings the asymptotic Floquet states of the system exhibit a kind of robustness against different system-bath coupling strength λ . It can be seen from Figs. 1(a), 2(a), 3(a), and 4(a) that in simulations with the same $\Delta\mu$ but different λ asymptotic $\langle\sigma_z\rangle(t)$ almost coincide. In other words, the asymptotic $\langle\sigma_z\rangle(t)$ is insensitive to λ . Meanwhile another dissipative parameter $\Delta\mu$ would greatly affect the asymptotic $\langle\sigma_z\rangle(t)$. This shows that, although both system-bath coupling strength λ and chemical potential difference $\Delta\mu$ both contribute to dissipation, their effects are distinct.

Such robustness breaks down when increasing Ω to 1 in a linearly driving field in the z direction case (see Fig. 5). The robustness also breaks down when increasing Ω to 1 in Fig. 2 (not shown in this article). However, we cannot conclude directly that the robustness breaks down under fast driving since simulations in Fig. 1 (circularly polarized field in the x - y plane) are also under fast driving ($\Omega = 1$) but the robustness still holds. Thus, for this moment we could only propose a hypothesis that the robustness holds at least under slow driving.

In addition, in Fig. 5(a) quantum stochastic resonance is demonstrated in which the amplitude of coherent oscillations is enhanced, rather than suppressed, by stronger system-bath coupling (stronger dissipation). Quantum stochastic resonance in the driven spin-fermion model needs further theoretical and numerical investigations and future study may be devoted to this issue.

These phenomena indicate that there exists a complex interplay between the driving, bath electronic structure, and system-bath coupling. According to our simulations the robustness against different λ holds at least under slow driving,

but what plays the essential role may be ratios of all parameters and the form of the driving field.

In conclusion, the spin-fermion model shows rich phenomena under monochromatic driving. An interesting phenomenon is that under certain drivings asymptotic Floquet states of the system exhibit robustness against a range of system-bath coupling strength λ : the asymptotic behaviors of the system are insensitive to different λ while the chemical potential difference of baths $\Delta\mu$ greatly affects them. Further simulations indicate that the robustness may be essentially a result of the interplay between the driving, bath electronic structure, and system-bath coupling and thus can break down depending on the characteristics of the interplay. The interplay can also lead to quantum stochastic resonance.

The property of robustness indicates that under certain drivings the asymptotic behaviors of the system are dominated by the electronic structure of baths regardless of system-bath coupling strength. In other words, we may extract information of the electronic structure of baths without knowing the exact system-bath coupling strength. Such a property may be useful in designing nanodetectors for the electronic structure of the fermionic environment. Unlike the common sense detector, the driving field is also a part of the detector and the time-dependent parameters of the driving field are of essential importance. Moreover, the observed result is also not merely a static quantity but a time-dependent $\langle\sigma_z\rangle(t)$. In this sense, we may say it is a detector in time.

Open questions remain whether this property commonly exists in dissipative systems and if the form of system-bath coupling changes or the system is no longer a simple TLS. Since parameters of numerical simulations are limited by practical computation sources and convergence conditions, theoretical analyses may give a more general physical picture about the interplay between driving and dissipation. Further numerical investigation on the driven spin-boson model and theoretical investigation on the driven spin-fermion model may be our future works.

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APPENDIX A: CONVERGENCE AND ERROR ANALYSIS

There are three parameters relevant to accuracy and convergence of simulations: bath size, the time step δt , and the truncated memory time τ_c . In this article, we have set the bath size as 80. With the identical fermionic bath, converged results are reported for a bath size of 40 [47]. For a more complicated system with both fermionic and bosonic baths, it has been reported that bath size of 30 is sufficient to obtain converged results [49].

Trotter error due to finite time step δt can be reduced by a smaller δt and the result would become exact when $\delta t \rightarrow 0$. However, with fixed N_s smaller δt means smaller τ_c and larger overall errors. Therefore, smaller δt needs larger N_s ,

to ensure enough length of τ_c . As we discussed in Sec. II, the computational memory increases exponentially with respect to N_s and thus the value of N_s is also restricted according to the available memory.

In principle one can extrapolate final results to the $\delta t \rightarrow 0$ limit and the Trotter error is then eliminated [47,53]. However, since we are dealing with the time-dependent driving case, with different δt the driving field is sampled in different time grids. This brings an extra error in extrapolation. To extrapolate to the $t \rightarrow 0$ case, we need to do simulations with a range of δt with a fixed τ_c . When δt is large, the time grid may miss important points, while small δt needs large N_s which may be not acceptable in practical computation. Therefore, in this article extrapolation is not employed.

It is shown that [47,53] τ_c roughly corresponds to $1/\Delta\mu$ for the spin-fermion model without driving. And $\delta t = 0.25$ and $N_s = 9$ are enough to ensure convergence for small $\Delta\mu = 0.6$ and intermediate coupling strength $\lambda = 0.2$. For strong interaction the time step δt needs to be small to guarantee convergence and correspondingly a large N_s is needed. That is, if $\delta t = 0.1$ and $\Delta\mu \sim 0.6$ then an extensive computation effort as $N_s > 16$ is required.

In this article we set $\delta t = 0.25$ and $N_s = 10$. Since for strong interaction small δt is needed for small $\Delta\mu$ (otherwise numerical divergence may be encountered), for safety we keep $\Delta\mu \geq 1$ and λ in the weak and intermediate interaction region ($\lambda = 0.05, 0.1, \text{ and } 0.15$) in this article.

Figure 6 shows a convergence test for increasing N_s with parameters used in Sec. V A 2. Basically it is a reexamination of Fig. 2 under circularly polarized fields in the y - z plane with $B = 1$, $\Delta = 1$, $\Omega = 0.1$, $\lambda = 0.1$, and $\Delta\mu = 2$. In Fig. 6(a) the dynamics of $\langle\sigma_z\rangle(t)$ is shown. Besides, the dynamics of $\langle\sigma_x\rangle(t)$ is also demonstrated in Fig. 6(b). It can be seen that convergence is well reached at $N_s = 10$.

APPENDIX B: COMPARISON TO STEADY-STATE SOLUTION OF MASTER EQUATION IN FLOQUET STATES REPRESENTATION

For comparison we employ a Born-Markov master equation in Floquet states representation to study the case in Sec. V A 1 where the driving field is circularly polarized in the x - y plane. In this case, the Floquet states of the isolated driven TLS can be analytically solved [6,70] such that the Floquet states and corresponding quasienergies are

$$\varphi_1(t) = \sqrt{\frac{\omega - \Lambda}{2\omega}} \begin{pmatrix} e^{-i\Omega t} \\ \frac{\Delta}{\omega - \Lambda} \end{pmatrix}, \quad \varepsilon_1 = \frac{1}{2}(\omega - \Omega) \quad (\text{B1})$$

$$\varphi_2(t) = \sqrt{\frac{\omega + \Lambda}{2\omega}} \begin{pmatrix} 1 \\ -\frac{\Delta}{\omega + \Lambda} e^{i\Omega t} \end{pmatrix}, \quad \varepsilon_2 = -\frac{1}{2}(\omega - \Omega),$$

where $\Lambda = \Omega - B$ and $\omega = \sqrt{\Delta^2 + \Lambda^2}$.

Here following the derivation in Ref. [52] we give a brief review about the Floquet master equation technique. For conciseness, we denote the system reduced density matrix $\rho_S(t)$ by $\varrho(t)$. After a standard Born-Markov master equation procedure [78], we obtain an integrodifferential

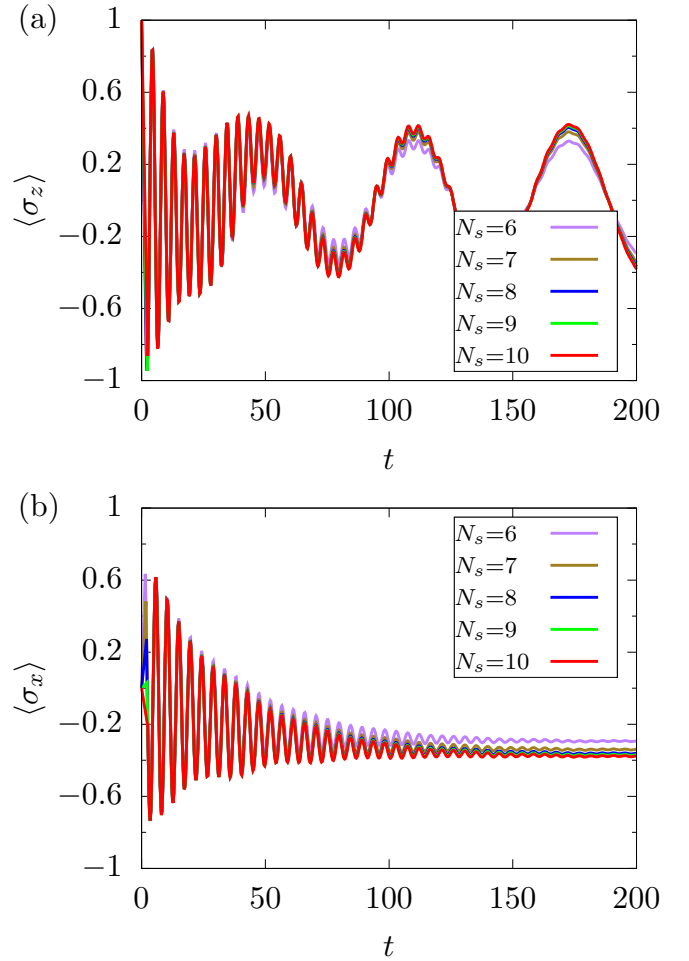


FIG. 6. Convergence behavior with increasing N_s . It is a reexamination of Fig. 2 and other parameters are the same as those in Fig. 2.

equation for $\varrho(t)$,

$$\begin{aligned} \frac{\partial \varrho(t)}{\partial t} = & -i[H_S(t), \varrho(t)] \\ & - \int_0^t [\sigma_z, \sigma_z(t - \tau, t)\varrho(\tau)]C(\tau) d\tau \\ & + \int_0^t [\sigma_z, \varrho(\tau)\sigma_z(t - \tau, t)]C^*(\tau) d\tau, \quad (\text{B2}) \end{aligned}$$

where $\sigma_z(t - \tau, t)$ stands for $U_S(t, t - \tau)\sigma_z U_S^\dagger(t, t - \tau)$.

Unlike most of the existing works based on two-body system-bath interaction, our system-bath coupling given by Eq. (15) contains nonadditive interactions. Therefore, our bath correlation function $C(\tau)$ is

$$\begin{aligned} C(\tau) = & \sum_{\alpha\beta:\alpha'\beta'} \sum_{kq:k'q'} V_{\alpha\beta} V_{\alpha'\beta'} \langle c_{\alpha k}^\dagger(\tau) c_{\beta q}(\tau) c_{\alpha' k'}^\dagger c_{\beta' q'} \rangle \\ = & \frac{\lambda^2}{g^2} \sum_{\alpha \neq \beta} \sum_{kq} n_{\alpha k} (1 - n_{\beta q}) e^{i(\varepsilon_k - \varepsilon_q)\tau}, \quad (\text{B3}) \end{aligned}$$

where g is the density of states of each Fermi bath and $n_{\alpha k} = \langle c_{\alpha k}^\dagger c_{\alpha k} \rangle = \text{Tr}_B [c_{\alpha k}^\dagger c_{\alpha k}]$. It is convenient to define the quantity

$C(E)$ as

$$C(E) = \int_0^\infty C(\tau) e^{iE\tau} d\tau \approx \frac{\lambda^2}{g^2} \sum_{\alpha \neq \beta} \sum_{kq} n_{\alpha k} (1 - n_{\beta q}) \delta(\varepsilon_k + E - \varepsilon_q), \quad (\text{B4})$$

where principal value contributions are neglected. Note that a continuous, not discretized, energy spectrum is used here to calculate $C(E)$.

Now we define the reduced density matrix of the system in Floquet representation as $\varrho_{ij}(t) = \langle \varphi_i(t) | \varrho(t) | \varphi_j(t) \rangle$. Denoting $\langle \varphi_i(t) | \sigma_z | \varphi_j(t) \rangle$ by $\sigma_{ij}(t)$ and expanding it in Fourier series such that

$$\sigma_{ij}(t) = \sum_m \sigma_{ij}(m) e^{im\Omega t} \quad (\text{B5})$$

yields the master equation for $\varrho_{ij}(t)$,

$$\left(\frac{\partial}{\partial t} + i\varepsilon_{ij} \right) \varrho_{ij}(t) = - \sum_{kl} [R_{ik;kl}(t) \varrho_{lj}(t) - R_{lj;ik}(t) \varrho_{kl}(t) - R_{ki;jl}^*(t) \varrho_{kl}(t) + R_{jl;lk}^*(t) \varrho_{ik}(t)], \quad (\text{B6})$$

where [noticing that $\sigma_{ij}^*(m) = \sigma_{ji}(-m)$]

$$R_{ij;kl}(t) = \sum_{mn} e^{i(m+n)\Omega t} \sigma_{ij}(m) \sigma_{kl}(n) \times \int_0^t e^{-i(\varepsilon_{kl} + n\Omega)\tau} C(\tau) d\tau, \quad (\text{B7})$$

and

$$R_{ij;kl}^*(t) = \sum_{mn} e^{i(m+n)\Omega t} \sigma_{ji}(m) \sigma_{lk}(n) \times \int_0^t e^{-i(\varepsilon_{lk} + n\Omega)\tau} C^*(\tau) d\tau \quad (\text{B8})$$

with $\varepsilon_{ij} = \varepsilon_i - \varepsilon_j$.

In the steady states where $t \rightarrow \infty$, only terms satisfying $m + n = 0$ survive due to the vanishing of oscillating factors $e^{i(m+n)\Omega t}$. The master equation for steady states ϱ_{ij} then reads

$$i\varepsilon_{ij} \varrho_{ij} = - \sum_{kl} [R_{ik;kl} \varrho_{lj} - R_{lj;ik} \varrho_{kl} - R_{kl;jl}^* \varrho_{kl} + R_{jl;lk}^* \varrho_{ik}], \quad (\text{B9})$$

where

$$R_{ij;kl} = \sum_m \sigma_{ij}(m) \sigma_{kl}(-m) \int_0^\infty e^{-i(\varepsilon_{kl} - m\Omega)\tau} C(\tau) d\tau, \quad (\text{B10})$$

and

$$R_{ij;kl}^* = \sum_m \sigma_{ji}(m) \sigma_{lk}(-m) \int_0^\infty e^{-i(\varepsilon_{lk} - m\Omega)\tau} C^*(\tau) d\tau. \quad (\text{B11})$$

According to Eq. (B1) the matrix element $\sigma_{ij}(t)$'s are

$$\begin{aligned} \sigma_{11}(t) &= -\frac{\Lambda}{\omega}, & \sigma_{12}(t) &= \frac{\Delta}{\omega} e^{i\Omega t} \\ \sigma_{21}(t) &= \frac{\Delta}{\omega} e^{-i\Omega t}, & \sigma_{22}(t) &= \frac{\Lambda}{\omega}. \end{aligned} \quad (\text{B12})$$

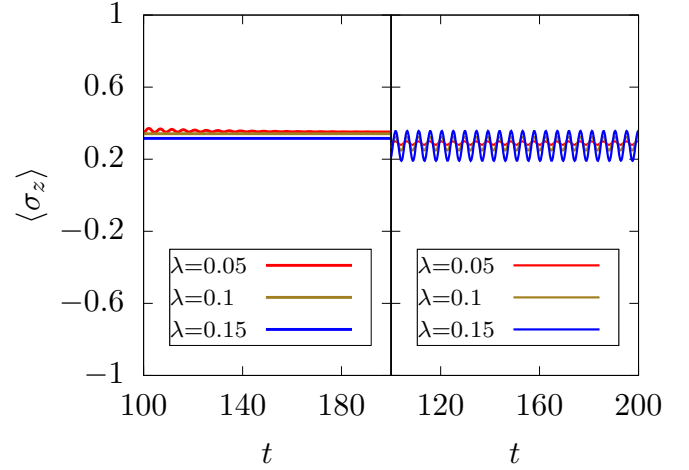


FIG. 7. $\langle \sigma_z \rangle(t)$ in asymptotic Floquet states with different λ calculated by different methods. Left: Results of the path integral method. Right: Results of the Floquet master equation. Other parameters are the same as those in Fig. 1(a).

Therefore, only the following $\sigma_{ij}(m)$'s are nonzero:

$$\begin{aligned} \sigma_{11}(0) &= -\frac{\Lambda}{\omega}, & \sigma_{12}(1) &= \frac{\Delta}{\omega} \\ \sigma_{21}(-1) &= \frac{\Delta}{\omega}, & \sigma_{22}(0) &= \frac{\Lambda}{\omega}. \end{aligned} \quad (\text{B13})$$

Accordingly only six $R_{ij;kl}$'s are nonzero: $R_{11;11}$, $R_{11;22}$, $R_{22;11}$, $R_{22;22}$, $R_{12;21}$, and $R_{21;12}$. The asymptotic Floquet states can be found by solving Eq. (B9). The comparisons between $\langle \sigma_z \rangle(t)$ in asymptotic Floquet states calculated by the path integral method and by the Floquet master equation are shown in Figs. 7 and 8.

Figure 7 shows $\langle \sigma_z \rangle(t)$ in asymptotic states with different λ when $\Delta\mu = 2$. Most parameters are the same as those in Fig. 1(a): $B = 0.1$, $\Delta = 1$, and $\Omega = 1$. The right-hand panel

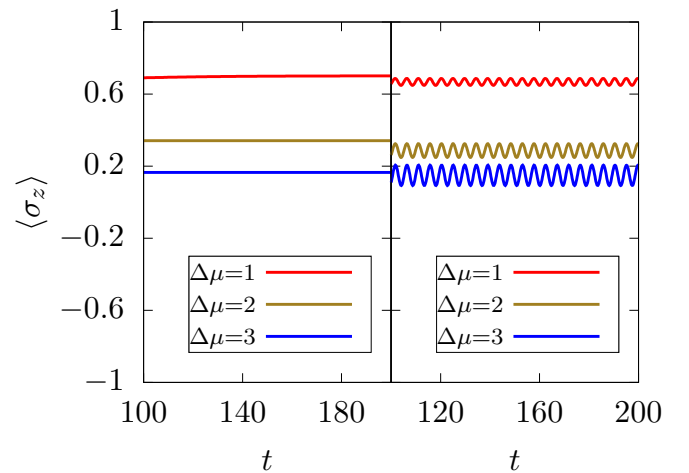


FIG. 8. $\langle \sigma_z \rangle(t)$ in asymptotic Floquet states with different $\Delta\mu$ calculated by different methods. Left: Results of the path integral method. Right: Results of the Floquet master equation. Other parameters are the same as those in Fig. 1(b).

shows the results calculated by the Floquet master equation. It could be seen that although oscillation amplitudes of $\langle\sigma_z\rangle(t)$ are different with different λ , their mean positions are almost the same. In other words, the Floquet master equation calculation reproduces the robustness against different λ . The left-hand panel shows the results calculated by the path integral method for comparison. It is shown that the results of the two methods are in agreement except that the results of the Floquet master equation are weakly oscillating.

Figure 8 shows $\langle\sigma_z\rangle(t)$ in asymptotic states with different $\Delta\mu$ when $\lambda = 0.1$. The mean positions of $\langle\sigma_z\rangle(t)$ calculated by two methods are in agreement. Both methods show that the results are significantly altered by different chemical potential bias $\Delta\mu$.

The results in Figs. 7 and 8 show that the dynamics of $\langle\sigma_z\rangle(t)$ by the path integral method are in agreement with those of the Floquet master equation. However, the Floquet master equation gives weakly oscillating $\langle\sigma_z\rangle(t)$, whereas the oscillation decays to zero when the path integral method is used. Since some other results of the path integral method still show small oscillations (for example, Fig. 3), the vanishing of the oscillation is unlikely due to the numerical feature of the path integral method. The origin of such oscillations by the master equation is possibly the perturbative nature of the master equation. However, since these two methods adopt different approximation schemes and numerical error mechanisms, further analysis is needed to account for their numerical difference.

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