Effect of critical fluctuations on the spin transport in liquid ^{3}He

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The contribution of pair fluctuations to the spin current in liquid ^{3}He in isotropic aerogel near the critical temperature of transition to the superfluid state is calculated.

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I. INTRODUCTION

The superfluid state of liquid ^{3}He is formed by means of the Cooper pairing with spin and orbital angular momentum equal to 1 [1]. Investigation of superfluid phases in high porosity aerogel allows one to study the influence of impurities on superfluidity with p pairing [2,3]. In the normal state, the spin-diffusion coefficient of ^{3}He in aerogel is determined both by elastic and inelastic scattering of ^{3}He quasiparticles. At low temperatures the collisions between the Fermi-liquid quasiparticles induce negligibly small correction to the spin diffusion due to the scattering on aerogel strands [4,5]. The field theoretical approach to the calculation of the spin-diffusion coefficient in the normal ^{3}He in an anisotropic aerogel has been developed in Ref. [6] in analogy with the calculation of electric current in an isotropic metal with randomly distributed impurities performed in Ref. [7]. Close to the superfluid transition temperature in line with regular spin transport limited by scattering of quasiparticles on the aerogel there is an additional mechanism determined by the Cooper pair fluctuations accelerating spin transport as the critical point is neared. The effects of fluctuations on the thermodynamics and kinetics of a superconductor near the transition point are well known [8]. The theoretical studies of these phenomena have acquired a firm basis since the publication of the seminal paper by Aslamazov and Larkin, where the field theoretical approach to the problem has been developed [9]. The corresponding theory in application to d-wave pairing in layered metals has been developed in two papers [10]. The low-temperature (quantum) limit has been considered in Ref. [11]. In the present paper, I apply this approach to calculation of the contribution of pair fluctuations to the spin current density above the critical temperature in normal ^{3}He in isotropic aerogel.

To make easy comparison with calculation of paraconductivity in superconductors with s pairing [9], I begin with definition of the fluctuating propagator for p-wave superfluid. An electric current presents a response to the em vector-potential. Similarly a spin current is given by the response to the nonuniform rotation of the spin space. This allows one to perform the derivation of para spin diffusion in liquid ^{3}He in the same spirit as paraconductivity of a metal near transition to the s-wave superconducting state.

II. SPIN DIFFUSION OF A FLUCTUATING PAIR

The order parameter of superfluid phases of ^{3}He is given [1] by the complex 3×3 matrix $A_{\alpha i}$, where α and i are the indices numerating the Cooper pair wave-function projections on spin and orbital axes, respectively. The second-order term in the Landau free-energy density is

$$F^{(2)}=\left\{\frac{1}{3g}\delta_{rs}-T\sum_n\int\frac{d^3p}{(2\pi)^3}G_{\mathbf{p},\varepsilon_n}G_{-\mathbf{p},-\varepsilon_n}\hat{p}_r\Lambda^s_{\mathbf{p},\varepsilon_n}\right\}A^\star_{\alpha r}A_{\alpha s},\tag{1}$$

where g is the constant of p-wave triplet pairing and $\hat{p}_r$ is the r component of the momentum unit vector $\mathbf{p}/|\mathbf{p}|$. Here,

$$G_{\mathbf{p},\varepsilon_n}=\frac{1}{i\tilde{\varepsilon}_n-\xi_{\mathbf{p}}}\tag{2}$$

is the normal-state quasiparticle Green's function and $\Lambda^s_{\mathbf{p},\varepsilon_n}$ is the vertex part. $\xi_{\mathbf{p}}=\epsilon_{\mathbf{p}}-\mu$ is the quasiparticle energy counted from the chemical potential, $\varepsilon_n=\pi T(2n+1)$ are the fermion Matsubara frequencies, $\tilde{\varepsilon}_n=\varepsilon_n+\frac{1}{2\tau}\mathrm{sgn}\varepsilon_n$, and τ is the mean free time scattering of quasiparticles in an isotropic aerogel. The Planck constant $\hbar$ was everywhere set equal to 1. Correspondingly the matrix of the fluctuation propagator is

$$L^{rs}(\mathbf{q},\Omega_k)=\left(\frac{1}{3g}\delta_{rs}-T\sum_n\int\frac{d^3p}{(2\pi)^3}G_{\mathbf{p},\varepsilon_n}G_{-\mathbf{p}+\mathbf{q},\Omega_k-\varepsilon_n}\hat{p}_r\Lambda^s_{\mathbf{p},\mathbf{q},\varepsilon_n}\right)^{-1},\tag{3}$$

where $\Omega_k=2\pi Tk$ are the boson Matsubara frequencies.

As it was pointed out in Ref. [9] the largest contribution to the conductivity of a fluctuating pair is given by the diagram shown in Fig. 1, where wavy lines are the fluctuating propagators, the straight lines are the Green's functions, and the shaded triangles are the vertex parts. In contrast to s pairing, due to momentum dependent pairing interaction, all the vertices are not scalar but vector functions. To find the corresponding analytic expression one must define the spin current.

The spin current in neutral Fermi liquid can be found [12,13] as a response to the gradient of the angle of rotation

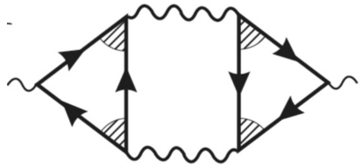

FIG. 1. The Aslamazov-Larkin diagram.

of the spin space $\boldsymbol{\omega}_i = \nabla_i \boldsymbol{\theta}$:

$$\mathbf{j}_i = -\frac{\delta H}{\delta \boldsymbol{\omega}_i}, \tag{4}$$

where

$$H = \frac{1}{2m}\int d^3r \left(D_i^{\alpha\lambda}\psi_\lambda\right)^\dagger D_i^{\alpha\mu}\psi_\mu + H_{\text{int}}, \tag{5}$$

$$D_i^{\alpha\beta} = -i\delta_{\alpha\beta}\nabla_i + \frac{1}{2}\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\omega}_i, \tag{6}$$

$\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices, and H_{int} includes the Fermi-liquid interaction and the interaction with impurities. The response to the gauge field $\boldsymbol{\omega}_i$ is calculated [6] in analogy with the response to the usual vector potential A_i [7]. The contribution of pair fluctuations to the spin current density above the critical temperature corresponding to the diagram shown in Fig. 1 is

$$\mathbf{j}_i(\omega_\nu) = \int \frac{d^3q}{(2\pi)^3} T \sum_k \mathbf{B}^{lr}_{i,\alpha\beta}(\mathbf{q}, \Omega_k, \omega_\nu) L^{rs}(\mathbf{q}, \Omega_k) \times \left[\mathbf{B}^{st}_{j,\beta\alpha}(\mathbf{q}, \Omega_k, \omega_\nu)\cdot \boldsymbol{\omega}_j\right] L^{tl}(\mathbf{q}, \Omega_k + \omega_\nu). \tag{7}$$

Here, $\omega_\nu = 2\pi T\nu$ are the boson Matsubara frequencies and $L^{rs}(\mathbf{q}, \Omega_k)$ is the fluctuation propagator. The triangle block

$$\mathbf{B}^{lr}_{i,\alpha\beta}(\mathbf{q}, \Omega_k, \omega_\nu) = T\sum_n \int \frac{d^3p}{(2\pi)^3} v_i \boldsymbol{\sigma}_{\alpha\beta} \Lambda^l_{\mathbf{p},\mathbf{q},\varepsilon_n+\omega_\nu,\Omega_k-\varepsilon_n} \Lambda^{r\star}_{\mathbf{p},\mathbf{q},\varepsilon_n,\Omega_k-\varepsilon_n} \times G_{\mathbf{p},\varepsilon_n+\omega_\nu} G_{\mathbf{p},\varepsilon_n} G_{\mathbf{q}-\mathbf{p},\Omega_k-\varepsilon_n} \tag{8}$$

is expressed through three Green's functions (2) and the impurity vertex functions $\Lambda^l_{\mathbf{q},\varepsilon_n,\Omega_k-\varepsilon_n}$ and $\Lambda^{r\star}_{\mathbf{q},\varepsilon_n,\Omega_k-\varepsilon_n}$. The sign of the complex conjugation in the second vertex function in Eq. (8) corresponds to the opposite direction of the arrows of the Green's-function lines in Fig. 1 with respect to the first vertex (time inversion). The impurity vertex functions are determined by the integral equation

$$\Lambda^l_{\mathbf{p},\mathbf{q},\varepsilon_n,\Omega_k-\varepsilon_n} = \hat{p}_l + \frac{1}{2\pi N_o \tau}\int \frac{d^3k}{(2\pi)^3} \times G_{\mathbf{p},\varepsilon_n} G_{-\mathbf{p}+\mathbf{q},\Omega_k-\varepsilon_n} \Lambda^l_{\mathbf{p},\mathbf{q},\varepsilon_n,\Omega_k-\varepsilon_n}. \tag{9}$$

Near critical temperature the main frequency dependence arises from the fluctuation propagators L having the pole structure. Due to this reason one can neglect the frequency dependence of the blocks $\mathbf{B}$ and the vertices Λ. In the integral Eq. (7) are essentially only small values of q. Then, the solution Eq. (9) is

$$\Lambda^l_{\mathbf{p},\mathbf{q},\varepsilon_n} = \hat{p}_l + \frac{i q_l v_F \operatorname{sgn}\tilde{\varepsilon}_n}{6|\tilde{\varepsilon}_n|(1-2|\tilde{\varepsilon}_n|\tau)} + \hat{q}_l \mathcal{O}(q^3), \tag{10}$$

where v_F is the Fermi velocity. The integral of the product of three Green's functions in the linear with respect to wave vector $\mathbf{q}$ approximation is

$$N_0 \int d\xi\, G_{\mathbf{p},\varepsilon_n} G_{\mathbf{p},\varepsilon_n} G_{\mathbf{q}-\mathbf{p},-\varepsilon_n} = -\frac{\pi N_0}{2}\left(\frac{\operatorname{sgn}\tilde{\varepsilon}_n}{\tilde{\varepsilon}_n^2} + \frac{(\mathbf{q}\cdot\mathbf{v})}{|\tilde{\varepsilon}_n|^3}\right), \tag{11}$$

where N_0 is the density of states per one spin projection. Substituting Eqs. (10) and (11) in Eq. (8) and performing the integration over angles we obtain

$$\mathbf{B}^{lr}_{i,\alpha\beta} = -\frac{\pi}{15} N_0 v_F^2 (\delta_{il}q_r + \delta_{ir}q_l + \delta_{lr}q_i)\boldsymbol{\sigma}_{\alpha\beta} T \sum_{n\geqslant 0}\frac{1}{\tilde{\varepsilon}_n^3} = \frac{1}{60}\frac{N_0 v_F^2}{(2\pi T)^2}\psi''\left(\frac{1}{2}+\frac{1}{4\pi T\tau}\right)(\delta_{il}q_r + \delta_{ir}q_l + \delta_{lr}q_i)\boldsymbol{\sigma}_{\alpha\beta}, \tag{12}$$

where $\psi''(z)$ is the second derivative of the digamma function.

The matrix of the fluctuation propagator is given by Eq. (3). The off-diagonal elements of this matrix can be omitted because they are proportional to higher-order terms in components of vector $\mathbf{q}$: $(\delta_{rs} + bq_rq_s)^{-1} = \delta_{rs} - bq_rq_s + \dots$. Performing integration over momenta in Eq. (3) we obtain at small q and Ω

$$L^{rs}(\mathbf{q}, \Omega_k) = \frac{3}{N_0}\frac{\delta_{rs}}{\epsilon + a\Omega + \xi^2 q^2}. \tag{13}$$

Here,

$$\epsilon = \ln\frac{T}{T_c}. \tag{14}$$

The critical temperature T_c is suppressed with respect to the temperature T_{c0} of the superfluid transition in pure helium and determined from the equation

$$\ln\frac{T_{c0}}{T_c} = \psi\left(\frac{1}{2}+\frac{1}{4\pi T_c\tau}\right) - \psi\left(\frac{1}{2}\right). \tag{15}$$

The coefficient

$$a = \frac{\pi T}{(2\pi T)^2}\psi'\left(\frac{1}{2}+\frac{1}{4\pi T\tau}\right) = \begin{cases}\frac{\pi}{8T}, & 4\pi T\tau \gg 1, \quad |\Omega| \ll 4\pi T,\\ \tau, & 4\pi T\tau \ll 1, \quad |\Omega|\tau \ll 1.\end{cases} \tag{16}$$

The first line here corresponds to the limit of weak scattering when the critical temperature is slightly suppressed by impurities $(T_{c0} - T_c)/T_{c0} \approx (\pi/8T_{c0}\tau) \ll 1$ and the typical frequencies of fluctuations $|\Omega| \approx (T - T_c) \ll T$. This is the quasistatic or classic fluctuation region. With respect to the second line one must remark that impurities completely suppress superfluidity at $\tau_c = \frac{\gamma}{\pi T_{c0}}$, where $\gamma \approx 1.8$ is the Euler constant. Hence, $4\pi T\tau > \frac{4\gamma T}{T_{c0}}$ and for fulfillment of inequality $4\pi T\tau \ll 1$ the temperature must be at least $1/4\gamma$ times lower than the critical temperature in pure helium. Still, at such low temperatures there are two different situations. First, this is again the region of classic fluctuations $|\Omega|\tau \ll 4\pi T\tau \ll 1$. The second is the region of quantum fluctuations

$4\pi T\tau \ll |\Omega|\tau \ll 1$ when the frequencies of fluctuations exceed the temperature.

The coefficient

$$\xi^2 = -\frac{1}{40}\frac{v_F^2}{(2\pi T)^2}\psi''\left(\frac{1}{2}+\frac{1}{4\pi T\tau}\right) = \begin{cases} \frac{7\zeta(3)}{20}\frac{v_F^2}{(2\pi T)^2}, & 4\pi T\tau \gg 1, \\ \frac{1}{10}v_F^2\tau^2, & 4\pi T\tau \ll 1. \end{cases} \quad (17)$$

It is convenient to rewrite these expressions in terms of zero-temperature coherence length $\xi_0 = \frac{v_F}{2\pi T_c}$. Hence, at temperatures near T_c

$$\xi \simeq \begin{cases} 0.65\xi_0, & 4\pi T\tau \gg 1, \\ 2T_c\tau\xi_0, & 4\pi T\tau \ll 1. \end{cases} \quad (18)$$

Thus, unlike the case of s-wave pairing [8], both in the clean case and in the dirty enough $T_c\tau \approx 1$ case, $\xi \approx \xi_0$.

Using this notation Eq. (12) acquires the following form:

$$\mathbf{B}^{lr}_{i,\alpha\beta} = -\tfrac{2}{3}N_0\xi^2(\delta_{il}q_r + \delta_{ir}q_l + \delta_{lr}q_i)\boldsymbol{\sigma}_{\alpha\beta}. \quad (19)$$

Substituting Eqs. (13) and (19) into Eq. (7) and making use of the analytical continuation from the discrete frequencies to the complex plane [9] we obtain the linear in frequency term in the spin current:

$$\mathbf{j}_i(\omega) = -\frac{\omega}{4\pi i}\left(\frac{2}{3}N_0\xi^2\right)^2\int\frac{d^3q}{(2\pi)^3} \times \int d\Omega\frac{(L^R-L^A)^2}{2T\sinh^2\frac{\Omega}{2T}}(9q_iq_j + 2q^2\delta_{ij})\boldsymbol{\omega}_j, \quad (20)$$

where

$$L^{R/A}(\mathbf{q},\Omega_k) = \frac{3}{N_0}\frac{1}{\epsilon \mp ia\Omega + \xi^2q^2} \quad (21)$$

are retarded and advanced fluctuation propagators. Performing the integration in the classic (static) $(T-T_c) \ll T$ limit we have

$$\mathbf{j}_i(\omega) = i\omega\frac{45}{16\xi\sqrt{\epsilon}}\boldsymbol{\omega}_i. \quad (22)$$

The quantum limit can be reached at low temperatures when $\epsilon \gg T\tau$ and the corresponding current expression is

$$\mathbf{j}_i(\omega) = i\omega\frac{5(\tau T)^2}{\xi\epsilon^{3/2}}\boldsymbol{\omega}_i. \quad (23)$$

Still, the temperature is limited from below by the critical temperature T_c and the critical fluctuations in close vicinity of critical temperature are always classical $\epsilon \ll T\tau$, to the displeasure of fans of quantum phase transitions.

Making use the Larmor theorem

$$\gamma\mathbf{H} = \frac{\partial\boldsymbol{\theta}}{\partial t} = -i\omega\boldsymbol{\theta}, \quad (24)$$

where $\gamma = 2\mu$ is the gyromagnetic ratio and μ is the magnetic moment of ^{3}He atoms, one can rewrite Eq. (22) for the fluctuation current as

$$\mathbf{j}_i^{\text{fl}} = -\frac{45}{8\xi\sqrt{\epsilon}}\mu\nabla_i\mathbf{H}. \quad (25)$$

III. CONCLUSION

In conclusion it is reasonable to compare the spin current due to the Cooper pair fluctuations with the diffusion current [6] determined by impurity scattering. The latter in dimensional units is

$$\mathbf{j}_i^{\text{dif}} = -\hbar N_0 D\mu\nabla_i\mathbf{H}. \quad (26)$$

Here, $D = \frac{1}{3}\tau v_F^2$ is the spin-diffusion coefficient. Thus, the ratio of two currents is

$$\frac{j^{\text{fl}}}{j^{\text{dif}}} = \frac{45}{8\hbar\xi N_0 D}\frac{1}{\sqrt{\epsilon}}. \quad (27)$$

In dense aerogel the diffusion coefficient can be small enough, $D \approx 10^{-3}$ cm^2/s [5]; the coherence length at ambient pressure [4] is $\xi_0 = 2\times10^{-6}$ cm; and the density of states at ambient pressure [14] is $N_0 \approx 0.5$ (erg cm^3)$^{-1}$. Thus,

$$\frac{j^{\text{fl}}}{j^{\text{dif}}} \approx 5\times10^{-2}\frac{1}{\sqrt{\epsilon}}. \quad (28)$$

With respect to experimental detection of fluctuation spin current this result is not encouraging. However, it will be perhaps useful for exact determination of the temperature of transition at the measurement of the coefficient of diffusion near the critical temperature.

The temperature dependence of spin-diffusion current in ^{3}He due to pair fluctuations (25) turns out to be the same as the temperature dependence of paraconductivity of a normal three-dimensional metal near transition to the s-wave superconducting state [9]. This is not astonishing because the structure of the theory is similar despite some particular features typical for p-wave pairing. The temperature dependence of the fluctuation spin-diffusion current in the quantum limit (23) also coincides with the temperature dependence of paraconductivity in d-wave superconductors in the quantum limit found in Ref. [11].

[1] D. Vollhardt and P. Wölfle, *The Superfluid Phases of Helium-3* (Tylor & Francis, London, 1990).

[2] J. V. Porto and J. M. Parpia, Phys. Rev. Lett. **74**, 4667 (1995).

[3] D. T. Sprague, T. M. Haard, J. B. Kycia, M. R. Rand, Y. Lee, P. J. Hamot, and W. P. Halperin, Phys. Rev. Lett. **75**, 661 (1995).

[4] J. A. Sauls, Yu. M. Bunkov, E. Collin, H. Godfrin, and P. Sharma, Phys. Rev. B **72**, 024507 (2005).

[5] V. V. Dmitriev, L. A. Melnikovsky, A. A. Senin, A. A. Soldatov, and A. N. Yudin, JETP Lett. **101**, 808 (2015).

[6] V. P. Mineev, Phys. Rev. B **98**, 014501 (2018).

[7] A. A. Abrikosov and L. P. Gor'kov, Zh. Exp. Teor. Fiz. **35**, 1558 (1958) [Sov. Phys. JETP **8**, 1090 (1959)].
[8] A. Larkin and A. Varlamov, *Theory of Fluctuations in Superconductors*, International Series of Monographs on Physics (Oxford University, New York, 2009).
[9] L. G. Aslamazov and A. I. Larkin, Fiz. Tverd. Tela **10**, 1104 (1968) [Sov. Phys. Solid State **10**, 875 (1968)].
[10] S. Yip, Phys. Rev. B **41**, 2612 (1990); J. Low Temp. Phys. **81**, 129 (1990).
[11] V. P. Mineev and M. Sigrist, Phys. Rev. B **63**, 172504 (2001).
[12] V. P. Mineev and Yu. G. Makhlin, Zh. Eksp. Teor. Fiz. **102**, 876 (1992) [Sov. Phys. JETP **75**, 479 (1992)].
[13] V. P. Mineev and G. E. Volovik, J. Low Temp. Phys. **89**, 823 (1992).
[14] J. C. Wheatley, Rev. Mod. Phys. **47**, 415 (1975).