Universal collective modes in two-dimensional chiral superfluids

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In this work, we utilize semiclassical kinetic equations to investigate the order parameter collective modes of a class of two-dimensional superfluids. Extending the known results for p-wave superfluids, we show that for any chiral ground state of angular momentum $L \geqslant 1$, there exists a subgap mode with a mass $\sqrt{2}\,\Delta$ in the BCS limit, where Δ is the magnitude of the ground-state gap. We determine the most significant Landau parameter that contributes to the mass renormalization and show explicitly that the renormalized modes become massless at the Pomeranchuk instability of the fermion vacuum. Particularly for $L = 1$, we propose a continuous field theory to include the Fermi liquid effect in a quadrupolar channel and produce the same result under consistent approximations. They provide potential diagnostics for distinguishing two-dimensional chiral ground states of different angular momenta with the order parameter collective modes and reveal another low-energy degree of freedom near the nematic transition.

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I. INTRODUCTION

In the studies of interacting quantum many-body systems, collective modes allow physicists to explore the correlated motions of underlying degrees of freedom. Especially in a superfluid phase, the order parameter component enriches the nature of collective excitations. Paradigmatic examples include the A and B phases of superfluid ^{3}He in (3 + 1)D [1], where massive subgap modes exist owing to the triplet pairing structure. They manifest themselves in terms of resonant signatures of transport properties when coupled to a particle-hole channel [2].

These developments permit various extensions. Natural questions include: (i) do subgap massive collective modes also exist in finite angular momentum pairing channels and in (2 + 1)D space-time? (ii) Do these bosonic degrees of freedom acknowledge the underlying fermionic state or the property of the Fermi surface? We pay special attention to these questions mainly because of the puzzle of $\nu = 5/2$ fractional quantum Hall state. Three of the most prominent candidates of the ground state, Pfaffian state, T-Pfaffian state, and anti-Pfaffian state, are understood as $p + ip$, $p - ip$, and $f - if$ chiral superconductors of nonrelativistic composite fermions, respectively [3]. Moreover, both experimental [4–6] and numerical [7] studies have revealed the importance of nematic fluctuations and quantum criticality in the second Landau level. As a consequence, a understanding of chiral superfluids/superconductors including these effects is pursued.

Regarding (i), it is known that the two-dimensional analog of the B phase hosts four modes of a mass $\sqrt{2}\ \Delta$ with angular momenta $\ell = \pm 2$. Δ is the magnitude of mass of the Bogoliubov quasiparticle. Similarly, the analog of the A phase, whose fermionic spectrum is fully gapped in two dimensions, hosts six modes of mass $\sqrt{2}\,\Delta$ [8–10]. On the other hand, (ii) it has been investigated in the context of (3 + 1)D ^{3}He superfluid with the Fermi liquid theory [1,11–13], where the corrections to the masses of massive subgap modes and the sound speeds of the Goldstone modes can be expressed in terms of the Landau parameters. In addition, for Sr_2RuO_4 [14], it has been shown that the strong-coupling effect and gap anisotropy are able to modify the magnitude of the masses and break the spectrum degeneracy.

This work intends to address the complementary faces of (i) and (ii). We specifically focus on superfluids in (2 + 1)D with general pairing channels of angular momenta $L = 0, 1, \dots$ For $L = 1$, the two-dimensional analogs of the A and B phases are considered, whereas for higher L, we concentrate on chiral ground states. We look for massive subgap modes and investigate the mechanisms that may correct their masses in the long-wavelength limit $q = 0$.

We find that in the limit with weak coupling and exact particle-hole symmetry, there is at least one pair of bosonic modes of universal mass $\sqrt{2}\,\Delta$ for all $L \geqslant 1$. We investigate corrections to these degenerate modes owing to fermionic vacuum in a phenomenological manner and determine the angular momentum channels substantial for mass renormalization. For a given chiral ground state of angular momentum L, the order parameter fluctuations longitudinal to the ground state are renormalized by the Landau parameter in the angular momentum channel $2L$, F_{2L}, and thus correspond to a type of spin-$2L$ mode.

The Fermi liquid correction is especially intriguing in (2 + 1)D. As we will show shortly in Sec. IV, it implies the subgap modes soften when F_{2L} is negative. Explicitly, as $F_{2L} \to -1$, the mass of the collective modes vanishes as

$$\sqrt{\frac{12(1 + F_{2L})}{6 + F_{2L}}}\,\Delta \to 0. \tag{1}$$

In particular, taking $L = 1$, the limit $F_2 \to -1$ serves as one of the mechanisms behind nematic electronic phases [15,16]. On top of previous studies on unconventional superconductors [17] and quantum Hall nematic phases [4–6], this is another

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example where the Pomeranchuk instability in a quadrupolar channel influences the nature of a paired phase [18] and it allows us to probe the high-frequency spin-2 mode usually omitted in literature. We thereby propose a toy model and compute its effective action in a Gaussian approximation and show that the kinetic result can be captured after implementing exact particle-hole symmetry.

In our work, (i) we generalize the known high-frequency subgap modes in p-wave superfluids in $(2+1)$D to higher angular momentum channels and compute their mass renormalizations in terms of the Landau parameters. (ii) Moreover, for p-wave chiral superfluids, we propose a continuous field-theory model to include the Landau parameter effect in the quadrupole channel. In addition to confirming the kinetic theory result, this model could easily be generalized when loosening particle-hole symmetry and provides an understanding of the underlying nature of spin-2 modes and nematic fluctuations.

This paper is organized as follows. In Sec. II, we review the semiclassical equation approach for the computation of collective excitations. The equations derived are used in Sec. III to compute collective excitations for various ground states. In Sec. IV, we calculate the Fermi liquid ground-state effect upon the bare bosonic spectra. Finally, Sec. V presents a field-theory model for a p-wave chiral superfluid with a continuous quadrupole interaction. We demonstrate that the results in Secs. III and IV can be produced in the limit of the exact particle-hole symmetry. Finally, a summary and several open directions are composed. The full solutions to the kinetic equation (2) without assuming $q=0$ and $\Delta\in\mathbb{R}$, and the computational method for the effective field theory are present along with the method in the Appendixes.

II. KINETIC THEORY

Bosonic collective modes in superfluids or superconductors [19] can be computed with various approaches. In this section, we start off with the time-dependent mean-field approximation to include the Fermi liquid corrections. This approach can be formulated in terms of the generalized Landau-Boltzmann kinetic equations [1] or the linearized nonequilibrium Eilenberger equation [12,20]. Though we will not repeat the derivations of the formalism, which we refer the readers to Refs. [12,20], we will give a complete elaboration of the workflow.

In the semiclassical limit, physical quasiparticle distribution is related to the Keldysh Green's function $\widehat{g}(\varepsilon,\hat{\mathbf{p}};\omega,\mathbf{q})$. In our computation, it is a 4×4 matrix function. We use two sets of Pauli matrices $\boldsymbol{\tau}=(\tau_1,\tau_2,\tau_3)$ and $\boldsymbol{\sigma}=(\sigma_1,\sigma_2,\sigma_3)$ to span the particle-hole and spin space, respectively. In its argument $(\varepsilon,\mathbf{p}=p_F\hat{\mathbf{p}})$ are the Fourier transformed variables of the fast coordinates, where $(\omega,\mathbf{q})$ are the Fourier transformed variables of the coordinates of the center of mass [21]. p_F is the magnitude of the Fermi momentum and v_F is the Fermi velocity. In clean limit, the linear response of a nonrelativistic fermion without spin-orbital coupling is given by the following kinetic equation:

$$\varepsilon_+\tau_3\delta\widehat{g}-\delta\widehat{g}\tau_3\varepsilon_- - v_F\hat{\mathbf{p}}\cdot\mathbf{q}\delta\widehat{g}-[\widehat{\sigma}_0,\delta\widehat{g}]=\delta\widehat{\sigma}\,\widehat{g}_0(\varepsilon_-)-\widehat{g}_0(\varepsilon_+)\,\delta\widehat{\sigma}, \quad (2)$$

where $\varepsilon_\pm$ denotes $\varepsilon\pm\omega/2$. The operator $\widehat{\sigma}_0(\hat{\mathbf{p}})$ is the molecular mean field or the self-energy at equilibrium, while $\delta\widehat{\sigma}(\omega,\mathbf{q})$ is the linear perturbation of $\widehat{\sigma}_0$. Similarly, $\widehat{g}_0(\varepsilon,\hat{\mathbf{p}})$ represents the Keldysh Green's function at equilibrium. It is related to the retarded and advanced Green's functions via $\widehat{g}_0=(g_0^R-g_0^A)\tanh(\varepsilon/2T)$, which yields [20]

$$\widehat{g}_0=\frac{-2\pi i(\tau_3\varepsilon-\widehat{\Delta})}{\sqrt{\varepsilon^2-|\Delta|^2}}\Theta(\varepsilon^2-|\Delta|^2)\mathrm{sgn}(\varepsilon)\tanh\frac{\varepsilon}{2T}. \quad (3)$$

The low-energy fluctuation of quasiparticles and the deduced physical quantities are given by the ε-integrated $\widehat{g}$, $\int_{-\infty}^{\infty}\frac{d\varepsilon}{2\pi i}\widehat{g}(\varepsilon,\hat{\mathbf{p}};\omega,\mathbf{q})$. In particular, the perturbation $\delta\widehat{\sigma}$ is self-consistently determined by the convolution of the interparticle potentials and $\delta\widehat{g}$.

To further elaborate, we note that $\delta\widehat{g}$ has a general structure in particle-hole space

$$\delta\widehat{g}=\begin{pmatrix}\delta g+\delta\mathbf{g}\cdot\boldsymbol{\sigma} & (\delta f+\delta\mathbf{f}\cdot\boldsymbol{\sigma})i\sigma_2\\ i\sigma_2(\delta f'+\delta\mathbf{f}'\cdot\boldsymbol{\sigma}) & \delta g'+\delta\mathbf{g}'\cdot\boldsymbol{\sigma}^t\end{pmatrix}, \quad (4)$$

and accordingly so does $\delta\widehat{\sigma}$

$$\delta\widehat{\sigma}=\begin{pmatrix}\delta\varepsilon+\delta\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma} & (d+\mathbf{d}\cdot\boldsymbol{\sigma})i\sigma_2\\ i\sigma_2(d'+\mathbf{d}'\cdot\boldsymbol{\sigma}) & \delta\varepsilon'+\delta\boldsymbol{\varepsilon}'\cdot\boldsymbol{\sigma}^t\end{pmatrix}, \quad (5)$$

where the primed variables are

$$\delta g'(\hat{\mathbf{p}};\omega,\mathbf{q})=\delta g(-\hat{\mathbf{p}};\omega,\mathbf{q}), \quad (6a)$$

$$\delta\varepsilon'(\hat{\mathbf{p}};\omega,\mathbf{q})=\delta\varepsilon(-\hat{\mathbf{p}};\omega,\mathbf{q}), \quad (6b)$$

$$\delta f'(\hat{\mathbf{p}};\omega,\mathbf{q})=\delta f^*(\hat{\mathbf{p}};-\omega,-\mathbf{q}), \quad (6c)$$

$$d'(\hat{\mathbf{p}};\omega,\mathbf{q})=d^*(\hat{\mathbf{p}};-\omega,-\mathbf{q}). \quad (6d)$$

We would like to explain the notations here before moving forward. The diagonal parts of $\delta\widehat{g}$ refer to the normal, or particle-hole, correlation functions $\langle\psi\psi^\dagger\rangle$ and $\langle\psi^\dagger\psi\rangle$, while the off-diagonal parts denote the anomalous or particle-particle correlation functions $\langle\psi\psi\rangle$ and $\langle\psi^\dagger\psi^\dagger\rangle$. $\boldsymbol{\sigma}$ denotes the Pauli matrices $(\sigma_1,\sigma_2,\sigma_3)$ in spin space as defined earlier and $\boldsymbol{\sigma}^t$ denotes the transposed Pauli matrices. Looking at the Green's function $\delta\widehat{g}$, in the particle-hole channel, δg and $\delta\mathbf{g}$ denote spin-independent and spin-dependent correlations, respectively. In the particle-particle channel, δf represent the spin-singlet pairing amplitude and $\delta\mathbf{f}$ the spin-triplet one. Correspondingly, the diagonal part of the self-energy $\delta\widehat{\sigma}$ is the particle-hole self-energy, including the spin-independent $\delta\varepsilon$ and spin-dependent part $\delta\boldsymbol{\varepsilon}$. The off-diagonal part of the self-energy is the superfluid gap induced by anomalous correlations. d is the spin-singlet gap, and $\mathbf{d}$ denotes the spin-triplet gap.

Note that physical observables are usually expressed in terms of the symmetric and anti-symmetric combination of δg, δf, and their primed partners. In this work, we define $(+)$ and $(-)$ combinations of a function f as

$$f^{(\pm)}=f\pm f'. \quad (7)$$

The eigenvalues (±1) represent the parity under charge conjugation. As we will see, the charge density and energy stress tensor correspond to the scalar and quadrupole modes of $\delta g^{(+)}$, respectively, whereas the current density is proportional to the vector mode of $\delta g^{(-)}$. Similarly, $\delta f^{(+)}$ and $\delta f^{(-)}$ stand for the amplitude and phase fluctuations of the anomalous correlation functions.

To complete the equations, the correction to the self-energy is determined by the two-body vertex. Evaluating the internal momentum integral over the Fermi surface, we have, in the particle-hole channel [22–24],

$$\delta\varepsilon(\hat{\mathbf{p}};\omega,\mathbf{q}) = \delta\varepsilon_{\rm ext}(\hat{\mathbf{p}};\omega,\mathbf{q}) + \int \frac{d\theta'}{2\pi} A^s(\theta,\theta') \times \int \frac{d\varepsilon'}{4\pi i}\delta g(\varepsilon',\hat{\mathbf{p}}';\omega,\mathbf{q}), \tag{8a}$$

$$\delta\boldsymbol{\varepsilon}(\hat{\mathbf{p}};\omega,\mathbf{q}) = \delta\boldsymbol{\varepsilon}_{\rm ext}(\hat{\mathbf{p}};\omega,\mathbf{q}) + \int \frac{d\theta'}{2\pi} A^a(\theta,\theta') \times \int \frac{d\varepsilon'}{4\pi i}\delta \mathbf{g}(\varepsilon',\hat{\mathbf{p}}';\omega,\mathbf{q}). \tag{8b}$$

These two equations state that at 1-loop, the particle-hole self-energy consists of external perturbation $\delta\varepsilon_{\rm ext}$ or $\delta\boldsymbol{\varepsilon}_{\rm ext}$ and a fermion loop is closed by a two-body interaction vertex. An example of $\delta\varepsilon_{\rm ext}$ is a background inhomogeneous chemical potential, whereas an example of $\delta\boldsymbol{\varepsilon}_{\rm ext}$ could be a weak external magnetic field. A^s (A^a) is the spin-independent (exchange) forward scattering amplitude, which can be rewritten in terms of Landau parameters F via the relation

$$A(\theta,\theta') = F(\theta,\theta') - \int \frac{d\theta''}{2\pi} F(\theta,\theta'')A(\theta'',\theta'). \tag{9}$$

Similar expressions arise in the particle-particle channel. Since the fluctuations of the superfluid gaps directly come from the anomalous correlation functions, the off-diagonal components are related by the linearized gap equations:

$$d(\hat{\mathbf{p}};\omega,\mathbf{q}) = \int \frac{d\theta'}{2\pi} V_{\rm e}(\theta,\theta') \int \frac{d\varepsilon'}{4\pi i}\delta f(\varepsilon',\hat{\mathbf{p}}';\omega,\mathbf{q}), \tag{10a}$$

$$\mathbf{d}(\hat{\mathbf{p}};\omega,\mathbf{q}) = \int \frac{d\theta'}{2\pi} V_{\rm o}(\theta,\theta') \int \frac{d\varepsilon'}{4\pi i}\delta \mathbf{f}(\varepsilon',\hat{\mathbf{p}}';\omega,\mathbf{q}), \tag{10b}$$

where $V_{\rm e}$ ($V_{\rm o}$) is the pairing potentials in even (odd) angular momentum channel.

In two dimensions, the scattering amplitudes and pairing potentials yield the approximate angular expansions:

$$A = \sum_{\ell=-\infty}^{\infty} A_\ell e^{-i\ell(\theta-\theta')},\ A_\ell = A_{-\ell}, \tag{11a}$$

$$V_{\rm e} = \sum_{\ell\in\{\rm even\}} V_\ell [e^{-i\ell(\theta-\theta')} + {\rm H.c.}], \tag{11b}$$

$$V_{\rm o} = \sum_{\ell\in\{\rm odd\}} V_\ell [e^{-i\ell(\theta-\theta')} + {\rm H.c.}], \tag{11c}$$

from which and (9) we can derive $A_\ell = \frac{F_\ell}{1+F_\ell}$, where F_ℓ is the conventional dimensionless Landau parameter of angular momentum channel ℓ. Notationwise, for other functions, $f(\hat{\mathbf{p}})$ evaluated at a point on the Fermi surface $\hat{\mathbf{p}}$, the angular decomposition is defined as

$$f = \sum_{\ell=-\infty}^{\infty} e^{-i\ell\theta} f_\ell. \tag{12}$$

We can then provide a recipe for the computation. We first invert (2) to obtain the perturbed Green's function $\delta\widehat{g}$ as a function of the equilibrium Green's function $\widehat{g}_0$, equilibrium self-energy $\widehat{\sigma}_0$, and perturbed self-energy $\delta\widehat{\sigma}$. Taking the convolution as in (8a), (8b), (10a), and (10b) establishes integral equations for $\delta\widehat{\sigma}$. Projecting equations (10a) and (10b) to different angular modes ℓ gives us the coupled equations of d_ℓ, $\mathbf{d}_\ell$, $\delta\varepsilon$, and $\delta\boldsymbol{\varepsilon}$. The bare bosonic collective modes are given by the normal modes of the homogeneous part of the equations. To include the Fermi liquid corrections, we project (8a), and (8b) to their ℓth angular modes as well and solve $\delta\varepsilon_\ell$ and $\delta\boldsymbol{\varepsilon}_\ell$ in terms of $\delta\varepsilon_{\rm ext}$, $\delta\boldsymbol{\varepsilon}_{\rm ext}$, d, and $\mathbf{d}$. Plugging the results back into the equations for d_ℓ and $\mathbf{d}_\ell$ yields inhomogeneous equations sourced solely by external fields. The renormalized mass spectrum is solved as the poles of the solution kernels.

In the rest of this section, we use the above formulation to derive the integral equation for two-dimensional spin-singlet and spin-triplet superfluids and compute the collective modes and Fermi liquid corrections in the sections following. While in the main text only the equations in the long-wavelength limit are presented, the complete set of dynamical equations are given in Appendix B.

A. Spin-singlet pairing

In a spin-singlet pairing channel, the equilibrium self-energy is characterized by a complex gap field Δ:

$$\widehat{\sigma}_0 = \widehat{\Delta} = \begin{pmatrix} 0 & \Delta i\sigma_2 \\ \Delta^* i\sigma_2 & 0\end{pmatrix}. \tag{13}$$

The fluctuation of the spin-singlet order parameter can be parametrized by a complex number d. It transforms as a scalar under spin rotation $SO_S(3)$ and can have internal structures, i.e., tensor indices under orbital rotation $SO_L(2)$ depending on pairing symmetries. In the absence of magnetic field, the spin-triplet fluctuations $\mathbf{d}$ are decoupled from d. Hence we consider them separately in the present work.

Plugging (13) into (2), inverting it using the variables defined in (4) and (5), and taking the convolution as in (10a) and (10b) give us, in the long-wavelength limit, the off-diagonal components of the molecular fields

$$d(\hat{\mathbf{p}};\omega) = \int \frac{d\theta'}{2\pi} V_{\rm e}(\theta,\theta')\bigg[\bigg(\gamma + \frac{1}{4}\bar{\lambda}[\omega^2 - 2|\Delta|^2]\bigg)d - \frac{\bar{\lambda}}{2}\Delta^2 d' - \frac{\omega}{4}\bar{\lambda}\Delta\delta\varepsilon^{(+)}\bigg], \tag{14a}$$

$$d'(\hat{\mathbf{p}};\omega) = \int \frac{d\theta'}{2\pi} V_{\rm e}(\theta,\theta')\bigg[\bigg(\gamma + \frac{1}{4}\bar{\lambda}[\omega^2 - 2|\Delta|^2]\bigg)d' - \frac{\bar{\lambda}}{2}(\Delta^*)^2 d + \frac{\omega\bar{\lambda}}{4}\Delta^*\delta\varepsilon^{(+)}\bigg]. \tag{14b}$$

γ is the BCS logarithm given explicitly in Appendix A. The function λ, often called the Tsunedo function, whose complete form is given in Appendix A. In $q \to 0$ limit,

$$\bar{\lambda} = \frac{\lambda(\hat{\mathbf{p}};\omega)}{|\Delta|^2} = \int_{|\Delta|}^{\infty} \frac{d\varepsilon}{\sqrt{\varepsilon^2 - |\Delta|^2}} \frac{\tanh\frac{\varepsilon}{2T}}{\varepsilon^2 - \omega^2/4}. \tag{15}$$

There could be angular dependence through the anisotropy in $|\Delta|^2$ even in the long-wavelength limit. Suppose only a single pairing channel L is significant, i.e., that $V = V_L(e^{-iL(\theta-\theta')} + \text{H.c.})$. Taking $\int \frac{d\theta}{2\pi} e^{iL\theta}$ on both sides of (14a) and (14b) eliminates γs. The dynamical equations of motion are then obtained

$$\langle e^{iL\theta}\bar{\lambda}([\omega^2 - 2|\Delta|^2]d - 2\Delta^2 d' - \omega\Delta\delta\varepsilon^{(+)})\rangle = 0, \quad (16a)$$

$$\langle e^{iL\theta}\bar{\lambda}([\omega^2 - 2|\Delta|^2]d' - 2(\Delta^*)^2 d + \omega\Delta^*\delta\varepsilon^{(+)})\rangle = 0, \quad (16b)$$

where we use the angle bracket $\langle\cdots\rangle$ to denote the angular average $\int_{-\pi}^{\pi}\frac{d\theta}{2\pi}\cdots$.

B. Spin-triplet pairing

In a spin-triplet pairing channel, the ground-state self-energy is characterized by the vector-valued gap function $\mathbf{\Delta}$

$$\widehat{\Delta} = \begin{pmatrix} 0 & \mathbf{\Delta}\cdot i\boldsymbol{\sigma}\sigma_2 \\ \mathbf{\Delta}^*\cdot i\sigma_2\boldsymbol{\sigma} & 0 \end{pmatrix}. \quad (17)$$

The fluctuation is encoded in the dynamics of the $\mathbf{d}$ vector, which transforms as a vector under $SO_S(3)$, and could contain internal structure depending on pairing symmetry as well. Taking two-dimensional p-wave superfluids, for example, it can be expanded as $d_\mu(\hat{\mathbf{p}}) = d_{\mu i}\hat{p}_i$, where $i = x, y$. Inverting the kinetic equations, the dynamical equations for $\mathbf{d}$ in $q \to 0$ limit are

$$\begin{aligned}\mathbf{d} = \int \frac{d\theta'}{2\pi} V_o(\theta,\theta')\bigg\{\Big[\gamma + \frac{1}{4}\bar{\lambda}\big(\omega^2 - 2|\Delta|^2\big)\Big]\mathbf{d} \\ + \frac{\bar{\lambda}}{2}\big[(\mathbf{\Delta}\cdot\mathbf{\Delta})\mathbf{d}' - 2(\mathbf{\Delta}\cdot\mathbf{d}')\mathbf{\Delta}\big] \\ - \frac{\omega\bar{\lambda}}{4}\big(\mathbf{\Delta}\delta\varepsilon^{(+)} - i\mathbf{\Delta}\times\delta\boldsymbol{\varepsilon}^{(+)}\big)\bigg\},\end{aligned} \quad (18a)$$

$$\begin{aligned}\mathbf{d}' = \int \frac{d\theta'}{2\pi} V_o(\theta,\theta')\bigg\{\Big[\gamma + \frac{1}{4}\bar{\lambda}(\omega^2 - 2|\Delta|^2)\Big]\mathbf{d}' \\ + \frac{\bar{\lambda}}{2}[(\mathbf{\Delta}^*\cdot\mathbf{\Delta}^*)\mathbf{d} - 2(\mathbf{\Delta}^*\cdot\mathbf{d})\mathbf{\Delta}^*] \\ + \frac{\omega\bar{\lambda}}{4}\big(\mathbf{\Delta}^*\delta\varepsilon^{(+)} + i\mathbf{\Delta}^*\times\delta\boldsymbol{\varepsilon}^{(+)}\big)\bigg\}.\end{aligned} \quad (18b)$$

Again multiplying (18a) and (18b) by $V = V_L[e^{-iL(\theta-\theta')} + e^{iL(\theta-\theta')}]$ and integrating over θ will give us

$$\begin{aligned}&\langle e^{iL\theta}\bar{\lambda}([\omega^2 - 2|\Delta|^2]\mathbf{d} + 2(\mathbf{\Delta}\cdot\mathbf{\Delta})\mathbf{d}' - 4(\mathbf{\Delta}\cdot\mathbf{d}')\mathbf{\Delta})\rangle \\ &\quad = \omega\langle e^{iL\theta}\bar{\lambda}(\mathbf{\Delta}\delta\varepsilon^{(+)} - i\mathbf{\Delta}\times\delta\boldsymbol{\varepsilon}^{(+)})\rangle,\end{aligned} \quad (19a)$$

$$\begin{aligned}&\langle e^{iL\theta}\bar{\lambda}([\omega^2 - 2|\Delta|^2]\mathbf{d}' + 2(\mathbf{\Delta}^*\cdot\mathbf{\Delta}^*)\mathbf{d} - 4(\mathbf{\Delta}^*\cdot\mathbf{d})\mathbf{\Delta}^*)\rangle \\ &\quad = -\omega\langle e^{iL\theta}\bar{\lambda}(\mathbf{\Delta}^*\delta\varepsilon^{(+)} + i\mathbf{\Delta}^*\times\delta\boldsymbol{\varepsilon}^{(+)})\rangle.\end{aligned} \quad (19b)$$

In the following section, we will solve (16a), (16b), (19a), and (19b) for the ground states of different pairing channels and symmetries.

III. COLLECTIVE MODES

In this section, we utilize the equations derived in the last section to compute the bare bosonic spectra for various superconducting ground states. We focus on the chiral ground states of angular momentum $L \neq 0$, in which the massive collective modes are interpreted as spin-$2L$ modes. The masses of order parameter collective modes appear as normal modes of the homogeneous part in (16a), (16b), (19a), and (19b). The self-energy $\delta\varepsilon$ and $\delta\boldsymbol{\varepsilon}$ in Landau channel are treated as external sources at the zeroth order, and they will *renormalize* the above bare masses in the next section as we conclude Fermi liquid effects.

A. *s*-wave pairing

For s-wave pairing, it is possible to choose a gauge such that $\Delta \in \mathbb{R}$, in such a limit the amplitude mode $d^{(+)}$ and phase mode $d^{(-)}$ decouple. The superscripts $(+)$ and $(-)$ are defined according to (7). The bosonic field has no internal structure and is simply a complex scalar. Two order parameter collective modes thus exist and obey the equations

$$(\omega^2 - 4\Delta^2)d^{(+)} = 0, \quad (20a)$$

$$\omega^2 d^{(-)} = 2\omega\Delta\delta\varepsilon_0^{(+)}, \quad (20b)$$

where the zero-angular momentum quasiparticle energy $\delta\varepsilon_0^{(+)}$ is obtained under the projection (12). The normal modes have masses 2Δ and 0 corresponding to the simplest example of *Higgs* and *Goldstone* bosons, respectively. Note that if we compute (20b) to the leading nonvanishing order in q^2, we would have obtain $(\omega^2 - \frac{1}{2}(v_F q)^2)d^{(-)}$, entailing the Goldstone boson moves at the speed $v_F/\sqrt{2}$. Another observation is that the Higgs mode receives no external force and consequently it would not be renormalized by particle-hole self-energy. On the other hand, the Goldstone boson is sourced by the density mode $\delta\varepsilon_0^{(+)}$, which would trigger the Higgs mechanism in the presence of the Coulomb interaction.

B. *p*-wave pairing

Owing to the triplet pairing and orbital structure, the p-wave pairing states have more degrees of freedom and thus more collective modes. In two dimensions, the fluctuation of p-wave superconductors can be represented by the complex tensor $d_{\mu i}$, which contains 3×2 complex degrees of freedom, leading to 12 collective modes in total. The number of the massless modes N_G, as we will see shortly, can be determined by ground-state symmetry breaking pattern. The rest $(6 - N_G)\times 2$ is number of subgap collective modes.

a. B phase. We first consider the two-dimensional analog of ^{3}He B phase, where the gap function assumes the form

$$\mathbf{\Delta} = \frac{\Delta}{p_F}(\hat{\mathbf{x}}p_x + \hat{\mathbf{y}}p_y), \ \Delta \in \mathbb{R}. \quad (21)$$

In this phase, the global symmetry breaks following the pattern $SO_S(3)\otimes SO_L(2)\otimes U(1) \to SO(2)$, which immediately indicates the existence of four Goldstone modes. Besides, the residual symmetry is SO(2) rotation and we expect the fluctuations can be characterized by total angular momentum J. Owing to this fact, it is convenient to first decompose d_μ into different angular momentum channels $d_\mu = \sum_{m=\pm 1} d_{\mu m}e^{-im\theta}$, where θ is the polar angle of $\hat{\mathbf{p}}$, and take the linear combinations as follows:

$$D_{\pm m} = d_{xm} \pm i d_{ym}, \quad (22)$$

$$D_{0m} = d_{zm}. \quad (23)$$

These $D_{\sigma\sigma'}$'s form a nice basis in which the dynamical equations can be solved. Moreover, as the gap function is real, modes transforming differently under charge conjugation again decouple. That is to say, we can further separate $d^{(\pm)} = d \pm d'$ degrees of freedom. We first look at the $\mathbf{d}^{(-)}$ modes governed by the equation

$$(\omega^2 - 4\Delta^2)\mathbf{d}^{(-)} + 4(\mathbf{\Delta} \cdot \mathbf{d}^{(-)})\mathbf{\Delta} = 2\omega\mathbf{\Delta}\delta\varepsilon^{(+)}. \quad (24)$$

Organizing the dynamical equations using the basis $D_{\sigma\sigma'}$, we could find

$$(\omega^2 - 4\Delta^2)D^{(-)}_{0\pm} = 0, \quad (25a)$$

$$(\omega^2 - 2\Delta^2)D^{(-)}_{\pm\pm} = 2\omega\Delta\delta\varepsilon^{(+)}_{\pm 2}, \quad (25b)$$

$$(\omega^2 - 4\Delta^2)(D^{(-)}_{+-} - D^{(-)}_{+-}) = 0, \quad (25c)$$

$$\omega^2(D^{(-)}_{+-} + D^{(-)}_{-+}) = 4\omega\Delta\delta\varepsilon^{(+)}_0. \quad (25d)$$

Consequently, $d^{(-)}$ has two subgap massive modes $J = \pm 2$ of the same mass $\sqrt{2}\,\Delta$, sourced by the spin-independent quadrupolar molecular field $\delta\varepsilon^{(+)}_{\pm 2}$.

Next we look at $d^{(+)}$, which obeys

$$[\omega^2\mathbf{d}^{(+)} - 4\mathbf{\Delta}(\mathbf{\Delta} \cdot \mathbf{d}^{(+)})] = -2i\omega\mathbf{\Delta} \times \delta\boldsymbol{\varepsilon}^{(+)}. \quad (26)$$

Following the same procedure to project each component to different J sectors, we would obtain

$$\omega^2 d^{(+)}_0 = -2i\omega(\mathbf{\Delta} \times \delta\boldsymbol{\varepsilon}^{(+)}) \cdot \hat{\mathbf{z}}, \quad (27a)$$

$$(\omega^2 - 2\Delta^2)D^{(+)}_{\pm\pm} = \mp 2\omega\Delta\delta\boldsymbol{\varepsilon}^{(+)}_{\pm 2} \cdot \hat{\mathbf{z}}, \quad (27b)$$

$$\omega^2(D^{(+)}_{-+} - D^{(+)}_{+-}) = 4\Delta\omega\delta\boldsymbol{\varepsilon}^{(+)}_0 \cdot \hat{\mathbf{z}}, \quad (27c)$$

$$(\omega^2 - 4\Delta^2)(D^{(+)}_{+-} + D^{(+)}_{+-}) = 0. \quad (27d)$$

Again modes $D^{(+)}_{\pm\pm}$ have the rest mass $\sqrt{2}\,\Delta$ and they are driven by the z component of the spin-dependent quadrupolar fields $\delta\boldsymbol{\varepsilon}^{(+)}_{\pm 2} \cdot \hat{\mathbf{z}}$.

b. A phase. Considering only the continuous symmetry, thetwo-dimensional A phase has a different symmetry breaking pattern $\mathrm{SO}_S(3) \otimes \mathrm{SO}_L(2) \otimes \mathrm{U}(1) \to \mathrm{U}_{L-N/2}(1) \otimes \mathrm{U}_{S_z}(1)$. The residual symmetry contains two parts. $\mathrm{U}_{L-N/2}(1)$ refers to the combination of orbital and phase rotation. The order parameter is symmetric when an orbital rotation of angle α is followed by a phase rotation $-\alpha/2$. The $\mathrm{U}_{S_z}(1)$ is the residual spin rotation about the direction of the ground state $\mathbf{\Delta}$.

Let us now consider a $p + ip$ ground state described by

$$\mathbf{\Delta} = \frac{p_x + ip_y}{p_F}\Delta\hat{\mathbf{z}} = e^{i\theta}\Delta\hat{\mathbf{z}}, \ \Delta \in \mathbb{R}. \quad (28)$$

The dynamic equations for $\mathbf{d}_\ell$ and $\mathbf{d}'_\ell$ ($\ell = \pm 1$) are now coupled and given as follows:

$$\langle e^{i\ell\theta}[(\omega^2 - 2\Delta^2)\mathbf{d} + 2\Delta^2\mathbf{d}'e^{2i\theta} - 4\Delta^2\hat{\mathbf{z}}(\hat{\mathbf{z}} \cdot \mathbf{d}')e^{i2\theta}]\rangle$$
$$= \omega\Delta\langle e^{i\ell\theta}e^{i\theta}[\hat{\mathbf{z}}\delta\varepsilon^{(+)} - i\hat{\mathbf{z}} \times \delta\boldsymbol{\varepsilon}^{(+)}]\rangle, \quad (29a)$$

$$\langle e^{i\ell\theta}[(\omega^2 - 2\Delta^2)\mathbf{d}' + 2\Delta^2\mathbf{d}e^{-2i\theta} - 4\Delta^2\hat{\mathbf{z}}(\hat{\mathbf{z}} \cdot \mathbf{d})e^{-i2\theta}]\rangle$$
$$= -\omega\Delta\langle e^{i\ell\theta}e^{-i\theta}[\hat{\mathbf{z}}\delta\varepsilon^{(+)} + i\hat{\mathbf{z}} \times \delta\boldsymbol{\varepsilon}^{(+)}]\rangle. \quad (29b)$$

We first look at the angular modes $\mathbf{d}_{\ell=1}$ and $\mathbf{d}'_{\ell=-1}$. They obey the equations

$$(\omega^2 - 2\Delta^2)\mathbf{d}_1 = \omega\Delta\langle\hat{\mathbf{z}}\delta\varepsilon^{(+)}_2 - i\hat{\mathbf{z}} \times \delta\boldsymbol{\varepsilon}^{(+)}_2\rangle, \quad (30a)$$

$$(\omega^2 - 2\Delta^2)\mathbf{d}'_{-1} = -\omega\Delta\langle\hat{\mathbf{z}}\delta\varepsilon^{(+)}_{-2} + i\hat{\mathbf{z}} \times \delta\boldsymbol{\varepsilon}^{(+)}_{-2}\rangle, \quad (30b)$$

and have the same mass $\sqrt{2}\,\Delta$. The external forces consist of both spin-dependent and spin-independent molecular fields, both of which are projected to quadrupolar channels. On the other hand, equations for $\mathbf{d}_{\ell=-1}$ and $\mathbf{d}'_{\ell=1}$ are coupled. Solving these equations, one can find three massless modes and three modes with the mass 2Δ. The external forces on the right-hand sides of (30a) and (30b) consist of both spin-dependent and spin-independent molecular fields, both of which are projected to the quadrupolar channels.

C. *d*-wave pairing

The d-wave gap fluctuation is captured by the complex field $d_{ij}\hat{p}_i\hat{p}_j$ with the irreducible complex degrees of freedom 1×2, represented by the modes $d_{\pm 2}e^{\mp i2\theta}$. In this work, we consider the chiral ground state

$$\frac{\Delta}{p_F^2}(p_x + ip_y)^2 = \Delta e^{i2\theta}, \ \Delta \in \mathbb{R}. \quad (31)$$

Equations (16a) and (16b) then become

$$(\omega^2 - 2\Delta^2)d_2 = \omega\Delta\delta\varepsilon^{(+)}_4, \quad (32a)$$

$$(\omega^2 - 2\Delta^2)d'_{-2} = -\omega\Delta\varepsilon^{(+)}_{-4}, \quad (32b)$$

$$(\omega^2 - 4\Delta^2)(d_{-2} + d'_2) = 0, \quad (32c)$$

$$\omega^2(d_{-2} - d'_2) = 2\omega\Delta\delta\varepsilon^{(+)}_0. \quad (32d)$$

Clearly d_2 and d'_{-2} have masses $\sqrt{2}\,\Delta$ and the external driving forces have angular momenta ± 4.

D. Higher *L* chiral ground states

Extending the analyses for p and d channels, we could actually consider a more general ground state

$$\text{singlet}: \Delta e^{iL_s\theta}, \ L_s = \text{even}, \quad (33a)$$

$$\text{triplet}: \hat{\mathbf{z}}\Delta e^{iL_t\theta}, \ L_t = \text{odd}. \quad (33b)$$

Modes d_{L_s}, d'_{-L_s}, $\mathbf{d}_{L_t}$, and $\mathbf{d}'_{-L_t}$ would automatically satisfy

$$(\omega^2 - 2\Delta^2)d_{L_s} = \omega\Delta\delta\varepsilon^{(+)}_{2L_s}, \quad (34a)$$

$$(\omega^2 - 2\Delta^2)d_{-L_s} = -\omega\Delta\delta\varepsilon^{(+)}_{-2L_s}, \quad (34b)$$

$$(\omega^2 - 2\Delta^2)\hat{\mathbf{z}} \cdot \mathbf{d}_{L_t} = \omega\Delta\delta\varepsilon^{(+)}_{2L_t}, \quad (34c)$$

$$(\omega^2 - 2\Delta^2)\hat{\mathbf{z}} \cdot \mathbf{d}'_{-L_t} = -\omega\Delta\delta\varepsilon^{(+)}_{-2L_t}. \quad (34d)$$

In this sense, $\sqrt{2}\Delta$ is a universal order parameter collective mode for any chiral ground state of the angular momentum L, each of which is sourced by quasiparticle self-energy $\delta\varepsilon_{2L}$. Since the right-hand sides belong to specific angular momentum channels, the collective modes could be regarded as generalized spin-$2L$ modes.

IV. FERMI LIQUID CORRECTIONS

In the previous section, we found that for chiral ground states of given L, spin-$2L$ bosonic modes d_L and d'_{-L} have the finite mass $\sqrt{2}\Delta$. In this section, we compute the Fermi liquid corrections to the mass spectra. Before presenting quantitative details, we point out some general features. Those modes with the mass 2Δ, e.g., Eq. (20a), in general, are not sourced by fermionic self-energy, and consequently these modes are not renormalized. On the other hand, for those massless modes, e.g., Eq. (20b), short-range fermionic self-energy can at most renormalize the sound speed and the magnitude of external source fields instead of generating a gap. We will therefore focus on the spin-$2L$ modes of mass $\sqrt{2}\,\Delta$.

A. Massless modes

Let us first look at the massless modes in the s-wave channel (20b). The right-hand side $\delta\varepsilon_0^{(+)}$ consists of pure external perturbation and the renormalization coming form the integral part of (8a). Since we have rewritten Eqs. (8a) and (8b) using $F(\theta,\theta')$ instead of $A(\theta,\theta')$, we substitute the properly normalized external perturbations $\delta\varepsilon_{\rm ext}$ and $\delta\boldsymbol{\varepsilon}_{\rm ext}$ with new symbols δu and $\delta\mathbf{u}$. In the long-wavelength limit,

$$\delta\varepsilon^{(+)}(\theta) = \delta u^{(+)} + \int \frac{d\theta'}{2\pi} F^s(\theta,\theta')\left[-\lambda\delta\varepsilon^{(+)} + \frac{\omega\lambda}{2\Delta}d^{(-)}\right]. \tag{35}$$

Projecting out $\ell = 0$ component, we obtain

$$\big(1+\lambda(\omega)F_0^s\big)\delta\varepsilon_0^{(+)} = \delta u_0^{(+)} + \frac{\omega\lambda}{2\Delta}F_0 d^{(-)}, \tag{36}$$

plugging which back into (20b) yields

$$\omega^2 d^{(-)} = 2\omega\Delta\delta u_0. \tag{37}$$

It entails that $d^{(-)}$ remains massless. To demonstrate a triplet-pairing example, we look at B phase (21) and (25c). For triplet-pairing states, the diagonal term of (8a) reads

$$\delta\varepsilon^{(+)}(\theta) = \delta u^{(+)} + \int \frac{d\theta'}{2\pi} F^s(\theta,\theta') \times \left[-\lambda(\omega)\delta\varepsilon^{(+)} + \frac{1}{2}\omega\bar{\lambda}\boldsymbol{\Delta}\cdot\mathbf{d}^{(-)}\right], \tag{38}$$

whose projection to ℓth mode is

$$\delta\varepsilon_\ell^{(+)} = \frac{\delta u_\ell^{(+)} + \frac{1}{2}\bar{\lambda}\omega F_\ell^s(\boldsymbol{\Delta}\cdot\mathbf{d}^{(-)})_\ell}{1+\lambda(\omega)F_\ell^s}. \tag{39}$$

For $\ell = 0$,

$$\mathrm{B}: \delta\varepsilon_0^{(+)} = \frac{\delta u_0 + \frac{\lambda\omega}{4\Delta}F_0^s(D_{+-}^{(-)} + D_{-+}^{(-)})}{1+\lambda F_0^s} \tag{40}$$

and we again find

$$\omega^2(D_{+-}^{(-)} + D_{-+}^{(-)}) = 4\omega\Delta\delta u_0. \tag{41}$$

The dynamical equations for d_0 and $D_{\{+-\}}$ are not modified by F_0^s, which implies that short-range interactions are not capable of gapping the Goldstone mode.

B. Massive subgap modes

Let us continue to examine how Landau parameters renormalize massive modes. We start with the B phase (25b). Take $\ell = \pm 2$ component of (39):

$$\mathrm{B}: \delta\varepsilon_{\pm 2}^{(+)} = \frac{\delta u_{\pm 2} + \frac{\omega\lambda}{4\Delta}F_2^s D_{\pm\pm}^{(-)}}{1+\lambda F_2^s}. \tag{42}$$

Plugging this back into (25b) renormalizes the solutions as

$$D_{\pm\pm}^{(-)} = \frac{2\omega\Delta\delta u_{\pm 2}}{(\omega^2 - 2\Delta^2) + \frac{1}{2}\lambda F_2^s(\omega^2 - 4\Delta^2)}. \tag{43}$$

The new mass is given by the zero of the denominator. In the limit $|F_2^s| \ll 1$,

$$\omega^2 \simeq 2\Delta^2\big(1 + \tfrac{1}{2}\lambda F_2^s\big). \tag{44}$$

λ is a positive number of order 1. We can see modes get heavier for repulsive interactions $F_2^s > 0$ and soften for attractive interactions $F_2^s < 0$.

Next let us look at the mode in the A phase (27b) sourced by spin-dependent quasiparticle energy:

$$\delta\varepsilon_z^{(+)}(\theta) = \delta u_z + \int \frac{d\theta'}{2\pi} F^a(\theta,\theta') \times \left[-\lambda\delta\varepsilon_z^{(+)} - \frac{i\omega}{2}\bar{\lambda}(\boldsymbol{\Delta}\times\mathbf{d}^{(+)})_z\right] \tag{45}$$

with $\delta u_z = \delta\mathbf{u}\cdot\hat{\mathbf{z}}$. Projecting it to $\ell = \pm 2$ modes,

$$\delta\varepsilon_{z,\pm 2}^{(+)} = \frac{\delta u_{z,\pm 2} \mp F_2^a \frac{\omega\lambda}{4\Delta}D_{\pm\pm}^{(+)}}{1+\lambda F_2^a}. \tag{46}$$

Substituting this back into (27b) yields

$$D_{\pm\pm}^{(+)} = \frac{\pm 2\omega\Delta\delta u_{z,\pm 2}}{(\omega^2 - 2\Delta^2) + \frac{1}{2}\lambda F_2^a(\omega^2 - 4\Delta^2)}. \tag{47}$$

Therefore the mass correction is given by the same transcendental equation with the replacement $F_2^s \to F_2^a$.

We are now ready to repeat the above computation for general chiral ground states. As it can be inferred from the previous analyses, the equations for singlet-pairing states are identical to ones for the longitudinal components $(\mathbf{d}\cdot\boldsymbol{\Delta})$ of the triplet-pairing states. Moreover, higher L states also have the same algebraic forms. Hence, we will concentrate on triplet-pairing states and take $L = 1$ without loss of generality.

The main difference between the preceding analyses and the one for general chiral states is that the gap function can no longer be chosen real and $d^{(\pm)}$ are no longer a good basis. Consequently, the scalar self-energy would satisfy the equation

$$\delta\varepsilon^{(+)} = \delta u^+ + \int \frac{d\theta'}{2\pi} F(\theta,\theta') \times \left[-\lambda\delta\varepsilon^{(+)} + \frac{1}{2}\omega\bar{\lambda}(\boldsymbol{\Delta}\cdot\mathbf{d} - \boldsymbol{\Delta}\cdot\mathbf{d}')\right]. \tag{48}$$

Let us take the z component of (30a) and (30b)

$$(\omega^2 - 2\Delta^2)d_{1z} = \omega\Delta\delta\varepsilon_2^{(+)}, \tag{49}$$

$$(\omega^2 - 2\Delta^2)d'_{-1z} = -\omega\Delta\delta\varepsilon_{-2}^{(+)}. \tag{50}$$

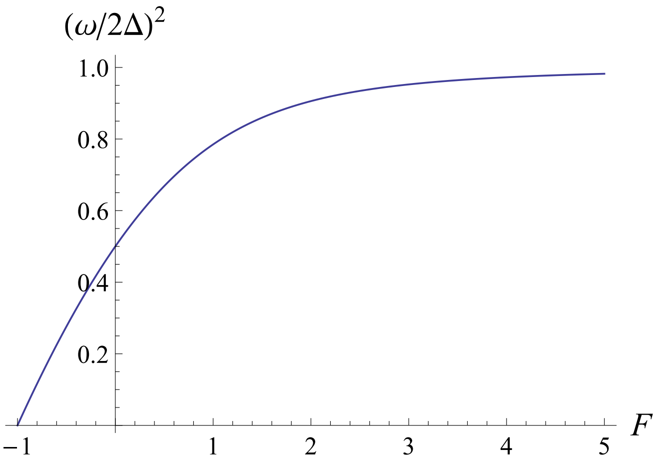

FIG. 1. The root to (55) depending on the value of Landau parameter F.

Renormalizing $\delta\varepsilon^{(+)}_{\pm 2}$ with (48), we find

$$d_{1z} = \frac{\omega\Delta\delta u^{(+)}_2}{(\omega^2 - 2\Delta^2) + \frac{1}{2}\lambda F^s_2(\omega^2 - 4\Delta^2)}, \quad (51a)$$

$$d'_{-1z} = \frac{\omega\Delta\delta u^{(+)}_{-2}}{(\omega^2 - 2\Delta^2) + \frac{1}{2}\lambda F^s_2(\omega^2 - 4\Delta^2)}. \quad (51b)$$

Finally we look at the transverse fluctuation by looking at the x component:

$$(\omega^2 - 2\Delta^2)d_{1x} = -i\omega\Delta(\hat{\mathbf{z}} \times \delta\boldsymbol{\varepsilon}^{(+)}_2)_x, \quad (52a)$$

$$(\omega^2 - 2\Delta^2)d'_{-1x} = -i\omega\Delta(\hat{\mathbf{z}} \times \delta\boldsymbol{\varepsilon}^{(+)}_{-2})_x. \quad (52b)$$

The spin-dependent self-energy now takes the form

$$\hat{\mathbf{z}} \times \delta\boldsymbol{\varepsilon}^{(+)} = \hat{\mathbf{z}} \times \delta\mathbf{u}^{(+)} + \int \frac{d\theta'}{2\pi} F^a(\theta,\theta') \times \left[-\lambda\hat{\mathbf{z}} \times \delta\boldsymbol{\varepsilon}^{(+)} - i\frac{\omega}{2}\bar{\lambda}\hat{\mathbf{z}}\times[(\boldsymbol{\Delta}^* \times \mathbf{d}) + (\boldsymbol{\Delta}\times\mathbf{d}')]\right]. \quad (53)$$

Projecting it to $\ell = \pm 2$ allows to solve

$$d_{1x} = \frac{-i\omega\Delta(\hat{\mathbf{z}} \times \delta\mathbf{u}^{(+)})_2}{(\omega^2 - 2\Delta^2) + \frac{1}{2}\lambda F^a_2(\omega^2 - 4\Delta^2)}. \quad (54)$$

To sum up, the analyses in this section have shown the following. (i) The Goldstone modes are not gapped by short-range interaction parametrized by Landau parameters. (ii) For p-wave superconductors in both B and A phases, the subgap modes $\sqrt{2}\,\Delta$ receive renormalization from quadrupolar Landau parameters F^s_2 or F^a_2. (iii) For all chiral ground states of finite orbital momenta L, the subgap modes parallel to their ground states receive mass renormalization from the channel F^s_{2L}. The mass corrections referred to in (ii) and (iii) are all determined by the following equation:

$$(\omega^2 - 2\Delta^2) + \tfrac{1}{2}\lambda(\omega)F(\omega^2 - 4\Delta^2) = 0. \quad (55)$$

In Fig. 1, we plot the numerical solution to (55) as a function of F, which stands for the Landau parameter of the channel of interest. Intuitively, we acquired, from the small F expansion, that a strong repulsive interaction in particle-hole channel increases the magnitude of the gap, which asymptotically approaches the pair-breaking threshold 2Δ. On the other hand, an attractive interaction softens the mass of order parameter. In particular, we see the mode would become massless as $F = -1$, at which the Pomeranchuk instability of two-dimensional Fermi liquid is triggered.

We can then look at the region $F = -1 + \epsilon$ with $\epsilon \ll 1$. At $T = 0$, we can expand the equation (55) around $\omega^2 \approx 0$ and extract its dependence on F near the instability. Using the closed form (A5a), we can deduce that Eq. (55) has the zero at

$$\omega^2 = \frac{3(1+F)}{6+F} \times 4\Delta^2 \approx \frac{12}{5}\epsilon\Delta^2. \quad (56)$$

This expression allows us to study how this mode becomes massless as we approached the instability.

V. FIELD-THEORY MODEL FOR CHIRAL p-WAVE SUPERFLUID

A complete kinetic theory treatment for both spin-singlet and spin-triplet chiral superfluids in the previous section has been conducted. However, it is still tempting to acquire an effective theory formulation, which would allow us to investigate the problem with techniques and insights across communities. Here, we propose a toy microscopic model for $L = 1$ p-wave chiral superfluid at $T = 0$. The corresponding Pomeranchuk instability in $2L = 2$ channel triggers the charge nematic order. In the approximation consistent with the kinetic theory approach, it reproduces exactly the same result, and moreover reveals the spin-2 nature of the subgap modes of interest.

Let us consider a two-dimensional spin-polarized nonrelativistic fermion ψ with the kinetic term $H_F[\psi]$, a short range pairing potential in $L = 1$ channel H_V [25], and a quadrupolar density interaction H_Q devised in Ref. [16]. Note that the spin degree of freedom is frozen in this regard, and therefore we no longer have the $SO_S(3)$ symmetry to start with. The model thus describes a minimalistic p-wave superfluid. The system presented in the previous section in this sense is considered to be three copies of the model here. Nonetheless, these ingredients suffice to produce the subgap modes and their renormalization. The exact form of the models read

$$H_F = \int \frac{d^2k}{(2\pi)^2}\psi^\dagger_{\mathbf{k}}\left(\frac{\mathbf{k}^2}{2m} - \epsilon_F\right)\psi_{\mathbf{k}} := \int \frac{d^2\mathbf{k}}{(2\pi)^2}\psi^\dagger_{\mathbf{k}}\xi_{\mathbf{k}}\psi_{\mathbf{k}}, \quad (57a)$$

$$H_V = -\frac{1}{p_F^2}\int \frac{d^2k}{(2\pi)^2}\frac{d^2k'}{(2\pi)^2} V_1 \psi^\dagger_{-\mathbf{k}}\psi^\dagger_{\mathbf{k}}\mathbf{k}\cdot\mathbf{k}'\psi_{\mathbf{k}'}\psi_{-\mathbf{k}'}, \quad (57b)$$

$$H_Q = \frac{1}{2}\int \frac{d^2\mathbf{q}}{(2\pi)^2} f_2(\mathbf{q})\mathbf{M}(-\mathbf{q})\cdot\mathbf{M}(\mathbf{q}), \quad (57c)$$

where $\mathbf{M} = (M_1, M_2)^T$ is defined by the quadrupole moment of particle density

$$\begin{pmatrix} M_1 & M_2 \\ M_2 & -M_1 \end{pmatrix} = -\frac{1}{p_F^2}\psi^\dagger \begin{pmatrix} k_x^2 - k_y^2 & 2k_xk_y \\ 2k_xk_y & k_y^2 - k_x^2 \end{pmatrix}\psi \quad (58)$$

and

$$f_2(\mathbf{q}) = \frac{f_2}{1+\kappa\mathbf{q}^2} = \nu_{2D}^{-1}\frac{F_2}{1+\kappa\mathbf{q}^2}. \tag{59}$$

$\nu_{2D} = \frac{m}{2\pi}$, denoting the density of state of 2D electron gas, and F_2 is the conventional Landau parameter. Owing to the frozen spin degree of freedom, the F_2 here corresponds to the spin-independent Landau parameter, F_2^s in the previous section. κ characterizes the interaction range and is irrelevant in the long-wavelength properties explored below. We introduce Hubbard-Stratonovich fields $(\phi, \phi^\dagger)$, $(\Phi, \Phi^\dagger)$, and $(\bar{Q}, Q)$ to decouple the two-body terms and rewrite the full action as

$$\mathcal{S} = \int d^3x\, \Psi^\dagger(i\partial_t - H)_{\text{BdG}}\Psi - \frac{1}{2V_1}\int d^3x\,(\phi^\dagger\phi + \Phi^\dagger\Phi) + \int d^3x\bar{Q}\, f_2^{-1}(-i\nabla)Q, \tag{60}$$

where $\Psi = \frac{1}{\sqrt{2}}(\psi, \psi^\dagger)^T$ and

$$(i\partial_t - H)_{\text{BdG}} = \begin{pmatrix} i\partial_t - \xi_{(-i\nabla)} & 0 \\ 0 & i\partial_t + \xi_{(-i\nabla)}\end{pmatrix} + p_F^{-2}\begin{pmatrix} Q(\partial_x - i\partial_y)^2 + \bar{Q}(\partial_x + i\partial_y)^2 & 0 \\ 0 & -[(\partial_x - i\partial_y)^2 Q + (\partial_x + i\partial_y)^2\bar{Q}]\end{pmatrix}$$
$$- \frac{i}{p_F}\begin{pmatrix} 0 & \phi(\partial_x - i\partial_y) + \Phi(\partial_x + i\partial_y) \\ \phi^\dagger(\partial_x + i\partial_y) + \Phi^\dagger(\partial_x - i\partial_y) & 0\end{pmatrix}. \tag{61}$$

The derivatives are understood to act on all quantities on their right. From the structure of the action, we see ϕ and Φ represent the $p - ip$ and $p + ip$ pairing amplitudes, respectively. In the mean-field limit,

$$p - ip : \langle\phi\rangle = \Delta,\ \langle\Phi\rangle = 0, \tag{62a}$$

$$p + ip : \langle\phi\rangle = 0,\ \langle\Phi\rangle = \Delta. \tag{62b}$$

On the other hand, Q and $\bar{Q}$ represent the nematic order parameters.

To proceed, we consider a ground state with a gapped fermion spectrum from either of the above choices, integrate the fermion sector, and compute the bosonic Gaussian fluctuation. In the explicit computation following, we choose the $p - ip$ ground state (62a) and shift $\phi \to \Delta + \phi$ so that ϕ presents purely the fluctuation. In this scenario, ϕ and Φ would be playing the role of $\mathbf{d}\cdot\hat{\mathbf{z}}$ in our kinetic approach. Note that the particle-hole symmetry is usually assumed in kinetic theory, whereas it is exact only on the Fermi surface. The effective field theory respecting this symmetry would acquire an emergent relativistic covariant form [26], even though the microscopic origin might rather respect Galilean symmetry. In the long-wavelength limit $\mathbf{q} \to 0$, the particle-hole symmetry can be implemented by evaluating loop momentum on the Fermi surface and extending the depth of the Fermi sea to infinity. Using the method and Feynman rules summarized in Appendix C, the resulting effective action has the form

$$\mathcal{S}_{\text{eff}} = \mathcal{S}_0[\Delta_0, \phi, \phi^\dagger] + \int\frac{d^3q}{(2\pi)^3}\,(\bar{Q}(q)M_{\bar{Q}Q}(q)Q(q) + \Phi^\dagger(q)M_{\Phi^\dagger\Phi}(q)\Phi(q) + M_{Q\Phi}(q)Q(-q)\Phi(q) + M_{\bar{Q}\Phi^\dagger}(q)\bar{Q}(-q)\Phi^\dagger(q)). \tag{63}$$

The leading part $\mathcal{S}_0$ contains the mean-field free energy, Goldstone fluctuations $(\phi^\dagger - \phi)/i$, and the amplitude mode $\phi^\dagger + \phi$. The bare masses of Q and Φ are given by the zeros of $M_{\bar{Q}Q}$ and $M_{\Phi^\dagger\Phi}$, whose explicit forms are given by

$$M_{\bar{Q}Q} = \nu_{2D}\left(\lambda(\omega) + \frac{1}{F_2}\right), \tag{64}$$

$$M_{\Phi^\dagger\Phi} = \frac{\nu_{2D}\lambda(\omega)}{8\Delta^2}(\omega^2 - 2\Delta^2). \tag{65}$$

λ is again the Tsunedo function (A5a). Hence, the bare mass of Q depends on the parameter F_2 and becomes soft as $F_2 \to -1$. On the other hand, the mass of Φ is shown to be $m_\Phi = \sqrt{2}\,\Delta$, in agreement with the result (30a) and (30b). The Fermi-liquid correction can now be understood in terms of the coupling $Q\Phi$ and $\bar{Q}\Phi^\dagger$

$$M_{Q\Phi} = M_{\bar{Q}\Phi^\dagger} = -\frac{\nu_{2D}\omega\lambda(\omega)}{4\Delta}, \tag{66}$$

indicating Φ and Q are actually not independent modes. As $F_2 \neq -1$ and Q has a finite bare mass, we are able to integrate out Q to obtain a more compact effective theory.

$$\mathcal{S}_{\text{eff}} = \mathcal{S}_0 + \int\frac{d^3q}{(2\pi)^3}\frac{\nu_{2D}\bar{\lambda}(\omega)}{8(1+F_2\lambda(\omega))}\Phi^\dagger(\omega) \times\left[(\omega^2 - 2\Delta^2) + \frac{1}{2}\lambda(\omega)F_2(\omega^2 - 4\Delta^2)\right]\Phi(\omega), \tag{67}$$

reproducing explicitly the result (55). Alternatively, one could integrate out Φ to derive an effective theory of Q.

$$\mathcal{S}_{\text{eff}} = \mathcal{S}_0 + \int\frac{d^3q}{(2\pi)^3}\nu_{2D}\bar{Q}(\omega) \times\left[\frac{\omega^2 - 4\Delta^2}{2(\omega^2 - 2\Delta^2)}\lambda(\omega) + \frac{1}{F_2}\right]Q(\omega). \tag{68}$$

Straightforward investigation shows the renormalized mass of Q in the above action is still given by (55).

In addition to reproducing the known result, the field-theory approach already offers some implications beyond the semiclassical kinetic theory approach.

(1) The exact value $\sqrt{2}\,\Delta$ is closely related to the assumption of particle-hole symmetry, or the approximate relativistic nature of the fermionic superfluid on the Fermi surface. Loosening this approximation allows corrections of order Δ/ϵ_F. Moreover, a term $\Phi^\dagger i\partial_t\Phi$ appears in the action if we breaks particle-hole symmetry during computation, which in turn modifies the value of the bare mass m_Φ as well. In this computation, we impose the particle-hole symmetry in order to be consistent with the assumptions of the kinetic theory. While one could compute nonuniversal corrections to the value of m_Φ by breaking the particle-hole symmetry, we comment that in the weak-coupling computation, terms odd in frequency merely change the mass slightly and the magnitude of m_Φ would remain $\mathcal{O}(\Delta)$. The qualitative fact that this mass is reduced by negative F_2 is not affected.

(2) In the presence of a condensate, operators are classified by the residual symmetry respected by the ground state. Taking the $p - ip$ ground state, for instance,

$$\Delta_{\mathbf{p}} \sim (p_x - ip_y)\langle\psi(-\mathbf{p})\psi(\mathbf{p})\rangle. \tag{69}$$

$\Delta_{\mathbf{p}}$ is symmetric under a combination of U(1) charge transformation and orbital rotation

$$\psi \to e^{i\alpha/2}\psi, \tag{70a}$$

$$\begin{pmatrix} p_x \\ p_y \end{pmatrix} \to \begin{pmatrix} p_x\cos\alpha - p_y\sin\alpha \\ p_y\cos\alpha + p_x\sin\alpha \end{pmatrix}. \tag{70b}$$

In this example, the operators are classified using the *angular momentum* ℓ defined by this combined transformation $\mathcal{O} \to e^{i\ell\alpha}\mathcal{O}$. In particular, the fluctuation of $p + ip$ condensate transforms as

$$(p_x + ip_y)\langle\psi(-\mathbf{p})\psi(\mathbf{p})\rangle \to e^{2i\alpha}(p_x + ip_y)\langle\psi(-\mathbf{p})\psi(\mathbf{p})\rangle \tag{71}$$

and has angular momentum $\ell = 2$. Similarly, the nematic order parameter transforms as

$$(p_x + ip_y)^2\langle\psi^\dagger(\mathbf{p})\psi(\mathbf{p})\rangle \to e^{2i\alpha}(p_x + ip_y)^2\langle\psi^\dagger(\mathbf{p})\psi(\mathbf{p})\rangle, \tag{72}$$

indicating both operators possess the spin-2 nature under the residual symmetry group. The spin-2L states can also be understood from this perspective. Besides, that Φ and Q are not independently fluctuating can be explained in terms of the notion of emergent geometry [27,28]. The nematic order parameters $Q := (Q_1 + iQ_2)/2$ and $\bar{Q} := (Q_1 - iQ_2)/2$, under a proper normalization [29], also parametrize an emergent unimodular metric $\mathfrak{g}_{ij}$ via

$$\mathfrak{g} := \exp\begin{pmatrix} Q_1 & Q_2 \\ Q_2 & -Q_1 \end{pmatrix}. \tag{73}$$

Similarly, the order parameters of a p-wave superfluid Δ^i also define an emergent geometric degree of freedom $\mathfrak{G}^{ij} \sim \Delta^{*i}\Delta^j$. The subgap modes, in this language, correspond to the spin-2 sector of $\mathfrak{G}$. Placed on a flat space and close to equilibrium, both $\mathfrak{g}$ and $\mathfrak{G}$ favor the Euclidean metric δ_{ij}. Hence, their fluctuations, both being spin-2, are indistinguishable from this geometric perspective.

(3) In previous studies, these modes are usually overlooked regarding low-energy physics [30], even though they are responsible for electromagnetic response at high frequency [2]. That the spin-2 mode becomes soft as $F_2 = -1$ suggests that an effective low-energy theory different from one in Ref. [30] should be formulated to incorporate a spin-2 mode close to a nematic critical point. While different microscopic models could produce different results depending on model dependent parameters, symmetry principle together with the above geometric picture suggests an effective action that replaces the background geometry with the internal fluctuating geometry. A proper microscopic model is then responsible for correctly producing effects such as the analogous Hall viscosity, which comes from a $\bar{Q}\partial_t Q$ term in effective action. The existence of this term breaks time reversal symmetry and distinguish a $p - ip$ ground state from a $p + ip$ one. This issue is beyond the scope of current work and will be addressed in detail separately in the future.

VI. CONCLUSION

In conclusion, we revisit a class of two-dimensional superfluids. Using the semiclassical kinetic equation, we compute the order parameter collective modes for the two-dimensional B phase and general chiral ground states of angular momenta $L \geqslant 0$. Extending the known results for $L = 1$, we show that the subgap modes of the universal mass value $\sqrt{2}\,\Delta$ exist for all chiral ground states $L \geqslant 1$ in the limit with exact particle-hole symmetry. By renormalizing the fermionic self-energy, we calculate the correction of these subgap modes from Fermi liquid corrections and discover those subgap modes, sourced by F_{2L}, could be regarded as spin-2L modes, where L is the angular momentum of their underlying ground state. The masses increase for repulsive fermionic interactions and soften for attractive ones. Remarkably, renormalized subgap modes become gapless when the Pomeranchuk instability in the corresponding channel is triggered.

Moreover, we proposed a toy model for the case $L = 1$, whose effective bosonic action is able to reproduce the kinetic result under the consistent approximations. This model could describe a p-wave chiral superfluid near a nematic critical point, and furthermore allows us to loosen the common assumptions made in semiclassical approaches and utilize the insights from field-theory communities to understand the nature of the subgap modes. We hope that the approaches and conclusions drawn from this work could provide the studies of quantum Hall nematic physics and nematic unconventional superfluid a complementary perspective and new insights.

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APPENDIX A: γ AND THE TSUNEDO FUNCTION λ

The integral γ is

$$\int_{|\Delta|}^{\infty}\frac{d\varepsilon}{2\pi i}\frac{1}{\sqrt{\varepsilon^2-|\Delta|^2}}\tanh\frac{\varepsilon}{2T}=\gamma. \tag{A1}$$

It is formally divergent, but can be regularized and identified with $1/V_\ell$ [or $1/(2V_0)$] using linearized gap equation.

The function λ was first introduced by Tsunedo as a kind of Cooper pair susceptibility:

$$\begin{aligned}\bar{\lambda}(\hat{\mathbf{p}};\omega,q)&=\frac{\lambda}{|\Delta(\hat{\mathbf{p}})|^2}\\&=\int_{-\infty}^{\infty}\frac{d\varepsilon}{2\pi i}\frac{\mathsf{n}(\varepsilon_-)(2\varepsilon\omega-\eta^2)-\mathsf{n}(\varepsilon_+)(2\varepsilon\omega+\eta^2)}{(4\varepsilon^2-\eta^2)(\omega^2-\eta^2)+4|\Delta|^2\eta^2},\end{aligned} \tag{A2}$$

where $\eta=v_F\mathbf{q}\cdot\hat{\mathbf{p}}$ and

$$\mathsf{n}(\varepsilon)=-\frac{2\pi i\,\mathrm{sgn}(\varepsilon)}{\sqrt{\varepsilon^2-|\Delta|^2}}\Theta(\varepsilon^2-|\Delta|^2)\tanh\frac{\varepsilon}{2T}. \tag{A3}$$

In $q\to 0$ limit, the integral reduces to

$$\lambda=|\Delta|^2\int_{|\Delta|}^{\infty}\frac{d\varepsilon}{\sqrt{\varepsilon^2-|\Delta|^2}}\frac{\tanh\frac{\varepsilon}{2T}}{\varepsilon^2-\omega^2/4}. \tag{A4}$$

These expressions can be used in numerical evaluation. This function actually has an analytic closed form in the limit $T\to 0$. Writing $x=\omega/(2|\Delta|)$,

$$\lambda(\omega)=\frac{\sin^{-1}x}{x\sqrt{1-x^2}},\ |x|<1, \tag{A5a}$$

where as for $|x|>1$,

$$\lambda(\omega)=\frac{1}{2x\sqrt{x^2-1}}\left[\ln\left|\frac{\sqrt{x^2-1}-x}{\sqrt{x^2-1}+x}\right|+i\pi\,\mathrm{sgn}(x)\right]. \tag{A5b}$$

APPENDIX B: FULL DYNAMICAL EQUATIONS

In this section, we sketch the steps for inverting the kinetic equation and give the full dynamical equations at finite wavelength. Expanding (2) with respect to the ground state of interest, we could found components of the Keldysh Green's function satisfy a general equation

$$\mathbf{\Omega}|\widehat{g}\rangle=\mathsf{M}|\widehat{\sigma}\rangle. \tag{B1}$$

The quasiclassical Green's functions can thus be obtained via

$$\int\frac{d\varepsilon}{2\pi i}|\widehat{g}\rangle=\int\frac{d\varepsilon}{2\pi i}\mathbf{\Omega}^{-1}\mathsf{M}|\widehat{\sigma}\rangle. \tag{B2}$$

We note that when performing ε integral in this work, the particle-hole symmetry $\varepsilon\leftrightarrow-\varepsilon$ is assumed.

The defined in (B1) the matrices are

$$\mathbf{\Omega}=\begin{bmatrix}-\eta&\omega&2i\Delta_I&-2\Delta_R\\\omega&-\eta&0&0\\2i\Delta_I&0&-\eta&2\varepsilon\\2\Delta_R&0&2\varepsilon&-\eta\end{bmatrix} \tag{B3}$$

and

$$\mathsf{M}=\begin{bmatrix}0&-\mathsf{m}_a&-i\mathsf{n}_s\Delta_I&\mathsf{n}_s\Delta_R\\-\mathsf{m}_a&0&-\mathsf{n}_a\Delta_R&i\mathsf{n}_a\Delta_I\\-i\Delta_I\mathsf{n}_s&\Delta_R\mathsf{n}_a&0&-\mathsf{m}_s\\-\mathsf{n}_s\Delta_R&i\mathsf{n}_a\Delta_I&-\mathsf{m}_s&0\end{bmatrix}. \tag{B4}$$

Δ_R and Δ_I are the real and imaginary parts of the gap function. In terms of the n defined by (A3), the elements in M are

$$\mathsf{n}_s=\mathsf{n}(\varepsilon_+)+\mathsf{n}(\varepsilon_-), \tag{B5a}$$

$$\mathsf{n}_a=\mathsf{n}(\varepsilon_+)-\mathsf{n}(\varepsilon_-), \tag{B5b}$$

$$\mathsf{m}=\varepsilon\mathsf{n}(\varepsilon), \tag{B5c}$$

$$\mathsf{m}_s=\mathsf{m}(\varepsilon_+)+\mathsf{m}(\varepsilon_-), \tag{B5d}$$

$$\mathsf{m}_a=\mathsf{m}(\varepsilon_+)-\mathsf{m}(\varepsilon_-). \tag{B5e}$$

In the rest of the section, we give the proper combinations $|\widehat{g}\rangle$ and $|\widehat{\sigma}\rangle$ and the complete dynamical equations.

1. Singlet-pairing ground state

For a singlet-pairing state, the bosonic fluctuation couples only to spin independent fermionic self-energies, and the relevant equations are those which δg, $\delta g'$, d, and d' obey. These equations can be easily solved by taking

$$|\widehat{g}\rangle=\begin{pmatrix}\delta g^{(-)}\\\delta g^{(+)}\\\delta f^{(+)}\\\delta f^{(-)}\end{pmatrix},\ |\widehat{\sigma}\rangle=\begin{pmatrix}\delta\varepsilon^{(-)}\\\delta\varepsilon^{(+)}\\d^{(+)}\\d^{(-)}\end{pmatrix}. \tag{B6}$$

Expressing $|\widehat{g}\rangle$ in terms of $|\widehat{\sigma}\rangle$ and performing convolutions with suitable potentials would imply the following equations:

$$\begin{aligned}\delta\varepsilon^{(-)}(\hat{\mathbf{p}};\omega,\mathbf{q})-\delta\varepsilon^{(+)}_{\rm ext}&=\int\frac{d\theta'}{2\pi}A^s(\theta,\theta')\left\{\left[1+(1-\lambda(\hat{\mathbf{p}}'))\frac{\eta'^2}{\omega^2-\eta'^2}\right]\delta\varepsilon^{(-)}(\hat{\mathbf{p}}')\right.\\&\left.\quad+\frac{\omega\eta'}{\omega^2-\eta'^2}(1-\lambda(\hat{\mathbf{p}}'))\delta\varepsilon^{(+)}(\hat{\mathbf{p}}')+\frac{\bar{\lambda}(\hat{\mathbf{p}}')\eta'}{2}[d(\hat{\mathbf{p}}')\Delta^*(\hat{\mathbf{p}}')-\Delta(\hat{\mathbf{p}}')d(\hat{\mathbf{p}}')]\right\},\end{aligned} \tag{B7a}$$

$$\begin{aligned}\delta\varepsilon^{(+)}(\hat{\mathbf{p}};\omega,\mathbf{q})-\delta\varepsilon^{(+)}_{\rm ext}&=\int\frac{d\theta'}{2\pi}A^s(\theta,\theta')\left\{\frac{\omega\eta'}{\omega^2-\eta'^2}(1-\lambda(\hat{\mathbf{p}}'))\delta\varepsilon^{(-)}(\hat{\mathbf{p}}')\right.\\&\left.\quad+\frac{\omega^2}{\omega^2-\eta'^2}(1-\lambda(\hat{\mathbf{p}}'))\delta\varepsilon^{(+)}(\hat{\mathbf{p}}')-\frac{1}{2}\omega\bar{\lambda}(\hat{\mathbf{p}}')[d'(\hat{\mathbf{p}}')\Delta(\hat{\mathbf{p}}')-d(\hat{\mathbf{p}}')\Delta^*(\hat{\mathbf{p}}')]\right\},\end{aligned} \tag{B7b}$$

$$d(\hat{\mathbf{p}};\omega,\mathbf{q}) = \int \frac{d\theta'}{2\pi} V_{\mathrm{e}}(\theta,\theta')\bigg\{\bigg[\gamma + \frac{1}{4}\bar{\lambda}(\hat{\mathbf{p}}')(\omega^2 - \eta'^2 - 2|\Delta(\hat{\mathbf{p}}')|^2)\bigg]d(\hat{\mathbf{p}}') - \frac{\bar{\lambda}(\hat{\mathbf{p}}')}{2}\Delta^2(\hat{\mathbf{p}}')d'(\hat{\mathbf{p}}') - \frac{\Delta(\hat{\mathbf{p}}')\bar{\lambda}(\hat{\mathbf{p}}')}{4}[\eta'\delta\varepsilon^{(-)}(\hat{\mathbf{p}}') + \omega\delta\varepsilon^{(+)}(\hat{\mathbf{p}}')]\bigg\}, \tag{B7c}$$

$$d'(\hat{\mathbf{p}};\omega,\mathbf{q}) = \int \frac{d\theta'}{2\pi} V_{\mathrm{e}}(\theta,\theta')\bigg\{\bigg[\gamma + \frac{1}{4}\bar{\lambda}(\hat{\mathbf{p}}')(\omega^2 - \eta'^2 - 2|\Delta(\hat{\mathbf{p}}')|^2)\bigg]d'(\hat{\mathbf{p}}') - \frac{\bar{\lambda}(\hat{\mathbf{p}}')}{2}(\Delta^*(\hat{\mathbf{p}}'))^2 d(\hat{\mathbf{p}}') + \frac{\Delta^*(\hat{\mathbf{p}}')\bar{\lambda}(\hat{\mathbf{p}}')}{4}[\eta'\delta\varepsilon^{(-)}(\hat{\mathbf{p}}') + \omega\delta\varepsilon^{(+)}(\hat{\mathbf{p}}')]\bigg\}. \tag{B7d}$$

2. Triplet-pairing ground state

For a triplet-pairing ground state, the vectors $\mathbf{d}$ and $\mathbf{d}'$ couple to both spin-dependent and independent self-energies. We denote the direction of the ground-state condensate $\boldsymbol{\Delta}$ as $\hat{\mathbf{n}}$. To solve vector quantities $\mathbf{d}$ and $\boldsymbol{\varepsilon}$, we decompose them into the longitudinal L and transverse T components with respect to $\hat{\mathbf{n}}$, that is, a vector $\mathbf{v}$ is decomposed as $\mathbf{v} = \mathbf{v}_L + \mathbf{v}_T$, where $\mathbf{v}_L = \hat{\mathbf{n}}(\mathbf{v}\cdot\hat{\mathbf{n}})$. The complete set of equations can be solved by considering the following combinations of $\{|\widehat{g}\rangle, |\widehat{\sigma}\rangle\}$. The part coupled with spin-independent δg is the longitudinal modes

$$\left\{\begin{pmatrix}\delta g^{(-)}\\ \delta g^{(+)}\\ \delta\mathbf{f}_L^{(+)}\\ \delta\mathbf{f}_L^{(-)}\end{pmatrix}, \begin{pmatrix}\delta\varepsilon^{(-)}\\ \delta\varepsilon^{(+)}\\ \mathbf{d}_L^{(+)}\\ \mathbf{d}_L^{(-)}\end{pmatrix}\right\}. \tag{B8a}$$

The part coupled with the spin-dependent $\delta\mathbf{g}$, on the other hand, includes the transverse and binormal parts of the anomalous Green's function.

$$\left\{\begin{pmatrix}\hat{\mathbf{n}}\times\delta\mathbf{g}^{(-)}\\ \hat{\mathbf{n}}\times\delta\mathbf{g}^{(+)}\\ i\delta\mathbf{f}_T^{(-)}\\ i\delta\mathbf{f}_T^{(+)}\end{pmatrix}, \begin{pmatrix}\hat{\mathbf{n}}\times\delta\boldsymbol{\varepsilon}^{(-)}\\ \hat{\mathbf{n}}\times\delta\boldsymbol{\varepsilon}^{(+)}\\ i\mathbf{d}_T^{(-)}\\ i\mathbf{d}_T^{(+)}\end{pmatrix}\right\}; \tag{B8b}$$

$$\left\{\begin{pmatrix}\delta\mathbf{g}_T^{(-)}\\ \delta\mathbf{g}_T^{(+)}\\ i\delta\mathbf{f}^{(-)}\times\hat{\mathbf{n}}\\ i\delta\mathbf{f}^{(+)}\times\hat{\mathbf{n}}\end{pmatrix}, \begin{pmatrix}\delta\boldsymbol{\varepsilon}_T^{(-)}\\ \delta\boldsymbol{\varepsilon}_T^{(+)}\\ i\mathbf{d}^{(-)}\times\hat{\mathbf{n}}\\ i\mathbf{d}^{(+)}\times\hat{\mathbf{n}}\end{pmatrix}\right\}. \tag{B8c}$$

The above two sets of vectors give only the binormal and transverse information about $\delta\mathbf{g}$. It turns out the spin-singlet components δf and d are required to access the longitudinal information of $\delta\mathbf{g}$ using the combination below:

$$\left\{\begin{pmatrix}\delta\mathbf{g}_L^{(+)}\\ \delta\mathbf{g}_L^{(-)}\\ \delta f^{(+)}\\ \delta f^{(-)}\end{pmatrix}, \begin{pmatrix}\delta\boldsymbol{\varepsilon}_L^{(+)}\\ \delta\boldsymbol{\varepsilon}_L^{(-)}\\ d^{(+)}\\ d^{(-)}\end{pmatrix}\right\}. \tag{B8d}$$

These spin-singlet degrees of freedom δf and d are treated as external sources and turned off at the end of computation. After solving all above $|\widehat{g}\rangle$ in terms of $|\widehat{\sigma}\rangle$, we could again make use of (8a), (8b), (10a), and (10b) to obtain the following equations:

$$\delta\varepsilon^{(-)}(\hat{\mathbf{p}};\omega,\mathbf{q}) - \delta\varepsilon^{(-)}_{\mathrm{ext}} = \int \frac{d\theta}{2\pi} A^s(\theta,\theta')\bigg\{\bigg[1 + (1-\lambda(\hat{\mathbf{p}}'))\frac{\eta'^2}{\omega^2-\eta'^2}\bigg]\delta\varepsilon^{(-)}(\hat{\mathbf{p}}') + \frac{\omega\eta'}{\omega^2-\eta'^2}(1-\lambda(\hat{\mathbf{p}}'))\delta\varepsilon^{(+)}(\hat{\mathbf{p}}') + \frac{1}{2}\eta'\bar{\lambda}(\hat{\mathbf{p}}')[\boldsymbol{\Delta}^*(\hat{\mathbf{p}}')\cdot\mathbf{d}(\hat{\mathbf{p}}') - \boldsymbol{\Delta}(\hat{\mathbf{p}}')\cdot\mathbf{d}'(\hat{\mathbf{p}}')]\bigg\}, \tag{B9a}$$

$$\delta\varepsilon^{(+)}(\hat{\mathbf{p}};\omega,\mathbf{q}) - \delta\varepsilon^{(+)}_{\mathrm{ext}} = \int \frac{d\theta'}{2\pi} A^s(\theta,\theta')\bigg\{\frac{\omega\eta'}{\omega^2-\eta'^2}\big(1-\lambda(\hat{\mathbf{p}}')\big)\delta\varepsilon^{(-)}(\hat{\mathbf{p}}') + \frac{\omega^2}{\omega^2-\eta'^2}(1-\lambda(\hat{\mathbf{p}}'))\delta\varepsilon^{(+)}(\hat{\mathbf{p}}') + \frac{1}{2}\omega\bar{\lambda}(\hat{\mathbf{p}}')[\boldsymbol{\Delta}^*(\hat{\mathbf{p}}')\cdot\mathbf{d}(\hat{\mathbf{p}}') - \boldsymbol{\Delta}(\hat{\mathbf{p}}')\cdot\mathbf{d}'(\hat{\mathbf{p}}')]\bigg\}, \tag{B9b}$$

$$\delta\boldsymbol{\varepsilon}^{(-)}(\hat{\mathbf{p}};\omega,\mathbf{q}) = \int \frac{d\theta'}{2\pi}A^{a}(\theta,\theta')\bigg\{\bigg(1+(1-\lambda(\hat{\mathbf{p}}'))\frac{\eta'^2}{\omega^2-\eta'^2}\bigg)\delta\boldsymbol{\varepsilon}^{(-)}(\hat{\mathbf{p}}') + \frac{\omega\eta'}{\omega^2-\eta'^2}(1-\lambda(\hat{\mathbf{p}}'))\delta\varepsilon^{(+)}(\hat{\mathbf{p}}') - \lambda(\hat{\mathbf{p}}')[\delta\boldsymbol{\varepsilon}^{(-)}(\hat{\mathbf{p}}')\cdot\hat{\mathbf{n}}(\hat{\mathbf{p}}')]\hat{\mathbf{n}}(\hat{\mathbf{p}}') - \frac{i\eta'\bar{\lambda}(\hat{\mathbf{p}}')}{2}[\boldsymbol{\Delta}^*(\hat{\mathbf{p}}')\times\mathbf{d}(\hat{\mathbf{p}}')+\boldsymbol{\Delta}(\hat{\mathbf{p}}')\times\mathbf{d}'(\hat{\mathbf{p}}')]\bigg\}, \tag{B9c}$$

$$\delta\boldsymbol{\varepsilon}^{(+)}(\hat{\mathbf{p}};\omega,\mathbf{q}) = \int \frac{d\theta'}{2\pi}A^{a}(\theta,\theta')\bigg\{\frac{\omega^2}{\omega^2-\eta'^2}(1-\lambda(\hat{\mathbf{p}}'))\delta\boldsymbol{\varepsilon}^{(+)}(\hat{\mathbf{p}}') + \frac{\eta'\omega}{\omega^2-\eta'^2}(1-\lambda(\hat{\mathbf{p}}'))\delta\boldsymbol{\varepsilon}^{(-)}(\hat{\mathbf{p}}') + \lambda(\hat{\mathbf{p}}')[\delta\boldsymbol{\varepsilon}^{(+)}(\hat{\mathbf{p}}')\cdot\hat{\mathbf{n}}(\hat{\mathbf{p}}')]\hat{\mathbf{n}}(\hat{\mathbf{p}}') - \frac{i\omega}{2}\bar{\lambda}(\hat{\mathbf{p}}')[\boldsymbol{\Delta}^*(\hat{\mathbf{p}}')\times\mathbf{d}(\hat{\mathbf{p}}')+\boldsymbol{\Delta}(\hat{\mathbf{p}}')\times\mathbf{d}'(\hat{\mathbf{p}}')]\bigg\}, \tag{B9d}$$

$$\mathbf{d}(\hat{\mathbf{p}};\omega,\mathbf{q}) = \int \frac{d\theta'}{2\pi}V_{\rm o}(\theta,\theta')\bigg\{\bigg(\gamma+\frac{1}{4}\bar{\lambda}(\hat{\mathbf{p}}')(\omega^2-\eta'^2-2|\Delta(\hat{\mathbf{p}}')|^2)\bigg)\mathbf{d}(\hat{\mathbf{p}}') - \frac{1}{4}\bar{\lambda}(\hat{\mathbf{p}}')\boldsymbol{\Delta}(\hat{\mathbf{p}}')[\eta'\delta\varepsilon^{(-)}(\hat{\mathbf{p}}')+\omega\delta\varepsilon^{(+)}(\hat{\mathbf{p}}')] + \frac{1}{4}\bar{\lambda}(\hat{\mathbf{p}}')i\boldsymbol{\Delta}(\hat{\mathbf{p}}')\times(\eta'\delta\boldsymbol{\varepsilon}^{(-)}(\hat{\mathbf{p}}')+\omega\delta\boldsymbol{\varepsilon}^{(+)}(\hat{\mathbf{p}}')) + \frac{1}{2}\bar{\lambda}(\hat{\mathbf{p}}')[(\boldsymbol{\Delta}(\hat{\mathbf{p}}')\cdot\boldsymbol{\Delta}(\hat{\mathbf{p}}'))\mathbf{d}'(\hat{\mathbf{p}}')-2(\boldsymbol{\Delta}(\hat{\mathbf{p}}')\cdot\mathbf{d}'(\hat{\mathbf{p}}'))\boldsymbol{\Delta}(\hat{\mathbf{p}}')]\bigg\}, \tag{B9e}$$

$$\mathbf{d}'(\hat{\mathbf{p}};\omega,\mathbf{q}) = \int \frac{d\theta'}{2\pi}V_{\rm o}(\theta,\theta')\bigg\{\bigg(\gamma+\frac{1}{4}\bar{\lambda}(\hat{\mathbf{p}}')(\omega^2-\eta'^2-2|\Delta(\hat{\mathbf{p}}')|^2)\bigg)\mathbf{d}'(\hat{\mathbf{p}}') + \frac{1}{4}\bar{\lambda}(\hat{\mathbf{p}}')\boldsymbol{\Delta}^*(\hat{\mathbf{p}}')[\eta'\delta\varepsilon^{(-)}(\hat{\mathbf{p}}')+\omega\delta\varepsilon^{(+)}(\hat{\mathbf{p}}')] + \frac{1}{4}\bar{\lambda}(\hat{\mathbf{p}}')i\boldsymbol{\Delta}^*(\hat{\mathbf{p}}')\times(\eta'\delta\boldsymbol{\varepsilon}^{(-)}(\hat{\mathbf{p}}')+\omega\delta\boldsymbol{\varepsilon}^{(+)}(\hat{\mathbf{p}}')) + \frac{1}{2}\bar{\lambda}(\hat{\mathbf{p}}')[(\boldsymbol{\Delta}^*(\hat{\mathbf{p}}')\cdot\boldsymbol{\Delta}^*(\hat{\mathbf{p}}'))\mathbf{d}(\hat{\mathbf{p}}')-2(\boldsymbol{\Delta}^*(\hat{\mathbf{p}}')\cdot\mathbf{d}(\hat{\mathbf{p}}'))\boldsymbol{\Delta}^*(\hat{\mathbf{p}}')]\bigg\}. \tag{B9f}$$

APPENDIX C: 1-LOOP ACTION COMPUTATION

The system concerning us in this note is a spin polarized p-wave chiral superfluid. We adapt the simplest pairing model induced by a contact pairing, which we can decouple by introducing a dynamical auxiliary field via Hubbard-Stratonovich transformation. We also introduce the nematic fluctuation in the particle-hole channel. Those interactions can also be decoupled by introducing more auxiliary/collective fields, which we will denote as ϕ_I in the following.

The fermionic part of the action after possibly multiple Hubbard-Stratonovich transformations can be written as $S = \int(dx)\,\Psi^\dagger iD^{-1}\Psi$, where Ψ is the Nambu spinor $\Psi^T=(\psi,\psi^\dagger)$. The partition function of the fermion sector is then

$$\int \mathscr{D}\Psi^\dagger\mathscr{D}\Psi\ \exp\bigg(-\int(dx)\Psi^\dagger D^{-1}\Psi\bigg) := \exp\bigg(iS_{\rm eff}\bigg). \tag{C1}$$

Since we are only considering a single species of fermions, the effective action of auxiliary fields $\{\phi_I\}$ reads

$$S_{\rm eff} = -\frac{i}{2}{\rm Tr}\ln D^{-1}. \tag{C2}$$

Formally, we can expand the action with respect to a classical solution $\phi_J^0(k)=\langle\phi_J^0\rangle\times(2\pi)^3\delta(k)$. In terms of the deviation $\delta\phi_J=\phi_J-\phi_J^0$,

$$S = S_{\rm eff}+S_{\rm aux}[\phi_I] = S[\phi_I^0] + \int(dq)\,\frac{\delta S}{\delta\phi_J(q)}[\phi^0]\delta\phi_J(q) + \frac{1}{2}\int(dq)(dq')\,\delta\phi_I(q)\frac{\delta^2 S}{\delta\phi_I(q)\delta\phi_J(q')}[\phi^0]\delta\phi_J(q')+\cdots \tag{C3}$$

The first-order condition

$$\frac{\delta S}{\delta\phi_I(k)}[\phi^0]=0 \tag{C4}$$

is often used to specify the information of a certain uniform ground state $\phi_I^0(2\pi)^3\delta(k)$. The key ingredient of the Gaussian effective theory is the second derivative of the action evaluated with respect to the ground state. To extract the contribution from $S_{\rm eff}$, we have to compute

$$\begin{aligned}\frac{\delta^2}{\delta\phi_I(q)\delta\phi_J(q')}S_{\rm eff} &= -\frac{i}{2}\frac{\delta^2}{\delta\phi_I(q)\delta\phi_J(q')}{\rm Tr}\ln D^{-1} = -\frac{i}{2}\frac{\delta}{\delta\phi_I(q)}{\rm Tr}D\frac{\delta D^{-1}}{\delta\phi_J(q')}\\ &= -\frac{i}{2}{\rm Tr}\frac{\delta D}{\delta\phi_I(q)}\frac{\delta D^{-1}}{\delta\phi_J(q')} - \frac{i}{2}{\rm Tr}D\frac{\delta^2 D^{-1}}{\delta\phi_I(q)\delta\phi_J(q')}\\ &= \frac{i}{2}{\rm Tr}\bigg[D\frac{\delta D^{-1}}{\delta\phi_I(q)}D\frac{\delta D^{-1}}{\delta\phi_J(q')}\bigg] - \frac{i}{2}{\rm Tr}\bigg[D\frac{\delta^2 D^{-1}}{\delta\phi_I(q)\delta\phi_J(q')}\bigg].\end{aligned} \tag{C5}$$

Thus the two-point function of our interest is then

$$M^{IJ}(q,q') = \frac{i}{2}\mathrm{Tr}\left[D\frac{\delta D^{-1}}{\delta\phi_I(q)}D\frac{\delta D^{-1}}{\delta\phi_J(q')}\right] - \frac{i}{2}\mathrm{Tr}\left[D\frac{\delta^2 D^{-1}}{\delta\phi_I(q)\delta\phi_J(q')}\right]\Bigg|_{\phi=\phi^0}. \tag{C6}$$

The second term in (C6) is usually referred to as the contact term. Example includes the diamagnetic current term of electromagnetic response. For the model concerning us in this paper, we only have to focus on the first term in (C6).

To compute the two-point functions presented in the main text, we will need the ground-state propagator of a $p-ip$ ground state and the variation of the sum (57a)+(57b)+(57c) with respect to fields $\bar{Q}$, Q, $\Phi^\dagger$, and Φ. The propagators are

$$iD_0^{-1}(p,p') = (2\pi)^3\delta(p-p')\begin{pmatrix} p_0-\xi_{\mathbf{p}} & \Delta p_F^{-1}(p_x-ip_y) \\ \Delta p_F^{-1}(p_x+ip_y) & p_0+\xi_{\mathbf{p}}\end{pmatrix}, \tag{C7a}$$

$$D_0(p,p') = (2\pi)^3\delta(p-p')\frac{i}{p_0^2-E_{\mathbf{p}}^2+i\eta}\begin{pmatrix} p_0+\xi_{\mathbf{p}} & -\Delta p_F^{-1}(p_x-ip_y) \\ -\Delta p_F^{-1}(p_x+ip_y) & p_0-\xi_{\mathbf{p}}\end{pmatrix} \tag{C7b}$$

with $E_{\mathbf{p}}^2 = \xi_{\mathbf{p}}^2 + p_F^{-2}\Delta^2\mathbf{p}^2$. Finally, the vertices are

$$\frac{\delta iD^{-1}(p,p')}{\delta Q(q)} = \frac{1}{p_F^2}\begin{pmatrix} -(p'_x-ip'_y)^2 & 0 \\ 0 & (p_x-ip_y)^2\end{pmatrix}(2\pi)^3\delta(p-p'-q), \tag{C8a}$$

$$\frac{\delta iD^{-1}(p,p')}{\delta \bar{Q}(q)} = \frac{1}{p_F^2}\begin{pmatrix} -(p'_x+ip'_y)^2 & 0 \\ 0 & (p_x+ip_y)^2\end{pmatrix}(2\pi)^3\delta(p'-p-q), \tag{C8b}$$

$$\frac{\delta iD^{-1}(p,p')}{\delta \Phi(q)} = (2\pi)^3\delta(p-p'-q)\begin{pmatrix} 0 & p_F^{-1}(p'_x+ip'_y) \\ 0 & 0\end{pmatrix}, \tag{C8c}$$

$$\frac{\delta iD^{-1}(p,p')}{\delta \Phi^\dagger(q)} = (2\pi)^3\delta(p'-p-q)\begin{pmatrix} 0 & 0 \\ p_F^{-1}(p'_x-ip'_y) & 0\end{pmatrix}. \tag{C8d}$$

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