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Dynamic spin-charge coupling: ac spin Hall magnetoresistance in nonmagnetic conductors

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The dynamic coupling between spin and charge currents in nonmagnetic conductors is considered. As a consequence of this coupling, the spin dynamics is directly reflected in the electrical impedance of the sample, with a relevant frequency scale defined by spin relaxation and spin diffusion. This allows the observation of the electron spin resonance by purely electrical measurements.

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Introduction. It was predicted nearly half a century ago [1,2] that spin-orbit interaction results in the interconnection between electrical and spin currents: an electrical current produces a transverse spin current and *vice versa*. This leads respectively to the direct and inverse spin Hall effects. Following the proposal in Ref. [3], the inverse spin Hall effect was observed experimentally by Bakun *et al.* [4] in 1984, without causing much excitement at that time.

Twenty years later, after the first experimental observations of the (direct) spin Hall effect [5,6] this topic has become a subject of considerable interest with thousands of publications (see, e.g., a review in Ref. [7]).

Because of the interconnection between the spin and charge currents, anything that happens with spins will influence the charge current, i.e., result in corresponding changes of the electrical resistance, which can be measured with a very high precision. An example of this link is provided by the *spin Hall magnetoresistance* [8], the reason for which is the depolarization of spins accumulated at the sample boundaries by a transverse magnetic field and the resulting decrease of the driving electric current (for a given voltage) [9]. This effect was experimentally demonstrated in platinum by Vélez *et al.* [10].

Earlier, a similar effect was discovered and studied by Nakayama *et al.* [11] in layered structures ferromagnet–normal metal. The magnetization in the ferromagnet can be rotated by an applied magnetic field which results in a change in the normal metal resistivity.

In recent years, the ac spin Hall effect in ferromagnet–normal metal structures has also been studied both experimentally [12–15] and theoretically [16–19]. The precession of the magnetization in a ferromagnet leads to a time-varying injection of spin into the normal metal. Due to the inverse spin Hall effect, the resulting spin current in the normal metal generates the ac electric current.

In particular, the observed ac voltage resonantly depends on the Larmor frequency in the ferromagnet and the frequency of the external ac magnetic field, which excites the precession of magnetization. In this way, with the aid of the spin Hall effect in a normal metal, the ferromagnetic resonance was observed by electric measurements.

While these studies are quite important for achieving the ultimate goal of storing and manipulating information by the use of spin Hall effect for switching magnetic domains in magnetics (see the reviews [20,21]), the physics of the layered magnetic structures is quite complicated, and this makes the exact theory and the quantitative analysis of experimental data rather difficult.

Here, following Ref. [8], we develop a much more simple theory of ac electron magnetotransport controlled by the direct and inverse spin Hall effects in *nonmagnetic* materials, semiconductors, or metals. The theory is based on the phenomenological transport equations [1,2,7,8] describing the interconnection between spin and charge currents. We show that spin resonance in nonmagnetic materials can be observed by purely transport measurements.

Transport equations. Consider a conductor in an external ac electric field $\boldsymbol{E}(t) \sim \cos(\omega t)$ and a magnetic field $\boldsymbol{B}$ (see Fig. 1). We assume that the ac frequency ω is much lower than the cyclotron frequency ω_c, and that $\omega\tau \ll 1$, where τ is the momentum relaxation time. However, the spin Larmor frequency Ω and the spin relaxation time $\tau_s \gg \tau$ are such that $\omega \sim \Omega \sim 1/\tau_s$.

In this frequency range, the basic phenomenological equations for the electron flow density $\boldsymbol{q} = \boldsymbol{j}/e$, the spin current density tensor q_{ij}, and the spin density vector $\boldsymbol{P}$ are [7]

$$\boldsymbol{q} = \mu n \boldsymbol{E} + \gamma D \,\mathrm{rot}\boldsymbol{P}, \tag{1}$$

$$q_{ij} = -D\frac{\partial P_j}{\partial x_i} + \gamma \mu n \epsilon_{ijk} E_k, \tag{2}$$

$$\frac{\partial P_j}{\partial t} + \frac{\partial q_{ij}}{\partial x_i} + (\boldsymbol{\Omega} \times \boldsymbol{P})_j + \frac{P_j}{\tau_s} = 0, \tag{3}$$

where n is the electron density, μ is the electron mobility, D is the diffusion coefficient, $\gamma \ll 1$ is the dimensionless parameter proportional to the strength of the spin-orbit interaction and describing the interconnection between the particle and the spin currents, ϵ_{ijk} is the unit antisymmetric tensor, the vector $\boldsymbol{\Omega}$ is directed along the applied magnetic field, Ω being the Larmor frequency for electron spins, and τ_s is the spin-relaxation time.

The first term in Eq. (1) is the usual Drude contribution, while the second term expresses the interconnection (caused by spin-orbit interaction) between particle current and the spin current caused by the inhomogeneity of the spin density.

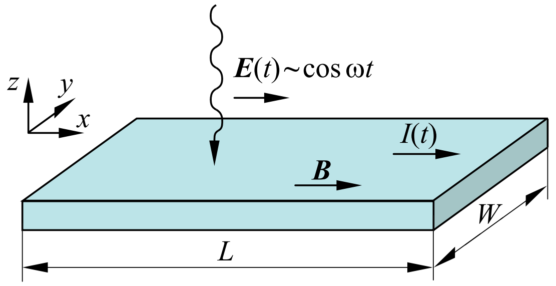

FIG. 1. A metal or semiconductor sample in a magnetic field $\boldsymbol{B}$ and ac electric field $\boldsymbol{E}(t)$. The length of the sample, L, is much greater than its width, W. It is assumed that the ac electric field penetrates everywhere into the sample, i.e., that the electron system is either two-dimensional, or three-dimensional, but with thickness less than the skin depth.

Equation (2) describes two contributions to the spin current density q_{ij}: the first one is due to diffusion of spin-polarized electrons and the second one is due to the transformation of particle current into spin current.

Equation (3) is the continuity equation for the spin density, taking into account spin diffusion, rotation of spin in magnetic field, and spin relaxation.

It should be noted that in the absence of inversion symmetry there may be additional terms describing spin-charge coupling. In particular, there is a spin current induced by a nonequilibrium spin polarization and a uniform spin polarization generated by electric current. A direct excitation of spin resonance by an ac electric field (the Rashba-Sheka effect [22]) also becomes possible. Here, we do not consider this type of effects due to inversion symmetry breaking.

In the geometry of Fig. 1, both the particle and the spin flows depend on the y coordinate only. We consider that the ac electric field, as well as the magnetic field, are directed along the x direction.

As follows from Eq. (2), the two nonzero components of the spin current density tensor are

$$q_{yy} = -D\frac{dP_y}{dy}, \quad q_{yz} = -D\frac{dP_z}{dy} + \gamma\mu nE\cos\omega t\,. \tag{4}$$

The absence of spin currents across the sample boundaries is described by the boundary conditions $q_{yy} = 0$ and $q_{yz} = 0$ at $y = \pm W/2$.

Solution of the transport equations. The nonzero components of the spin density vector $\boldsymbol{P}$ can be written in a complex form: $P_y(y,t) = [P_y(y)e^{-i\omega t} + \text{c.c.}]/2$, and similarly for $P_z(y,t)$. Then, from Eqs. (3) and (4) we obtain a system of coupled equations for $P_y(y)$ and $P_z(y)$:

$$D\frac{d^2P_y}{dy^2} = \left(-i\omega + \frac{1}{\tau_s}\right)P_y + \Omega P_z,$$
$$D\frac{d^2P_z}{dy^2} = \left(-i\omega + \frac{1}{\tau_s}\right)P_z - \Omega P_y, \tag{5}$$

with the boundary conditions

$$\left.\frac{dP_y}{dy}\right|_{y=\pm W/2} = 0, \quad \left.\frac{dP_z}{dy}\right|_{y=\pm W/2} = \gamma\frac{\mu nE}{D}\,. \tag{6}$$

The solution of Eqs. (5) with the boundary conditions (6) yields the spin density profile:

$$P_y(y) = -i\gamma\frac{\mu nE}{2D}[F_+(y) - F_-(y)],$$
$$P_z(y) = \gamma\frac{\mu nE}{2D}[F_+(y) + F_-(y)], \tag{7}$$

where

$$F_\pm(y) = \frac{\sinh(\lambda_\pm y)}{\lambda_\pm\cosh(\lambda_\pm W/2)}, \tag{8}$$

$$\lambda_\pm = \frac{\sqrt{1 + i(-\omega \pm \Omega)\tau_s}}{L_s}, \tag{9}$$

and $L_s = \sqrt{D\tau_s}$ is the spin-diffusion length.

Thus for narrow samples, $|\lambda_\pm|W \ll 1$, the spin density $\boldsymbol{P}$ depends linearly on the coordinate y, while for wide samples, $|\lambda_\pm|W \gg 1$, the spin density $\boldsymbol{P}$ is concentrated near the sample edges. In the last case, spin density exhibits spin resonance at $\omega = \pm\Omega$ provided that $\omega\tau_s \gtrsim 1$. The signs $\pm$ correspond to the contribution to $\boldsymbol{P}$ from the components of $\boldsymbol{E}(t)$ with the right and the left circular polarizations, respectively.

The current density $j_x = eq_x$ can now be calculated using Eq. (1): $j_x = e\mu nE\cos\omega t + \Delta j(y,t)$, where the first term is the normal Drude contribution (in the assumed limit $\omega\tau \ll 1$), while the second term is a correction which is of second order in the spin-orbit interaction: $\Delta j(y,t) = [\Delta j(y)e^{-i\omega t} + \text{c.c.}]/2$, where

$$\Delta j(y) = \gamma^2\frac{e\mu nE}{2}\left[\frac{dF_+}{dy} + \frac{dF_-}{dy}\right]. \tag{10}$$

For wide samples, $|\lambda_\pm|W \gg 1$, this correction to the ac density, like the spin density, is concentrated near the sample edges.

The spin-orbit correction ΔI to the main Drude part, $I_0 = e\mu nEW$, of the total current can be calculated from Eq. (10):

$$\Delta I = \int_{-W/2}^{W/2}\Delta j(y)dy. \tag{11}$$

Thus we obtain the final result for the correction to the sample impedance, $\Delta Z = \Delta Z(\omega,\Omega)$, caused by spin-orbit interaction:

$$\frac{\Delta Z}{Z_0} = -\gamma^2\left[\frac{\tanh(\lambda_+W/2)}{\lambda_+W} + \frac{\tanh(\lambda_-W/2)}{\lambda_-W}\right], \tag{12}$$

where $Z_0 = L/(e\mu nW)$ and $\lambda_\pm$ are defined by Eq. (9).

Results and discussion. We now analyze our results given by Eq. (12) for some special cases.

(i) Low frequencies: $\omega\tau_s \ll 1$. The results coincide with those of Ref. [8] for the dc spin Hall magnetoresistance.

(ii) Zero magnetic field, $B = 0$. In Fig. 2 we plot the ratio $\varrho(\omega) = \text{Re}\Delta Z(\omega,0)/\Delta Z_0$ of the real part of the spin-orbit correction ΔZ [Eq. (12)] in the absence of magnetic field to its value at zero frequency, $\Delta Z_0 = \Delta Z(0,0)$. It is seen that $\varrho(\omega)$ has a quasiuniversal behavior as a function of the parameter $\omega\tau^*$. Here $1/\tau^* = 1/\tau_s + 1/\tau_d$ is the effective total relaxation rate which is the sum of the bulk spin-relaxation rate $1/\tau_s$ and the diffusion rate for space inhomogeneity in the spin distribution $1/\tau_d = 4D/W^2$.

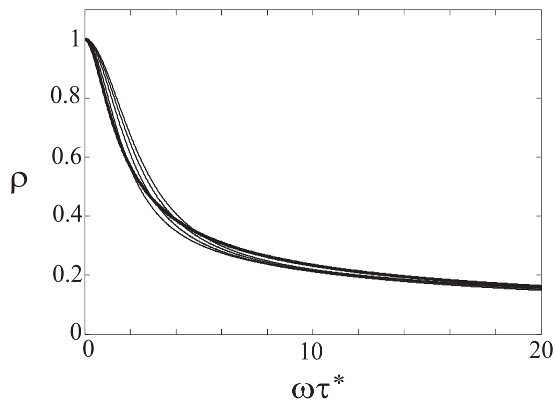

FIG. 2. The ratio ϱ of the real part of the spin-orbit correction to impedance ΔZ to its value at zero frequency ΔZ_0 as a function of ac frequency ω in the absence of magnetic field. Thin curves correspond to $W/L_s = 0.2,\ 0.8,\ 1.3,\ 2$, while the thick curve corresponds to $W/L_s = \infty$. It is seen that all curves practically coincide.

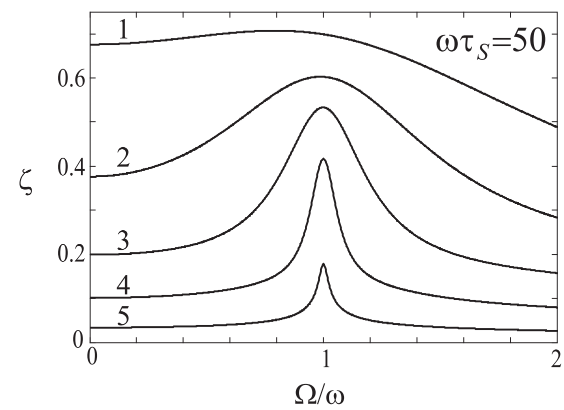

FIG. 3. The real part of the correction to the ac impedance ζ as a function of the Larmor frequency Ω at a fixed ac frequency ω for medium and large sample widths. For curves 1, 2, 3, 4, and 5 the parameter W/L_s is equal to 0.4, 0.6, 1, 2, and 6, respectively.

Thus there are the two relaxation processes: the bulk spin relaxation with the rate $1/\tau_s$ and the decay of spin inhomogeneity due to diffusion of spin-polarized electrons. The quasiuniversal results in Fig. 2 are quite similar to those obtained in Ref. [8] for the dc spin Hall magnetoresistance.

Indeed, from Eq. (12) one can obtain the relation between the corrections to the ac impedance in zero magnetic field and to the dc magnetoresistance:

$$\mathrm{Re}\,\Delta Z(\omega, 0) = \Delta Z(0, \Omega = -\omega). \tag{13}$$

Here $\Delta Z(0, \Omega)$ is, in fact, the spin Hall magnetoresistance calculated in Ref. [8] and denoted therein as $\Delta R(\Omega)$.

(iii) High frequencies: $\omega\tau_s \gg 1$. For narrow samples ($W \ll L_s/\sqrt{\omega\tau_s}$) the correction ΔZ depends neither on frequency, nor on magnetic field in the main order by the parameter $W/L_s \ll 1$ (at not too high magnetic fields when $\Omega \lesssim \omega$). With the small correction on the order of $(W/L_s)^2$ included, we obtain

$$\frac{\Delta Z}{Z_0} = -\gamma^2\left(1 - \frac{1 - i\omega\tau_s}{24}\frac{W^2}{L_s^2}\right) \tag{14}$$

For wide samples ($W \gg L_s$) Eq. (12) leads to the formula

$$\frac{\Delta Z}{Z_0} = -\gamma^2\frac{L_s}{W}\sum_{\pm}\frac{1}{\sqrt{1 + i(-\omega \pm \Omega)\tau_s}}, \tag{15}$$

displaying spin resonance at $\omega = \pm\Omega$ with a width $1/\tau_s$.

The general formula (12) is needed for medium sample widths ($L_s/\sqrt{\omega\tau_s} \ll W \ll L_s$). In this case Eq. (12) describes the crossover between the resonant dependence of ΔZ on Ω for wide samples [Eq. (15)] and the nonresonant dependence of ΔZ on Ω for narrow samples [Eq. (14)].

In Fig. 3 we plot the ratio $\zeta = \mathrm{Re}\,\Delta Z/(-\gamma^2 Z_0)$, given by the general equation (12), as a function of Ω at a fixed ω for different sample widths W. In the limit of very narrow samples, when $W \ll L_s$, this ratio is equal to 1 [see Eq. (14)]. The transition from the nonresonant to the resonant behavior of $\zeta(\Omega)$ with the increase of W is clearly seen. Equation (15) and Fig. 3 show that the wider the sample, the smaller both the amplitude and the width of the resonance peak.

It is interesting to study the behavior of the normalized ac magnetoresistance

$$\varrho(\omega, \Omega) = \frac{\mathrm{Re}\Delta Z(\omega, \Omega)}{\mathrm{Re}\Delta Z(\omega, \Omega = \omega)}. \tag{16}$$

An analysis similar to that performed above for the dependence $\varrho(\omega)$ in zero magnetic field, shows that $\varrho(\omega, \Omega)$ at a fixed ω has a quasiuniversal behavior as a function of the parameter $(\Omega - \omega)\tau^*$ at $|\Omega| > \omega$, similar to the behavior of $\varrho(\omega)$ displayed in Fig. 2. However, the dependencies of ϱ on Ω at $|\Omega| < \omega$, as seen from Fig. 3, are qualitatively different for wide and for narrow samples.

Conclusion. We have shown that the combination of the direct and inverse spin Hall effects in nonmagnetic metals and semiconductors offers an interesting possibility to study high-frequency spin phenomena, including spin resonance, by purely electrical measurements. The corresponding corrections to the sample impedance are of second order in the spin-orbit coupling parameter, γ.

In the absence of an external magnetic field, the frequency dependence of the electrical impedance is defined by the sum of the bulk spin-relaxation rate and the spin-diffusion rate. The interplay between the two corresponding relaxation times defines also the width and the amplitude of the electrically measured spin resonance.

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