

Dynamics of noncollinear antiferromagnetic textures driven by spin current injection

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We present a theoretical formalism to address the dynamics of textured, noncollinear antiferromagnets subject to spin current injection. We derive sine-Gordon type equations of motion for the antiferromagnets, which are applicable to technologically important antiferromagnets such as Mn₃Ir and Mn₃Sn, and enables an analytical approach to domain wall dynamics in those materials. We obtain the expression for domain wall velocity, which is estimated to reach ~ 1 km/s in Mn₃Ir by exploiting spin Hall effect with electric current density $\sim 10^{11}$ A/m².

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I. INTRODUCTION

Since the prediction of staggered magnetic order [1] and its experimental observation in MnO [2], antiferromagnetic (AFM) materials have occupied a central place in the study of magnetism. The absence of macroscopic magnetization in AFMs, however, indicates that they cannot be effectively manipulated and observed by external magnetic field. This fact has hindered active applications of AFMs in today's technology, despite consistent development of theoretical understanding on the nature of AFM dynamics [3]. Research in the emergent field of antiferromagnetic spintronics [4] has shown that electric and spin currents can access AFM dynamics through spin-transfer torques [5–13] and spin-orbit torques [14,15]. Similar to ferromagnets, AFMs can also accommodate topologically nontrivial textures such as domain walls (DWs) [16–18] and skyrmions [19,20], which play crucial roles in spintronics applications, e.g., racetrack memories [21]. The studies on current-driven dynamics of AFM textures have opened an avenue toward AFM-based technologies.

Recently, AFMs with noncollinear magnetic configurations are generating increasing attention as they exhibit large magnetotransport and thermomagnetic effects; e.g., anomalous Hall effect [22–24], anomalous Nernst effect [25,26] and magneto-optical Kerr effect [27,28]. These phenomena have their origins in the topological character of the electronic band structures, which in turn are associated with the noncollinear magnetism. To take full advantages of noncollinear AFMs in spintronics applications, it is also important to achieve efficient manipulation of magnetic textures, such as DWs, in those materials. The studies on current-driven dynamics of AFMs, however, have thus far mostly focused on collinear structures. Understanding the effects of electric and spin currents in noncollinear AFMs is a crucial issue in the community [29–31].

In this paper, we focus on the dynamics of noncollinear AFMs induced by spin current (SC) injection, which may be realized by exploiting spin Hall effect/spin-polarized electric current in an adjacent heavy-metal/ferromagnetic layer. We derive sine-Gordon type equations of motion for the AFMs, including effective forces due to SC injection, external

magnetic field, and internal dissipation. Our model can be applied to technologically important triangular AFMs such as Mn₃Ir and Mn₃Sn. We then study DW dynamics, where an analytical expression for the DW velocity is derived.

II. MODEL

We consider an antiferromagnet (AFM) composed of three equivalent magnetic sublattices (A, B, and C) with constant saturation magnetization M_S . In our coarse-grained model, the classical vector $\vec{m}_A(\vec{r}, t)$ ($|\vec{m}_A(\vec{r}, t)| = 1$) is a continuous field that represents the magnetization direction in sublattice A, with similar definitions for $\vec{m}_B(\vec{r}, t)$ and $\vec{m}_C(\vec{r}, t)$ (Fig. 1); all three sublattice magnetizations are defined as each spatial point \vec{r} . This continuous treatment is appropriate when the spatial variation of each sublattice magnetization is sufficiently slow compared to the atomistic length scale.

The magnetic energy density u of the AFM is modeled as follows,

$$\begin{aligned}
 u = & J_0 \sum_{\langle \zeta \eta \rangle} \vec{m}_\zeta \cdot \vec{m}_\eta + A_1 \sum_{x_i=x,y,z} \sum_{\zeta=A,B,C} \left(\frac{\partial \vec{m}_\zeta}{\partial x_i} \right)^2 \\
 & - A_2 \sum_{x_i=x,y,z} \sum_{\langle \zeta \eta \rangle} \frac{\partial \vec{m}_\zeta}{\partial x_i} \cdot \frac{\partial \vec{m}_\eta}{\partial x_i} + D_0 \vec{e}_z \cdot \sum_{\langle \zeta \eta \rangle} \vec{m}_\zeta \times \vec{m}_\eta \\
 & + u_{\text{ani}} - \mu_0 M_S \vec{H} \cdot \sum_{\zeta=A,B,C} \vec{m}_\zeta, \quad (1)
 \end{aligned}$$

where $J_0 (> 0)$ is the homogenous exchange coupling energy, A_1 and A_2 are the isotropic inhomogeneous exchange stiffnesses [32], D_0 characterizes the homogeneous Dzyaloshinskii-Moriya interaction (DMI), \vec{H} is the external magnetic field, and μ_0 is the vacuum permeability. The symbol $\langle \zeta \eta \rangle$ indicates the sum over the pairs $(\zeta, \eta) = (A, B)$, (B, C) , and (C, A) . For the anisotropy part u_{ani} we assume

$$u_{\text{ani}} = -K \sum_{\zeta=A,B,C} (\vec{e}_\zeta \cdot \vec{m}_\zeta)^2, \quad (2)$$

where $K (> 0)$ is the anisotropy constant, and the unit vectors \vec{e}_ζ indicate the easy axes for \vec{m}_ζ in the x - y plane;

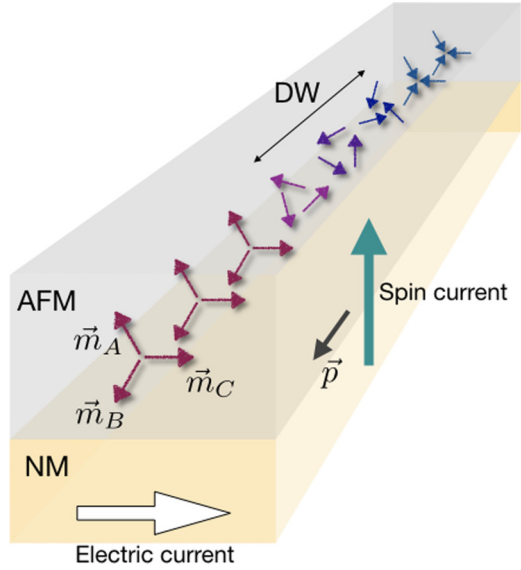


FIG. 1. Scheme of the studied system; bilayer of noncollinear antiferromagnet (AFM) and nonmagnetic (NM) heavy metal, where the spin current with polarization along \vec{p} is injected into the AFM. Spin current may be created via spin Hall effect (as shown) or by alternative techniques, such as spin pumping and injection of spin-polarized electric current from a ferromagnetic layer. The domain wall (DW), connecting the all-in (blue) and all-out (red) domains, is driven into motion by the spin current.

$\vec{e}_A = (-\vec{e}_x + \sqrt{3}\vec{e}_y)/2$, $\vec{e}_B = -(\vec{e}_x + \sqrt{3}\vec{e}_y)/2$, and $\vec{e}_C = \vec{e}_x$. The magnetic anisotropy of this form applies to triangular AFMs such as the $L1_2$ phase of Mn_3Ir [33] and the hexagonal phase of Mn_3Sn [31], with the (1,1,1) plane of their fcc crystals identified as our x - y plane. Although there can also be a smaller out-of-plane anisotropy in realistic materials, Eq. (2) suffices for our present purpose of understanding the fundamental response of triangular AFMs to SC injection.

The dynamics of \vec{m}_ζ ($\zeta = A, B, C$) are assumed to obey the coupled Landau-Lifshitz-Gilbert equations;

$$\frac{\partial \vec{m}_\zeta}{\partial t} = -\vec{m}_\zeta \times \gamma \vec{H}_\zeta + \alpha \vec{m}_\zeta \times \frac{\partial \vec{m}_\zeta}{\partial t} - \vec{m}_\zeta \times (\vec{m}_\zeta \times \vec{p}), \quad (3)$$

where γ and α are the gyromagnetic ratio and the Gilbert damping constant, respectively, which are assumed for simplicity to be sublattice independent, and $\vec{H}_\zeta = -(\mu_0 M_S)^{-1} \delta u / \delta \vec{m}_\zeta$ is the effective magnetic field for the sublattice ζ . The last term in Eq. (3) is the Slonczewski-Berger spin-transfer torque [34] due to SC injection. The vector \vec{p} represents the value and polarization of the SC, which depend on the way of SC injection, device materials, geometry, etc. We have assumed that the injected SC transfers the angular momentum equiprobably to each of the sublattices [30].

III. IN-PLANE TRIANGULAR APPROXIMATION

We here introduce

$$\vec{n}_1 = \frac{\vec{m}_A + \vec{m}_B - 2\vec{m}_C}{3\sqrt{2}}, \quad \vec{n}_2 = \frac{-\vec{m}_A + \vec{m}_B}{\sqrt{6}}, \quad (4)$$

$$\vec{m} = \frac{\vec{m}_A + \vec{m}_B + \vec{m}_C}{3}. \quad (5)$$

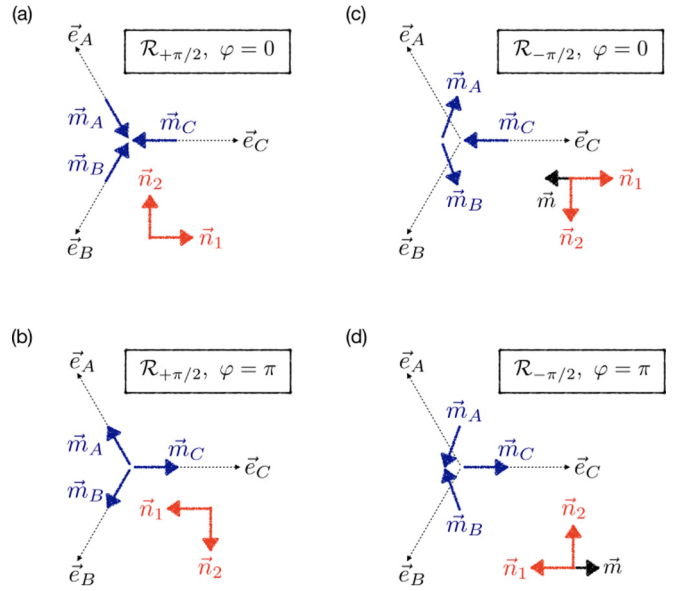


FIG. 2. Triangular magnetic configurations with (a),(b) $\mathcal{R}_{+\pi/2}$ and (c),(d) $\mathcal{R}_{-\pi/2}$ chiralities. They are parametrized by (a),(c) $\varphi = 0$ and (b),(d) $\varphi = \pi$. Dotted lines indicate the easy axes of the magnetic anisotropy.

Because the AFM exchange coupling responsible for the formation of triangular structure is usually dominant over the other energies, one can safely assume $|\vec{m}(\vec{r}, t)| \ll 1$. The vectors \vec{n}_1 and \vec{n}_2 are then approximated to be orthogonal to each other and have the fixed length as $|\vec{n}_1| \simeq |\vec{n}_2| \simeq 1/\sqrt{2}$. These two vectors can be regarded order parameters of the AFM [30], specifying the particular triangular configuration.

We further assume that the in-plane anisotropy is sufficiently large that the triangle is formed in the x - y plane with $|\vec{m}_\zeta^z| \ll 1, \forall \zeta$. This leads to an approximation where only the x and y components of \vec{n}_1 and \vec{n}_2 are nonzero (while \vec{m} can still have a finite z component). In this case the orientations of \vec{n}_1 and \vec{n}_2 in the x - y plane can be parameterized by a single azimuthal angle φ [3,30,31,33] as

$$\vec{n}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} \cos \varphi \\ \sin \varphi \\ 0 \end{bmatrix}, \quad (6)$$

$$\vec{n}_2 = \mathcal{R}_{\pm\pi/2} \vec{n}_1 \equiv \frac{1}{\sqrt{2}} \begin{bmatrix} \cos(\varphi \pm \pi/2) \\ \sin(\varphi \pm \pi/2) \\ 0 \end{bmatrix}. \quad (7)$$

In Eq. (7), $\mathcal{R}_{+\pi/2}$ and $\mathcal{R}_{-\pi/2}$ select the $+\pi/2$ and $-\pi/2$ rotations of \vec{n}_2 against \vec{n}_1 , respectively, corresponding to the two different chiralities of the triangular structure, defined by $\text{sgn}(\vec{e}_z \cdot \vec{n}_1 \times \vec{n}_2)$; in Fig. 2, four different triangular configurations are shown as examples. Which of $\mathcal{R}_{+\pi/2}$ and $\mathcal{R}_{-\pi/2}$ should be chosen is dictated by the DMI and magnetic anisotropy. The DMI favors the $\mathcal{R}_{+\pi/2}$ ($\mathcal{R}_{-\pi/2}$) chirality if the sign of D_0 is negative (positive). The magnetic anisotropy, on the other hand, can never be fully respected by $\mathcal{R}_{-\pi/2}$ [Figs. 2(c) and 2(d)], in contrast to $\mathcal{R}_{+\pi/2}$ where the anisotropy energy is minimized by taking $\varphi = 0, \pi$. [Figs. 2(a) and 2(b)] The $\mathcal{R}_{-\pi/2}$ chirality is thus favored when

the DMI satisfies the condition $2\sqrt{3}D_0 > K$. As a result of the competition between the anisotropy, exchange coupling, and DMI, the $\mathcal{R}_{-\pi/2}$ triangles carry the weak in-plane ferromagnetic moment \vec{m} [Figs. 2(c) and 2(d)]. Typical materials that host the $\mathcal{R}_{+\pi/2}$ triangles include the $L1_2$ phase of IrMn_3 [35,36], while the $\mathcal{R}_{-\pi/2}$ configurations are observed in, e.g., the hexagonal phase of Mn_3Z ($\text{Z} = \text{Sn, Ge, Ga}$) [37].

It turns out that, for either chirality, the AFM responds to the injected SC in a similar way. In the following we mostly focus on the $\mathcal{R}_{+\pi/2}$ case and later consider configurations with $\mathcal{R}_{-\pi/2}$.

With the parametrization in Eqs. (6) and (7), the state of an AFM is described by four variables (φ, \vec{m}) . By rewriting Eqs. (3) in terms of (φ, \vec{m}) and assuming $J_0 \gg |D_0| \gg K$, one obtains, up to the first order of \vec{m} , the closed equation of motion for φ ,

$$c^2 \square \varphi - \frac{3\omega_E \omega_K}{2} \sin 2\varphi = 3\omega_E p_z - \gamma \frac{\partial H_z}{\partial t} + \omega_\alpha \frac{\partial \varphi}{\partial t}, \quad (8)$$

and the explicit expression for \vec{m} ,

$$\vec{m} = \frac{1}{3\omega_E} \left(-\frac{\partial \varphi}{\partial t} \vec{e}_z + \gamma \vec{H} \right), \quad (9)$$

for the $\mathcal{R}_{+\pi/2}$ case. We have introduced $\omega_E = \gamma J_0 / \mu_0 M_S$, $\omega_K = 2\gamma K / \mu_0 M_S$, $\omega_\alpha = 3\alpha \omega_E$, and $\square = \nabla^2 - (1/c^2) \partial^2 / \partial t^2$ with $c = \sqrt{3\omega_E \gamma (2A_1 + A_2) / \mu_0 M_S}$ the group velocity of spin wave.

Equation (9) shows that \vec{m} is expressed in terms of φ and vanishes in the absence of magnetic dynamics ($\partial \varphi / \partial t = 0$) and external magnetic field. Equation (8) is one of our main results. The rhs of this equation contains the effective forces originating from SC injection, time-varying magnetic field, and internal damping. In the absence of these forces, Eq. (8) is reduced to a sine-Gordon equation, consistent with the work in Ref. [33]. In the limit of homogeneous systems ($\nabla \varphi = 0$) without external magnetic field, Eq. (8) then reproduces the result of Ref. [30]. Now, our Eq. (8) allows one to study inhomogeneous AFM textures under the external driving forces due to SC and magnetic field, and the internal damping.

Notice that only the out-of-plane (z) component of the SC polarization and of the magnetic field can induce the dynamics of φ . We should also remark that the DMI does not appear in Eqs. (8) and (9). This is because the DMI energy, which can be written as $3\sqrt{3}D_0 \vec{e}_z \cdot \vec{n}_1 \times \vec{n}_2$, is constant within the present approximation, and its contribution to the equations of motion is higher order. The DMI plays a crucial role, however, in lifting the degeneracy with respect to the chirality (Fig. 2) as discussed above. In the following, we discuss the translational motion of a DW driven by SC (setting $\vec{H} = 0$).

IV. DOMAIN WALL DYNAMICS

The doubly-degenerate ground states for the $\mathcal{R}_{+\pi/2}$ case are given such that the magnetic anisotropy energy is minimized by $\varphi = 0$ and π , corresponding to the all-in and all-out configurations, respectively [Figs. 2(a) and 2(b)]. A DW can be formed as a transition region connecting the two ground states (Fig. 1). Here let us consider a one-dimensional DW extending along the z axis. (Due to the isotropic character of the exchange stiffnesses, our conclusions will be independent

of the choice of the direction of DW extension, as long as the SC is polarized along the z axis so defined.) When the rhs of Eq. (8) is absent, a standard solution $\varphi_e(z)$ for a static DW with the boundary condition $\varphi_e(z = \pm\infty) = (0, \pm\pi)$ or $(\pm\pi, 0)$ (notice that φ is defined in $-\pi \leq \varphi \leq \pi$) is obtained as $\varphi_e(z) = 2F \tan^{-1} [\exp(Q \frac{z-z_0}{\Delta_0})]$; z_0 is the coordinate of the DW center, $\Delta_0 = \sqrt{(2A_1 + A_2)/2K}$ is the DW width parameter, and $(Q, F) = (\pm 1, \pm 1)$, satisfying $QF = (1/\pi) \int_{-\infty}^{\infty} dz (\partial \varphi_e / \partial z)$, specifies the boundary condition.

To study steady motion of the DW driven by SC, we employ the following ansatz

$$\varphi(z, t) = 2F \tan^{-1} \left[\exp \left(Q \frac{z - Vt}{\Delta} \right) \right], \quad (10)$$

where V is the velocity of DW center and Δ is the dynamical width parameter. By substituting this ansatz into Eq. (8), multiplying the subsequent equation by $\sin \varphi$, and integrating it along the z axis from $z = -\infty$ to $+\infty$, one finds the relation $V = (FQ\pi \Delta / 2\alpha) p_z$.

In the special case where the DW exhibits an inertial motion in the *absence* of the rhs of Eq. (8), the width parameter Δ_{in} is given by

$$\Delta_{\text{in}} = \Delta_0 \sqrt{1 - \frac{V^2}{c^2}}. \quad (11)$$

Equation (11) implies the Lorentz contraction of the DW, stemming from the Lorentz invariance of the sine-Gordon equation. The rhs of Eq. (8) may be regarded perturbation, if $3\omega_E p_z$ and $\omega_\alpha |\partial \varphi / \partial t| \sim \omega_\alpha V / \Delta$ are sufficiently small compared to each term in the lhs. In this perturbative regime one can use the approximation $\Delta = \Delta_{\text{in}}$, which leads to

$$V = FQ \frac{\mu p_z}{\sqrt{1 + (\mu p_z / c)^2}}, \quad (12)$$

where we have introduced the DW mobility (in the unit of length)

$$\mu = \frac{\pi \Delta_0}{2\alpha}. \quad (13)$$

Equation (12) is one of our central results, revealing important natures of the DW dynamics. The sign of V is determined by that of p_z , i.e., the polarization of the SC, and the factor FQ that characterizes the DW structure. For $\mu |p_z| / c \ll 1$, the DW velocity depends linearly on p_z as $V \simeq FQ \mu p_z$. Importantly, V monotonically increases with $|p_z|$, in a similar manner as in collinear AFMs [15]. Our result thus indicates that the absence of the so-called Walker breakdown [38] is ubiquitous for general AFMs, and a high DW velocity can be achieved by increasing the SC injection. The previous studies showed that noncollinear AFMs have an advantage over collinear ones in the large magnetotransport effects [22–24,27,28], which provide efficient ways to detect DWs. Now that Eq. (12) reveals that the noncollinear AFMs can accommodate DWs moving as fast as in collinear ones, the former are indeed a potential candidate for future spintronics applications. In Fig. 3, Eq. (12) is plotted by the solid line as a function of μp_z with $(F, Q) = (+1, +1)$.

To check the validity of Eq. (12), we compute the DW velocity by numerically solving Eq. (3) with the fourth order

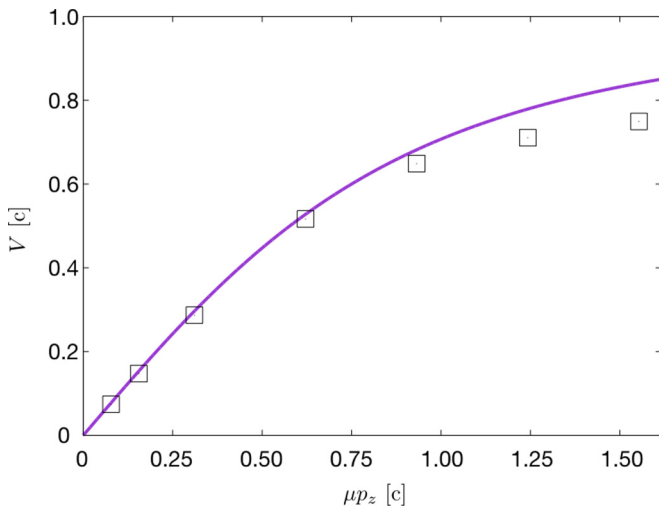


FIG. 3. The DW velocity V as a function of SC μp_z , calculated numerically from Eq. (3) (open symbols) and analytically from Eq. (12) (solid line). Both V and μp_z are measured in the unit of c ($\simeq 16$ km/s with the present parameter set shown below). The parameter values are typical for Mn_3Ir [35,36]: $J_0 = 2.4 \times 10^8$ J/m³, $A_1 = 0$ (corresponding to taking into only account the nearest-neighbor coupling in the kagome lattice), $A_2 = 2 \times 10^{-11}$ J/m, $D_0 = -2 \times 10^7$ J/m³, $K = 3 \times 10^6$ J/m³, $\mu_0 M_S = 1.63$ T, $\gamma = 1.76 \times 10^{11}$ Hz/T, and $\alpha = 10^{-2}$. When the SC is created via spin Hall effect, as in Fig. 1, the electric current density j_c , corresponding to $\mu p_z = 0.1c$ and $V \simeq 4.7$ km/s, is estimated as $j_c \simeq 8.5 \times 10^{11}$ A/m², using $p_z = (\gamma \hbar / 2e \mu_0 M_S) \theta_{\text{SHE}} j_c / d$ [34] with the spin Hall angle $\theta_{\text{SHE}} = 0.15$ for NM and the sample thickness $d = 1$ nm.

Runge-Kutta method. We consider a nanowire with dimensions of $1 \times 1 \text{ nm}^2 \times 20 \mu\text{m}$, dividing it into the unit cells of $1 \times 1 \times 1 \text{ nm}^3$. We employ the periodic boundary condition along the x direction, to mimic a thin (in the y direction), wide (in the x direction) nanowire. The results of the simulations are indicated by the open symbols in Fig. 3, which agree well with the analytical calculation in the relatively low current regime. The discrepancy starts visibly developing as μp_z is increased as large as $\sim c$, where the rhs and lhs of Eq. (8) become comparable (with our present choice of parameters) and the perturbative approach is invalid. The deviation of the numerical results from Eq. (12) in the high current regime may be attributed to several factors. First, the out-of-plane components of the magnetizations grow with p_z , thus reducing the accuracy of the in-plane approximation. Second, the homogeneous SC, represented by the spatial-independent p_z term in Eq. (8), acts not only within the DW region but also on each domain. The SC thus causes the rotation of the domains away from $\varphi = 0, \pm\pi$, and the ansatz (10) becomes inappropriate. In this regime, one may be able to find some quantitative difference in the detailed DW dynamics between collinear and noncollinear AFMs. We leave more systematic and thorough study on the high current regime for future work.

V. $\mathcal{R}_{-\pi/2}$ CHIRALITY

Lastly, we show that qualitatively similar conclusions are obtained for the $\mathcal{R}_{-\pi/2}$ case. The equations of motion are

derived with the same line of approximations used in deriving Eqs. (8) and (9). For the weak ferromagnetic moment \vec{m} one obtains

$$\vec{m} = \frac{1}{3\omega_E} \left(-\frac{\partial\varphi}{\partial t} \vec{e}_z + \gamma \vec{H} + \frac{\omega_K}{\sqrt{2}} \mathcal{M}\vec{n} \right), \quad (14)$$

where the external magnetic field \vec{H} is restored, and $\mathcal{M}\vec{n} = 2^{-1/2}(-\cos\varphi, \sin\varphi, 0)$. Equation (14) differs from Eq. (9) in the third term, which arises from the competition between the magnetic anisotropy, exchange coupling, and DMI, as discussed above.

The equation of motion for φ , up to the first order of \vec{m} , is

$$c^2 \square \varphi = 3\omega_E p_z - \gamma \frac{\partial H_z}{\partial t} + 3\alpha \omega_E \frac{\partial \varphi}{\partial t} - \frac{\omega_K \gamma}{2} (H_x \sin \varphi + H_y \cos \varphi). \quad (15)$$

There are two major differences between the magnetic dynamics for the $\mathcal{R}_{+\pi/2}$ [Eq. (8)] and $\mathcal{R}_{-\pi/2}$ [Eq. (15)] cases: First, for $\mathcal{R}_{-\pi/2}$, in-plane magnetic fields can create additional driving forces [the last terms in the rhs of Eq. (15)], which originates from the direct Zeeman coupling between the weak ferromagnetic moment [the last term in Eq. (14)] and the magnetic field. Second, the $\sin 2\varphi$ term does not appear in Eq. (15), which indicates the absence of effective anisotropy for φ . In the $\mathcal{R}_{-\pi/2}$ case, effective anisotropies arise from higher-order terms of \vec{m} [31]. For Mn_3Sn , indeed, a small anisotropy ~ 10 J/m³ of the form of $\cos 6\varphi$ has been predicted, which leads to formations of 60° DWs [31]. Although a DW is in general not a 180° wall depending on the symmetry of the effective anisotropy, the SC acts on the DW in essentially the same way as on the 180° walls in the $\mathcal{R}_{+\pi/2}$ case, since the p_z term is identical in Eqs. (8) and (15). Most of the conclusions on the DW motion derived before thus hold qualitatively, with renormalization $\varphi \rightarrow \frac{n}{2}\varphi$ for a n -fold anisotropy with n an even integer.

VI. CONCLUSIONS

We have derived sine-Gordon type equations of motion for the noncollinear antiferromagnets, with spin current injection, external magnetic field, and dissipative terms included. We have demonstrated that the injected spin current, when it is polarized perpendicular to the triangular plane, can drive a translational motion of a domain wall. When the spin current is injected by exploiting the spin Hall effect, the domain wall velocity as high as ~ 1 km/s can be achieved for typical noncollinear antiferromagnets, with realistic electric current density $\sim 10^{11}$ A/m². As the spin current injection into noncollinear antiferromagnets remains to be experimentally demonstrated, our findings provide a guideline for devising future experiments.

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APPENDIX: DERIVATION OF EQS. (8) AND (9)

Here we show a derivation of the effective equations of motion (8) and (9) for the case with $\mathcal{R}_{+\pi/2}$. Equation (3) can be rewritten by using Eqs. (4), (5), (6), and (7) as

$$\begin{aligned} \frac{\partial \vec{n}_1}{\partial t} = & -\vec{n}_1 \times (\gamma \vec{H} - 3\omega_E \vec{m} + \vec{\omega}_{K_m} + \mathcal{D}' \vec{m}) - \vec{m} \times (\vec{\omega}_D \times \mathcal{R}_{+\pi/2} \vec{n}_1 + \vec{\omega}_{K_{n1}} + \mathcal{D} \vec{n}_1) + \frac{\omega_K}{2\sqrt{2}} (m_x \sin \varphi + m_y \cos \varphi) \vec{e}_z \\ & + \frac{1}{2\sqrt{2}} [(p_x \cos 2\varphi + p_y \sin 2\varphi) \vec{e}_x + (p_x \sin 2\varphi - p_y \cos 2\varphi) \vec{e}_y], \end{aligned} \quad (\text{A1})$$

$$\begin{aligned} \frac{\partial \mathcal{R}_{+\pi/2} \vec{n}_1}{\partial t} = & -\mathcal{R}_{+\pi/2} \vec{n}_1 \times (\gamma \vec{H} - 3\omega_E \vec{m} + \vec{\omega}_{K_m} + \mathcal{D}' \vec{m}) - \vec{m} \times (-\vec{\omega}_D \times \vec{n}_1 + \vec{\omega}_{K_{n2}} + \mathcal{D} \mathcal{R}_{+\pi/2} \vec{n}_1) \\ & - \frac{\omega_K}{2\sqrt{2}} (m_x \cos \varphi - m_y \sin \varphi) \vec{e}_z + \frac{1}{2\sqrt{2}} [(p_x \sin 2\varphi - p_y \cos 2\varphi) \vec{e}_x - (p_x \cos 2\varphi + p_y \sin 2\varphi) \vec{e}_y], \end{aligned} \quad (\text{A2})$$

$$\frac{\partial \vec{m}}{\partial t} = -\vec{m} \times \gamma \vec{H} + \frac{p_x \vec{e}_x + p_y \vec{e}_y}{2} + \left[\left(-\frac{c^2}{3\omega_E} \nabla^2 + \alpha \frac{\partial}{\partial t} \right) \varphi + \frac{\omega_K \sin 2\varphi}{2} + p_z \right] \vec{e}_z, \quad (\text{A3})$$

up to the first order of \vec{m} , where

$$\mathcal{D} = \left[\frac{\gamma(2A_1 + A_2)}{\mu_0 M_S} \nabla^2 - \alpha \frac{\partial}{\partial t} - \vec{p} \times \right], \quad \mathcal{D}' = \left[\frac{2\gamma(A_1 - A_2)}{\mu_0 M_S} \nabla^2 - \alpha \frac{\partial}{\partial t} - \vec{p} \times \right], \quad (\text{A4})$$

$$\vec{\omega}_D = \frac{\sqrt{3}\gamma \vec{D}_0}{\mu_0 M_S}, \quad (\text{A5})$$

$$\vec{\omega}_{K_{n1}} = \omega_K \frac{(\vec{e}_A \cdot \vec{m}_A) \vec{e}_A + (\vec{e}_B \cdot \vec{m}_B) \vec{e}_B - 2(\vec{e}_C \cdot \vec{m}_C) \vec{e}_C}{3\sqrt{2}}, \quad (\text{A6})$$

$$\vec{\omega}_{K_{n2}} = \omega_K \frac{-(\vec{e}_A \cdot \vec{m}_A) \vec{e}_A + (\vec{e}_B \cdot \vec{m}_B) \vec{e}_B}{\sqrt{6}}, \quad (\text{A7})$$

$$\vec{\omega}_{K_m} = \omega_K \frac{(\vec{e}_A \cdot \vec{m}_A) \vec{e}_A + (\vec{e}_B \cdot \vec{m}_B) \vec{e}_B + (\vec{e}_C \cdot \vec{m}_C) \vec{e}_C}{3}. \quad (\text{A8})$$

Let us ignore the STTs and all the \vec{m} -dependent terms but the H_E term, assuming that they are sufficiently small;

$$\frac{\partial \vec{n}_1}{\partial t} = -\vec{n}_1 \times (\gamma \vec{H} - 3\omega_E \vec{m}), \quad (\text{A9})$$

$$\frac{\partial \mathcal{R}_{+\pi/2} \vec{n}_1}{\partial t} = -\mathcal{R}_{+\pi/2} \vec{n}_1 \times (\gamma \vec{H} - 3\omega_E \vec{m}). \quad (\text{A10})$$

Multiplying by $\vec{n}_1 \times$ the latter equation and using $\vec{n}_1 \times \partial \mathcal{R}_{+\pi/2} \vec{n}_1 / \partial t = 0$, one is led to $\vec{n}_1 \cdot \vec{m} = \vec{n}_1 \cdot \gamma \vec{H} / 3\omega_E$. Substitute this into the expression of $\vec{n}_1 \times \partial \vec{n}_1 / \partial t$ to obtain

$$\vec{m} = \frac{1}{3\omega_E} \left(\gamma \vec{H} - \frac{\partial \varphi}{\partial t} \vec{e}_z \right). \quad (\text{A11})$$

This is Eq. (9). It is then straightforward to obtain Eq. (8) by substituting Eq. (A11) into (A3).

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