Quantum computing with sine-Gordon qubits

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A universal quantum computing scheme, with a universal set of logical gates, is proposed based on networks of one-dimensional (1D) quantum systems. The encoding of information is in terms of universal features of gapped phases, for which effective field theories such as sine-Gordon field theory can be employed to describe a qubit. Primary logical gates are from twist, pump, glue, and shuffle operations that can be realized in principle by tuning parameters of the systems. Our scheme demonstrates the power of 1D quantum systems for robust quantum computing.

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I. INTRODUCTION

Finding qubits with robust properties is crucial to build a quantum computer. Robust quantum computing demands of good error-correction codes [1], which could accompany a large overhead, or a self-correcting quantum memory [2], which, as the analog of classical bits, with the two-dimensional (2D) Ising models as a physical cornerstone for classical computers, is still missing. Topological quantum computing (TQC) [3], with qubits usually carried by edge modes or anyonic excitations [4–6], has been one of the most promising schemes for quantum computing.

Topological orders [7], which underlie the physics of TQC, have seen significant progress in recent years. This motivates the search for new schemes of TQC with distinct features. With symmetry-protected topological (SPT) order [8,9], 1D quantum systems have been recognized as promising candidates of quantum computer hardware, both bosonic and fermionic [4,5,10–13]. In this work, we study universal quantum computing with qubits encoded in the bulk states of 1D gapped quantum spin systems. With valence-bond solids and bosonization theory [14–17], robust code properties have been demonstrated recently [12,13]. However, it is not known whether 1D gapped quantum spin systems, and in general systems that can be well described by sine-Gordon field theory and its equivalence [17] can support universal quantum computing. We achieve this by the design of a scheme with a universal set of gate operations on such sine-Gordon qubits.

In our scheme, a qubit is encoded in the universal property of the bulk states of gapped phases, with field variables described by the sine-Gordon theory. Logical gates follow from the so-called vertex algebra of field observable, which have a certain topological robustness. In particular, the logical phase flip Z_L is a flux insertion that can be realized by external global fields, bit flip X_L is a pump process of excitations in a cycle on the system. Hadamard gate H_L, which exchanges Z_L and X_L, can be induced by the unitary shuffle process from one gapped phase to its dual, or alternatively, by the quantum teleportation method [18]. Entangling gates are from glue operations of qubit states that can be realized by tuning of interaction parameters of a model. The scheme is scalable forming various networks, as shown in Fig. 1. With the implementation in a spin-ladder system, our scheme generalizes the classical magnetic logic, and also extends the interplay between spintronics and quantum computing [19–21]. With bosonization, our method will be adapted to other systems [22–27], including Josephson junctions arrays and lattice boson ladders, which can be treated as quantum simulators, hence it serves as a potential test bed with advanced control technique for the scheme we propose.

II. PRELIMINARY

We start from basic sine-Gordon field theory and explain how it can describe a qubit, and then study examples in 1D quantum spin system. A simple sine-Gordon Hamiltonian takes the form

$$H = H_0 + V(\phi), \tag{1}$$

for a free Gaussian part H_0 [16,17] and a sine-Gordon nonlinear term $V(\phi) = g\int dx \cos\beta\phi$ with real parameters g and β. Here x is the spatial direction along the system. It describes the dynamics of conjugate bosonic field operators ϕ and θ (with the hat symbol omitted) such that

$$[\phi(x), \theta(y)] = i\Theta(x-y) \tag{2}$$

for Heaviside step function Θ. The free part is massless while a mass can be induced if the nonlinear term is relevant under renormalization flow [16,17].

It is appropriate to understand the essence of the model as a harmonic oscillator with nonlinearity, while the fields ϕ and θ are compactified (i.e., periodic). As a result, ϕ and θ, or precisely, their values on the code space, can be treated as the angular coordinates for an encoded qubit, whose state can be expressed as

$$\rho \propto \mathbb{1} + \vec{n}\cdot\vec{\sigma}, \tag{3}$$

with Bloch vector $\vec{n} = (\sin\phi\cos\theta, \sin\phi\sin\theta, \cos\phi)|\vec{n}|$, and Pauli vector $\vec{\sigma} = (\sigma^x, \sigma^y, \sigma^z)$. The norm $|\vec{n}| \in (0, 1]$ and stays

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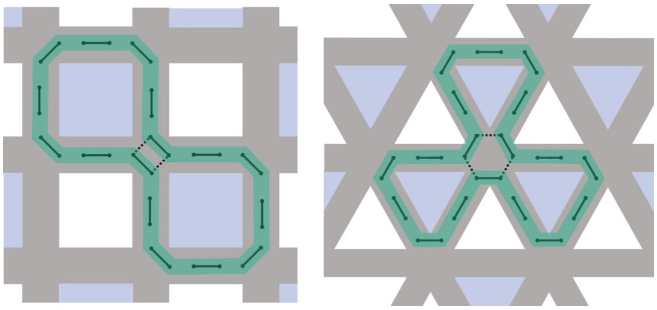

FIG. 1. Networks of sine-Gordon qubits. 1D systems (gray lines) are connected in networks forming square lattice (left) or triangular lattice (right). Each qubit is formed by edges of shaded plaquette. Unshaded plaquettes do not encode qubits. Single qubit gates are from operations on edges and plaquette, while entangling gates also involve glue operations on vertices. The (green) loops represent glued configurations of multiple qubits when the quantum switch at a shared corner converts the singlets from the initial ones (dashed) to the final ones (solid).

the same under unitary transformation. A state can be determined by the measurement of Pauli observables in $\vec{\sigma}$, each of which is both unitary and self-adjoint. Furthermore, in the framework of field theory physical observable are the so-called vertex operators [16,17] $e^{ia\phi(x)}$ and $e^{ib\theta(y)}$, $a, b \in \mathbb{R}$, and they satisfy

$$e^{ia\phi(x)}e^{ib\theta(y)} = e^{-iab\Theta(x-y)}e^{ib\theta(y)}e^{ia\phi(x)}, \tag{4}$$

which is a Weyl algebra serving as the physical foundation for logical operators on the qubit.

To proceed further, we recall the encoding of a qubit via dimerized states of spin-$\frac{1}{2}$ Heisenberg model with staggered or second nearest-neighbor (NN) exchange interactions with periodic boundary condition (PBC) [13]. A dimer is also known as a singlet, and a dimerized state is a product of NN singlets. The dimerization is due to breaking of lattice translation by odd number of sites, denoted by T. The ground-state degeneracy (GSD) is two, and a qubit can be encoded. The two primary logical operators are

$$X_L = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Z_L = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{5}$$

known as bit-flip and phase-flip gates, respectively. It is easy to see that the logical bit-flip X_L is T, as the generator of the broken translation symmetry. Physically, X_L can also be viewed as the pump of a spinon excitation along the system, which could be interpreted as a Wilson loop of spinon, and implementable by an adiabatic pumping cycle [28]. The logical phase flip Z_L is the so-called twist operator [29]

$$F = \otimes_{n=1}^{L} e^{i\frac{2\pi}{L}nS_n^z}, \tag{6}$$

which extracts the SPT order of the ground states. The F operator is equivalent to a vertex operator of ϕ [13,30,31], and can also be viewed as the insertion of a flux through the hole encircled by the system. The algebra of X_L and Z_L is due to (4), with T equivalent to a vertex operator of θ.

For universal quantum computing, a universal set of quantum gates are required. For sine-Gordon qubits, the apparent difficulty is that there exists a discord between X_L and Z_L, namely, they are from different mechanism of symmetries, as discussed above. This means there is no easy way to realize the logical Hadamard operator

$$H_L = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \tag{7}$$

which switches X_L and Z_L and generates superposition. We find that this difficulty can be resolved by employing more sophisticated spin systems and the mechanism of duality.

To put duality in the setting of quantum computing, first consider the encoding of a classical bit by the 2D Ising model. Below the critical temperature T_c, there are two subspaces, denoted $\mathcal{C}_0$, $\mathcal{C}_1$, due to the breaking of a global Z_2 symmetry. The bit 0 (1) is encoded as $\mathcal{C}_0$ ($\mathcal{C}_1$) with total magnetization M up (down), and the logical bit flip X_L is from the broken Z_2 symmetry. This code is known as a repetition code with code-words determined by the majority-vote rule. Furthermore, the code is a subsystem code [32] in the following sense. The whole space $\mathcal{H}$ can be decomposed as

$$\mathcal{H} \cong \mathcal{C}_0 \oplus \mathcal{C}_1 \cong \mathbb{C}^2 \otimes \mathcal{G}. \tag{8}$$

The code space is $\mathbb{C}^2$ and the rest is a so-called gauge space $\mathcal{G}$. As the dimension of $\mathcal{H}$ is even, states in $\mathcal{C}_0$ and $\mathcal{C}_1$ are one-to-one correspondent, leading to the code space $\mathbb{C}^2$ encoding the sign of M. Local thermal noises that do not flip the sign of M are described by $\mathcal{G}$.

Now, if we treat it as a qubit and consider the superposition $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$, M is basically zero. This indicates that $|\pm\rangle$ have different order from $|0, 1\rangle$. To be more concrete, consider the quantum version of the 2D Ising model

$$H = -\sum_n \sigma_n^x - \lambda \sum_n \sigma_n^z \sigma_{n+1}^z. \tag{9}$$

The critical point is $\lambda_c = 1$, and large (small) λ corresponds to the low- (high-)temperature phase. A notable feature is the duality [33–35] defined by $\tilde{\sigma}_n^x = \sigma_n^z\sigma_{n+1}^z$, $\tilde{\sigma}_n^z = \prod_{m<n}\sigma_m^x$ on the dual lattice, from which $H = -\lambda\sum_n \tilde{\sigma}_n^x - \sum_n \tilde{\sigma}_n^z\tilde{\sigma}_{n+1}^z$. The high-temperature phase has zero order $\sum_n \sigma_n^z$ while a nonzero disorder $\sum_n \tilde{\sigma}_n^z$, and the opposite for the low-temperature phase [36].

In fact, the duality serves as the logical Hadamard gate H_L that switches between the order and disorder. Namely, H_L : $X_L \leftrightarrow Z_L$ for $Z_L = \sigma_n^z$ and $X_L = \otimes_n \sigma_n^x$. This means the space of the high-temperature phase also divides into two parts, $\mathcal{C}_+$ and $\mathcal{C}_-$ for the even and odd parity of X_L. Note the parity can be viewed as the number of local states $|-\rangle$ in a configuration as a product of local $|+\rangle$ and $|-\rangle$ states, assuming the system size is odd without loss of generality. Suppose that there is no noise term σ_n^z on an odd number of sites and λ can be tuned properly, then the exchange of the two phases implements H_L. However, a phase flip σ_n^z on a local site n can be easily induced in practice, which is the key reason for it being only a good classical bit.

III. LOGICAL QUBIT GATES

Now we study models that can be described by the sine-Gordon field theory and provide qubit with robust logical X_L and Z_L, and H_L from duality. This would surpass the

TABLE I. Encoding of qubit in the phases of the two-leg spin-$\frac{1}{2}$ ladder. Logical states $|0, 1\rangle$ are encoded in the columnar (C) dimer phase, and $|\pm\rangle$ in the Rung-singlet (R) phase. The staggered (S) dimer phase and Haldane (H) phase form another equivalent code space. The C and S phases are specified by fields ϕ_+ and ϕ_-, and R and H phases by fields ϕ_+ and θ_-.

(ϕ_+, ϕ_-)		(ϕ_+, θ_-)	
$\|0, 1\rangle$	C	$\|\pm\rangle$	R
	S		H

encodings via Ising model or spin chains discussed above, and integrate SPT order with duality for better encoding of qubits. We find the two-leg spin-$\frac{1}{2}$ ladder is a system with duality property, as a natural extension of the single-leg case. With the new fields

$$\phi_\pm = \frac{1}{\sqrt{2}}(\phi_1 \pm \phi_2), \quad \theta_\pm = \frac{1}{\sqrt{2}}(\theta_1 \pm \theta_2), \tag{10}$$

and 1,2 labeling the two legs, we employ the Hamiltonian

$$H = H_0 + V(\phi_+) + V(\phi_-) + V(\theta_-) \tag{11}$$

for the free part of the ladder H_0, the symmetric part $V(\phi_+) = g_1 \int dx \cos 2\sqrt{\pi}\phi_+$, and antisymmetric parts $V(\phi_-) = g_2 \int dx \cos 2\sqrt{\pi}\phi_-$, $V(\theta_-) = g_3 \int dx \cos 2\sqrt{\pi}\theta_-$. Terms with θ_+ are forbidden by the global symmetry. Coupling constants g_i are functions of original exchange strengths, and the phase diagram is well established [25,37–42]. There are four gapped phases, as summarized in Table I. Columnar (C) dimer phase: Its ground states are dimerized with aligned dimers, and the GSD is two. Staggered (S) dimer phase: Its ground states are dimerized with staggered dimers, and the GSD is two. Haldane (H) phase: The ground state is unique, and each two aligned spins form an effective triplet, and there is a singlet between each two triplets. Rung-singlet (R) phase: The ground state is unique, and each two aligned spins form a singlet.

For the phases C and S, the two spin chains are decoupled, while for phases H and R, the two spin chains are coupled, and in fact, entangled. The phases C and S each have definite values of ϕ_+ and ϕ_-, and phases H and R each have definite values of ϕ_+ and θ_-. Viewed as a continuous-variable system [43] analog with harmonic oscillator or photonic states, low-energy states of phases H and R each are entangled with respect to $\phi_{1,2}$ and $\theta_{1,2}$. This motivates the encoding of a qubit as follows. The state space of phase C is divided into two parts $\mathcal{C}_0$ and $\mathcal{C}_1$ due to dimerization, and the codeword $|0\rangle := \mathcal{C}_0$ ($|1\rangle := \mathcal{C}_1$) with positive (negative) values of $\cos\sqrt{2\pi}\phi_-$. The codewords $|+\rangle := \mathcal{C}_+$ ($|-\rangle := \mathcal{C}_-$) with positive (negative) values of $\cos 2\sqrt{\pi}\theta_-$ of phase R.

Note that a field theory describes low-energy, including low-lying excitations, universal features of the spin system. To characterize the logic of quantum computing, it is a great advantage of using field theories since quantum information is encoded in the universal features of phases. The total space of a spin system is decomposed as

$$\mathcal{H} \cong \mathcal{C} \oplus \mathcal{C}^\perp, \quad \mathcal{C} \cong \mathcal{C}_0 \oplus \mathcal{C}_1 \cong \mathbb{C}^2 \otimes \mathcal{G}. \tag{12}$$

The space $\mathcal{C}^\perp$ represents the high-energy part that cannot be well described by sine-Gordon field theory, which plays a trivial role for the Ising case (8). The code space $\mathbb{C}^2$ is due to dimerization, and it also has symmetry-protected topological order that is absent for the Ising model. The gauge space $\mathcal{G}$ can be interpreted as the part for soliton excitations (and their bound states) that will not make a logical error.

The encoding via the ladder system can be viewed as a repetition code concatenated with an underlying code by a single chain. In the language of stabilizer code [32], which describes codes that are stabilized by a set of commuting operators, the codewords $|0\rangle$ and $|1\rangle$ are stabilized by Z_1Z_2 while Z_L is Z_1 or Z_2, and $|\pm\rangle$ are stabilized by Z_1Z_2 and $X_L = X_1X_2$. In the spin language, and let ϕ be ϕ_1 or ϕ_2, the logical operators are

$$X_L := e^{i2\sqrt{\pi}\theta_-}, \quad Z_L := e^{i\sqrt{2\pi}\phi}. \tag{13}$$

The logical bit flip X_L is a pump operation of spinons, and the logical phase flip Z_L is a twist or flux operation and can be realized by inserting electrical fields.

The logical Hadamard gate H_L is played by the duality mapping between X_L and Z_L, which can be realized by slowly tuning parameters, e.g., g_2 and g_3, in the Hamiltonian (11) as a unitary process shuffling between phases C and R. For large but finite system sizes, the shuffle operation can be engineered to be unitary in principle, and it will map between the corresponding eigenspaces of X_L and Z_L. In the thermodynamic limit, the gap closing during the shuffle labels the change of order parameters (see Table I), i.e., the second-order phase transition between phases C and R. The phase transition could jeopardize the exact reversibility of the shuffle operation in practice. However, as long as thermal noises do not lead to logical errors X_L or Z_L, the shuffle realizes the unitary gate H_L on the logical level. Also as the encoding is via low-lying subspaces instead of merely ground states, the system does not have to be maintained on ground states.

Another common method to realize gates is by gate teleportation [18]. For the Hadamard gate H_L, it requires an entangling gate and projective measurement. With the CZ gate, a scheme for which is explained below, the H_L can be realized as

$$\langle m|_S H_S CZ |\psi\rangle_S |+\rangle_A = X^m H |\psi\rangle_S, \quad m = 0, 1, \tag{14}$$

given an arbitrary qubit state $|\psi\rangle_S$, and a qubit ancilla prepared on state $|+\rangle_A$, and the projective measurement on X basis $\langle m|_S H_S$ on the qubit. The byproduct X^m can be corrected given the measurement outcome m, and the output is the state $H|\psi\rangle_S$.

Our encoding is similar but greatly generalizes that for the Ising model. The phases employed here not only support symmetry breaking, but also have symmetry-protected topological order. The global logical operators X_L and Z_L detect the topological order parameters of these phases, and hence, they are not easy to be mimicked by the noisy environment. In addition, phases S and H form another copy of code space, which differs from the original code space of phases C and R by the value of $e^{i2\sqrt{\pi}\phi_+}$, which has definite values for all the phases. The observable $e^{i2\sqrt{\pi}\phi_+}$ flips its sign when phases C and S (or R and H) are exchanged, by, e.g., T on one of the two spin chains, which is a first-order phase transition and can be detected. In practice, to locate a disturbed chain, a slight asymmetry can be introduced for the two legs of the ladder so that the two legs can be distinguished. The correction is then the operation T itself, or the pump of spinon along the disturbed chain. The measurement of $e^{i2\sqrt{\pi}\phi_+}$ also benefits initialization and the entangling gates. Preparation of a logical state can be done by cooling and energy splitting from staggered interaction, for instance. To identify a logical state, Hermitian interchain or intrachain dimer order parameters can be measured.

IV. ENTANGLING SCHEME

For universal quantum computing, entangling gates are required. Next we propose a method to realize the well-known CZ gate and CCZ gate. The CZ (CCZ) gate generates a minus sign when the two (three) qubits are on logical state $|1\rangle$. For convenience, we denote $CZ \equiv \Lambda_2$, $CCZ \equiv \Lambda_3$, and it will be clear that our method can also be employed to realize Λ_n for $n > 3$, which, however, are great challenges for control technique.

In the setting of TQC, our method to realize entangling gates is by the change of topology. This is to glue (or merge) loops of states for 1D systems with PBC. As the states of a qubit can be properly viewed as loops of singlets except a few excitations, states from different qubits can be glued together, which is a topological quantum operation and enables entangling gates. Therefore, qubits can be arranged on 2D lattices with point contact between all NN pairs, see Fig. 1 for the square lattice and triangular lattice. A controllable interaction at a corner, as a quantum switch, glues qubits together conditioned on special states of them. An entangling gate Λ_n is realized by the sequence of glue, a global twist, and then deglue.

We now show the details for the spin ladder system. The model (11) can be realized by two-leg spin-$\frac{1}{2}$ ladder with spin exchange interaction $\vec{S}_i \cdot \vec{S}_j$ for NN (and possible second NN) sites on each chain, each pair of sites on the rungs and along plaquette diagonals [25,37–42]. First, to illustrate the basic mechanism, consider the case when a single chain is used for a qubit. At a corner of a square lattice, there are four spins, for which the five proper singlet configurations are shown in Fig. 2(a), and the two qubits sit at the northwest and southeast plaquette. They represent logical states $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$ of the two qubits, and the last one is a glued state, denoted as $|\Phi\rangle$. Denote the sites of the four spins as u(p), d(own), l(eft), r(ight), an exchange interaction $\vec{S}_u \cdot \vec{S}_l$ as h_{ul}, we employ an antiferromagnetic interaction

$$H = J(h_{ul} + h_{dr}) + J_g(h_{ur} + h_{dl}). \tag{15}$$

The ratio J_g/J will be adiabatically tuned to a big value such that the state $|11\rangle \mapsto |\Phi\rangle$ while others stay the same. Now the glued system can be viewed as a whole system, with states $|00\rangle$ and $|\Phi\rangle$ serving as the two new degenerate dimerized logical states. A global twist similar with (6) on the two qubits will enable

$$|00\rangle \mapsto |00\rangle, |\Phi\rangle \mapsto -|\Phi\rangle. \tag{16}$$

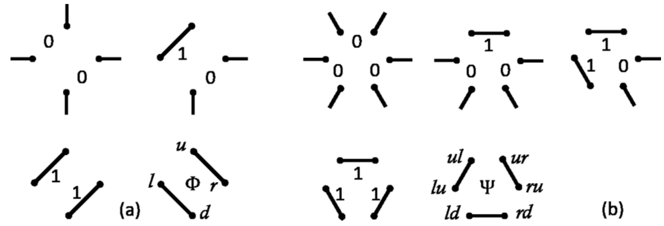

FIG. 2. The singlet configurations at the corner of the 2D square lattice (a) and triangular lattice (b). A spin-$\frac{1}{2}$ is shown as a dot, and a singlet as a bar. The 0 (1) labels logical state $|0\rangle$ ($|1\rangle$) of a qubit. The state $|01\rangle$ is a rotated version of $|10\rangle$ (a), states $|001\rangle$, $|010\rangle$ are rotated versions of $|100\rangle$, and $|011\rangle$, $|011\rangle$ are rotated versions of $|110\rangle$ (b). The glued states are $|\Phi\rangle$ (a) and $|\Psi\rangle$ (b). The corner sites are labeled as u(p), d(own), l(eft), r(ight) (a), ul, ur, lu, ld, ru, and rd (b).

The states $|01\rangle$ and $|10\rangle$ each has two domains and the two spinons at the domain walls forming a singlet at the corner, hence are low-lying excited states above $|00\rangle$ and $|\Phi\rangle$. The global twist will break the corner singlets leading to modified states $|01'\rangle$ and $|10'\rangle$, which will acquire the same dynamical phase $e^{i\delta}$, $\delta = Et$ for their energy E and free evolution time t. After the twist, the deglue operation will drive back to the code space, namely, $|\Phi\rangle \mapsto |11\rangle$, $|01'\rangle \mapsto |01\rangle$, $|10'\rangle \mapsto |10\rangle$, and $|00\rangle$ stays the same. If t is short enough such that $e^{i\delta} \approx 1$, the sequence of glue-twist-deglue (GTG) enables the gate Λ_2. Further, it is not hard to see for the spin-$\frac{1}{2}$ ladder, with the two legs arranged along the third dimension, i.e., vertically, and eight spins at a corner, the same mechanism works leading to the gate Λ_2.

The GTG scheme can be applied to the triangular lattice to implement the gate Λ_3, where there are twelve spins at a corner arranged as two diamonds overlapped vertically. For each six spins, labeled as ul, ur, lu, ld, ru, rd, we employ the interaction

$$H = J(h_{ul,ur} + h_{lu,ld} + h_{ru,rd}) + J_g(h_{ul,lu} + h_{ur,ru} + h_{ld,rd}). \tag{17}$$

Now there are nine proper singlet configurations, shown in Fig. 2(b). The glue interaction (17) will map between states $|111\rangle$ and $|\Psi\rangle$ by tuning the value J_g/J. The π phase shift on $|\Psi\rangle$ is induced by the global twist on the three qubits. During the GTG operation, state $|000\rangle$ stays the same, while $|100\rangle$, $|010\rangle$, and $|001\rangle$ obtain the same phase $e^{i\delta_1}$, $|110\rangle$, $|101\rangle$, and $|011\rangle$ obtain the same phase $e^{i\delta_2}$, both of which can be made trivial by reducing the time of free evolution. Overall, by the GTG operation the gate Λ_3 can be realized. It is also clear that the gate Λ_2 can be realized on this lattice. As the result, this system supports the universal gate set $\{H_L, \Lambda_3\}$ [44], hence can be used for universal quantum computation.

V. DISCUSSION AND CONCLUSION

Our study serves as a constructive proof of the universality (and scalability) of sine-Gordon qubits, demonstrating the power of (quasi-)1D quantum systems for quantum computation. To realize this, there are great practical challenges. As a qubit is encoded in the two-leg ladder system, the bit-flip and phase-flip operations may require controllability of any single leg. Although bit-flip and phase-flip gates can be realized by global operations, the current proposal of Hadamard gate and entangling gates rely on tunability of interaction terms. The shuffle between phases for the Hadamard gate may only be realized approximately due to noises. The teleportation scheme avoids this subtlety, yet it requires the entanglement with an ancilla and projective measurement. The entangling gates Λ_n require precise timing and local addressability (at the corner). A global scheme to realize entangling gates would be appealing, which remains as an interesting open question.

To summarize, a scalable topological quantum computing scheme based on spin ladder, and sine-Gordon qubits in general, is proposed. The computation will be robust against a certain perturbation of control parameters as qubits are encoded into phases instead of states. The lifetime of a qubit, although topological, will be affected by various control processes. Our scheme reveals a novel relation between quantum computing and phase transition. Our method can also be extended to multileg or high-spin ladders and other relevant systems.

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