Rapid Communications

Towards experimental observation of parametrically squeezed states of microwave magnons in yttrium iron garnet films

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We demonstrate theoretically and confirm experimentally that nonlinear spin waves excited in thin yttrium iron garnet films are good candidates for squeezing vacuum quantum noise. The experimental demonstration is in the form of a measurement of spin-wave induced modulation instability (IMI) conducted in the classical regime. The experiment evidences strong phase locking of an idler wave parametrically generated in the film with a deterministic small-signal wave launched into the film from an external source. The theory predicts that the same behavior will be observed for vacuum quantum noise, resulting in squeezing of the noise.

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Parametrically squeezed states of quantized fields [1,2] are of fundamental importance for the theory of quantum measurements [3,4] and have found applications in gravitational-wave detection, atom interferometers [1,5], and in true random number generators [6]. They are also crucial for quantum computing, more precisely for continuous-variable quantum computing (CVC) [7,8]. For instance, very recently, a proposition to employ CVC to build a quantum neural network has been put forward [9]. All these very important fields rely heavily on the quantum optics implementation of the quantum field theory.

The demonstration of the parametric squeezing in optics dates back to the 1980s [10,11]. Both three- [10] and four- [11] photon parametric processes were employed to generate squeezed visible light. The four-wave parametric squeezing of the vacuum noise [11] is a quantum counterpart of the classical effect of the spontaneous modulation instability (SMI). SMI represents parametric amplification of noise in a nonlinear medium resulting in amplitude modulation of the pump wave. In fiber optics, SMI was observed in Ref. [12].

Coherent photon states can also couple parametrically to a quasiclassical pump wave. The same phase relationships are valid for the parametric amplification of the vacuum noise and the complex mode amplitude of a coherent state. The classical counterpart of the four-photon parametric amplification of the complex mode amplitude is the induced modulation instability (IMI). IMI represents parametric coupling of a classical deterministic signal to a pump wave. The coupling leads to an exponential increase in the signal amplitude with time and distance of the signal wave propagation. Experimentally this is seen as an increase in the amplitude of the signal from the output of the experimental device in the presence of the pump wave.

In fiber optics, IMI was observed in [13]. As pointed out in [14], the evolution equations for creation and annihilation operators for four-photon parametric gain are the same as for amplitudes of classical waves. This implies that the same phase relationships must remain valid for IMI. Therefore, observation of those relationships for classical waves may be considered a prerequisite for a medium's ability to squeeze quantum noise. Due to the deterministic nature of all signals involved in IMI, technically, it is easier to observe phase relationships for IMI than for its "sister effect"—SMI.

The three- and four-wave parametric processes also take place in ferromagnets, but at microwave (MW) frequencies. They appear as interactions among quanta of spin waves (SWs), which are called magnons. The general theory of the four-wave processes in *bulk* (unbounded) magnetic media was developed long ago [15], but details relevant to the present work were not considered. Later on, nonlinear SWs were widely studied in thin ferromagnetic films [16–18]. In particular, SMI of SWs in films of yttrium iron garnet (YIG) has been reported [19]. Importantly, SWs in YIG films have much lower nonlinearity thresholds than light in the conventional optical fiber [20].

Experimentally, squeezed magnons were observed in antiferromagnets at terahertz frequencies [21]. Squeezing (mostly parametric) of microwave magnons in ferro- and ferrimagnets has been discussed in the literature, but only theoretically [22–25]. References [22,24] considered three- and four-magnon-based squeezing, respectively. Both works were carried out for a *bulk* ferro-/ferrimagnet. The works revealed that the squeezed states may exist in the medium for short periods of time. Contrary to Refs. [22–24], we consider a different ferrimagnetic medium. These are the YIG films. A unique practical advantage of the film geometry is the easiness of excitation and detection of SWs. Furthermore, the experimental observation of the spin-wave SMI was made using this particular medium and this particular type of ferrimagnet. This suggests that from all possible geometries, the thin films may represent the best candidates for the experimental investigation of the squeezed magnon states in ferro-/ferrimagnetic materials. Furthermore, single-crystal YIG films with thicknesses in the micrometer range grown with liquid-phase epitaxy are characterized by record-low magnetic losses and a very low

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SW nonlinearity threshold. In terms of this combination of parameters, they outperform by far any other known ferri-/ferromagnetic material.

The aim of this Rapid Communication is twofold. Firstly, we carry out a theoretical analysis of the four-magnon processes in *YIG films* demonstrating that parametrically coupled magnon states are squeezed. Secondly, we carry out observation of the SW IMI in a YIG film and study phase dependencies between the involved classical waves. We emphasize that we focus on the phase dependencies for the SW amplitude quadratures. They are central to the effect of squeezing, but have not been experimentally studied for the case of parametric four-magnon interactions. Our experiment confirms that thin YIG films represent good candidates for future observation of the squeezed quantum states [26].

The goal of the theoretical analysis below is to derive expressions for the quadratures of quantum noise starting from a suitable classical model for the spin-wave IMI in YIG films. Previously (see, e.g., [15,16,27], and references therein), it was shown that four-wave self-modulation processes in YIG, such as the formation of SW envelope solitons, are well described by the nonlinear Schrödinger (NLS) equation model. For SWs this equation takes the form

$$i\left(\frac{\partial}{\partial t}+V_g\frac{\partial}{\partial z}+\gamma\right)c+\frac{D}{2}\frac{\partial^2 c}{\partial z^2}-T|c|^2c=0, \tag{1}$$

where c is the scalar amplitude (envelope) of the wave, $V_g = \partial\omega(k)/\partial k$ is the wave's group velocity, T is the nonlinear four-wave (self-modulation) coefficient, γ is the SW damping coefficient, and $D = \partial^2\omega(k)/\partial k^2$ is the "dispersion coefficient." It represents the curvature of the SW dispersion curve $\omega(k)$, where ω is the SW frequency and k is the SW wave number. The coefficients are calculated for a point $\omega(k_0)$ in the SW dispersion curve. The total SW field (envelope+MW carrier) reads $c(t,z)\exp[i\omega(k_0)t + ik_0z)]$.

Bright envelope solitons are formed in a medium, provided $DT < 0$. As shown in [28], D and T depend on the film magnetic parameters and the direction of film magnetization. For instance, for a perpendicularly magnetized film, T is given by the product of the gyromagnetic ratio and the saturation magnetization of the film and is positive. For YIG $T = 3\times 10^{10}\,\mathrm{s}^{-1}$. In the same conditions, D is negative and hence $DT < 0$.

The same criterion, $DT < 0$, is valid for the existence of SMI and IMI [29]. Furthermore, experimental investigations of SW solitons in YIG films are in good quantitative agreement with the model (1) [30]. Therefore, we may start our theoretical analysis with Eq. (1).

The solution of the NLS equation, describing SMI/IMI, is $c = c_0 + c_1\exp(i\Omega t - i\kappa z) + c_2\exp(-i\Omega t + i\kappa z)$. This ansatz is a combination of an intense continuous-wave (cw) pump wave with an amplitude c_0 [and carrier frequency $\omega(k_0)$ and wave number k_0], and two small-signal cw waves with amplitudes c_1 and c_2 [$|c_1|, |c_2| \ll |c_0|$] shifted in frequency by $-\Omega$ and $+\Omega$, respectively, from the central frequency $\omega(k_0)$. The wave numbers of these two waves are shifted by $\pm\kappa$ from the carrier wave number k_0. Substituting the ansatz into (1) and linearizing the result yields

$$\begin{aligned} c_1(z) &= \mu_1(z)c_1(z=0)+\nu_1\bar{c}_2(z=0),\\ \bar{c}_2(z) &= \bar{\mu}_2(z)\bar{c}_2(z=0)+\bar{\nu}_2c_1(z=0), \end{aligned} \tag{2}$$

where $z = 0$ corresponds to the location where the three waves are launched into the film. This solution implies that the amplitudes of the small-signal waves scale linearly with the initial wave amplitudes $c_\alpha(z=0)$ (where $\alpha = 1, 2$). In the presence of magnetic losses in the film, there is no analytical solution to Eq. (1), and the coefficients $\mu_\alpha(z)$ and $\nu_\alpha(z)$ have to be found numerically. Note the presence of the nonvanishing cross-coupling coefficients $\nu_\alpha(z)$ in Eq. (2). It evidences phase synchronism between c_1 and c_2.

The synchronism establishes as the two waves propagate along the film (i.e., in the $+z$ direction) and become parametrically amplified in the course of the propagation, due to their coupling to the pump wave. We now consider the beat signal of the two small-signal waves:

$$\begin{aligned} A &= c_1\exp\{i[\omega(k_0)+\Omega]t - i[k_0+\kappa]z\}\\ &\quad + c_2\exp\{i[\omega(k_0)-\Omega]t - i[k_0-\kappa]z\}. \end{aligned} \tag{3}$$

We quantize Eqs. (3) and (2) and introduce its quadratures, $X = Ae^{i\varphi} + A^\dagger e^{-i\varphi}$, $Y = i(Ae^{i\varphi} - A^\dagger \exp^{-i\varphi})$, where φ is some phase. Note that the loss term is present in Eq. (1). Therefore, our quantization is analogous to solving a quantum Langevin equation for a medium with losses [Eq. (2) in [31]]. We also assume a vacuum state $|0\rangle$ at the input of the film $[c_1(0)|0_1\rangle = 0, c_2(0)|0_2\rangle = 0]$. This allows us to obtain fluctuations of the quadratures of the envelope signal at the frequency Ω [32]:

$$\begin{aligned} \langle 0|X^2, Y^2|0\rangle &= |\mu_1|^2+|\mu_2|^2+|\nu_1|^2+|\nu_2|^2\\ &\quad \pm 2|\nu_1||\mu_2|\cos(\Theta_1)\pm 2|\mu_1||\nu_2|\cos(\Theta_2), \end{aligned} \tag{4}$$

where

$$\Theta_{1(2)} = \arg(\mu_{2(1)}) + \arg(\nu_{1(2)}) - 2(\varphi + k_0z), \tag{5}$$

and the upper and lower signs correspond to the fluctuations of X and Y, respectively. Here we introduced shorthand notations $\mu_i = \mu_i(z)$ and $\nu_i = \nu_i(z)$, and $\arg(\ldots)$ denotes the phase angle of the respective complex-valued quantity.

The central result of this theory is shown in Fig. 1. This is an example calculation using Eq. (4), with μ_i and ν_i obtained from a numerical solution of Eq. (1) for a set of parameters used in the experiment described below: $D = -2\times 10^{-3}\,\mathrm{cm}^2/\mathrm{s}$, $T = 3\times 10^{10}\,\mathrm{s}^{-1}$, $V_g = 4.4\times 10^6\,\mathrm{s}^{-1}$, $\gamma = 7.0\times 10^6\,\mathrm{cm/s}$, and $\Omega/2\pi = 25\,\mathrm{MHz}$. The parameters were calculated based on the magnetic parameters of the film, microwave frequency, and magnetic field (applied perpendicular to the film plane) employed in the experiment. One sees that the quantum noise of one of the quadratures can be reduced to zero for some values of φ. Importantly, the fluctuations of the second quadrature are maximized for the same φ. This shows that the magnon quantum noise in a YIG film is squeezed. This is the main theoretical finding of this work.

The squeezing is due to the process of four-magnon coupling of the three waves. Following [32], in the absence of losses in the material, $\mu_2(z) = \mu_1(z) \equiv \mu(z)$, $\nu_2(z) = \nu_1(z) = \nu(z)$, and, accordingly, $\Theta_2 = \Theta_1 \equiv \Theta$. Then, from Eq. (2) one

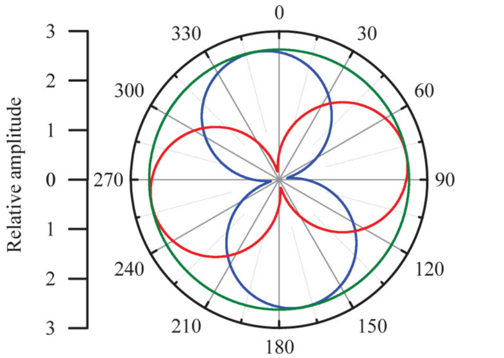

FIG. 1. Quadratures of the noise from the output of the film as a function of the phase angle φ. Red line: expectation value for fluctuations of the X quadrature ($\sqrt{\langle 0|X^2|0\rangle}$). Blue line: one for the Y quadrature ($\sqrt{\langle 0|Y^2|0\rangle}$). Green line: the sum of the expectation values ($\sqrt{\langle 0|X^2+Y^2|0\rangle}$).

finds that the phase angle dependence of the fluctuations is given by

$$\sqrt{\langle 0|X,Y|0\rangle} = \sqrt{\text{const}_1 \pm \text{const}_2 \, \cos\left[\Theta(\varphi)\right]} \quad (6)$$

[where $\Theta(\varphi)$ is given by Eq. (5)], and the fluctuations minimize for one quadrature and simultaneously maximize for the other one for $\Theta = \pi n$. As a result, the ratio $\langle 0|X|0\rangle/\langle 0|Y|0\rangle$ is not equal to one. Thus, the noise becomes squeezed. The same behavior is seen in Fig. 1, although losses are present in our model. Physically, the periodic dependence of the quadrature fluctuations [Eq. (6) and Fig. 1]) takes place because one quadrature of the input vacuum noise is deamplified in some 180°-wide range of the phase angle φ. This leads to suppression of the respective output noise quadrature. For the remainder of the phase angles, fluctuations of this quadrature are amplified. The amplification/deamplification is due to the phase synchronism of the three involved waves [see Eqs. (5) and (6)]. This demonstrates the central role of $\Theta(\varphi)$, Eq. (5), in the formation of the parametrically squeezed quantum states.

Now recall that Eqs. (2) have the same form for both creation-annihilation operators for magnons and classical SW amplitudes. This implies that if a small-signal classical wave is launched into the medium, the quadratures of the output signal will be functions of φ of the same form Eqs. (5), (6). This is confirmed by our simulations and will be shown analytically closer to the end of the Rapid Communication. Therefore, observation of a dependence of the same shape as in Fig. 1, for the quadratures of the output classical signal of IMI should represent an experimental evidence that the parametrically amplified/deamplified magnon quantum states are likely to be squeezed.

Quantum-optical experiments can be carried out at room temperature, because of the high energies of optical photons. This is not true for MW magnons—to exclude the effect of thermal noise, experiments with YIG have to be carried at millikelvin temperatures [33–36]. However, a study of IMI of classical SWs can be carried out at 300 K.

To observe IMI, we use a MW circuit, shown in Fig. 2. Its main parts are a 5.7-μm-thick YIG film grown with the liquid-phase epitaxy on top of a gadolinium gallium garnet

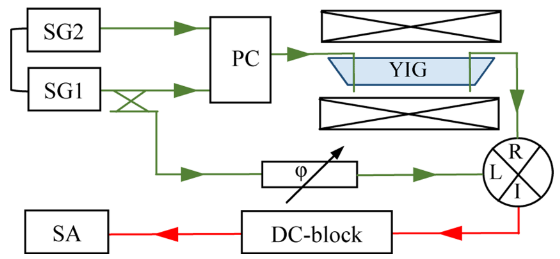

FIG. 2. Diagram of the experimental setup. SG1 and SG2 are MW generators generating signals at ω_0 and $\omega_0+\Omega$, respectively, YIG is the YIG film, φ is a variable phase shifter, and $L|R|I$ is a microwave balanced mixer ("L", "R," and "I" denote its local-oscillator, radio-frequency, and intermediate-frequency ports, respectively). SA is a spectrum analyzer, and PC is a power combiner.

(GGG) substrate and a MW balanced mixer. The distance of SW propagation in the film is 3.8 mm. The film is magnetized perpendicular to its plane by a field of 4338 Oe. The ferromagnetic resonance linewidth for the film measured with the method from [37] is 0.4 Oe. This translates into $\gamma = 7.0\times10^6$ cm/s mentioned above.

A small-amplitude signal, of frequency 7915 MHz and power of +5 dBm, is applied to one input port of the power combiner (PC) from the MW generator SG2 (Fig. 2). The pump signal is fed into the second PC port from SG1. It has a frequency of 7889 MHz and power of +25 dBm. The generators are *phase locked*. Thus, at the film input ($z=0$), we have a combination of a weak signal wave (S), with c_1 and at $\omega_0+\Omega = 7915$ MHz, and an intense pump wave (P), with c_0 and at $\omega_0 = 7889$ MHz ([$c_0(z=0) \gg c_1(z=0)$]. Spectra of those signals are shown in Fig. 3(a).

A variable phase shifter controls the phase shift φ between P and the signal fed into the local-oscillator port (L) of the mixer. The mixer-based receiver part of the circuit represents a MW analog of the optical homodyne detector widely used in quantum optics to observe the quadratures of single photons [11]. Therefore, it is able to register the quadratures of the output signal of the film. The measurements show that the signal from the intermediate-frequency port (I) of the mixer has two frequency components: a dc one (not shown) and one at Ω (26 MHz) [Fig. 3(b)]. We register the φ dependence of the amplitude of the component at Ω with a MW spectrum analyzer. The measurement yields the dependence shown in Fig. 3(c). The displayed quantity actually is one quadrature of the output signal of the film. The plot has the same shape as the one from Fig. 1. This is the main experimental finding of this work.

The physics behind the shape of the plot from Fig. 3(c) is as follows. Four-magnon parametric coupling of S to P generates an idler wave (I) at $\omega_0-\Omega = 7863$ MHz and with an amplitude c_2. Visualizing the frequency spectrum of the film output signal [Fig. 3(a)] with a MW spectrum analyzer confirms the presence of I. We expect I to be phase locked to S. Because SG1 and SG2 (Fig. 2) are also phase locked, the phase locking of S to I results in deterministic beat of the two linear harmonic signals at the film output. The form of the beat signal is given by Eqs. (2) and (3) written down for $z = 3.8$ mm and $c_2(z=0)=0$. In our experiment, we adjust the parametric gain (by carefully choosing the pump power and monitoring

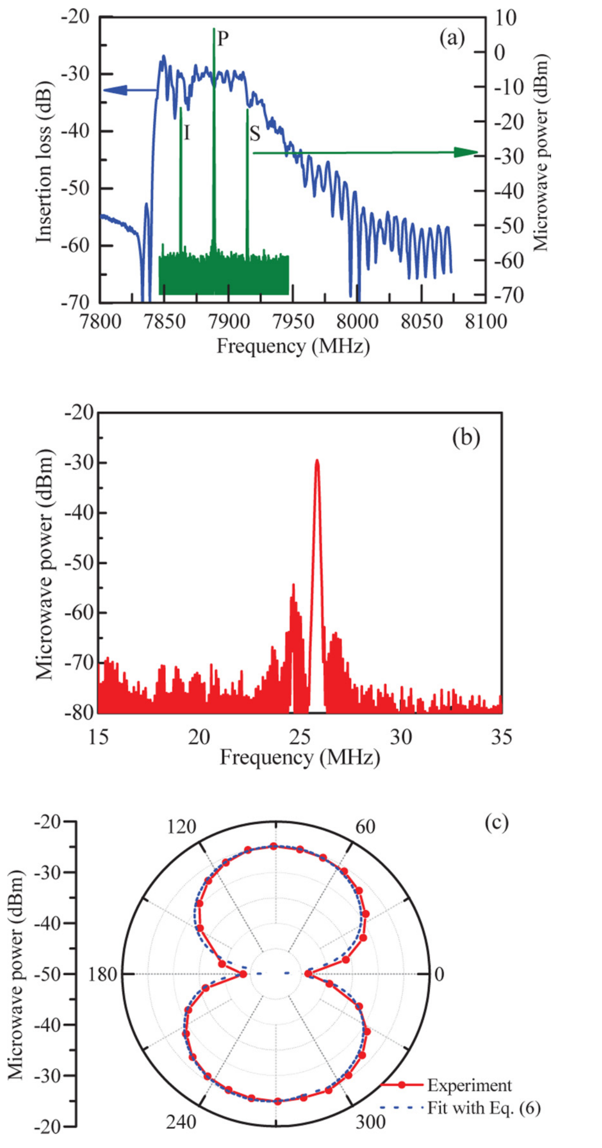

FIG. 3. (a) Blue line: transmission characteristic of the sample taken in the linear regime with a microwave network analyzer. Green line: spectrum of the output signal registered with a microwave spectrum analyzer. From the left to the right, the spectral lines are I, P, and S. (b) Frequency spectrum of the mixer output registered with the spectrum analyzer. (c) Dots and solid line: measured φ-angle dependence of the output of the balanced mixer at the frequency Ω (26 MHz). Dashed line: fit with the right-hand side of Eq. (6).

the film output spectrum with the spectrum analyzer) such that $|c_1(z = 3.8\,\text{mm})| = |c_2(z = 3.8\,\text{mm})|$ [Fig. 3(a)]. This implies that $|\mu_1(z = 3.8\,\text{mm})| = |\nu_2(z = 3.8\,\text{mm})|$. Accordingly, the amplitude of the Ω-frequency output of the mixer is given by $X = X_0(\Phi)\cos(\Omega t + \Upsilon)$, where $X_0(\Phi) = 2|c_1(z = 0)||\mu_1||\cos(\Phi/2)|$, $\Phi = \Theta_2$ from Eq. (5), $\Upsilon = [\arg(\mu_1) - \arg(\nu_2)]/2 - \kappa 3.8\text{ mm} + \arg[c_1(z = 0)]$, $\arg[c_1(z = 0)]$ is the initial phase of the pump wave with respect to the initial phase of S, and all quantities are taken at $z = 3.8$ mm. The Y quadrature is obtained by replacing $|\cos(\Phi/2)|$ with $|\sin(\Phi/2)|$ in the formula for X.

One sees that the signal from port I of the mixer is a sine wave of frequency Ω, which is the frequency of the beat of S and I. The amplitude of the beat may be recast in the form as follows: $X_0(\Phi) = 2|c_1(z = 0)||\mu_1|\sqrt{[\cos(\Phi) + 1]/2}$. Thus, $X_0(\Phi)$ scales as $\sqrt{\text{const}_1 \cos(\Phi) + \text{const}_2}$. Given that $\Phi = \Theta_2$ [Eq. (5)], this is the same as the right-hand side of Eq. (6). The IMI signal quadratures are characterized by the same φ dependence as the magnon quantum noise quadratures. This is the sought connection between the formation of a parametrically squeezed magnon state and the phase dependencies for the classical waves involved in the IMI process. The dashed line in Fig. 3(c) represents the fit of the experimental data with the right-hand side of Eq. (6). One sees that the data are well fitted by this function.

In conclusion, we showed theoretically and confirmed experimentally that magnons in YIG films are good candidates for the generation of parametrically squeezed states at MW frequencies. The process of nonlinear four-particle interaction can be employed to this end. A classical signature of the medium's capability to form the squeezed states is IMI of the respective classical waves. We observed IMI of SWs in a YIG film experimentally and measured quadratures of the beat of the signal and idler waves using an original experimental configuration. This measurement delivered strong evidence that the quantum magnon states would be squeezed if the experiment were conducted in the single-magnon regime and at cryogenic temperatures. It paves the way for the future low-temperature quantum-regime experiments and suggests the same mixer-based setup (Fig. 2) as the most appropriate tool for observing squeezing of the magnon quantum noise [38].

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[1] A. I. Lvovsky, arXiv:1401.4118.
[2] R. Slusher and B. Yurke, J. Lightwave Technol. **8**, 466 (1990).
[3] M. Mensky, *Quantum Measurements* (Cambridge University Press, Cambridge, UK, 1992).
[4] A. V. Belinsky, *Quantum Measurements* (Binom, Moscow, 2008) (in Russian).
[5] B. P. Abbott *et al.*, Phys. Rev. Lett. **116**, 061102 (2016).
[6] T. Symul, S. M. Assad, and P. K. Lam, Appl. Phys. Lett. **98**, 231103 (2011).
[7] S. L. Braunstein and P. van Loock, Rev. Mod. Phys. **77**, 513 (2005).
[8] N. Killoran, J. Izaac, N. Quesada, V. Bergholm, M. Amy, and C. Weedbrook, Quantum **3**, 129 (2019).
[9] N. Killoran, T. R. Bromley, J. M. Arrazola, M. Schuld, N. Quesada, and S. Lloyd, arXiv:1806.06871.

[10] L.-A. Wu, M. Xiao, and H. J. Kimble, J. Opt. Soc. Am. B **4**, 1465 (1987).
[11] R. E. Slusher, L. W. Hollberg, B. Yurke, J. C. Mertz, and J. F. Valley, Phys. Rev. Lett. **55**, 2409 (1985).
[12] K. Tai, A. Hasegawa, and A. Tomita, Phys. Rev. Lett. **56**, 135 (1986).
[13] K. Tai, A. Tomita, J. L. Jewell, and A. Hasegawa, Appl. Phys. Lett. **49**, 236 (1986).
[14] H. P. Yuen and J. H. Shapiro, Opt. Lett. **4**, 334 (1979).
[15] V. S. L'vov, *Wave Turbulence under Parametric Excitation* (Springer-Verlag, Berlin, 1994), Sec. 1.5, "Four-Wave Processes".
[16] D. D. Stancil and A. Prabhakar, *Spin Waves: Theory and Applications* (Springer, New York, 2009).
[17] B. A. Kalinikos and A. B. Ustinov, *Nonlinear Spin Waves in Magnetic Films and Structures: Physics and Devices, Solid State Physics Vol. 64* (Academic Press/Elsevier, Inc., Burlington, 2013).
[18] T. Brächer, F. Heussner, P. Pirro, T. Meyer, T. Fischer, M. Geilen, B. Heinz, B. Lägel, A. A. Serga, and B. Hillebrands, Sci. Rep. **6**, 38235 (2016).
[19] B. A. Kalinikos, N. G. Kovshikov, and A. N. Slavin, Tech. Phys. Lett. **10**, 392 (1984).
[20] Y. S. Kivshar and G. Agrawal, *Optical Solitons: From Fibers to Photonic Crystals* (Academic, New York, 2003).
[21] J. Zhao, A.V. Bragas, D. J. Lockwood, and R. Merlin, Phys. Rev. Lett. **93**, 107203 (2004).
[22] Z. Haghshenasfard and M. G Cottam, J. Phys.: Condens. Matter **29**, 045803 (2017).
[23] F. Peng, Physica B **334**, 183 (2003).
[24] J. Wang, Z. Cheng, Y. Ping, J. Wan, and Y. Zhang, Phys. Lett. A **353**, 427 (2006).
[25] A. Kamra and W. Belzig, Phys. Rev. Lett. **116**, 146601 (2016).
[26] Note the difference between the phase relations we study and the ones from [18]. In terms of the formalism of the present work, [18] investigated the $\arg[c_1(z=0)]$ dependence of the *amplitude* of the parametrically amplified signal. We study the φ dependence of its *quadratures*. The $\arg[c_1(z=0)]$ dependence shows how to maximize parametric gain for the specific three-magnon process used in [18]. For IMI, the dependence is a constant function. In either case, it does not tell anything about the medium's ability to form parametrically squeezed states.
[27] B. A. Kalinikos, N. G. Kovshikov, and A. N. Slavin, Sov. Phys. JETP **67**, 303 (1988).
[28] A. K. Zvezdin and A. F. Popkov, Sov. Phys. JETP **57**, 350 (1983).
[29] V. I. Karpman, *Non-Linear Waves in Dispersive Media*, revised ed. (Pergamon, Oxford, 1975). Translated from the Russian by F. F. Cap, International Series of Monographs in Natural Philosophy Vol. 71, MR 0455792.
[30] H. Y. Zhang, P. Kabos, H. Xia, A. Staudinger, P. A. Kolodin, and C. E. Patton, J. Appl. Phys. **84**, 3776 (1998).
[31] M. J. Collett and C. W. Gardiner, Phys. Rev. A **30**, 1386 (1984).
[32] B. Yurke, Phys. Rev. A **29**, 408 (1984).
[33] M. Goryachev, W. G. Farr, D. L. Creedon, Y. H. Fan, M. Kostylev, and M. E. Tobar, Phys. Rev. Appl. **2**, 054002 (2014).
[34] Y. Tabuchi, S. Ishino, T. Ishikawa, R. Yamazaki, K. Usami, and Y. Nakamura, Phys. Rev. Lett. **113**, 083603 (2014).
[35] Y. Tabuchi, S. Ishino, A. Noguchi, T. Ishikawa, R. Yamazaki, K. Usami, and Y. Nakamura, Science **349**, 405 (2015).
[36] R. G. E. Morris, A. F. van Loo, S. Kosen, and A. D. Karenowska, Sci. Rep. **7**, 11511 (2017).
[37] B. A. Kalinikos, N. G. Kovshikov, P. A. Kolodin, and I. P. Panchurin, IEEE Trans. Magn. **53**, 0700202 (2017).
[38] To measure the noise, the generator SG2 and the power combiner PC will not be needed.