PHYSICAL REVIEW B VOLUME 10, NUMBER 2 15 JULY 1974

Theory of surface effect in photoassisted field emission

Amitabha Bagchi
Battelle Memorial Institute, 505 King Avenue, Columbus, Ohio 43201
(Received 12 February 1974)

The influence of the surface on the photoassisted field emission from a metal is studied theoretically for two simple models of the solid. First, we consider the model of a half-space square well for the metal, and solve exactly for the current density after ignoring the Schottky effect. Next, we take into account the effect of the image charge, and calculate the current density in the WKB approximation. Numerical results based on the calculations are compared with the recent experimental data on photoassisted field emission from tungsten. Areas of agreement and discrepancy between the theory and experiment are pointed out and discussed.

I. INTRODUCTION

Field emission[1-4] and photoemission[5-8] have proved extremely useful in recent times for exploring the electronic density of states near the surface of a metal with and without adsorbates. The two methods are combined in photoassisted field emission (also called photofield emission), in which a metal is irradiated with photons of energy $\hbar\omega < \phi$, the target work function, and the excited or "hot" electrons are extracted by applying an external electric field F. The process is shown schematically in Fig. 1. Interest in photofield emission stems from the fact that it may be used to explore the electronic band structure of the target material in a region not normally accessible to photoelectrons. Also, the extreme surface sensitivity of field emission should make the process a powerful tool in chemisorption studies. Several experiments on photofield emission from tungsten samples have been reported recently.[9,10] We wish to explore, in this paper, the role of the surface in photoassisted field emission to see if surface makes an important contribution to the emission process. An electron can absorb a photon only in the presence of a potential center to take up momentum. The large potential variation near the surface of a metal can lead to the absorption of photons by electrons. By "surface effect," we denote the process where the potential variation near the surface is responsible for photon absorption. Such surface effect has been studied previously in connection with ordinary ($\hbar\omega > \phi$) photoemission. Here we investigate the influence of the surface on the current density and the electronic energy distribution in photoassisted field emission.

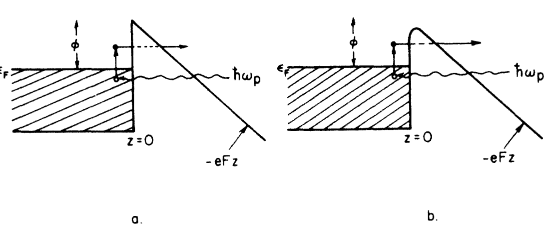

FIG. 1. Schematic representation of the process of photoassisted field emission. The incident photon has an energy $\hbar\omega_p$ which is less than the target work function ϕ. ϵ_F denotes the Fermi energy and F is the applied electric field. (a) Image-charge effect has been ignored. (b) Image-charge effect is included, and this leads to the rounding of the barrier at $z = 0$.

Our study is primarily motivated by the experiment of Lee,[10] who irradiated tungsten close to the (310) plane with photons of two different energies, and measured the total energy distribution (TED) of excited electrons tunneling in the applied field. Certain features of his experimental data are hard to understand in terms of the usual theory of photoassisted field emission.[11] Neumann's theory[11] regards photofield emission as a two-step process whereby an electron is first excited by a photon and subsequently tunnels out in the applied electric field. But the excitation of an electron to a real intermediate state, for a given photon energy, is possible only for electrons of very special initial energies. For photons of energy less than the work function of the target material, the excited electrons have relatively large mean free paths in the solid. Consequently, the crystal momentum k, i.e., electron momentum reduced to the first Brillouin zone, should be conserved for electronic excitation in the bulk. But an examination of the energy band structure of tungsten[12] shows that a direct interband transition, for a given excitation energy $\lesssim 3.5$ eV, can occur only for very specific electron momenta k. The experimental data,[10] on the other hand, indicate a fairly smooth TED for the excited electrons. To a first order, the TED of the excited electrons looks like the TED of the unexcited electrons shifted through the photon energy. The one difference is that, in the TED of the excited electrons, the leading or high-energy edge appears to be much less sharp than in ordinary field emission. It would seem, from the experiment, that electronic transitions not allowed by the k-conservation rule can possible occur in the photoassisted-field-emission process.

This suggests that surface may play an important role in photoassisted field emission, because its presence breaks the translational symmetry of the problem and permits nondirect transitions to take place. The possibility of having non-k-conserving or nondirect transitions in ordinary photoemission has been suggested previously,[13] but such transitions are presumably not needed to explain the experimental data.[14] Photoemission with photons of energy large compared to the work function of the target metal is dominated by the so-called "volume effect" as defined by Mahan.[15] Such processes conserve the electron momentum and give rise to direct transitions. But an additional contribution to the photoemitted current density comes from the surface effect,[15] where a photon is absorbed in the vicinity of the surface because of the large potential variation there. The surface effect in photoemission was calculated by Adawi.[16] It becomes increasingly important near the threshold for photoemission, and recent experiments[17] have confirmed the existence of the process. The present work is concerned with the calculation of a similar effect in photoassisted field emission. We should point out that a qualitative interpretation of his experimental data in terms of the surface photoelectric effect was proposed independently by Lee.[18]

The interesting point about the surface effect is that the momentum normal to the surface is no longer a good quantum number, and it need not be conserved in a transition. There is thus the possibility of certain nondirect transitions occurring through the mediation of the surface. If we assume the surface effect to be the dominant process in photofield emission, we can readily obtain a qualitative understanding of the recent experimental data,[10] which shows no dramatic difference between the TED curves of excited and unexcited electrons. Such an assumption, as we argued before, may not be physically unreasonable in view of the fact that few direct transitions are allowed at low photon energies. Furthermore, simple model calculations are possible for the surface effect in photoassisted field emission to determine its influence on the energy distribution of emitted electrons. A comparison of the theoretical line shapes with the experimental TED curves should indicate how important the role of the surface is in the photofield-emission process. This is the point of view we adopt in this paper.

Following Mahan,[15] we regard photoassisted field emission as a scattering process rather than a two-step process. The theory given in Sec. II can be looked upon formally as a two-step process, but the intermediate state of the electron must then be regarded as virtual. To simplify the problem, we assume that the potential of the solid varies only in the direction normal to the surface, which we take to be the z direction. The general theory of surface-induced photofield emission in this situation is worked out in Sec. II. In Sec. III, we consider the problem of a half-space square-well model for a metal with an applied electric field, and we ignore image-charge effects. We present an exact solution for the transition probability of photoassisted field emission in that case. In Sec. IV we take image charge into consideration, and we solve the problem in the WKB approximation. The current densities for the two models are worked out in Sec. V and compared with the experimental data. A discussion of our conclusions is contained in Sec. VI.

II. GENERAL THEORY

In order to study the surface effect in photoassisted field emission, we choose a simple model which does not allow for photon absorption in the bulk of the target. We consider a free-electron metal which occupies the half-space $-L \leq z \leq 0$, where L is the normalization length and the z axis is the direction normal to the surface. The metal potential with an applied electric field is assumed to depend only on z, and is written

$$V(\vec{r}) = -V_0\Theta(-z) + V_1(z)\Theta(z) , \qquad (2.1)$$

where $\Theta(z)$ is the step function and $V_1(z)$ will be specified later. Electrons are assumed to be noninteracting, and they fill up all states of the metal up to the Fermi energy ϵ_F. Measuring energy with respect to the field-free vacuum level, we have

$$\epsilon_F = \hbar^2 q_F^2/2m - V_0 < 0 , \qquad (2.2)$$

where q_F is the Fermi momentum.

Let us assume that light of frequency ω_p and wave vector $p = \omega_p/c$ is incident on the solid. We assume $\hbar\omega_p < \phi = -\epsilon_F$, the work function, so that direct photoemission is impossible. Our treatment of the electron excitation problem closely parallels that of Adawi.[16] Let us denote the initial unexcited state of an electron of energy $\mathcal{E}$ in the metal by

$$\psi_{\mathcal{E},\vec{K}}(\vec{r}) = e^{i\vec{K}\cdot\vec{\rho}}\phi_E(z)/L^{3/2} , \qquad (2.3)$$

where $\vec{\rho}$ is the radial vector in the xy plane, and $E = \mathcal{E} - \hbar^2K^2/2m$. The momentum $\vec{K}$ parallel to the surface is a good quantum number in this model. The state of (2.3) is allowed for a given $\vec{K}$ if $E > -V_0$. The unperturbed Hamiltonian $\mathcal{H}_0$ of the system can be written as the sum of an electronic part and a radiation part,

$$\mathcal{H}_0 = \mathcal{H}_e + \mathcal{H}_r = -\hbar^2\nabla^2/2m + V(\vec{r}) + \sum_p \hbar\omega_p a_p^\dagger a_p , \qquad (2.4)$$

where a_p ($a_p^\dagger$) denotes the annihilation (creation) operator for a photon of momentum p. The perturbation provided by the photon field is ($e > 0$)

$$\mathcal{H}' = (e/mc)\vec{A}\cdot\vec{p} = (-ie\hbar/mc)\sum_p (2\pi\hbar c/p)^{1/2}\Omega^{-1/2} \times (a_p e^{i\vec{p}\cdot\vec{r}} + \mathrm{H.c})(\hat{\epsilon}_p\cdot\nabla) , \qquad (2.5)$$

where Ω is the volume of normalization, and $\hat{\epsilon}_p$ stands for the polarization vector. Since the potential, by definition, varies only with z, we have to consider only the z component of the gradient operator. Accordingly,

$$\mathcal{H}' = \sum_p \gamma_p (a_p e^{i\vec{p}\cdot\vec{r}} + \mathrm{H.c})\frac{\partial}{\partial z} , \qquad (2.6)$$

with

$$\gamma_p = (-ie\hbar/m)(2\pi\hbar/\omega_p)^{1/2}\Omega^{-1/2}(\hat{\epsilon}_p\circ\hat{z}) , \qquad (2.7)$$

and $\hat{z}$ denotes the unit vector in the z direction.

Suppose that, in the initial state, there are n_p photons each of energy $\hbar\omega_p$, and an electron in the state $\psi_{\mathcal{E},\vec{K}}(\vec{r})$. Since the radiation momentum is much smaller than the electron momentum, we may set $\vec{p} = 0$ in the phase factor of (2.6). The parallel momentum of the electron state is then unaffected by the incident radiation. As a result of one-photon absorption, the electronic wave function, to first order in the perturbation, changes to

$$\psi^{+}_{\mathcal{E},\vec{K}}(\vec{r}) = \psi_{\mathcal{E},\vec{K}}(\vec{r}) + \psi_1(\vec{r}) , \qquad (2.8)$$

where

$$\psi_1(\vec{r}) = \gamma_p n_p^{1/2}\int d^3r'\, \mathcal{G}_{\mathcal{E}+\hbar\omega_p}(\vec{r},\vec{r}\,')\frac{\partial}{\partial z'}\psi_{\mathcal{E},\vec{K}}(\vec{r}\,'). \qquad (2.9)$$

Here $\mathcal{G}_{\mathcal{E}+\hbar\omega_p}(\vec{r},\vec{r}\,')$ is the Green's function for the Hamiltonian $\mathcal{H}_e$ at energy $\mathcal{E}+\hbar\omega_p$. It obeys the usual scattering-theory boundary condition[19] of outgoing waves far from the source of scattering. We may decompose it in the Fourier series

$$\mathcal{G}_{\mathcal{E}+\hbar\omega_p}(\vec{r},\vec{r}\,') = \frac{1}{L^2}\sum_{\vec{K}'} e^{i\vec{K}'\cdot(\vec{\rho}-\vec{\rho}\,')} G_{E+\hbar\omega_p}(z,z') , \qquad (2.10)$$

where $G_{E+\hbar\omega_p}(z,z')$ refers to the Green's function of the one-dimensional problem with the potential of (2.1). Using (2.3) and (2.10) in (2.9), we obtain

$$\psi_1(\vec{r}) = e^{i\vec{K}\cdot\vec{\rho}}\phi_1(z)/L^{3/2} , \qquad (2.11a)$$

where

$$\phi_1 = \gamma_p n_p^{1/2}\int dz'\, G_{E+\hbar\omega_p}(z,z')\frac{\partial}{\partial z'}\phi_E(z') . \qquad (2.11b)$$

For this simple model, then, the theoretical problem reduces essentially to a determination of the one-dimensional Green's function corresponding to the potential of (2.1). The Green's function satisfies the inhomogeneous Schrödinger equation

$$\left(\tilde{E} + \frac{\hbar^2}{2m}\frac{d^2}{dz^2} - V(z)\right)G_{\tilde{E}}(z,z') = \delta(z-z') , \qquad (2.12)$$

where $\tilde{E} = E + \hbar\omega_p$ and $V(z) \equiv V(\vec{r})$. The boundary condition on $G_{\tilde{E}}(z,z')$ is that it must give rise to outgoing waves for $z \to \pm\infty$. The Green's function clearly obeys the homogeneous Schrödinger equation for $z \neq z'$. Let $u(z)$ and $v(z)$ denote two linearly independent solutions of the homogeneous equation satisfying the boundary condition of outward propagating waves for large positive and negative z directions, respectively. Then,

$$G_{\tilde{E}}(z,z') = A(z')u(z), \quad z > z' \qquad (2.13a)$$

$$= B(z')v(z) , \quad z < z' . \qquad (2.13b)$$

The symmetry of the Green's function with respect to an interchange of its arguments means that we can write

$$G_{\tilde{E}}(z,z') = Cu(z_>)\,v(z_<) , \qquad (2.14)$$

where $z_>$ ($z_<$) refers to the greater (lesser) of the pair z, z'. The coefficient C is determined from the boundary condition on the slope implied by the δ function of (2.12):

$$C = (-2m/\hbar^2)[W(u,v)]^{-1} , \qquad (2.15a)$$

where $W(u,v)$ is the Wronskian

$$W(u,v) = u\frac{\partial v}{\partial z} - v\frac{\partial u}{\partial z} , \qquad (2.15b)$$

and it is easily seen to be independent of z.

The use of Eqs. (2.14) and (2.15) in Eqs. (2.11) shows that the correction to the electronic wave function due to one-photon absorption is given in the limit $z \to \infty$ by

$$\psi_1(\vec{r}) \underset{z\ \mathrm{large}}{=} \frac{-2m}{\hbar^2}\gamma_p n_p^{1/2}\frac{e^{i\vec{K}\cdot\vec{\rho}}u(z)}{L^{3/2}W(u,v)} \times\int dz'\, v(z')\frac{\partial}{\partial z'}\phi_E(z') . \qquad (2.16)$$

Since $v(z)$ and $\phi_E(z)$ obey the homogeneous Schrödinger equation in the potential $V(z)$ with energies $E+\hbar\omega_p$ and E, respectively, elementary calculation[15,16] gives

$$\int dz'\, v(z')\frac{\partial}{\partial z'}\phi_E(z') = -(\hbar\omega_p)^{-1} \times\int dz'\, v(z')\frac{\partial V(z')}{\partial z'}\phi_E(z') . \qquad (2.17)$$

The particle current density due to the excited electrons can be obtained from the relation

$$J_z(E,\vec{K}) = \frac{\hbar}{2mi}\left[\psi_1^*(\vec{r})\frac{\partial}{\partial z}\psi_1(\vec{r}) - \left(\frac{\partial}{\partial z}\psi_1^*(\vec{r})\right)\psi_1(\vec{r})\right] . \qquad (2.18)$$

To calculate the total particle current density at an energy $\epsilon > \epsilon_F$, we note that the excited electrons must come from states of initial energy $\mathcal{E} = \epsilon - \hbar\omega_p$, and we must sum over all such states after multiplying the contribution of each by the probability that an electron is present in the state prior to photon absorption. The resultant current density is readily seen to be given by

$$j(\epsilon) = 2 \sum_{\{E,\vec{K}\}} f(\epsilon - \hbar\omega_p) J_z(E,\vec{K}) \times \delta(\epsilon - \hbar\omega_p - \hbar^2 K^2/2m - E) \,. \tag{2.19}$$

Here, $f(\xi)$ is the Fermi occupation function

$$f(\xi) = 1/(e^{\beta(\xi - \epsilon_F)} + 1) \,, \tag{2.20}$$

$\beta = (k_B T)^{-1}$, k_B is Boltzmann's constant, and we have assumed that $\epsilon - \epsilon_F \gg k_B T$ so that no electron is initially present in a state of energy ϵ. The factor of 2 comes from spin. We now turn to the calculation of the matrix elements and the current density for specific models of the surface.

III. HALF-SPACE SQUARE WELL WITHOUT IMAGE CHARGE: EXACT SOLUTION

We assume the model of an electric field F applied to a metal which is described by a half-space square well of depth V_0, and we ignore the image-charge effect. The problem of photoassisted field emission can then be solved analytically. The potential of (2.1) in this case is given by

$$V(\vec{r}) = -V_0 \Theta(-z) - eFz\Theta(z) \,, \tag{3.1}$$

and it is shown in Fig. 1(a). In Eqs. (2.16) and (2.17), we need the functions $u(z)$ and $v(z)$. Since we wish to compute the current for large and positive z, we need $u(z)$ for $z > 0$, where it satisfies the Schrödinger equation in an applied electric field. The solutions of the latter are well known[20] to be the Airy functions Ai and Bi of appropriate arguments. The linear combination representing an expanding wave is

$$u(z) = \mathrm{Ai}(\xi) - i\,\mathrm{Bi}(\xi) \,, \quad z > 0 \,, \tag{3.2}$$

where $\xi = -(2meF/\hbar^2)^{1/3}(z + \tilde{E}/eF)$ and $\tilde{E} = E + \hbar\omega_p$. This function is proportional to the Hankel function of the first kind of order $\frac{1}{3}$, and has the asymptotic expansion[21]

$$u(z) \underset{z\to\infty}{\sim} \pi^{-1/2} e^{-i7\pi/12} (2meF/\hbar^2)^{-1/12} (z + \tilde{E}/eF)^{-1/4} \times \exp[i\tfrac{2}{3}(2meF/\hbar^2)^{1/2}(z + \tilde{E}/eF)^{3/2}] \,. \tag{3.3}$$

For the other, linearly independent, solution $v(z)$, we choose

$$v(z) = e^{-iqz} \,, \qquad z < 0 \tag{3.4a}$$

$$= \alpha\,\mathrm{Ai}(\xi) + \beta\,\mathrm{Bi}(\xi) \,, \quad z > 0 \,, \tag{3.4b}$$

with $q^2 = 2m(E + \hbar\omega_p + V_0)/\hbar^2$. The continuity of the wave function and its derivative at $z = 0$ determines the coefficients α and β as

$$\alpha = \pi[\mathrm{Bi}'(\xi_0) - \eta\,\mathrm{Bi}(\xi_0)] \tag{3.5a}$$

and

$$\beta = -\pi[\mathrm{Ai}'(\xi_0) - \eta\,\mathrm{Ai}(\xi_0)] \,, \tag{3.5b}$$

where $\xi_0 = -(2meF/\hbar^2)^{1/3}\tilde{E}/eF$, $\eta = iq/(2meF/\hbar^2)^{1/3}$, and the prime indicates differentiation with respect to the argument. In deriving this result, we have made use of the relation[22]

$$\mathrm{Ai}(\xi_0)\,\mathrm{Bi}'(\xi_0) - \mathrm{Bi}(\xi_0)\,\mathrm{Ai}'(\xi_0) = \pi^{-1} \,. \tag{3.6}$$

Note that $\xi_0 > 0$ in the geometry of photofield emission. The Wronskian is now easy to evaluate,[22] and is given by

$$W(u, v) = -(2meF/\hbar^2)^{1/3}(\beta + i\alpha)/\pi \,. \tag{3.7}$$

It is now straightforward to calculate the current in photofield emission in the asymptotic region $z \to \infty$. For the potential of (3.1), we have

$$\frac{\partial V}{\partial z} = V_0 \delta(z) - eF\Theta(z) \,. \tag{3.8}$$

Combining Eqs. (3.2)–(3.8) with Eqs. (2.16) and (2.17), we obtain

$$\psi_1(\vec{r}) \underset{z\ \mathrm{large}}{=} \Gamma e^{i\vec{K}\cdot\vec{\rho}} u(z)/L^{3/2} \,, \tag{3.9}$$

where

$$\Gamma = (2m/\hbar^2)(-ie\hbar/m)(2\pi\hbar/\omega_p)^{1/2}(n_p/\Omega)^{1/2}(\hat{\epsilon}_p \cdot \hat{z})(\hbar\omega_p)^{-1} \times \frac{-\pi}{(2meF/\hbar^2)^{1/3}(\beta + i\alpha)} \left(V_0 v(0)\phi_E(0) - eF\int_0^\infty dz'\, v(z')\phi_E(z') \right) . \tag{3.10}$$

Note that Γ is expressed entirely in terms of known functions. The current density arising from electrons initially in the state $\psi_{\mathcal{E},\vec{K}}(\vec{r})$ and excited by the absorption of photons is given by (2.18). The use of (3.9) in this formula gives

$$J_z(E,\vec{K}) \underset{z\to\infty}{=} |\Gamma|^2 L^{-3}\hbar(2meF/\hbar^2)^{1/3}/m\pi \,, \tag{3.11}$$

where the asymptotic form (3.3) of $u(z)$ has been utilized. The total current of excited particles in photoassisted field emission at $T=0$ can be obtained by multiplying (3.11) by the electronic charge $(-e)$ and integrating over the initial distribution of electron states. Letting $q_0=[2m(E+V_0)/\hbar^2]^{1/2}>0$, the result is

$$I=2L^3\int\frac{dq_0}{\pi}\int\frac{d\vec{K}}{(2\pi)^2}(-e)J_z(E,\vec{K}) . \tag{3.12}$$

The factor of 2 comes from sum over spin. The upper limit on q_0 is the Fermi momentum q_F and the lower limit is determined by the minimum energy E_m for which an unoccupied final state exists for electron excitation.

It is instructive to consider limiting cases where our results can be compared directly with the results of other theories. First consider surface effect in ordinary photoemission when $\hbar\omega>\phi$ and $F\to 0$. In this case, $\tilde{E}=E+\hbar\omega_p>0$ for photoemission to occur, and

$$\xi_0=-(2meF/\hbar^2)^{1/3}\tilde{E}/eF , \tag{3.13}$$

is very large and negative as $F\to 0$. The use of the asymptotic formulas for the Airy functions[21] shows that

$$\mathrm{Ai}(\xi_0)-i\,\mathrm{Bi}(\xi_0)\sim\pi^{-1/2}e^{-i7\pi/12}(2meF/\hbar^2)^{-1/12}\times(\tilde{E}/eF)^{-1/4}\exp[i\tfrac{2}{3}(2m\tilde{E}/\hbar^2)^{1/2}\tilde{E}/eF] \tag{3.14a}$$

and

$$\mathrm{Ai}'(\xi_0)-i\mathrm{Bi}'(\xi_0)\sim -i\pi^{-1/2}e^{-i7\pi/12}(2meF/\hbar^2)^{1/12}\times(\tilde{E}/eF)^{1/4}\exp[i\tfrac{2}{3}(2m\tilde{E}/\hbar^2)^{1/2}\tilde{E}/eF] . \tag{3.14b}$$

It follows from Eqs. (3.5) that

$$\beta+i\alpha=-\pi\{[\mathrm{Ai}'(\xi_0)-i\,\mathrm{Bi}'(\xi_0)]-\eta[\mathrm{Ai}(\xi_0)-i\,\mathrm{Bi}(\xi_0)]\} , \tag{3.15}$$

so that

$$(2meF/\hbar^2)^{1/3}(\beta+i\alpha)/(-\pi)\underset{F\to 0}{\sim}\pi^{-1/2}e^{-i7\pi/12}\times\exp[i\tfrac{2}{3}(2m\tilde{E}/\hbar^2)^{1/2}(\tilde{E}/eF)](2meF/\hbar^2)^{-1/12}\times(\tilde{E}/eF)^{-1/4}(-i)(k+q) , \tag{3.16}$$

where $k^2=2m\tilde{E}/\hbar^2$, and we made use of the definition of η. Substituting this in (3.10) and taking the square of the modulus, we obtain

$$|\Gamma|^2\underset{F\to 0}{\sim}(2m/\hbar^2)^2(e^2\hbar^2/m^2)(2\pi\hbar/\omega_p)(n_p/\Omega)\times(\hat{\epsilon}_p\cdot\hat{z})^2(\hbar\omega_p)^{-2}\pi(2meF/\hbar^2)^{1/6}(\tilde{E}/eF)^{1/2}\times(k+q)^{-2}V_0^2|v(0)|^2|\phi_E(0)|^2 . \tag{3.17}$$

It is shown in the appendix that this formula, in conjunction with Eqs. (3.11) and (3.12), leads exactly to Adawi's result.[16]

Let us consider next the case of photoassisted field emission in a relatively weak external field F. It is possible, then, to compare our result with that obtained from the WKB theory of barrier penetration.[23] Since $\tilde{E}=E+\hbar\omega_p<0$ in this case, we may write $\xi_0=\zeta$, where

$$\zeta=-(2meF/\hbar^2)^{1/3}\tilde{E}/eF=(2meF/\hbar^2)^{1/3}|\tilde{E}|/eF \tag{3.18}$$

and $\zeta\gg 1$ if F is weak. The use of asymptotic formulas[21] shows that when $\zeta\gg 1$, $\mathrm{Bi}(\zeta)$, $\mathrm{Bi}'(\zeta)\gg\mathrm{Ai}(\zeta)$, $\mathrm{Ai}'(\zeta)$, and

$$\mathrm{Bi}(\zeta)\underset{\zeta\gg 1}{\sim}(\zeta^{-1/4}/\pi^{1/2})e^{2\zeta^{3/2}/3} , \tag{3.19a}$$

$$\mathrm{Bi}'(\zeta)\underset{\zeta\gg 1}{\sim}(\zeta^{1/4}/\pi^{1/2})e^{2\zeta^{3/2}/3} . \tag{3.19b}$$

From (3.5) and (3.7) we obtain, in this situation,

$$W(u,v)\cong -i(2meF/\hbar^2)^{1/3}[\mathrm{Bi}'(\zeta)-\eta\,\mathrm{Bi}(\zeta)]\underset{\zeta\gg 1}{\sim}-i\pi^{-1/2}e^{2\zeta^{3/2}/3}\times[(2meF/\hbar^2)^{1/3}\zeta^{1/4}-iq\zeta^{-1/4}] . \tag{3.20}$$

Accordingly,

$$[1/W(u,v)]^2\underset{\zeta\gg 1}{\sim}\pi e^{-4\zeta^{3/2}/3}[(2meF/\hbar^2)^{2/3}(2meF/\hbar^2)^{1/6}\times(|\tilde{E}|/eF)^{1/2}+q^2(2meF/\hbar^2)^{-1/6}(|\tilde{E}|/eF)^{-1/2}]^{-1}=\pi e^{-4\zeta^{3/2}/3}(2meF/\hbar^2)^{-1/3}(2m|\tilde{E}|/\hbar^2)^{1/2}/(2mV_0/\hbar^2) . \tag{3.21}$$

Ignoring constant terms, we obtain from (3.10)

$$\Gamma^2\propto[1/W(u,v)]^2|\phi_E(0)|^2 , \tag{3.22}$$

and the use of (3.21) and (3.22) in (3.11) gives

$$J_z(E,\vec{K})\propto\frac{|\tilde{E}|^{1/2}}{V_0}\times\exp\left[-\frac{4}{3}\left(\frac{2meF}{\hbar^2}\right)^{1/2}\left(\frac{|\tilde{E}|}{eF}\right)^{3/2}\right]|\phi_E(0)|^2 . \tag{3.23}$$

In deriving this result, we have assumed that the largest contribution to the matrix element of (2.17) or (3.10) comes from the large potential discontinuity at $z=0$, and that the additional contribution from the weak external field can be neglected. Then $|\phi_E(0)|^2$ can be replaced to first order by its value in the absence of the field, which is given in the appendix as [cf. Eqs. (A4) and (A5)]

$$|\phi_E(0)|^2 \cong 2(E+V_0)/V_0 \,. \tag{3.24}$$

We therefore obtain

$$J_z(E,\vec{K}) \propto (E+V_0)\frac{(-E-\hbar\omega_p)^{1/2}}{V_0} \times \exp\left[-\frac{4}{3}\left(\frac{2m}{\hbar^2}\right)^{1/2}\frac{(-E-\hbar\omega_p)^{3/2}}{eF}\right] \,. \tag{3.25}$$

The above equation can be given a simple physical interpretation. Recalling the Fowler-Nordheim expression for the barrier-penetration probability[24] at the energy $E+\hbar\omega_p$ (<0) in the configuration of Fig. 1(a),

$$D=\frac{4[(E+\hbar\omega_p+V_0)(-E-\hbar\omega_p)]^{1/2}}{V_0} \times \exp\left[-\frac{4}{3}\left(\frac{2m}{\hbar^2}\right)^{1/2}\frac{(-E-\hbar\omega_p)^{3/2}}{eF}\right] \,, \tag{3.26}$$

we may rewrite (3.25) as

$$J_z(E,\vec{K}) \propto (E+V_0)^{1/2}[(E+V_0)/(E+\hbar\omega_p+V_0)]^{1/2}D \,. \tag{3.27}$$

The current density in photoassisted field emission is therefore the product of an incident electron flux [$\propto (E+V_0)^{1/2}$] and the Fowler-Nordheim factor D, multiplied by a kinematical factor $[(E+V_0)/(E+V_0+\hbar\omega_p)]^{1/2}$. The last factor owes its origin to the conservation of tangential momentum in our model. For a given total energy $\mathcal{E}$ of an electron, this factor is the largest for motion in the forward direction when $E\cong\mathcal{E}$ and $\vec{K}\cong 0$. This should make the distribution of excited electrons appear peaked in the forward or tunneling direction, and some experimental evidence exists[10] in support of such an effect. We shall return to Eq. (3.25) when we calculate the current densities for TED curves in Sec. V.

IV. MODEL CALCULATION WITH IMAGE CHARGE: WKB APPROXIMATION

In this section, we include the effect of the image charge in our model and study how the current density in photoassisted field emission is modified as a result of that. The potential of (2.1) is now assumed to be

$$V(z) = -e^2/4z - eFz \,, \quad z>0 \tag{4.1a}$$

$$= -V_0 \,, \quad z<0 \,. \tag{4.1b}$$

In the region near $z=0$, it is assumed that the potential is regular and connects smoothly between the limiting forms of Eq. (4.1a) and (4.1b). The shape of the potential is shown in Fig. 1(b); it has a maximum value of $V_{max} = -(e^3F)^{1/2}$. The same model was studied in connection with field emission by Murphy and Good,[25] who found that their results were insensitive to details of the potential shape close to $z=0$.

The problem of photoassisted field emission can no longer be solved analytically, but WKB approximation[26] is easy to carry out. Let us consider first an electron having an energy E for motion in the z direction such that $E+\hbar\omega_p < V_{max} = -(e^3F)^{1/2}$. The electron can escape from the metal after photon absorption by tunneling through the potential barrier. Let z_1 and z_2 denote the classical turning points in the surface potential barrier at the energy $E+\hbar\omega_p$ with $z_1<z_2$. The independent solutions of the one-dimensional Schrödinger equation at this energy, $u(z)$ and $v(z)$, can be expressed in the following form:

$$u(z)=\frac{A}{[k(z)]^{1/2}}\exp\left(i\int_{z_1}^{z}k(z)\,dz\right) + \frac{B}{[k(z)]^{1/2}}\exp\left(-i\int_{z_1}^{z}k(z)\,dz\right), \quad z<z_1 \tag{4.2a}$$

$$=\frac{1}{[k(z)]^{1/2}}\exp\left(i\int_{z_2}^{z}k(z)\,dz\right), \quad z>z_2 \tag{4.2b}$$

and

$$v(z)=\frac{1}{[k(z)]^{1/2}}\exp\left(-i\int_{z_1}^{z}k(z)\,dz\right), \quad z<z_1 \tag{4.3a}$$

$$=\frac{F}{[k(z)]^{1/2}}\exp\left(i\int_{z_2}^{z}k(z)\,dz\right) + \frac{G}{[k(z)]^{1/2}}\exp\left(-i\int_{z_2}^{z}k(z)\,dz\right) \,, \quad z>z_2 \,, \tag{4.3b}$$

where

$$k(z)=\{2m[\tilde{E}-V(z)]/\hbar^2\}^{1/2} \tag{4.4}$$

and

$$\tilde{E}=E+\hbar\omega_p \,. \tag{4.5}$$

With the help of the connecting formulas across the classical turning points, we obtain[27]

$$A=G=(\Theta+1/4\Theta) \,, \tag{4.6a}$$

$$B=F=-i(\Theta-1/4\Theta) \,, \tag{4.6b}$$

with

$$\Theta=\exp\left(-i\int_{z_1}^{z_2}k(z)\,dz\right)=\exp\left(\int_{z_1}^{z_2}K(z)\,dz\right) \,, \tag{4.7}$$

$$K(z)=\{2m[V(z)-\tilde{E}]/\hbar^2\}^{1/2} \,. \tag{4.8}$$

The Wronskian is now easily evaluated, e.g., in the region $z>z_2$, as

$$W(u,v)=[G/k(z)][-2ik(z)]=-2i(\Theta+1/4\Theta). \tag{4.9}$$

Note that the Wronskian is independent of z.

The correction to the electronic wave function due to photon absorption is given by Eq. (2.16). In the current density $J_z(E, \vec{K})$ of (2.18), the most significant energy dependence comes from the barrier penetration factor[27] which is contained in $W(u, v)$. The energy dependence of the integral in (2.16) is weak, and for the purpose of the numerical calculation of TED of photofield-emitted electrons, we shall omit this energy dependence. We then find

$$J_z(E, \vec{K}) \propto |\gamma_p|^2/[W(u,v)]^2 \cong (\tfrac{1}{4}|\gamma_p|^2)(\Theta^2 + \tfrac{1}{2})^{-1} . \tag{4.10}$$

Since $|\gamma_p|^2$ is proportional to $(\hat{\epsilon}_p \cdot \hat{z})^2$ [cf. Eq. (2.7)], the current density depends on the polarization vector of the incident photon through the factor $(\hat{\epsilon}_p \cdot \hat{z})^2$. This is characteristic of surface photoelectric effect.[16] Thus, a study of the polarization dependence of the current in photoassisted field emission will be a good test to see the importance of the surface effect in the emission process.

For the model that we are considering here, the factor Θ^2 has been evaluated by Murphy and Good.[25] With the definitions (4.1) and (4.8), we have

$$2\int_{z_1}^{z_2} K(z)\,dz = 2\int_{z_1}^{z_2} \left[\left(-\tilde{E} - \frac{e^2}{4z} - eFz\right)\right]^{1/2} dz . \tag{4.11}$$

We define

$$y = (e^3 F)^{1/2}/|\tilde{E}| = -(e^3F)^{1/2}/\tilde{E} . \tag{4.12}$$

If we let $\rho = 2z/(|\tilde{E}|/eF)$, we obtain

$$2\int_{z_1}^{z_2} K(z)\,dz = \frac{4}{3}\left(\frac{2m}{\hbar^2}\right)^{1/2} \frac{|\tilde{E}|^{3/2}}{eF}\, v(y) , \tag{4.13a}$$

where

$$v(y) = \frac{3}{4\sqrt{2}} \int_{1-(1-y^2)^{1/2}}^{1+(1-y^2)^{1/2}} (2-\rho-y^2\rho^{-1})^{1/2}\, d\rho . \tag{4.13b}$$

This integral can be expressed in terms of complete elliptic integrals.[25] It has been tabulated by Burgess, Kroemer, and Houston[28] for the range $0 \le y \le 1$. A plot of $v(y)$ for y in this range is shown in Fig. 2. We finally obtain

$$\Theta^2 = \exp\left(2\int_{z_1}^{z_2} K(z)\,dz\right)$$

$$= \exp\left[\frac{4}{3}\left(\frac{2m}{\hbar^2}\right)^{1/2} \frac{(-E-\hbar\omega_p)^{3/2}}{eF}\, v\left(-\frac{(e^3F)^{1/2}}{E+\hbar\omega_p}\right)\right] \tag{4.14}$$

for $y \le 1$. Note that it depends only on E and is independent of $\vec{K}$.

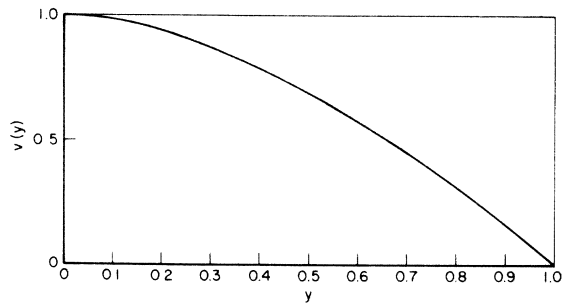

FIG. 2. Plot of the function $v(y)$ vs y in the region $0 \le y \le 1$. See Eqs. (4.12) and (4.13b) for definition.

When the energy E of an electron for motion normal to the surface is such that $E + \hbar\omega_p > V_{max}$, the electron will go over the top of the barrier after photon absorption, and y will be greater than unity. The WKB approximation in that case gives unit transmission[29] and $W(u,v) = 1$. This is certainly an error of the approximation scheme, and improved approximations have been discussed by Miller and Good.[29] For the present purpose, however, we believe the WKB approximation to be adequate, and we assume the transmission coefficient to be unity for electrons with energy above the hump of the potential. We turn to the numerical calculation of the current density in Sec. V.

V. EVALUATION OF THE CURRENT DENSITY

The particle current density in photoassisted field emission for electrons of energy ϵ in the final state is given by Eq. (2.19). We introduce the variables k_0 and q_0 through the relations

$$k_0^2 = 2m(\epsilon - \hbar\omega_p + V_0)/\hbar^2 \tag{5.1}$$

and

$$q_0^2 = 2m(E + V_0)/\hbar^2 . \tag{5.2}$$

We note that $0 \le q_0 \le k_0$ because $\epsilon - \hbar\omega_p = \mathcal{E} = E + \hbar^2K^2/2m$. Thus, (2.19) may be rewritten

$$j(\epsilon) = 2f(\epsilon - \hbar\omega_p) \sum_{q_0} \sum_{\vec{K}} J_z(q_0, \vec{K}) \times \delta\left(\frac{\hbar^2}{2m}(k_0^2 - K^2) - \frac{\hbar^2 q_0^2}{2m}\right) . \tag{5.3}$$

Let us consider first the model shown in Fig. 1(b). Note that $J_z(E, \vec{K})$ of (4.10) depends only on q_0 through Θ^2. Hence,

$$j(\epsilon) \propto 2f(\epsilon - \hbar\omega_p) \sum_{q_0} \sum_{\vec{K}} \frac{2m/\hbar^2}{\Theta^2(q_0) + \frac{1}{2}}\, \delta(k_0^2 - K^2 - q_0^2) . \tag{5.4}$$

We define

$$\Omega = V_0 - \hbar\omega_p \tag{5.5}$$

so that

$$|\tilde{E}| = -E - \hbar\omega_p = \Omega - \hbar^2 q_0^2/2m \ . \quad (5.6)$$

From Eq. (4.14), we obtain

$$\Theta^2(q_0) = \exp\left[\frac{4}{3}\left(\frac{2m}{\hbar^2}\right)^{1/2} \frac{(\Omega - \hbar^2 q_0^2/2m)^{3/2}}{eF} \times v\left(\frac{(e^3F)^{1/2}}{\Omega - \hbar^2 q_0^2/2m}\right)\right] \ . \quad (5.7)$$

Changing the sums over momenta in Eq. (5.4) to integrals, we readily find

$$j(\epsilon) \propto f(\epsilon - \hbar\omega_p) I(\epsilon) \ , \quad (5.8)$$

where

$$I(\epsilon) = \int dq_0 \int d\vec{K}[\Theta^2(q_0) + \tfrac{1}{2}]^{-1} \delta(k_0^2 - K^2 - q_0^2) \quad (5.9a)$$

$$= \pi \int_0^{k_0} dq_0 [\Theta^2(q_0) + \tfrac{1}{2}]^{-1} \ , \quad (5.9b)$$

after carrying out the integration over $\vec{K}$. We introduce the definitions

$$\nu = q_0/k_0 \ , \quad (5.10a)$$

$$E_F = \epsilon_F + V_0 = \hbar^2 q_F^2/2m \ , \quad (5.10b)$$

and

$$\alpha = \hbar\omega_p/E_F \ . \quad (5.10c)$$

Then (5.1) shows that

$$k_0^2 = (2mE_F/\hbar^2)(X - \alpha) \ , \quad (5.11)$$

where

$$X = (\epsilon + V_0)/E_F \ . \quad (5.12)$$

It is clear from (2.20) that

$$f(\epsilon - \hbar\omega_p) = 1/(e^{\beta(\epsilon - \hbar\omega_p + V_0 - E_F)} + 1) = 1/(e^{\beta E_F(X - \alpha - 1)} + 1) \ . \quad (5.13)$$

Equations (5.10)–(5.12), when substituted in (5.9b), yields

$$I(\epsilon) = \pi k_0 \int_0^1 d\nu\, [\Theta^2(k_0\nu) + \tfrac{1}{2}]^{-1} = \pi\left(\frac{2mE_F}{\hbar^2}\right)^{1/2} (X - \alpha)^{1/2} \times \int_0^1 d\nu \left\{\exp\left[\frac{4}{3}\left(\frac{2m}{\hbar^2}\right)^{1/2} \frac{[\Omega - E_F(X-\alpha)\nu^2]^{3/2}}{eF}\, v\left(\frac{(e^3F)^{1/2}}{\Omega - E_F(X-\alpha)\nu^2}\right)\right] + \frac{1}{2}\right\}^{-1} \ . \quad (5.14)$$

The integral was numerically evaluated on a computer with the help of Simpson's rule.[30] For comparison with Lee's experiment,[10] we chose the external electric field $F = 0.23$ V/Å. Then $|V_{max}| = (e^3F)^{1/2} = 1.82$ eV. We also chose the following parameters as being approximately characteristic of tungsten[12]: $V_0 = 10.7$ eV, $E_F = 6.2$ eV, and ϕ = work function = $V_0 - E_F = 4.5$ eV. When the argument of the function v in the integrand exceeded unity, we replaced the integrand by 1, following the prescription of the WKB approximation as explained in Sec. IV. The temperature was taken to be $T = 300$ °K. A plot of $j(\epsilon)$ against the electron energy relative to the Fermi level is shown in Fig. 3(a) for $\hbar\omega_p = 2.6$ eV. Figure 3(b) shows a smooth line drawn through the experimental points[10] for the current density in photoassisted field emission with photons of this energy. Figure 4 shows a similar comparison between the theory and experiment for $\hbar\omega_p = 3.53$ eV. The photon energies were chosen for a comparison with the experimental data.[10]

An inspection of the curves of Figs. 3 and 4 shows that, although the theoretical line shapes qualitatively reproduce the experimental curves, quantitative agreement is far from striking. There are three major disagreements between theory and experiment. The theoretical peaks are narrower than the experimental ones; the leading or high-energy edges of the curves are steeper in the theory; and the theoretical peaks lie at somewhat higher energies than in the experiment. The widths of the theoretical curves will presumably increase when factors like instrumental resolution are taken into account. But the disagreement in peak location—of the order of 0.25 eV when $\hbar\omega_p = 2.6$ eV, and 0.4 eV when $\hbar\omega_p = 3.53$ eV—appears to be far more serious. This probably shows that the simple model considered here is an inadequate representation of a realistic situation with tungsten. Possible reasons for the discrepancy are discussed in Sec. VI.

The numerical calculations reported so far were carried out within the WKB approximation when we ignored the energy dependence of the transition matrix element for photon absorption. Although that energy dependence is definitely weak, it is of interest to see what role it plays in modifying the curves for the current density. To study this, we considered the model of Sec. III where the effect of the image charge is ignored, and the essentially exact value of $J_z(E, \vec{K})$ is written in Eq. (3.25). The total energy distribution of photofield electrons in this case is $j_0(\epsilon)$, where

$$j_0(\epsilon) \propto f(\epsilon - \hbar\omega_p) \sum_{q_0} \sum_{\vec{K}} J_z(q_0) \delta(k_0^2 - K^2 - q_0^2)$$

$$\propto f(\epsilon-\hbar\omega_p)\int_0^{k_0} dq_0\, q_0^2(\Omega-\hbar^2q_0^2/2m)^{1/2}$$
$$\times\exp\left[-\frac{4}{3}\left(\frac{2m}{\hbar^2}\right)^{1/2}\frac{(\Omega-\hbar^2q_0^2/2m)^{3/2}}{eF}\right]. \quad (5.15)$$

Using the definitions (5.6) and (5.10)–(5.12), we obtain

$$j_0(\epsilon)\propto f(\epsilon-\hbar\omega_p)(2mE_F/\hbar^2)^{3/2}(X-\alpha)^{3/2}$$
$$\times\int_0^1 d\nu\,\nu^2[\Omega-E_F(X-\alpha)\nu^2]^{1/2}$$
$$\times\exp\left[-\frac{4}{3}\left(\frac{2m}{\hbar^2}\right)^{1/2}\frac{[\Omega-E_F(X-\alpha)\nu^2]^{3/2}}{eF}\right]. \quad (5.16)$$

This integral was also performed numerically on a computer. The current densities are much smaller in this case, but the general line shapes are unchanged from our previous WKB calculation with the model of Fig. 1(b). Plots of $j_0(\epsilon)$ vs $\epsilon-\epsilon_F$ are shown in Fig. 5 for $\hbar\omega_p = 2.6$ and 3.53 eV. The

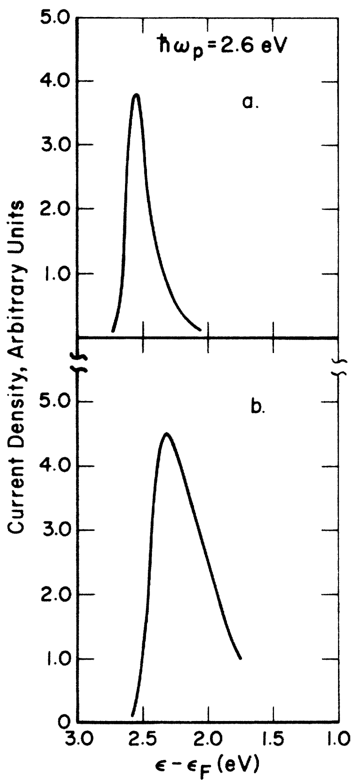

FIG. 3. (a) Theoretical total energy distribution of electrons emitted due to the surface effect in photoassisted field emission for photons of energy $\hbar\omega_P = 2.6$ eV. The image-charge effect is included. Parameters for the model calculation are: $V_0 = 10.7$ eV, $\phi = 4.5$ eV, and $F = 0.23$ V/Å. (b) Solid line drawn through the experimental data points of Lee (Ref. 10) for photons of this energy.

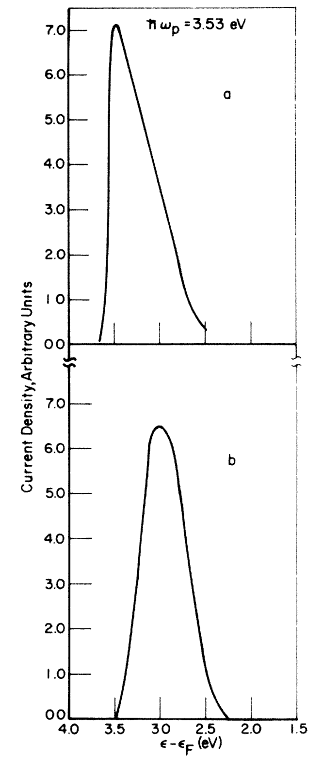

FIG. 4. (a) Theoretical total energy distribution of electrons emitted due to the surface effect in photoassisted field emission for photons of energy $\hbar\omega_P = 3.53$ eV. The image-charge effect is included. Parameters for the theoretical calculation are shown in the caption to Fig. 3. (b) Solid line drawn through the experimental data points of Lee (Ref. 10) for photons of this energy.

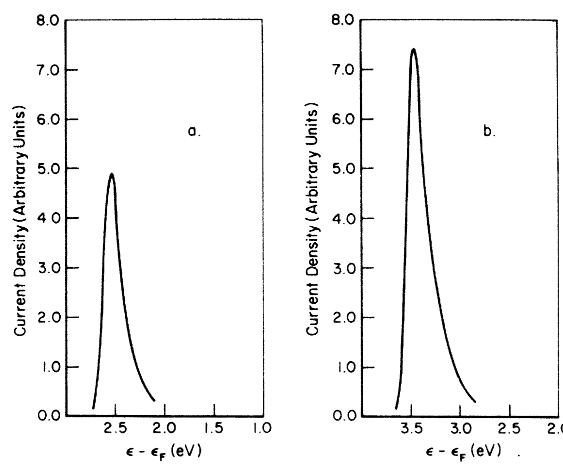

FIG. 5. Theoretical current density plotted against the electron energy for surface effect in photoassisted field emission, using the model of Fig. 1(a) where the image charge is ignored. Parameters for the model calculation are shown in the caption to Fig. 3. Calculations for two photon energies are shown: (a) $\hbar\omega_P = 2.6$ eV, (b) $\hbar\omega_P = 3.53$ eV.

peaks look much sharper than, and are located at about the same energies as, those in Figs. 3(a) and 4(a). The leading edge seems to be even steeper in this model. In sum, the energy dependence of matrix elements does not seem to affect the theoretical curves in any material way.

VI. DISCUSSION

In this paper, we have worked out the theory of surface effect in photoassisted field emission for the simple model of a metal represented by a half-space square well filled up to the Fermi energy with noninteracting electrons. We first ignore the image-charge effect and solve the problem exactly. Next we include the image charge and compute the current density in the WKB approximation. The results of a numerical calculation based on the theory are compared with the available experimental data on the photoassisted field emission from a tungsten target.[10]

Such a comparison is really meaningful if it is assumed that the surface effect makes the dominant contribution to the current density in photoassisted field emission. Based on our knowledge of the band structure of tungsten,[12] we feel that the assumption is probably justified. The peak in the experimental current density of photofield electrons[10] must then be interpreted as arising basically from the Fermi distribution. An alternative explanation might be that this is a bulk effect, with the peak originating in some definite interband transition which conserves the crystal momentum. Such an interpretation, however, will be inconsistent with Fig. 6, where we show the band structure of tungsten in the [310] direction by interpolating through three points given by Mattheiss's calculation.[12] The dotted curve and the dash-dotted curve represent electron energies for direct transition from band 4 with photons of energy 2.6 eV (= 0.1912 Ry) and 3.53 eV (= 0.2596 Ry), respectively. A direct transition is allowed when these curves intersect the upper bands 5 and 6. An inspection of Fig. 6 fails to identify any direct transition that can explain the experimental peak; the peak arises from electrons which lie very close to the Fermi level in initial energy. The argument is not conclusive without the examination of the band structure in other directions as well, even though the external electric field preferentially selects out electrons moving normal to the surface. But a study of the tungsten band structure[12] in other directions shows that, for photons of energy lower than the work function, a peak coming from interband transition close to the Fermi energy is unlikely.

This point can certainly be clarified with further measurements using photons of different energies, and by employing a tungsten target oriented along

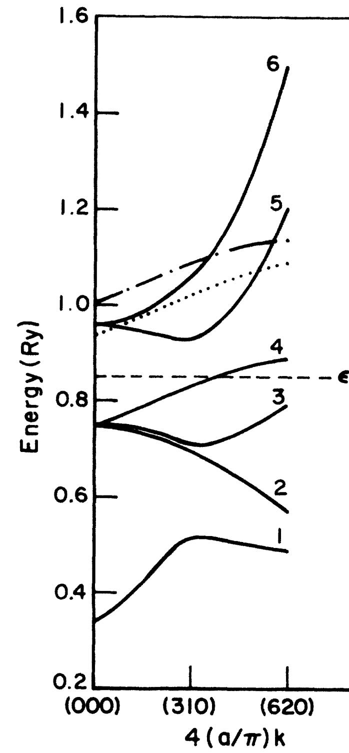

FIG. 6. Band structure in the [310] direction of tungsten, obtained by interpolating through three points given in Mattheiss's calculation (Ref. 12). The dotted curve and the dash-dotted curve are obtained by adding to band 4, for each k, 2.6 eV ($\cong$ 0.19 Ry) and 3.53 eV ($\cong$ 0.26 Ry), respectively. The Fermi level is indciated by the dashed line.

various crystal planes other than (310). Also, the polarization dependence of the surface effect, viz., $(\hat{\epsilon}_p \cdot \hat{z})^2$, is specific, and an experimental investigation of the polarization dependence should conclusively determine the importance of surface effect in photoassisted field emission.

Our comparison of the model theoretical calculation with experiment indicates that the theoretical curves resemble the experimental curves to first order, but the former are shifted to somewhat higher energies with respect to the latter. Since there are electrons present in the metal above ϵ_F due to thermal distribution at a finite T, it is difficult to understand why the current density in photoassisted field emission should cut off exactly at $\epsilon_F + \hbar\omega_p$. There may be some question here regarding the *location* of the Fermi level. A closer experimental investigation of this point seems to us to be worthwhile.

The theoretical curves will certainly be broadened by instrumental resolution in a measurement. But the steep high-energy edge found in our calculation is probably a failure of the model. There are two important sources of possible error in the theoretical computation. Tungsten is not a free-electron metal, and the exact electron states in tungsten must have a bearing on the energy dependence of the transition matrix element. The effect is hard to estimate, but our calculation in Sec. V with the simple model of Fig. 1(a) suggests that matrix elements may not have a profound influence on the calculated current density. The second problem with our model is that the metal potential is assumed to have a sharp edge at $z=0$. A more realistic, smoother potential at the surface may have a large influence on the theoretical line shape. This point is currently under investigation.

In conclusion, then, we feel that the experimental determination of the importance of surface effect in photoassisted field emission is of great interest. Such a determination can be based on the polarization dependence of the current density, and also on the study of emission from various crystal faces with photons of different energies, to see if an identifiable Fermi-energy peak appears in all cases. The question of the exact location of the Fermi level in the experimental data should also be clarified.

ACKNOWLEDGMENTS

The author has profited greatly from conversations with Professor Martin Lee on various aspects of the project. Helpful conversations with Dr. M. L. Glasser and Professor H. Ehrenreich are also gratefully acknowledged.

APPENDIX

We wish to calculate the total current due to surface effect in ordinary photoemission as a limiting case of our theory when $\hbar\omega_p > \phi$, $\tilde{E}=E+\hbar\omega_p>0$, and $F\to 0$. Using (3.17) in (3.11), the particle current density for electrons initially in the state $\psi_{\mathcal{E},\vec{K}}(\vec{r})$ is found to be

$$\begin{aligned} J_z(E,\vec{K}) &= (\hbar/m\pi)L^{-3}(2meF/\hbar^2)^{1/3}\,|\Gamma|^2 \\ &\underset{F\to 0}{\sim} (2m/\hbar^2)^2(e^2\hbar^2/m^2)(2\pi\hbar/\omega_p)(n_p/\Omega) \\ &\quad\times(\hat{\epsilon}_p\cdot\hat{z})^2(\hbar\omega_p)^{-2}(\hbar/m\pi)L^{-3}\pi(2m\tilde{E}/\hbar^2)^{1/2} \\ &\quad\times[V_0^2/(k+q)^2]\,|v(0)|^2\,|\phi_E(0)|^2\,, \end{aligned} \tag{A1}$$

and this is independent of F, as it ought to be. Note that $J_z(E,\vec{K})$ is independent of $\vec{K}$ and depends only on E. From (3.4) we find $v(0)=1$. To find $\phi_E(0)$, we note that the properly normalized solution of the appropriate one-dimensional Schrödinger equation for the energy E can be written

$$\phi_E(z)=\sqrt{2}\,\sin q_0(z-\delta)\,,\quad z<0 \tag{A2a}$$

$$\qquad = ae^{-p_0 z}\,,\quad z>0\,, \tag{A2b}$$

where $p_0^2=-2mE/\hbar^2$ and $q_0^2=2m(E+V_0)/\hbar^2$. The continuity of the logarithmic derivative at $z=0$ gives

$$\cot q_0\delta = p_0/q_0\,. \tag{A3}$$

This implies that, to within a phase factor,

$$\phi_E(0)=a=-\sqrt{2}\,\sin q_0\delta=-\sqrt{2}\,q_0/(p_0^2+q_0^2)^{1/2}\,, \tag{A4}$$

and the definition of p_0 and q_0 shows that

$$p_0^2+q_0^2=2mV_0/\hbar^2\,. \tag{A5}$$

Using the above results in (3.12), we obtain the total current density at absolute zero as

$$\begin{aligned} I &= -e\left(\frac{\hbar}{m\pi}\right)\frac{2L^3}{4\pi^3}\int_0^{q_F} dq_0\,\pi(q_F^2-q_0^2)\left(\frac{2meF}{\hbar^2}\right)^{1/3}|\Gamma|^2 \\ &= -\frac{16e^3\pi^2}{4\pi^3 m\hbar^2\omega_p^3}\left(\frac{n_p}{\Omega}\right)(\hat{\epsilon}_p\cdot\hat{z})^2\int_0^{q_F} dq_0\,(q_F^2-q_0^2) \\ &\quad\times\left[\frac{2m(E+\hbar\omega_p)}{\hbar^2}\right]^{1/2}\frac{V_0^2}{(k+q)^2}\,\frac{2q_0^2}{p_0^2+q_0^2} \end{aligned} \tag{A6}$$

Let us introduce a dimensionless variable $\epsilon=q_0^2/q_F^2$. We also define $\bar{\Omega}=\hbar\omega_p/E_F$ and $\mu=V_0/E_F$, where $E_F=\hbar^2q_F^2/2m$. Then

$$[2m(E+\hbar\omega_p)/\hbar^2]^{1/2}=q_F(\epsilon-\mu+\bar{\Omega})^{1/2}\,, \tag{A7}$$

$$(q_F^2-q_0^2)=q_F^2(1-\epsilon)\,, \tag{A8}$$

$$2q_0^2\,dq_0=q_F^3\epsilon^{1/2}\,d\epsilon\,, \tag{A9}$$

and

$$-k^2+q^2=2mV_0/\hbar^2\,. \tag{A10}$$

Substituting all this in (A6), we obtain

$$\begin{aligned} I &= -\frac{4e^3}{\pi m\hbar^2\omega_p^3}\left(\frac{n_p}{\Omega}\right)(\hat{\epsilon}_p\cdot\hat{z})^2 q_F^6\int^1 d\epsilon\,(1-\epsilon) \\ &\quad\times(\epsilon+\bar{\Omega}-\mu)^{1/2}\epsilon^{1/2}\,\frac{V_0^2}{(k+q)^2}\,\frac{\hbar^2}{2mV_0}\,. \end{aligned} \tag{A11}$$

But

$$\begin{aligned} \frac{V_0}{(k+q)^2} &= \frac{\hbar^2}{2m}\,\frac{q^2-k^2}{(q+k)^2} \\ &= \frac{\hbar^2}{2m}\left(\frac{(\epsilon+\bar{\Omega})^{1/2}-(\epsilon+\bar{\Omega}-\mu)^{1/2}}{(\epsilon+\bar{\Omega})^{1/2}+(\epsilon+\bar{\Omega}-\mu)^{1/2}}\right)\,. \end{aligned} \tag{A12}$$

The total current density can therefore be written

$$I=-PJ_1\,, \tag{A13}$$

where

$$\begin{aligned} J_1 &= \frac{1}{2}\int^1 d\epsilon\,(1-\epsilon)(\epsilon+\bar{\Omega}-\mu)^{1/2}\epsilon^{1/2} \\ &\quad\times\left(\frac{(\epsilon+\bar{\Omega})^{1/2}-(\epsilon+\bar{\Omega}-\mu)^{1/2}}{(\epsilon+\bar{\Omega})^{1/2}+(\epsilon+\bar{\Omega}-\mu)^{1/2}}\right) \end{aligned} \tag{A14}$$

and

$$P=\frac{8e^3}{\pi m\hbar^2\omega_p^3}\left(\frac{n_p}{\Omega}\right)(\hat{\epsilon}_p\cdot\hat{z})^2 q_F^6\,\frac{\hbar^4}{4m^2}$$

$$= \alpha^4 a_B^3 q_F^3 (6\pi Nec)(n_p/\Omega)(c/\omega_p)^3(\hat{\epsilon}_p \cdot \hat{z})^2 , \tag{A15}$$

with $\alpha = e^2/\hbar c$, $a_B = \hbar^2/me^2$, $q_F^3 = 3\pi^2 N$, and N is the number density of electrons in the target. The lower limit of integration in (A11) is determined by the condition $\epsilon + \overline{\Omega} - \mu > 0$. Equations (A13)–(A15) exactly reproduce Adawi's result[16] for the current density in one-photon surface photoelectric effect.

Note added in proof. After the completion of this work, we came across a paper by Caroli *et al.*,[31] where they study photoassisted field emission by employing the Keldysh nonequilibrium Green's-function formalism. They apply their microscopic theory to a model calculation involving a semi-infinite, free electron metal and normally incident light. The simple "surface effect" considered in this paper then disappears, and the photoemission they obtain comes from a phenomenological damping of the electromagnetic wave inside the metal, which can lead to nonmomentum conserving excitation of electrons.

[1]E. W. Plummer and R. D. Young, Phys. Rev. B 1, 2088 (1970).

[2]E. W. Plummer and J. W. Gadzuk, Phys. Rev. Lett. 25, 1493 (1970).

[3]E. W. Plummer and A. E. Bell, J. Vac. Sci. Technol. 9, 583 (1972).

[4]P. L. Young and R. Gomer, Phys. Rev. Lett. 30, 955 (1973); P. L. Young and R. Gomer (unpublished).

[5]B. J. Waclawski and E. W. Plummer, Phys. Rev. Lett. 29, 783 (1972).

[6]B. Feuerbacher and B. Fitton, Phys. Rev. Lett. 29, 786 (1972).

[7]J. M. Baker and D. E. Eastman, J. Vac. Sci. Technol. 10, 223 (1973).

[8]B. Feuerbacher and B. Fitton, Phys. Rev. Lett. 30, 923 (1973).

[9]H. Neumann and Ch. Kleint, Ann. Phys. (Leipz.) 27, 237 (1971).

[10]M. J. G. Lee, Phys. Rev. Lett. 30, 1193 (1973).

[11]H. Neumann, Physica (Utr.) 44, 587 (1969).

[12]L. F. Matthiess, Phys. Rev. 139, 1893 (1965); I. Petroff and C. R. Viswanathan, Phys. Rev. B 4, 799 (1971).

[13]C. N. Berglund and W. E. Spicer, Phys. Rev. 136, A1030 (1964); 136, A1044 (1964).

[14]N. V. Smith, Phys. Rev. B 3, 1862 (1971); N. V. Smith and M. M. Traum, Phys. Rev. Lett. 29, 1243 (1972).

[15]G. D. Mahan, Phys. Rev. B 2, 4334 (1970).

[16]I. Adawi, Phys. Rev. 134, A788 (1964).

[17]P. O. Gartland, S. Berge, and B. J. Slagsvold, Phys. Rev. Lett. 30, 916 (1973); S. A. Flodström and J. G. Endriz, *ibid.* 31, 893 (1973).

[18]M. J. G. Lee, in Proceedings of the Twentieth Field Emission Symposium, University Park, Pennsylvania, 1973 (unpublished).

[19]See, e.g., M. L. Goldberger and K. M. Watson, *Collision Theory* (Wiley, New York, 1964), pp. 75 and 300.

[20]L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Pergamon, Oxford, England, 1965), p. 73.

[21]M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1965), Chaps. 9 and 10. See, in particular, Sec. 10.4.

[22]M. Abramowitz and I. A. Stegun, in Ref. 21, Sec. 10.4.10.

[23]See, e.g., R. Gomer, *Field Emission and Field Ionization* (Harvard University Press, Cambridge, Mass., 1961), Chap. 1.

[24]See, e.g., R. Gomer, in Ref. 23, Chap. 1, p. 9.

[25]E. L. Murphy and R. H. Good, Jr., Phys. Rev. 102, 1464 (1956).

[26]E. Merzbacher, *Quantum Mechanics* (Wiley, New York, 1961), Chap. 7.

[27]E. Merzbacher, in Ref. 26, p. 122.

[28]R. E. Burgess, H. Kroemer, and J. M. Houston, Phys. Rev. 90, 515 (1953).

[29]S. C. Miller, Jr. and R. H. Good, Jr., Phys. Rev. 91, 174 (1953).

[30]M. Abramowitz and I. Stegun, in Ref. 21, p. 886.

[31]C. Caroli, D. Lederer-Rozenblatt, B. Roulet, and D. Saint-James, Phys. Rev. B (to be published).