Self-resonant modes in high-Q Josephson tunnel junctions

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Our tin-tin oxide-tin tunnel junctions, when excited in their resonant electromagnetic cavity modes, exhibit quality factors Q as high as 660. This has allowed us to test Kulik's extension of Fiske-mode theory. We have verified his prediction that the dc current associated with a Fiske mode goes through a peak as Q increases, and we find his numerical value for the height of this peak to be correct. We find a small disagreement with his prediction of the value of the magnetic field that will maximize the dc current. We have also made some measurements of zero-field modes and of small, anomalous modes occuring at half-integral mode positions.

INTRODUCTION

We have studied self-resonant modes in tin-tin oxide-tin Josephson tunnel junctions by observation of their dc current-voltage characteristics. In our experiment, the modes appear as current steps at constant voltage. We have measured the step height, or step critical current, as a function of applied magnetic field and temperature.

Fiske modes,[1-14] which occur only when a field is applied, involve the nonlinear interaction of the ac Josephson current with the associated electromagnetic wave. The junction acts like an open-ended microwave cavity, and the current steps appear at voltages such that the electromagnetic wave is resonant in this cavity.

Fiske modes are one of the few aspects of the ac Josephson effect that can be studied by simple dc methods. A correct theory requires a correct understanding of the ac effect, and it is gratifying that we find agreement in most (but not all) details. We will also see that the measurements yield values for the quality factors Q of the resonances.

A number of authors have developed the theory of Fiske modes. The most detailed work is due to Kulik,[11] which involves an extension to relatively high Q's . The new predictions of Kulik's theory are tested for the first time in the work reported here.

There are also modes that appear in a zero field, for which the theoretical situation is not as clear. A picture proposed by Fulton and Dynes,[15] involving moving vortices in the junction, is consistent with all observations, and we are inclined to accept it. However, there does not yet exist a detailed theory with which we can compare all of our measurements.

We hope that the results reported here will stimulate the development of such a theory. We also hope that the differences between our results and Kulik's predictions will stimulate further work on the theory of Fiske modes. It seems particularly important to find what changes would occur if the $V\sigma \cos\phi$ term in the Josephson equation[16] were included.

THEORY

Fiske modes

The theory of Fiske modes has been discussed in many places,[14] so our discussion will be brief. The basic idea is that two waves are present in the oxide and the immediately adjacent metal. Electromagnetic waves are generated by the ac supercurrent that is present whenever the voltage is nonzero. When there is also an applied magnetic field, the ac supercurrent distribution is in the form of a traveling wave, with velocity proportional to the field strength. The equations governing a junction are highly nonlinear, and these two waves interact to produce, among other things, a dc current. This current is only large enough to observe when the ac electromagnetic field is intense. This in turn requires this field to be resonant in the junction. Because the frequency is proportional to the voltage, we have resonances at specific voltages, giving rise to current spikes. Since we use a constant current source, the higher voltage side of the spike is not accessible to us. Instead of a spike we see a step, and we shall refer to "current steps" in this paper.

The most detailed theory of Fiske modes is due to Kulik.[11] His specific advance is to extend the analysis to large values of the parameter z_n, $z_n \equiv Q_n(l/\pi n\lambda_J)^2$. Here Q_n is the quality factor of the nth electromagnetic resonance; l is the length of the junction in the direction of propagation of the electromagnetic waves, i.e., the length perpendicular to the applied magnetic field; and λ_J is the Josephson penetration depth and is equal to $(\hbar c^2/16\pi e\,\lambda_L j_0)^{1/2}$, where λ_L is the London penetration depth of the tin film and j_0 is the maximum zero-field supercurrent density. (All equations in this paper are in Gaussian units.) Although z_n is unrestricted in size, the condition $(l\pi n\lambda_J)^2 \ll 1$ must be maintained, as in previous work. The principal new feature that arises is that as Q_n (or z_n) increases, due, say, to decreasing the temperature, the height of the current step goes through a maximum and then decreases. The peak occurs when z_n is about 4. As Q_n approaches infinity, the step height goes to zero. (This must be true because a dc current step I at finite voltage V implies power dissipation at a rate IV. If Q_n is infinite, there are no losses, so there can be no power dissipation or dc current.)

Kulik considers a rectangular junction with voltage V across it and field H in the plane of the oxide layer, perpendicular to the edge of length l. The phase difference across the junction, ϕ, obeys the equation

$$\frac{\partial^2\phi}{\partial x^2} - \frac{1}{\bar{c}^2}\left(\frac{\partial^2\phi}{\partial t^2} + \gamma\frac{\partial\phi}{\partial t}\right) = \frac{1}{\lambda_J^2}\sin\phi \ . \tag{1}$$

Here x is the coordinate along the side l, γ is the damping coefficient, and $\bar{c}$ is the velocity of electromagnetic waves in the junction. $\bar{c}$ was shown to be given to a good approximation as $\bar{c} = c[(d/2\epsilon\lambda_L)^{1/2}]$ by Swihart.[17] In this expression c is the velocity of light in free space, d is the oxide thickness, and ϵ is the dielectric constant of the oxide. We can imagine that the damping is due to some phenomenological resistance R, so that $\gamma = 1/RC$, where C is the junction capacitance.

Equation (1) has formed the basis of all discussions of Fiske modes, but it should be recognized that it is fairly crude. Treating γ as a constant, independent of voltage or frequency, is certainly not correct, and the pair-quasiparticle interference term $V\sigma(V)\cos\phi$ in the expression for the Josephson current[16] has not been included. Nevertheless we shall see that in many respects the theory is in good agreement with our results.

Kulik solves Eq. (1) subject to the assumptions $V \cong$ constant, $H \cong$ constant, and $Q > 1$ to get the dc average supercurrent density of the nth step, j_n. His result is

$$j_n = j_0 J_0\left(\frac{a}{2}\right) J_1\left(\frac{a}{2}\right) F_n(r)\Psi_n(\omega), \tag{2}$$

where the J_n are Bessel functions, and ω is $2eV/h$. We use r to denote the ratio of the field H in the junction to that value H_0 required to give one flux quantum in the junction. In other words,

$$r \equiv H_{\text{appl}}/H_0 \ . \tag{3}$$

The magnetic-field-dependent parameter a is given by

$$a/J_0^2(\tfrac{1}{2}a) = z_n F_n(r)\Psi_n(\omega) \ , \tag{4}$$

where Ψ_n is defined by

$$\Psi_n(\omega) \equiv \frac{\omega\gamma}{[(\omega_n^2 - \omega^2)^2 + \omega^2\gamma^2]^{1/2}}, \tag{5}$$

with

$$\omega_n = \pi n\bar{c}/l, \tag{6}$$

and

$$F_n(r) \equiv \frac{2r\,|\sin(\pi r - \frac{1}{2}\pi n)|}{\pi\,|r^2 - \frac{1}{4}n^2|} . \tag{7}$$

The current depends on both V and H. We will consider only the resonance case, where the wavelength of the electromagnetic wave in the junction satisfies $\lambda = 2l/n$, so that ω equals ω_n, and Ψ_n is unity. Equations (2) and (4) then become

$$j_n = j_0 J_0(\tfrac{1}{2}a) J_1(\tfrac{1}{2}a) F_n(r) \tag{8}$$

and

$$a/J_0^2(\tfrac{1}{2}a) = z_n F_n(r) . \tag{9}$$

For small z_n, i.e., Q_n not too large, Eq. (8) can be simplified to

$$j_n = (\tfrac{1}{4}z_n) j_0 F_n^2(r). \tag{10}$$

This relation corresponds to the predictions of previous theories. The field dependence is given by F_n^2, a function which rises from zero at $r = 0$ and which has maxima whose position and value depend upon n. If z_n is not small, Eqs. (8) and (9) must be used. Then the dependence of j_n on H is not simple, but it is qualitatively the same as for small z. The difference in shape for different z_n values is illustrated for $n = 1$ in Fig. 1. The "small z" curve is just F_1^2, and both curves are

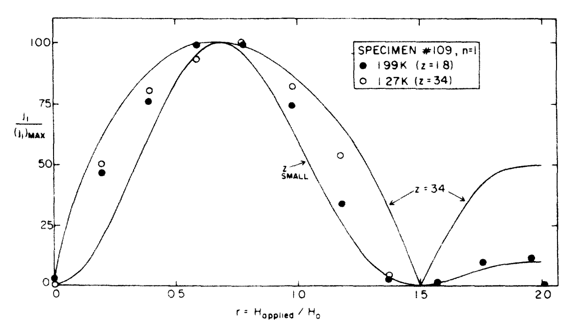

FIG. 1. Step height against applied field for the $n = 1$ Fiske mode of junction 109 at two temperatures. The data and theoretical curves are normalized by dividing by the maximum value, so that each has a peak value of unity. The theoretical curves and z values were determined from Kulik's theory, as described in the text. H_0 is the field that gives one flux quantum in the junction. Note that the "z small" and "$z = 34$" curves differ little for $r < 1.5$.

normalized by dividing by their maximum value. Also shown are measured values for junction No. 109 at two temperatures. The values of z determined from the data, as explained below, are shown in the key. The z of 1.8 corresponds to a Q of 40 and the 34 to $Q=660$. These z values represent fairly extreme cases from the point of view of our measurements, yet the difference between the two theoretical curves is not great. Our data are not good enough to distinguish between these shapes, and so our analysis will focus on the maximum step height.

We have adopted the following procedure to measure Q for a given Fiske mode at a given temperature. The step height versus field is measured and the maximum value determined. This maximum is much more sensitive to Q than is any other aspect of the data. We compute j_{max}/j_0F_{max}, where j_0 is the zero-field, zero-voltage supercurrent density at the same temperature, and F_{max} is the maximum value of $F_n(r)$. (The maximum value of F_n is close to unity for all n values, and Kulik took it to be unity in his computations.) Inspection of Eqs. (8) and (9) shows that j_n/j_0F_n is a universal function of z_nF_n, for all n and H. If we know the value of F_n and of j_n/j_0, we can solve for z_n. We suppose that, at whatever applied field the experimental j_n is maximized, F_n has its theoretical maximum value. This is not strictly correct because j_n depends on J_0J_1, as well as on F_n. However, for our range of parameters, J_0J_1 is a weak function of the field near the maximum of F_n, and F_n should determine the point at which j_n is maximum. Experimentally we find that the maxima of the j_n tend to occur at somewhat lower fields than is predicted by the theory. (But the zeroes of j_n tend to fall very accurately where expected.) We ignore this small discrepancy. If instead we had used the theoretical value of F_n at the actual field where j_n is maximum, our results for Q would not be changed substantially.

In Fig. 2 we show the universal function j/j_0F vs zF, computed numerically from Eqs. (8) and (9). From the experimental j_{max}/j_0 we calculate j_{max}/j_0F_{max}, taking the F_n appropriate to the n value of the Fiske mode in question. From Fig. 2 we get two values of zF and thus two values of z and Q. We select between them by plotting both results against temperature for the given mode. We assume that Q is a monotonically decreasing function of temperature. If enough different temperatures are represented, this assumption leads to unambiguous choices for Q. The two sets of choices intersect at the temperature at which j_{max} is maximum. This corresponds to the peak in Fig. 2 and explains why the step height falls off at lower temperatures even though Q continues to increase.

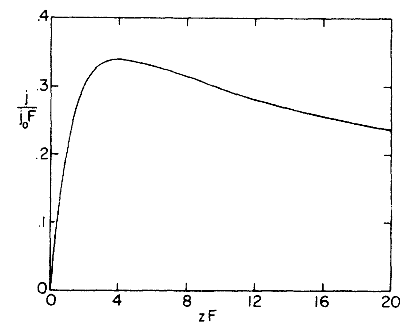

FIG. 2. The universal function j/j_0F against zF, computed from Eqs. (8) and (9).

In a sense, Fig. 2 embodies the main result of Kulik's work. One simple prediction that comes from it is the numerical value of j_n/j_0 when it is maximized with respect to temperature and field. Since all of the F_n are near unity at the peak in the figure, all of these maximum values of j_n/j_0 will be near 0.34. For $n=1$, for example, it is 0.37.

Zero-field modes

For the current steps that occur in zero field, there is no detailed theory. Fulton and Dynes[15] have advanced a model involving vortex motion which seems to be in accord with the facts. However, this picture has not been carried far enough to predict field dependences or to allow one to compute Q from the measured step height. Briefly, they attribute the step to one or more vortices that have become trapped in the junction. These vortices are driven along the junction by the tunneling current and are reflected at the edges. Resonance occurs when the time for the vortex pattern to repeat itself equals the period of the ac supercurrent. The model predicts that steps will occur at voltages corresponding to the even-n Fiske modes. It also predicts a cutoff given by $V \cong f_J \Phi_0$, where Φ_0 is the flux quantum and f_J is the Josephson plasma frequency $f_J = \bar{c}/\lambda_J$. For voltages near or beyond this value, Fulton and Dynes expect the vortex mode to be unstable and the step structure to disappear.

EXPERIMENTAL DETAILS

Our tunnel junctions were made by the same techniques as in earlier work by our group.[18] They were "in-line" as opposed to crossed strips. The tin films were roughly 1000 Å thick and were deposited on cooled sapphire substrates. The oxides were grown in an oxygen glow discharge. Other characteristics of the specimens are listed in Table I.

For the measurements, the specimens were im-

TABLE I. Specimens and modes observed.

Specimen No.	Normal state resistance (Ω)	Maximum super-current[a] (mA)	Josephson penetration depth λ_J[a] (mm)	Length L (mm)	n_L	V_n[b] (mV)	Width W (mm)	n_W	V_n[b] (mV)
21	0.527	1.75	0.086	0.05	1	0.28	0.11	1	0.125
					2	0.55		2	0.25
								4	0.5
22	0.624	1.39	0.090	0.044	$\frac{1}{2}$	~0.18	0.11	$\frac{1}{2}$	0.065
					1	0.356		1	0.13
								2	0.26
107	0.716	1.30	0.145	0.10	$\frac{1}{2}$	0.062	0.10	c	
					1	0.125			
					$\frac{3}{2}$	0.19			
					2	0.25			
					$\frac{5}{2}$	0.31			
					3	0.37			
109	0.7	1.41	0.135	0.09	$\frac{1}{2}$	0.066	0.10	c	
					1	0.13			
					$\frac{3}{2}$	0.20			
					2	0.27			
					3	0.40			

[a]Measured at roughly 1.3 K.

[b]Estimated accuracy of V is 3%.

[c]For 107 and 109, which were square or nearly so, we did not apply a field perpendicular to the side W, and so, the corresponding Fiske modes were not studied.

mersed directly in the liquid helium. The specimen region was magnetically shielded by a high-permeability metal can, and the Dewar tail was entirely nonmagnetic. A vertical magnetic field was applied with a solenoid, and a horizontal field with a Helmholtz pair. The circuits were carefully shielded, and rf filters separated them from the specimen. The junctions were judged to contain trapped flux if the maximum zero-voltage supercurrent was not at zero field, for fields applied in any direction in the plane of the oxide. When this was observed the specimen was warmed to above its transition temperature and cooled again.

The current-voltage characteristic of a junction was displayed on an oscilloscope using an ac signal generator. In order to observe Fiske modes the current must not pass through zero, so we added a dc bias voltage so that the current was always positive. The current would decrease to its minimum value and then rise again. When the voltage arrived at the value corresponding to a Fiske mode, the current would increase at essentially constant voltage to some maximum value. Our power supply was a constant current source, so when the maximum current was reached, the voltage would jump to some higher value. This higher voltage might correspond to another Fiske mode, if there happened to be one of sufficient maximum current, or it would be the energy gap. Thus not every mode is seen simultaneously. By adjusting the dc bias to various values, all modes that did not happen to have zero step height could be studied.

RESULTS

Fiske modes

Our specimens are listed in Tables I and II. They showed the same near-ideal properties as others that have been reported by our group.[18] In particular, their measured supercurrents were within a few percent of the values predicted by the

TABLE II. Values of electromagnetic phase velocity in the junction $\bar{c}$ divided by the free space velocity c_0. $\Delta V_n/\Delta n$ was determined from a plot of Fiske-mode voltage V_n against mode number n. c/c_0 was computed from this. The results are believed accurate to about 13%.

Specimen No.	Dimension (mm)	$\Delta V_n/\Delta n$ (mV)	$\bar{c}/c_0$
21	0.050	0.278	0.045
	0.11	0.125	0.044
22	0.044	0.356	0.050
	0.11	0.13	0.042
107	0.10	0.124	0.040
109	0.090	0.133	0.034

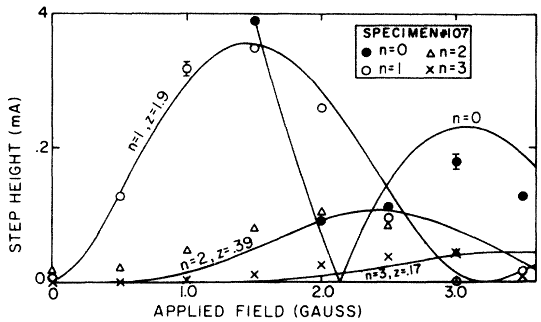

FIG. 3. Current step height against field for the zero-voltage supercurrent ($n=0$) and the Fiske modes ($n=1,2,3$) of junction 107 at 2.03 K. The solid lines and corresponding z values were determined using Kulik's theory. The z value was chosen so that the theoretical curve would have the same maximum step height as the data.

BCS theory.

We agree with other workers in that Fiske modes are not seen at voltages beyond about half the superconducting energy gap. Presumably, beyond this voltage losses are too great to permit the modes to develop. For our specimens, this means that the largest n expected is in the range from 2 to 4. All of the steps that we observed are listed in Table I. The steps having half-integral n values were very small. They will be discussed later.

Our principal measurement was step height (maximum dc current for the mode) versus applied field. Examples are shown in Fig. 3. Also shown is a portion of the zero-voltage ($n=0$) supercurrent versus field. This was used to determine H_0, the field giving one quantum of flux in the junction. This same H_0 was then used for the theoretical curves drawn for the Fiske modes. Each of the Fiske mode curves then has only one adjustable

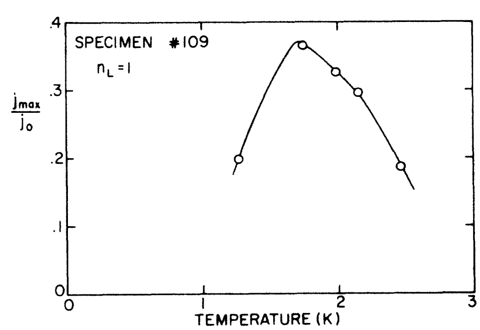

FIG. 4. Temperature dependence of the $n_L=1$ Fiske mode of junction 109. j_{max} is the height of the step at whatever applied field maximizes it, and j_0 is the zero-field supercurrent density at the same temperature. The rather sharp peak near 1.7 K ($T/T_c=0.5$) corresponds to the peak in Fig. 2. The solid line is intended merely to be suggestive.

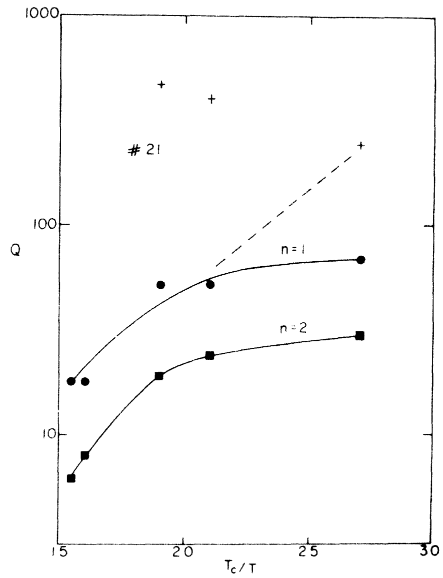

FIG. 5. Quality factor Q against T_c/T for the Fiske modes of junction 21. The crosses and dashed line show alternate choices for $n=1$. This ambiguity is explained in the text. The solid lines are intended merely to be suggestive.

parameter, the maximum current. The theoretical curve was chosen to give the measured maximum current, even though the experimental maximum does not occur at exactly the same field as does the theoretical maximum. We regard the agreement shown as reasonable in the light of the approximations of the theory.

For any mode, we call the maximum current density that occurs at any field j_{max}. For example, from Fig. 3 we see that for specimen 107, $n=1$, $T=2.03$ K, the maximum current is about 0.36 mA. We studied the temperature dependence of j_{max} for the various modes for each specimen. Figure 4 shows an example. As was anticipated earlier, we see that j_{max} goes through a peak. (Some modes did not show a peak, but presumably they would have at some lower temperature.) In every case that such a peak was found, its value agreed with the theoretical prediction. For example, for $n=1$, the theory says that j_{max}/j_0 should have a peak value of 0.37, which agrees with the results shown in Fig. 4. In this respect, we find excellent numerical agreement between theory and experiment.

As explained earlier, the j_{max} values were used to compute Q values. The results are shown in Figs. 5–8. For specimen 109, Fig. 8, the pairs of values for Q that the theory gives are shown for $n=1$. We selected the points shown by circles as the correct ones by requiring Q to be a monotonic decreasing function of temperature. For $n=1$, we see a linear dependence of $\ln Q$ on T_c/T, with Q

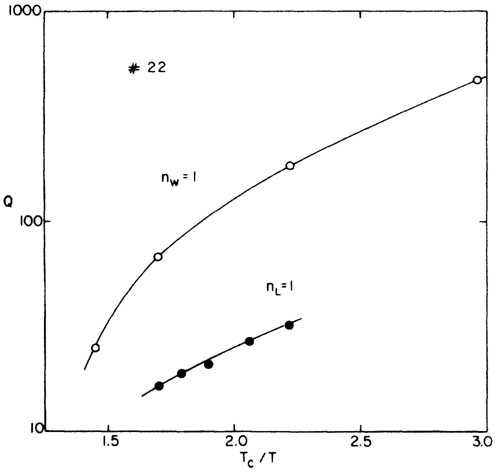

FIG. 6. Quality factor Q against T_c/T for some of the Fiske modes of junction 22. The solid lines are intended to be merely suggestive

reaching 660 at $T_c/T = 2.9$ ($T = 1.27$ K). This is the largest Q value that we have observed for any of our specimens. The linear dependence suggests Q proportional to $e^{-\alpha T_c/T}$ with $\alpha = 2.5$. If we suppose that the losses determining Q are due to thermally excited quasiparticles, we would expect Q to be roughly proportional to $e^{-\Delta/kT}$. However, this gives $\alpha = 1.8$, which is well outside our experimental error. In any case, we see from the figures that such an exponential temperature dependence was not exhibited by all the modes.

For specimen 21, Fig. 5, we also show both values for Q for $n = 1$. This shows that some care is required in the choice of temperatures. We have interpreted the data to mean that j_{max}/j_0 does not reach its peak in the range of temperature studied. However, it is equally plausible to suppose that the pairs of values cross each other near $T_c/T = 2.5$. Then the value of Q would be 240 at $T_c/T = 2.7$, instead of 69. Since there are no data near $T_c/T = 2.5$, we do not know which is really correct. Unfortunately, the specimen was de-

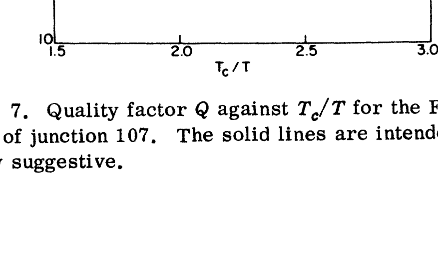

FIG. 7. Quality factor Q against T_c/T for the Fiske modes of junction 107. The solid lines are intended to be merely suggestive.

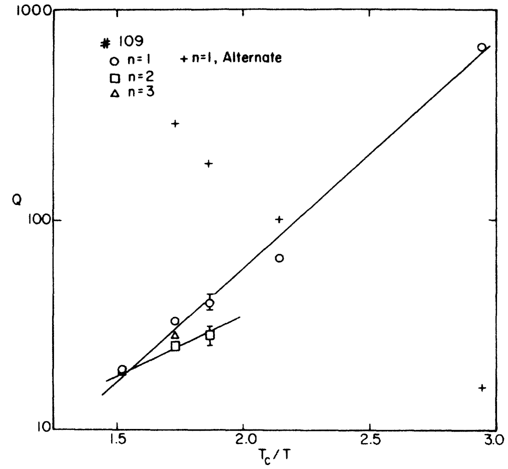

FIG. 8. Quality factor Q against T_c/T for the Fiske modes of junction 109. The crosses show alternate choices for $n = 1$. The circles were selected in preference to the crosses by requiring Q to decrease as T increases. One can see that j_{max}/j_0 for $n = 1$ has a peak at $T_c/T \cong 2.2$, or $T \cong 1.7$ K because a line drawn through the crosses intersects the line through the circles at this point. This peak is shown in Fig. 4. The straight lines are intended merely to be suggestive.

stroyed before the ambiguity was discovered. The choice that we made makes the $n = 1$ curve have the same shape as the $n = 2$ one. (The latter is not ambiguous.) The curve that we would have obtained from the other choice is suggested by the dashed line in Fig. 5. If this other curve is taken to be the right one, then all of the results for $n = 1$ modes for all the specimens are at least consistent with an exponential dependence of Q on T_c/T with about the same exponent. However, such a dependence is definitely not correct for the higher n modes.

Since the Q values cannot rise indefinitely as T is lowered, these curves must flatten off at some value of T_c/T. Therefore, we feel that the results shown are quite plausible. More study will be required before conclusions can be drawn regarding the nature of the losses. We should note that if we try to estimate Q from the dc quasiparticle resistance implied by the dc current-voltage characteristics of the junctions, we get values that are much too high.

Wilson *et al.*[19] have studied the Q's of lead-lead and lead-niobium junctions by applying microwaves. For Pb-Pb, they find $Q \cong 100$ at 4.2 K, rising to roughly 350 at 2.2 K. For Pb-Nb, they find $Q \cong 20$ at 6 K, rising to roughly 70 at 2 K. Thus their results are at least qualitatively similar to ours. It would be interesting to make both kinds of measurements on the same specimen.

Another aspect of Kulik's theory that is of interest is the prediction of the dependence on n of the value of field that maximizes j. For example, Fig. 3 shows some theoretical curves. H_0 for this specimen was 2.14 G. If we convert the field axis to $r \equiv H_{appl}/H_0$, we find the theoretical peaks to lie at $r = 0.675$, 1.126, and 1.592 for $n = 1$, 2, and 3, respectively. If we plot these r values against n we get very nearly a straight line, but it does not go through the origin. This linear behavior agrees with our findings (although we do not measure precisely the same values of r) and also with the results of others.[2,3,5] (Since V_n is closely proportional to n, similar statements apply to a plot of the r that gives the maximum j against V_n.) It is interesting to note that since the line does not pass through the origin, it cannot really be interpreted as corresponding to simple "velocity matching" between electromagnetic waves of speed $\bar{c}$ and supercurrent density waves of speed proportional to H. In any case, Kulik correctly predicts the form of this curve. On the other hand, one of the small but consistent differences between his theory and our measurements is in the value of r that maximizes j. We find smaller than predicted values, with the discrepancy being larger for larger n values. In Fig. 3, the $n = 2$ data show a peak at 2.0 G which corresponds to $r = 0.93$ instead of 1.13. For $n = 3$ the peak is at about 1.4 instead of 1.6. (One might question the choice of H_0 used in this figure, since the data for $n = 0$ are rather sparse. However, measurements at other temperatures established the value of H_0 to be 2.14 G.)

Because of this discrepancy, and also because it is hard to locate these broad peaks precisely, we have not used measurements of the field that maximizes j to determine $\bar{c}$. Instead, we used Eq. (6) and $\omega_n = 2eV_n/\hbar$ to compute $\bar{c}$. For the set of integral modes, we plotted V_n against n, computed the slope $\Delta V_n/\Delta n$, and from this obtained $\bar{c}/c$. The results are shown in Table II. We can use Swihart's formula for $\bar{c}$, given earlier, to determine the dielectric constant ϵ of the oxide, but since we do not know the thickness accurately it will only be an estimate. Taking the thickness to be 15 Å we find ϵ to be roughly equal to 7, while 10 Å gives an ϵ of about 5.

As Table I shows, we also observed a number of half-integral modes. With the exception of $n = \frac{3}{2}$ for specimen 107, these appeared only when a field was applied. Their voltages corresponded accurately to half-integral values. The currents were always small ($j/j_0 \lesssim 0.03$), but they appeared quite consistently when they were large enough to be clearly seen. One might attribute them to flux trapped in the junction. However, they showed no correlation with the two effects which were clearly related to trapped flux—reduced zero-field supercurrent and asymmetric field dependence of the supercurrent. Another possibility is to imagine

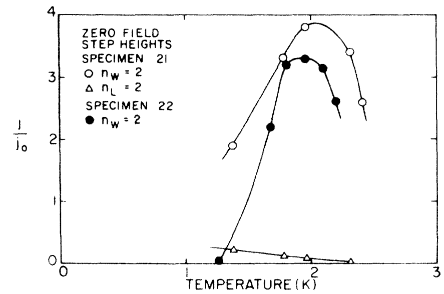

FIG. 9. Some zero-field step heights, divided by the supercurrent at the same temperature, against temperature. The solid lines are intended merely to be suggestive. The open and solid circles represent "large" steps, and the triangular points are for a "small" step with voltage beyond cutoff. For the circles, note the qualitative similarity with Fiske-mode behavior (as illustrated in Fig. 4).

that harmonic generation is occurring, somewhat as was predicted by Werthamer and Shapiro.[20] In other words, we can suppose that at, say, $V = \frac{1}{2}V_1$, the second harmonic of the ac supercurrent is exciting the $n = 1$ Fiske mode, but we have no evidence that this is what occurs. In particular, the observed field dependence for $n = \frac{1}{2}$ is not the same as for $n = 1$.

Zero-field steps

We distinguish between "small" and "large" zero-field steps. Small corresponds roughly to $j/j_0 \lesssim 0.03$ for all temperatures, while large steps have j/j_0 up to roughly 0.3 at some temperature. This distinction is somewhat arbitrary, since a "small" step might become large at a still lower temperature. Nevertheless our results do seem to fall into two groups.

Large zero-field steps have been reported by Fulton and Dynes[15] and by Chen *et al.*[21] The explanation given in the former work in terms of a moving vortex was mentioned in the Theory section. Our results are consistent with the previous measurements. Examples of temperature dependences are shown in Fig. 9. Note that the temperature dependence of the step height is qualitatively the same as for Fiske modes. In fact, in our two cases where a peak shows clearly, the maximum value of j/j_0 is not far from the Fiske mode value of 0.34. However this numerical correspondence is probably an accident since Chen *et al.* found values of j/j_0 as high as 0.55 for Pb-Pb junctions.

In Fig. 10 we show the effect of applying a magnetic field parallel to the direction of propagation of the vortex. In this case the corresponding Fiske mode is not excited, and we see the step height driven to zero rather sharply at r close to 0.4. Note, however, that for one case, the lowest tem-

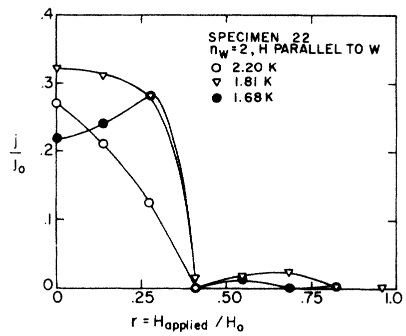

FIG. 10. Effect of a field on a zero-field vortex mode. The field is parallel to the presumed direction of propagation of the vortex and should not excite the corresponding Fiske mode. (Applying a field in the perpendicular direction gives similar results, but the behavior is confused by the appearance of the Fiske mode).

perature, the step height first rose and then fell. For the same conditions with specimen 21, the step went to zero at $r = 1$.

Our "large" zero-field steps were all consistent with the Fulton and Dynes model. Every even-n Fiske-mode voltage that was less than the predicted cutoff showed a large zero-field step. All zero-field steps that contradicted the model, namely, steps at odd n values or beyond cutoff, were small.

Figure 9 also shows an example of a "small" zero-field step, for $n_L = 2$. The voltage of this step was 0.55 mV. (This is close to the 0.50 mV corresponding to $n_W = 4$. However, when a field is applied, the Fiske mode at 0.55 mV peaks at about H_0, as it should for $n = 2$.) This voltage is well beyond the predicted cutoff voltage, which for this specimen is 0.30 mV. We do not know the origin of the small steps.

CONCLUSIONS

We feel that our results give strong support to Kulik's extension of Fiske-mode theory to high Q values. In particular, the prediction that, for a given mode, the maximum dc current passes through a maximum as Q increases is shown to be correct. Further, the predicted value of the maximum is verified, for the low n values studied, to within our accuracy of about 5%.

We find that the zero-field modes that are presumably due to vortex motion show a temperature dependence that is qualitatively similar to Fiske modes.

We have also observed two kinds of weak modes of unknown origin. These are (a) zero-field modes that do not fit the pattern of previous observations and of the Fulton and Dynes vortex picture—that is, they occur at non-even-integral positions or beyond the cutoff voltage; (b) modes at half-integral voltage positions. We cannot rule out the possibility that these modes are due to specimen flaws or to some unanticipated magnetic field.

We have shown that dc Fiske-mode studies can be used to accurately measure values of Q and $\bar{c}$ for junction resonances. However, the range of temperature over which this method can be applied is somewhat limited. Near the transition temperature and also at very low temperatures the dc current steps are very small.

Note added in proof. After this manuscript was accepted, we learned that K. Schwidtal and C. F. Smiley have also reported some comparisons of Kulik's theory with measurements on Pb-Pb and Nb-Pb junctions [in *Low Temperature Physics*-LT 13, edited by K. D. Timmerhaus, W. J. O'Sullivan, and E. F. Hammel (Plenum, New York, 1974), Vol. 4, p. 575]. They too report agreement with the predicted maximum step height.

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[1]M. D. Fiske, Rev. Mod. Phys. 36, 221 (1964).
[2]R. E. Eck, D. J. Scalapino, and B. N. Taylor, Phys. Rev. Lett. 13, 15 (1964).
[3]I. M. Dmitrenko, I. K. Yanson, and V. M. Svistunov, Zh. Eksp. Teor. Fiz. Pis'ma Red. 2, 17 (1965) [Sov. Phys. JETP Lett. 2, 10 (1965)].
[4]D. D. Coon and M. D. Fiske, Phys. Rev. 138, A744 (1965).
[5]I. M. Dmitrenko and I. K. Yanson, Zh. Eksp. Teor. Fiz. 49, 1741 (1965) [Sov. Phys.-JETP 22, 1190 (1965)].
[6]I. K. Yanson, V. M Svistunov, and I. M. Dmitrenko, Zh. Eksp. Teor. Fiz. 48, 976 (1965) [Sov. Phys.-JETP 21, 650 (1965)].
[7]D. N. Langenberg, D. J. Scalapino, B. N. Taylor, and R. E. Eck, Phys. Rev. Lett. 15, 294 (1965).
[8]I. M. Dmitrenko and I. K. Yanson, Zh. Eksp. Teor. Fiz. Pis'ma Red. 2, 242 (1965) [Sov. Phys. JETP Lett. 2, 154 (1965)].
[9]I. O. Kulik, Zh. Eksp. Teor. Fiz. Pis'ma Red. 2, 134 (1965) [Sov. Phys. JETP Lett. 2, 84 (1965)].
[10]R. E. Eck, D. J. Scalapino, and B. N. Taylor, in *Proceedings of the 9th International Conference on Low*

Temperature Physics, edited by J. G. Daunt (Plenum, New York, 1965), p. 415.
[11]I. O. Kulik, Zh. Tek. Fiz. 37, 157 (1967) [Sov. Phys.-Tech. Phys. 12, 111 (1967)].
[12]Ye. M. Ivanchenko, A. V. Svidzinskii, and V. A. Slyusarev, Zh. Eksp. Teor. Fiz. 51, 194 (1966) [Sov. Phys.-JETP 24, 131 (1967)].
[13]J. A. Blackburn, J. D. Leslie, and H. J. Smith, J. Appl. Phys. 42, 1047 (1971).
[14]The theory of Fiske modes is discussed in I. O. Kulik and I. K. Yanson, *The Josephson Effect in Superconductive Tunneling Structures*, translated by P. Gluck (Keter, Jerusalem, (1972)), as well as in Refs. 1–13.
[15]T. A. Fulton and R. C. Dynes, Solid State Commun. 12, 57 (1973).
[16]B. D. Josephson, Adv. Phys. 14, 419 (1965).
[17]J. C. Swihart, J. Appl. Phys. 32, 461 (1961).
[18]S. Paley, J. Wilson, and R. I. Gayley, Phys. Lett. A 41, 311 (1972).
[19]J. Wilson, T. F. Finnegan, and J. Toots, Bull. Am. Phys. Soc. 19, 204 (1974), and (unpublished).
[20]N. R. Werthamer and S. Shapiro, Phys. Rev. 164, 523 (1967).
[21]J. T. Chen, T. F. Finnegan, and D. N. Langenberg, Physica (Utr.) 55, 413 (1971).