Current-induced resistive state in cylindrical type-I superconductors*

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We have investigated the current-induced resistive state in superconducting indium wires through simultaneous measurements of the dc resistive voltage and the electrical noise power as the electrical transport current was increased from zero to beyond the critical value. The experiments were performed in zero applied magnetic field using indium wires with diameters of 0.17–0.52 mm. The measurements were made below the λ point to discriminate against contributions from thermal fluctuations in the specimens due to nucleate boiling of liquid helium. The noise power was measured in the frequency range 20 Hz–20 kHz. For two specimens with diameters of 0.17 mm, a peak in the noise power was observed at the critical value of the transport current. The noise-power spectrum of one of the smallest-diameter samples showed very weak dependence of noise power on frequency at low frequencies and approximately ω^{-2} behavior at high frequencies. The magnitude of the noise voltage at the noise-power peak was found to be at most 2.4×10^{-3}% of the dc resistive voltage. No ac voltages larger than 10^{-5}% of the dc voltage were detectable for samples with diameters of 0.27 and 0.52 mm. Our results cast doubt on the validity of the Gorter dynamic model of the current-induced intermediate state and tend to support a static model of the type first proposed by London.

I. INTRODUCTION

The destruction of superconductivity by an electrical current in a cylindrical type-I superconductor with circular cross section was first treated by London.[1] Considering a wire with radius r_0 large compared to the coherence length, electrical resistance is expected to appear as soon as the current-generated magnetic field at the surface of the wire exceeds the critical field H_c; i.e., for currents exceeding the critical value,

$$I_c = (\tfrac{1}{2}\, c r_0) H_c. \tag{1}$$

In London's analysis, the stable state of such a superconductor carrying a current $I > I_c$ consists of an outer shell of normal resistive metal surrounding an inner core of metal in the intermediate state. It is assumed that the core consists of alternate layers of normal and superconducting phase arranged perpendicular to the current. At $I = I_c$, the intermediate state extends to the surface of the wire, and, as the current is increased above the critical value, the diameter of the intermediate-state core shrinks, causing a rise in the electrical resistance according to

$$R = (\tfrac{1}{2} R_n)\{1 + [1 - (I_c/I)^2]^{1/2}\}, \tag{2}$$

where R_n is the normal-state resistance. More recently, London's model has been refined by Baird and Mukherjee[2,3] and Andreev[4] in order to account for the detailed manner[4–8] in which the resistance appears with increasing current, and to explain the fact that, at $I = I_c$, the resistance jumps to about 70% of its normal value rather than to 50% of its normal value as predicted by the London model. Recent measurements by Mukherjee[9] on pure thick indium and thallium wires were in good agreement with the predictions of the Baird-Mukherjee model.

A dynamic model of the current-induced resistive state in a wire, consisting of closed, circular collapsing flux tubes has been proposed by Gorter.[10,11] The fundamental frequency of the periodic mode is given by

$$f = \left(\frac{c^2 \rho_n}{4\pi r_0^2}\right) \frac{1}{\tau_f}, \tag{3}$$

for a wire of radius r_0 and normal-state resistivity ρ_n. The term $c^2\rho_n/4\pi r_0^2$ in Eq. (3) arises from the regulation of the speed of flux-tube collapse by eddy current damping, and τ_f is the reduced time between successive formations of flux tubes. Assuming that all flux tubes along the wire move in phase, Gorter calculated τ_f to be between 0.248 and 0.392 for $I/I_c = 1.1$, and estimated the amplitude of the ac component to be between 1.8% and 4.8% of the dc voltage. A search by Meissner[12] for such an oscillatory voltage component in two tin wires yielded an effect. However, he found the ac component to be not larger than (0.015–0.018)% of the dc voltage, indicating that, if such flux tube motion exists at all, it would be largely uncorrelated along the wire.

Interesting investigations of the electric and magnetic instabilities in the current-induced resistive state of type-I and type-II superconducting wires were reported by Lalevic.[13–15] He observed rather abrupt variations of the electrical resistance between metastable resistance levels. These effects can be understood in terms of abrupt rearrangements of the distribution of currents and magnetic fields within the specimens. Similar magnetic and electric instabilities are a general property of materials with a high value of the magnetoresistance,[16] and a superconductor is here the most extreme case.

In the following we report electrical-resistance and noise-power measurements on a series of superconducting indium wires in the current-induced resistive state. The experiments were performed in zero applied magnetic field, the only field present being that associated with the transport current itself. Our experiments were stimulated by recent results[17,18] on the current-induced resistive state in film strips of type-I superconductors, indicating a sharp peak in the electrical noise power at the onset of resistance. Further, a careful search was to be made for an ac voltage component resulting from a dynamic character of the current-induced resistive state, such as the one contained in the Gorter model. Our results raise doubt about the validity of the dynamic model by Gorter and tend to support a static model of the current-induced resistive state.

II. EXPERIMENTAL

The samples were indium wires of different diameters and lengths. The characteristic dimensions, which were measured with a traveling microscope, are listed in Table I. The indium wires were extruded through steel dies using In with 99.9999% purity. The diameters of the extruded wires were constant to within 0.01 mm. The specimens were mounted on glass slides, and voltage leads consisting of copper wires with diameters of 0.18 mm were attached with indium solder. The separation between the voltage probes and the ends of the specimens was approximately 0.5 cm. The samples were placed in direct contact with the liquid-helium bath and were enclosed in a copper can surrounded by a superconducting lead foil. The temperature of the helium bath could be controlled by pumping. The axes of the samples were aligned approximately parallel with the earth's magnetic field. The measurements were performed in zero applied magnetic field.

All electrical-resistance measurements were performed using the four-probe technique. The current source was an Intermagnetics Model 150-M constant current supply, whose current output is controlled by an electronic ramp generator providing adjustable rates of current increase or decrease. The current was raised at the rate of 600 mA/min until a dc voltage across the sample was detectable, at which point the rate of current increase was reduced to 150 mA/min. The current ramp could be interrupted at any time during the

TABLE I. Sample characteristics, including the temperatures at which the measurements were made, critical currents, observed critical fields obtained from Eq. (1), calculated critical fields from Ref. 19, normal-state resistivities obtained by linearly extrapolating $R(H)$ to $R(H=0)$, the ratios of $R(1\text{ kOe})$ to $R(0)$, the ranges of the fundamental frequency of the periodic mode expected from the Gorter dynamic model at $I/I_c = 1.1$ from Eq. (3), and normalized resistivity jumps at $I = I_c$.

Sample	Wire Diameter $2r_0$(mm)	Length l(mm)	$\frac{R(295\text{ K})}{R(4.2\text{ K})}$	T(K)	I_c(A)	$H_{c\,\text{obs}}$(Oe)	$H_{c\,\text{calc}}$(Oe)	$\rho_n(10^{-9}\ \Omega\text{ cm})$	$\frac{R(1\text{ kOe})}{R(0)}$	f(Hz)	$(R/R_n)_{I_c}$
In wire 6	0.52	8.90	3150	2.024	22.4	172	179	2.14	1.17	640–1020	0.67
In wire 3	0.27	8.89	870	2.000	12.1	179	182	11.72	1.04	13100–20600	0.75
In wire 4	0.17	9.38	2320	2.027	8.15	192	179	2.83	1.08	7960–12600	0.74
In wire 7	0.17	0.51	2320	2.035	7.88	185	178	2.83	1.08	7960–12600	0.74

sweep to hold the current at a particular value.
In order to reduce the amount of 60-Hz ripple current, a π-type filter consisting of two capacitors with $C = 0.21$ F and an inductor with $L = 0.8$ mH was inserted in the output of the current supply. The inductor was immersed in liquid nitrogen during a run. A 0.01-Ω standard resistor was placed in series with the specimen. The voltage across this resistor was fed to the X channel of an X-Y recorder. The voltage across the specimen was amplified by a Keithley Model 140 Precision Nanovoltmeter dc Amplifier and fed to the Y channel of the X-Y recorder to provide a continuous V-I plot. Simultaneously, the oscillatory voltage component across the sample was monitored using a step-up transformer (ratio 60:1) within the liquid-helium bath and a selective amplifier. The selective amplifier consisted of the type-C preamplifier and the intermediate amplifier and tuned amplifier section of a Princeton Applied Research Model HR-8 operated at 1-μV sensitivity. The signal then passed through a rectifier and entered an integrator which was controlled manually. Noise measurements were made between 20 Hz and 20 kHz using a constant quality factor Q of 25 in the tuned-amplifier section. The measurements were made at a constant temperature below the λ point to eliminate any noise contribution from nucleate boiling of the liquid helium at the surface of the samples. The background rms noise voltage $V_0(f)$ was at most a few nanovolts. The background noise level was determined during each run for all frequencies by reducing the output of the current source to zero, but still keeping the current source turned on. The maximum variation of the background voltage V_0 during a run, $\Delta V_0(f)$, was less than 1.5% of V_0 for all frequencies in the range 20 Hz–20 kHz. The background voltages were subtracted from the measured rms signal before the noise-power values were calculated. A calibration of the complete ac detection system and its frequency dependence was carried out for each individual run. For this purpose a small alternating current of known amplitude and various frequencies was superimposed on a direct current large enough to drive the sample normal at the temperature at which the experimental measurements were performed. Further details relating to the noise power measurements can be found elsewhere.[17] The primary of the step-up transformer served as a shunt in parallel with the sample. This shunt resistance was always at least three orders of magnitude larger than the sample resistance in the current range investigated.

III. EXPERIMENTAL RESULTS AND DISCUSSION

The temperatures at which the simultaneous measurements of the dc resistive voltage and the noise voltage were made and the critical currents for the specimen at the given temperature are listed in columns 5 and 6, respectively, of Table I. The value of the critical current and the radius of each sample were substituted into Eq. (1) to obtain the value of the critical field, $H_{c\,\text{obs}}$. Critical-field values were also calculated for each sample using $H_0 = 282.66$ Oe and $T_c = 3.407$ K and assuming a deviation from parabolicity of 2% in this temperature regime.[19] The values of the critical field obtained in this manner, $H_{c\,\text{calc}}$, are listed in column 8. The two values are in reasonable agreement for the specimens with a maximum disagreement of about 7% for indium wire 4.

The normal-state resistivities ρ_n for the specimens listed in Table I were obtained in the following manner. Each sample was placed in an applied magnetic field, which was increased from 250 to 1250 Oe in 250-Oe steps, at the same temperatures for which the dc resistive voltages and the noise voltages were measured. A V-I curve was plotted and the resistance was determined for each magnetic-field value. For each sample, the resistance was found to increase with increasing applied magnetic field. This magnetic-field dependence could be fitted linearly in the range investigated, and the normal-state resistance $R(H=0)$ was obtained by a linear extrapolation of the resistance to zero applied magnetic field. These values of $R(0)$ were then used to calculate the ρ_n values listed in column 9 of Table I. Values of $R(1\text{ kOe})/R(0)$ for the samples are listed in column 10. The normal-state resistivity and the radius for each sample were substituted into Eq. (3) to obtain the range of the fundamental frequency f of the periodic mode expected from the Gorter dynamic model at $I/I_c = 1.1$. This frequency range is indicated in column 11 of Table I.

The normalized resistance and the electrical noise power at different frequencies versus the normalized current of In wire 4 at $T = 2.027$ K and In wire 7 at $T = 2.035$ K are shown in Figs. 1 and 2, respectively. The normalized resistance and current values were obtained directly from the continuous V-I plot for each specimen. The resistance of all specimens showed a small increase before the occurrence of the rapid rise at the critical current. The electrical noise-power data were obtained with the selective amplifier of the lock-in detector tuned to the given frequencies with $Q = 25$. A sharp peak in the noise power at the critical current is observed for all frequencies measured in the range 20 Hz–20 kHz, with the largest value occurring at $f = 20$ Hz, the lowest frequency of the range investigated. These data suggest the occurrence of electric and magnetic fluctuations at the onset of the current-induced resistive state. Similar results have been reported for superconducting strips of lead and indium.[17,18]

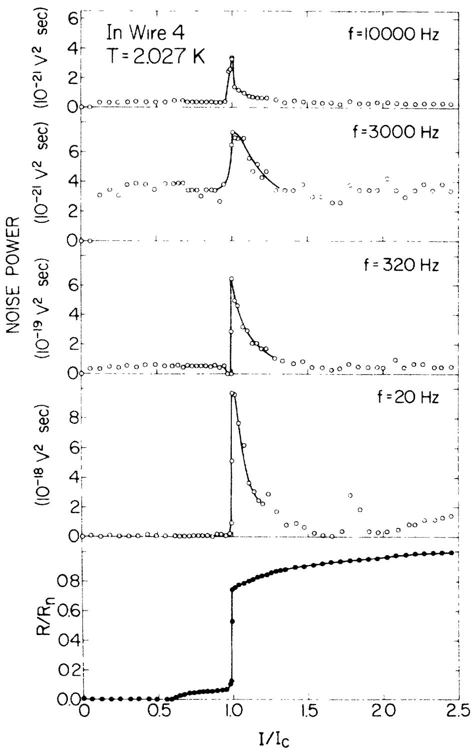

FIG. 1. Normalized resistance (bottom) and electrical noise power at different frequencies (top) vs normalized current of indium wire 4 at 2.027 K.

The noise voltage data for the two specimens are also presented in Tables II and III to allow a more direct comparison with the Gorter model. The background rms noise voltages V_0 and the maximum variation of the background rms noise voltages during a run ΔV_0 are listed in columns 3 and 4, respectively, for each frequency. The rms noise voltages obtained after the subtraction of the background rms noise voltages V_{ac} are given for $I = I_c$ and $I = 1.1\ I_c$ in column 5 of each table. Column 6 of the tables lists the values of the dc resistive voltage V_{dc} for $I = I_c$ and $I = 1.1\ I_c$, and, in the last column the ratio V_{ac}/V_{dc} is expressed in units of $10^{-3}\%$. We observe that the magnitude of the rms noise voltage at the noise-power peak is at most $2.4\times10^{-3}\%$ of the dc resistive voltage for each sample and that the maximum value of V_{ac}/V_{dc} occurs for $f = 20$ Hz, the lowest frequency of the range investigated.

Our noise data for In wire 4 and In wire 7 are in disagreement with the predictions of the Gorter dynamic model of the current-induced resistive state. The largest observed noise voltages occur at much lower frequencies than those given by Eq. (3) and listed in Table I. In addition, the ratio of the noise voltage to the dc resistive voltage is only about 10^{-3} of the value predicted by the model.

Such a small ac voltage component might be explained by a lack of phase coherence along the wire among the collapsing flux tubes. If the dynamic model of collapsing flux tubes would be applicable with phase coherence only within short sections of the wire, the relative magnitude of the ac voltage component would be expected to increase with decreasing specimen length. However, similar results were obtained for the two samples, although In wire 7 is only about 5% as long as In wire 4. No noise voltages larger than $10^{-5}\%$ of the dc voltage, the detection limit of the experiment, were obtained for the two larger diameter samples, In wire 6 and In wire 3. Additional measurements made on these two samples to within 0.1 K of T_c, where any phase coherence would be expected to be enhanced, also yielded noise voltages of similarly low magnitude.

The noise-power spectrum for indium wire 4 at $T = 2.027$ K and $I = I_c$ is given in Fig. 3. The spectrum shows very weak dependence of the noise power on frequency at low frequencies and approximately ω^{-2} behavior at high frequencies. Such a frequency dependence of the noise-power spectrum at high frequencies can arise from the following model. Consider statistically independent rectangular voltage pulses of height Φ and average

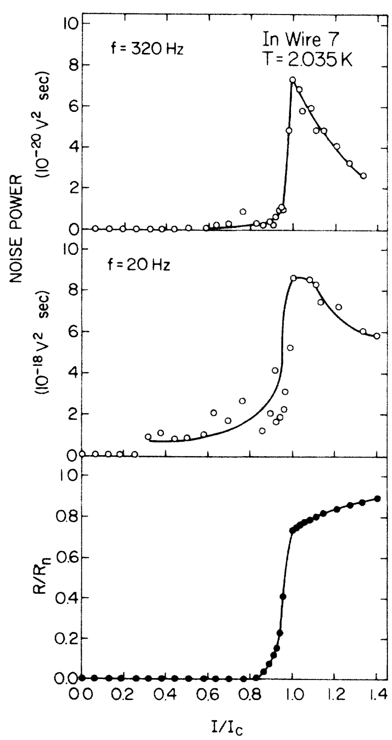

FIG. 2. Normalized resistance (bottom) and electrical noise power at different frequencies (top) vs normalized current of indium wire 7 at 2.035 K.

TABLE II. Noise-voltage results at different frequencies for indium wire 4 at 2.027 K for $I/I_c=1.0$ and $I/I_c=1.1$. V_0 is the background rms noise voltage. ΔV_0 is the maximum variation of the background rms noise voltage during a run. The rms noise voltage after subtraction of the background rms noise voltage is given by V_{ac}. V_{dc} is the value of the dc resistive voltage.

f (Hz)	I/I_c	V_0 (10^{-10} V)	ΔV_0 (10^{-11} V)	V_{ac} (10^{-10} V)	V_{dc} (10^{-6} V)	V_{ac}/V_{dc} (10^{-3}%)
20	1.0	2.6	0.2	10.8	71	2.42
	1.1			5.9	84	0.70
320	1.0	6.3	0.8	7.7	71	1.42
	1.1			4.3	84	0.51
3 000	1.0	5.0	0.7	1.4	71	0.43
	1.1			1.2	84	0.14
10 000	1.0	3.4	0.3	2.8	71	0.36
	1.1			2.9	84	0.11

pulse rate ν constituting the time-dependent voltage $V(t)$. If the voltage pulses have an exponential distribution of their lifetime τ as indicated by the autocorrelation function

$$\langle V(t)V(t+\tau)\rangle_{av} = \Phi^2 \nu \tau_0 e^{-\tau/\tau_0}, \quad (4)$$

the Wiener-Khintchine theorem yields the noise-power spectrum[20]

$$w(f) = 4\langle V\rangle_{av}\Phi \frac{\tau_0}{1+4\pi^2 f^2 \tau_0^2} . \quad (5)$$

The data shown in Fig. 3 have the frequency dependence of Eq. (5) and indicate voltage fluctuations characterized by a single time constant $\tau_0 = 2.6\times10^{-3}$ sec. It is likely that the voltage fluctuations are caused by rearrangements of the distribution of currents and magnetic fields within the wire. The speed of such rearrangements is, of course, limited by eddy current damping. The lifetime $\tau_0 = 2.6\times10^{-3}$ sec, taken from Fig. 3, is about 50 times longer than the time required for a flux change to propagate through the wire, as given by the first term in Eq. (3).

One can use Eq. (5) further for calculating the height Φ of the voltage pulses with lifetime τ_0 given above. Using the data contained in Fig. 3 and Table II, we find $\Phi = 2.4\times10^{-11}$ V. This value of Φ would correspond to an equivalent normal section of the wire with a length of 2.4×10^{-3} μm. The temporary formation of a normal domain of such small size, extending across the total cross section of the sample, appears unlikely for an indium wire of 0.17-mm diameter.[21] Therefore our interpretation of the noise-power data in terms of the statistical nucleation of normal domains would require the formation of domains occupying only a small fraction of the cross section of the wire.

Recently Landau[22] reported interesting results on the paramagnetic effect in a type-I superconductor. Here, the destruction of superconductivity by a current has been investigated in the presence of a magnetic field H_e applied parallel to the axis of the cylindrical specimen. In the magnetic-field regime $0.1 < H_e/H_c < 0.4$ Landau observed current-induced periodic oscillations in both the magnetization and the electrical resistance. These oscillations strongly suggest a dynamic state consisting of coaxial layers of a two-dimensional mixed state[23,24] moving toward the sample axis, conceptually similar to the Gorter dynamic model. In the magnetic field regimes $H_e/H_c < 0.1$ and $H_e/H_c > 0.4$, Landau did not observe any oscillations. This appears to be consistent with our results, which refer to the limit $H_e/H_c = 0$.

Figure 4 shows the normalized resistance versus normalized current as predicted by the London

TABLE III. Noise-voltage results at two different frequencies for indium wire 7 at $T = 2.035$ K. The column headings are the same as in TABLE II.

f (Hz)	I/I_c	V_0 (10^{-10} V)	ΔV_0 (10^{-11} V)	V_{ac} (10^{-10} V)	V_{dc} (10^{-6} V)	V_{ac}/V_{dc} (10^{-3}%)
20	1.0	17.2	0.2	2.5	10.4	2.40
	1.1			2.4	13.7	1.75
320	1.0	2.8	0.4	1.5	10.4	1.42
	1.1			1.4	13.7	1.02

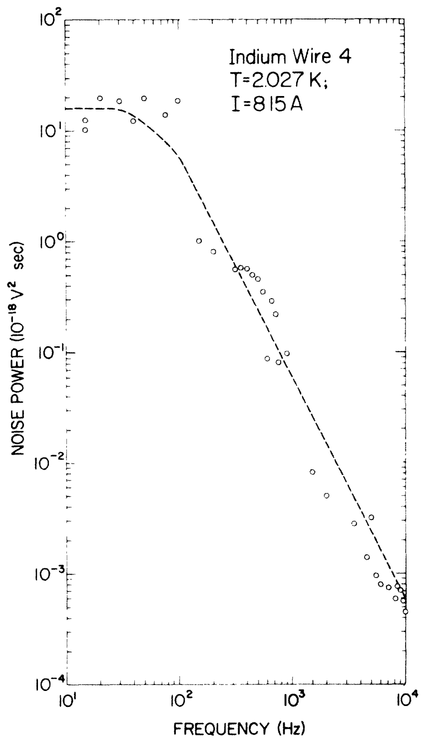

FIG. 3. Noise-power spectrum at 2.027 K of indium wire 4. The data were taken at the peak of the noise power at $I_c = 8.15$ A.

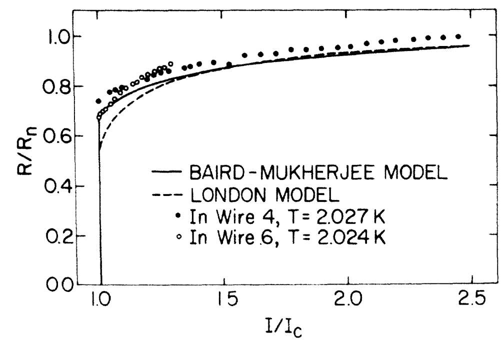

FIG. 4. Normalized resistance vs normalized current of indium wire 4 at 2.027 K and indium wire 6 at 2.024 K compared with theoretical curves for the Baird-Mukherjee and London models.

and Baird-Mukherjee static models of the current-induced resistive state. Also included in the figure are experimental points for In wire 4 and In wire 6. The quantity $(R/R_n)_{I_c}$ for each sample is given in the last column of Table I. The resistance jumps at $I = I_c$ are in close agreement with the Baird-Mukherjee model. Above about $I = 1.3\ I_c$ the predictions of the models by London and by Baird and Mukherjee are nearly identical, and our results are in reasonable agreement with each model.

IV. CONCLUSIONS

In the frequency range 20 Hz–20 kHz noise voltages associated with the current-induced resistive state have been detected only in the two indium wires with 0.17-mm diameter. No noise voltages larger than $10^{-5}\%$ of the dc voltage were detected in the larger diameter samples. For the two samples with smallest diameter a distinct peak in the noise power was observed at the critical current. These noise-voltage results conflict with the predictions of the dynamic Gorter model of the current-induced resistive state in two respects. First, maximum noise was observed at a frequency of 20 Hz, the lowest frequency in the range investigated and nearly three orders of magnitude smaller than the dominant frequency predicted by the Gorter model. Second, the magnitude of the noise voltage at the noise-power peak is at most $2.4 \times 10^{-3}\%$ of the dc resistive voltage, in contrast to the prediction of a value of a few percent by the Gorter model. A possible explanation of such a small ac voltage component by a lack of phase coherence along the wire among the collapsing flux tubes of the Gorter model seems unlikely because of the similarity of the results obtained with two specimens which differed in length by a factor of 20. The noise-power spectrum in the smallest diameter wire showed a frequency dependence consistent with statistically independent voltage fluctuations of constant amplitude and lifetime. These fluctuations are likely to be attributed to rearrangements of currents and magnetic fields in the wire. Our results cast doubt on the validity of the Gorter model. Finally, we note that the value of the normalized resistance jumps at the critical current are in close agreement with the predictions of the Baird-Mukherjee model. Above about $1.3\ I_c$, the experimental data are in good agreement with either the London or the Baird-Mukherjee model of the current-induced resistive state.

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[1]F. London, *Une Conception Nouvelle de la Superconductibilité* (Hermann et Cie, Paris, 1937); *Superfluids* (Wiley, New York, 1950), Vol. I.

[2]D. C. Baird and B. K. Mukherjee, Phys. Rev. Lett. 21, 996 (1968).

[3]D. C. Baird and B. K. Mukherjee, Phys. Rev. B 3, 1043 (1971).

[4]A. F. Andreev, Zh. Eksp. Teor. Fiz. 54, 1510 (1968)

[Sov. Phys.-JETP 27, 809 (1968)].
[5]R. B. Scott, J. Res. Natl. Bur. Stand. (U.S.) 41, 581 (1948).
[6]L. Rinderer, Helv. Phys. Acta 29, 339 (1956).
[7]H. Meissner and R. Zdanis, Phys. Rev. 109, 681 (1958).
[8]R. Freud, Cz. Sulkowski, and B. Makiej, Phys. Lett. A 27, 187 (1968).
[9]B. K. Mukherjee, J. Low Temp. Phys. 12, 181 (1973).
[10]C. J. Gorter, Physica (Utr.) 23, 45 (1957).
[11]C. J. Gorter and M. L. Potters, Physica (Utr.) 24, 169 (1958).
[12]H. Meissner, Phys. Rev. 113, 1183 (1959).
[13]B. Lalevic, Phys. Rev. 128, 1070 (1962).
[14]B. Lalevic, J. Appl. Phys. 35, 1785 (1964).
[15]B. Lalevic, Phys. Status Solidi 9, 63 (1965).
[16]M. Ya. Azbel, Zh. Eksp. Teor. Fiz. Pis'ma Red. 10, 550 (1969) [JETP Lett. 10, 351 (1969)].
[17]R. P. Huebener and D. E. Gallus, Phys. Rev. B 7, 4089 (1973).
[18]R. P. Huebener and H. L. Watson, Phys. Rev. B 9, 3725 (1974).
[19]D. K. Finnemore and D. E. Mapother, Phys. Rev. 140, A507 (1965).
[20]D. K. C. MacDonald, *Noise and Fluctuations* (Wiley, New York, 1962).
[21]R. P. Huebener and R. T. Kampwirth, Phys. Status Solidi 13, 255 (1972).
[22]I. L. Landau, Zh. Eksp. Teor. Fiz. 64, 557 (1973) [Sov. Phys.-JETP 37, 285 (1973)].
[23]I. L. Landau and Yu. V. Sharvin, Zh. Eksp. Teor. Fiz. Pis'ma Red. 10, 192 (1969) [JETP Lett. 10, 121 (1969)].
[24]I. L. Landau and Yu. V. Sharvin, Zh. Eksp. Teor. Fiz. Pis'ma Red. 15, 88 (1972) [JETP Lett. 15, 59 (1972)].