Propagation and instability of microwaves in extrinsic InSb subject to crossed static electric and magnetic fields

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The propagation of an electromagnetic wave in a direction perpendicular to both the applied static magnetic field $\vec{B}_0$ and the static electric field $\vec{E}_0$ in extrinsic indium antimonide (InSb), and wave instability are studied theoretically. The dispersion equation $D(\omega, k)$ relating the wave angular frequency ω and the wave number k is derived from Maxwell's equations, the equation of momentum transfer, and the continuity equation, using the magnetohydrodynamic approach and a one-dimensional linearized theory. With the aid of the dispersion relationship, the propagation characteristics of the slow electromagnetic wave in a collision-dominated semiconductor plasma is examined in detail, for both n-type and p-type materials. The range of parameters considered are $1 \leq f \leq 9$ GHz, $1 \leq B_0 \leq 10$ kG, and $0 \leq E_0 \leq 30$ V/cm, and the variation of the phase velocity of the wave, v_φ, and the amplitude constant of the wave, $\alpha \equiv \mathrm{Im}(k)$, with the parameters B_0, E_0, and the wave frequency f, are investigated. It is shown that under proper conditions wave amplification, defined by $\beta\alpha > 0$, where $\tilde{k} = \beta + i\alpha$, and the wave instability, defined by $\omega_r\omega_i < 0$, where $\tilde{\omega} = \omega_r + i\omega_i$, is possible. For example, under the conditions $(\omega_R/\omega)^2 < 1 + \mu_s^2 B_0^2$ and $|\nabla n_0/n_0| \ll |k|$, the threshold condition for the wave amplification, the threshold condition for wave instability, the spatial growth rate ($\alpha > 0$), the phase velocity v_φ, and the threshold oscillation angular frequency $\omega_0 = \omega_r$ (for which $\omega_i = 0$) as functions of E_0, B_0, f, and n_0 are derived, where ω_R and n_0 denote the dielectric-relaxation angular frequency and the carrier density of the material, respectively. μ_s denotes the carrier drift mobility which takes a negative value for the electron. The effect of the carrier density gradient on v_φ, α, and E_{th} (the threshold electric field for instability) is also briefly discussed.

INTRODUCTION

In recent years, a great deal of attention has been given to the study of microwave-emission phenomena from InSb subject to crossed static electric and magnetic fields, by various authors.[1-8] It is generally believed[9-11] that some sort of instabilities in the semiconductor plasma might be responsible for these emissions. However, it appears that little attention has been given to the question of how an electromagnetic wave in the microwave-frequency range may travel once it is excited within the materials subject to the crossed-static-fields configuration.

The purpose of this paper is to report a study of the propagation characteristics and the possibility of instability of a microwave traveling in a direction perpendicular to both a static applied electric field $\vec{E}_0$ and magnetic field $\vec{B}_0$, where $\vec{E}_0 \perp \vec{B}_0$. It should be noted that this particular choice of static-field configuration is of interest because it permits the coupling of electromagnetic-wave energy into and out of the semiconductor plasma. Using the one-dimensional small-signal linearized theory, the dispersion equation for the electromagnetic wave under consideration is derived in Sec. II from the Maxwell equations, the equation of momentum transfer, and the continuity equation in the magnetohydrodynamic approach. With the aid of the dispersion relationship, the propagation characteristics of the electromagnetic wave is studied in detail for the collision-dominated case in extrinsic InSb. The possibility of instability of the slow electromagnetic wave is investigated and the threshold condition for the wave instability is also derived.

BASIC EQUATIONS AND DISPERSION EQUATIONS

The electromagnetic fields in the semiconductor are governed by Maxwell's equations:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \tag{1a}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}, \tag{1b}$$

$$\nabla \cdot \vec{D} = \rho, \tag{1c}$$

$$\nabla \cdot \vec{B} = 0, \tag{1d}$$

where the current density $\vec{J}$ and the carrier charge density ρ are given by

$$\vec{J} = \sum_s q_s n_s \vec{v}_s \tag{2a}$$

and

$$\rho = \sum_s q_s n_s, \tag{2b}$$

in which n_s, $\vec{v}_s$ and q_s denotes the carrier concentration, carrier velocity, and carrier charge, respectively. The subscripts s take either e for the electron or h for the hole, and $q_e=-e$ and $q_h=+e$, where $e>0$.

The motion of carrier type s is governed by the equation of momentum transfer, and written in the following form:

$$\frac{d\vec{v}_s}{dt}=\frac{q_s}{m_s}(\vec{E}+\vec{v}_s\times\vec{B})-\nu_s\vec{v}_s-\frac{v_{Ts}^2}{2}\frac{\nabla n_s}{n_s}, \tag{3}$$

where v_{Ts} denotes the carrier thermal velocity.

The carrier concentration and carrier velocity are related by the continuity equation

$$\nabla\cdot(n_s\vec{v}_s)+\frac{\partial n_s}{\partial t}=0. \tag{4}$$

Each of the variables, electric field intensity $\vec{E}$, magnetic flux density $\vec{B}$, velocity $\vec{v}$, and the carrier concentration n, is assumed to be the sum of an equilibrium or the time-invariant part (subscript 0) and a small time-varying part (subscript 1), e.g.,

$$n=n_0+n_1e^{i(\omega t-\vec{k}\cdot\vec{r})}.$$

Here ω and $\vec{k}$ denote, respectively, the wave angular frequency and the propagation vector. The static system of equations is given by

$$\nabla\times\vec{E}_0=0, \tag{5a}$$

$$\nabla\times\vec{H}_0=\vec{J}_0, \tag{5b}$$

$$\nabla\cdot\vec{D}_0=\rho_0, \tag{5c}$$

$$\nabla\cdot\vec{B}_0=0, \tag{5d}$$

$$\vec{J}_0=\sum_s q_s n_{0s}\vec{v}_{0s}, \tag{6}$$

$$\frac{q_s}{m_s}(\vec{E}_0+\vec{v}_{0s}\times\vec{B}_0)-\nu_s\vec{v}_{0s}-\frac{v_{Ts}^2}{2}\frac{\nabla n_{0s}}{n_{0s}}=0, \tag{7}$$

and

$$\nabla\cdot(n_{0s}\vec{v}_{0s})=0. \tag{8}$$

On the other hand, under the linearized theory, the time-varying system of equations is given as follows:

$$\vec{k}\times\vec{E}_1=\omega\mu_0\vec{H}_1, \tag{9a}$$

$$k\times\vec{H}_1=i\vec{J}_1-\omega\epsilon_l\epsilon_0\vec{E}_1, \tag{9b}$$

$$\vec{k}\cdot\vec{D}_1=i\rho_1, \tag{9c}$$

$$\vec{k}\cdot\vec{B}_1=0, \tag{9d}$$

$$\vec{J}_1=\sum_s q_s(n_{0s}\vec{v}_{1s}+\vec{v}_{0s}n_{1s}), \tag{10a}$$

$$\rho_1=\sum_s q_s n_{1s}, \tag{10b}$$

$$[i(\omega-\vec{k}\cdot\vec{v}_{0s})+\nu_s]\vec{v}_{1s}+\vec{v}_{1s}\times\vec{\omega}_{cs}=\frac{q_s}{m_s}(\vec{E}_1+\vec{v}_{0s}\times\vec{B}_1)+i\frac{v_{Ts}^2}{2}\left(\frac{n_{1s}}{n_{0s}}\right)\vec{k}, \tag{11}$$

where $\vec{\omega}_{cs}\equiv-q_s\vec{B}_0/m_s$ in which m_s denotes the carrier effective mass, and

$$\nabla\cdot[(n_{0s}\vec{v}_{1s}+n_{1s}\vec{v}_{0s})e^{i(\omega t-\vec{k}\cdot\vec{r})}]+i\omega n_{1s}e^{i(\omega t-\vec{k}\cdot\vec{r})}=0. \tag{12}$$

From Eqs. (8) and (12),

$$\frac{n_{1s}}{n_{0s}}=\frac{(\vec{k}\cdot\vec{v}_{1s})+i\vec{\gamma}_s\cdot\vec{v}_{1s}}{\omega-\vec{k}\cdot\vec{v}_{0s}+i(\vec{\gamma}_s\cdot\vec{v}_{0s})}, \tag{12a}$$

where

$$\vec{\gamma}_s\equiv\nabla n_{0s}/n_{0s}. \tag{12b}$$

The combination of Eqs. (9a) and (9b) gives

$$\vec{k}\times(\vec{k}\times\vec{E}_1)+k_0^2\vec{E}_1=i\omega\mu_0\vec{J}_1. \tag{13}$$

By combining Eqs. (10a) and (12a),

$$\vec{J}_1=\sum_s q_s n_{0s}\left[\vec{v}_{1s}+\vec{v}_{0s}\left(\frac{(\vec{k}+i\vec{\gamma}_s)\cdot\vec{v}_{1s}}{\omega'+i\vec{\gamma}_s\cdot\vec{v}_{0s}}\right)\right], \tag{14}$$

and the combination of Eqs. (9a), (11), and (12b) yields

$$(i\omega'+\nu_s)\vec{v}_{1s}+\vec{v}_{1s}\times\vec{\omega}_{cs}-i\frac{v_{Ts}^2}{2}\vec{k}\left(\frac{(\vec{k}+i\vec{\gamma}_s)\cdot\vec{v}_{1s}}{\omega'+i\vec{\gamma}_s\cdot\vec{v}_{0s}}\right)=\frac{q_s}{m_s}\left(\frac{\omega'}{\omega}\right)\vec{E}_1+\frac{q_s}{m_s}\frac{\vec{k}(\vec{v}_{0s}\cdot\vec{E}_1)}{\omega}, \tag{15}$$

where

$$\omega'\equiv\omega-\vec{k}\cdot\vec{v}_{0s},$$

$$k_0^2\equiv\omega^2\mu_0\epsilon_0\epsilon_l=\omega^2/c^2,$$

and c = the speed of light in the material.

Upon the elimination of $\vec{J}_1$ and $\vec{v}_{1s}$ from Eqs. (13)–(15), a vector equation governing $\vec{E}_1$ can be obtained, which is expressible in the form $\overleftrightarrow{G}\cdot\vec{E}_1=0$, where $\overleftrightarrow{G}=\{G_{ij}\}$ is a square matrix and $\vec{E}_1$ is a column matrix. Consequently, the desired dispersion relation is given by setting the determinant of $\overleftrightarrow{G}$ equal to zero, i.e., $\|G_{ij}\|=0$.

The element of the determinant G_{ij} depends upon the static velocity of carrier which is determined by the static electric and magnetic field configuration. From Eq. (7),

$$\vec{v}_{0s}+\vec{v}_{0s}\times\vec{\eta}_s=\mu_s\vec{E}_0-D_s\vec{\gamma}_s, \tag{16}$$

where

$$\mu_s=q_s/m_s\nu_s,$$

$$\eta_s=\omega_{cs}/\nu_s,$$

$$D_s=v_{Ts}^2/2\nu_s=KT_s/m_s\nu_s,$$

in which $q_s = +e$ for hole and $q_s = -e$ for electron. μ_s and D_s denote the carrier drift mobility and carrier diffusion coefficient, respectively. The components of the static-carrier velocity, $\vec{v}_{0s}$, can be given by solving Eq. (16), in a Cartesian coordinate system:

$$v_{0x} = \frac{1}{1+\eta_0^2}[u_x(1+\eta_x^2) + u_y(\eta_x\eta_y - \eta_z) + u_z(\eta_y + \eta_z\eta_x)],$$

$$v_{0y} = \frac{1}{1+\eta_0^2}[u_x(\eta_z + \eta_x\eta_y) + u_y(1+\eta_y^2) + u_z(\eta_y\eta_z - \eta_x)], \quad (17a)$$

$$v_{0z} = \frac{1}{1+\eta_0^2}[u_x(\eta_z\eta_x - \eta_y) + u_y(\eta_x + \eta_y\eta_z) + u_z(1+\eta_z^2)],$$

where

$$\vec{u} = \mu_s \vec{E}_0 - D_s \vec{\gamma} \quad (17b)$$

and

$$\eta_0^2 = |\vec{\eta}|^2 = \eta_x^2 + \eta_y^2 + \eta_z^2 . \quad (17c)$$

Suppose that the spatial rate of change of the static quantity is much smaller than that of the time-varying quantity, i.e., $|\vec{\gamma}_s| \ll |\vec{k}|$. Then by taking $\vec{k} = (0, 0, k)$, and $\vec{B}_0 = (B_{0x}, B_{0y}, 0)$, for the case of an extrinsic semiconductor, either p type or n type, after some algebraic manipulation the elements of the determinant, G_{ij}, are obtained as follows:

$$G_{xx} = (k_0^2 - k^2) - \frac{k_p^2}{\Delta}\left[\left(\frac{\omega'}{\omega}\right)(PQ - Y_x^2) + \left(\frac{kv_{0x}}{\omega'}\right)\left(\frac{kv_{0x}}{\omega}\right)P^2\right],$$

$$G_{xy} = \frac{k_p^2}{\Delta}\left\{\left(\frac{\omega'}{\omega}\right)Y_xY_y + iP\left[Y_y\left(\frac{kv_{0y}}{\omega}\right) + Y_x\left(\frac{kv_{0x}}{\omega}\right)\right] - \left(\frac{kv_{0x}}{\omega'}\right)\left(\frac{kv_{0y}}{\omega}\right)P^2\right\},$$

$$G_{xz} = \frac{k_p^2 P}{\Delta}\left[iY_y - \left(\frac{kv_{0x}}{\omega'}\right)P\right],$$

$$G_{yx} = \frac{k_p^2}{\Delta}\left\{\left(\frac{\omega'}{\omega}\right)Y_xY_y - iP\left[Y_y\left(\frac{kv_{0y}}{\omega}\right) + Y_x\left(\frac{kv_{0x}}{\omega}\right)\right] - \left(\frac{kv_{0y}}{\omega'}\right)\left(\frac{kv_{0x}}{\omega}\right)P^2\right\}, \quad (18)$$

$$G_{yy} = (k_0^2 - k^2) - \frac{k_p^2}{\Delta}\left[\left(\frac{\omega'}{\omega}\right)(PQ - Y_y^2) + \left(\frac{kv_{0y}}{\omega'}\right)\left(\frac{kv_{0y}}{\omega}\right)P^2\right],$$

$$G_{yz} = -\frac{k_p^2 P}{\Delta}\left[iY_x + \left(\frac{kv_{0y}}{\omega'}\right)P\right],$$

$$G_{zx} = -\frac{k_p^2 P}{\Delta}\left[iY_y + \left(\frac{kv_{0x}}{\omega'}\right)P\right],$$

$$G_{zy} = \frac{k_p^2 P}{\Delta}\left[iY_x - \left(\frac{kv_{0y}}{\omega'}\right)P\right],$$

$$G_{zz} = k_0^2 - \frac{k_p^2 P^2}{\Delta}\left(\frac{\omega}{\omega'}\right),$$

where

$$\Delta \equiv P(PQ - Y_x^2 - Y_y^2),$$

$$P \equiv \omega'/\omega - jZ, \quad k_p^2 = \omega_p^2/c^2,$$

$$Q \equiv P - \frac{v_{Ts}^2}{2}\left(\frac{k^2}{\omega\omega'}\right), \quad \omega_p^2 = \left(\frac{q_s^2 n_0}{m_s \epsilon_0 \epsilon_l}\right),$$

$$\omega' \equiv \omega - kv_{0z},$$

$$Y_x = Y_0 \cos\varphi, \quad Y_y = Y_0 \sin\varphi, \quad Y_0 = \omega_{cs}/\omega,$$

$$B_{0x} = B_0\cos\varphi, \quad B_{0y} = B_0 \sin\varphi, \quad Z = \nu_s/\omega .$$

For the case of the crossed-static-electric and magnetic fields configuration of interest, by taking $\vec{E}_0 = (E_0, 0, 0)$, and $\vec{B}_0 = (0, B_0, 0)$, i.e., $\varphi = \frac{1}{2}\pi$, the expressions in Eqs. (17a) and (18) are simplified considerably. In this case, $Y_x = 0$, $Y_y = Y_0$, $\eta_x = 0$, $\eta_y = \eta_0$, $\eta_z = 0$, so that the static velocity is given by

$$v_{0x} = \frac{1}{1+\eta_0^2}(u_x + u_z\eta_0),$$

$$v_{0y} = u_y, \quad (19a)$$

$$v_{0z} = \frac{1}{1+\eta_0^2}(-u_x\eta_0 + u_z),$$

where

$$u_x = \mu_s E_0 - D_s\gamma_x = \mu_s E_0 - D_s \frac{1}{n_0}\frac{\partial n_0}{\partial x},$$

$$u_y = -D_s\gamma_y = -D_s\frac{1}{n_0}\frac{\partial n_0}{\partial y}, \quad (19b)$$

$$u_z = -D_s\gamma_z = -D_s\frac{1}{n_0}\frac{\partial n_0}{\partial z}.$$

Assuming that $\partial n_0/\partial y = 0$, then $u_y = 0$ so that $v_{0y} = 0$. For this case, the desired dispersion relationship is given by

$$\begin{vmatrix} G_{xx} & 0 & G_{xz} \\ 0 & G_{yy} & 0 \\ G_{zx} & 0 & G_{zz} \end{vmatrix} = 0, \quad (20)$$

where

$$G_{xx} = (k_0^2 - k^2) - \frac{k_p^2 P}{\Delta}\left[\left(\frac{\omega'}{\omega}\right)Q + \left(\frac{k v_{0x}}{\omega'}\right)\left(\frac{k v_{0x}}{\omega}\right)P\right],$$

$$G_{xz} = -\frac{k_p^2 P}{\Delta}\left[\left(\frac{k v_{0x}}{\omega'}\right)P - iY\right],$$

$$G_{yy} = (k_0^2 - k^2) - \frac{k_p^2}{\Delta}\left[\left(\frac{\omega'}{\omega}\right)(PQ - Y^2)\right], \qquad (21)$$

$$G_{zx} = -k_p^2 \frac{P}{\Delta}\left[\left(\frac{k v_{0x}}{\omega'}\right)P + iY\right],$$

$$G_{zz} = k_0^2 - \frac{k_p^2 P^2}{\Delta}\left(\frac{\omega}{\omega'}\right).$$

Expansion of Eq. (20) gives

$$G_{yy}(G_{xx}G_{zz} - G_{xz}G_{zx}) = 0\,, \qquad (22a)$$

which implies that

$$G_{yy} = 0 \qquad (22b)$$

or

$$G_{xx}G_{zz} - G_{xz}G_{zx} = 0\,. \qquad (22c)$$

This suggests that the mode linearly polarized in the y direction, i.e., in the $\vec{B}_0$ direction is uncoupled to the mode polarized in the x or z direction. Using the fact that $\Delta = P(PQ - Y^2)$ for the case under consideration, Eq. (22b) gives the dispersion equation for the mode linearly polarized in the direction of $\vec{B}_0$, as

$$\frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2(\omega - k v_{0z})}{\omega^2(\omega - k v_{0z} - i\nu_s)}, \qquad (23)$$

on the other hand, under the assumption

$$|\omega'/\omega| \ll |\nu/\omega|\,, \quad \text{and } v_{Ts} = 0\,. \qquad (24)$$

Equation (22c) gives the dispersion equation of the mixed mode (hybrid mode) as follows:

$$\left[k_p^2 Z\left(\frac{\omega'}{\omega}\right)^2 + i(k_0^2 - k^2)(Z^2 + Y^2)\left(\frac{\omega'}{\omega}\right) + k_p^2 Z\left(\frac{k v_{0x}}{\omega}\right)^2\right] \times \left[Z k_p^2 + i k_0 (Z^2 + Y^2)\left(\frac{\omega'}{\omega}\right)\right] + k_p^4\left[Y^2\left(\frac{\omega'}{\omega}\right)^2 - Z^2\left(\frac{k v_{0x}}{\omega}\right)^2\right] = 0\,. \qquad (25)$$

PROPAGATION CHARACTERISTICS OF AN ELECTROMAGNETIC WAVE IN A COLLISION-DOMINATED PLASMA

For the collision-dominated plasma, we can take

$$|\omega - k v_{0z}| \ll \nu_s\,, \qquad (26)$$

so that Eq. (23) becomes a quadratic equation in in k or in ω. It is not difficult to show that there is no spatial growth or instability of wave possible for this mode. Consequently, no further consideration is given to this mode. On the other hand, for the hybrid mode described by Eq. (25), it can be shown that under a proper condition an instability of wave can exist, which is to be shown in Sec. IV. However, it is of interest to examine the propagation characteristic of the hybrid mode here. For convenience of discussion, Eq. (25) is written in the following dimensionless form:

$$A_4\xi^4 + A_3\xi^3 + A_2\xi^2 + A_1\xi + A_0 = 0\,, \qquad (27)$$

where

$$A_4 = \delta_z^2(Z^2 + Y^2)\,,$$

$$A_3 = -\delta_z[2(Z^2 + Y^2) - iZX(1 - \delta_x^2 - \delta_z^2)]\,,$$

$$A_2 = [(Z^2 + Y^2)(1 - \delta_z^2) + \delta_z^2 X^2] - iZX(1 - \delta_x^2 - 3\delta_z^2)\,,$$

$$A_1 = -2\delta_z\{[X^2 - (Z^2 + Y^2)] + i2ZX\}\,,$$

$$A_0 = [X^2 - (Z^2 + Y^2)] + i2ZX\,,$$

where

$$\delta_x \equiv v_{0x}/c\,, \quad X = \omega_p^2/\omega^2\,,$$

$$\delta_z \equiv v_{0z}/c\,,$$

$$\xi \equiv ck/\omega\,.$$

Equation (27), being a quartic equation in ξ or k, has four roots, which are written in the form,

$$k_l = \beta_l + j\alpha_l\,,$$

where $l = 1$, 2, 3, and 4.

Once the values of the static-field strengths E_0, B_0, and the wave frequency f are specified, and if ∇n_0 is also known, then the coefficients A are determined so that Eq. (27) can be solved numerically.

The physical parameters for n-type indium antimonide used in the present sample calculation are taken as follows:

$$\epsilon_l = 17.5\,, \quad m_e = 0.013 m_0\,, \quad m_h = 0.40 m_0\,, \quad T = 77\,^\circ\text{K}\,,$$

$$\mu_e = -5 \times 10^5\ \text{cm}^2\text{V}^{-1}\,\text{sec}^{-1}\,,$$

$$n_e = 2.8 \times 10^{14}\ \text{cm}^{-3}\,.$$

The calculation was made for the range of parameters $E_0 \leq 30$ V cm^{-1}, $1 \leq B_0 \leq 10$ kG, and $1 \leq f \leq 9$ GHz, in the absence of carrier density gradient. The result of the calculation reveals that three roots have the attenuation constant α so large that wave propagation is not possible. Consequently, they are of no interest. However, there is one root which has a large phase constant β, but a small $|\alpha| < 4\ \text{cm}^{-1}$, which will permit the propagation of the wave in the material. For this root, $\beta > 0$, and the amplitude constant, α may be positive or negative, but its magnitude is small compared with the other three roots. Positive α represents the spatial growth of the wave while negative α represents the attenuation of the wave. For the range of parameter considered, β is approximately proportional to f so that the phase velocity of the wave $v_\varphi = \omega/\beta$, is constant with respect to f. The variation of v_φ with E_0 and B_0 are illustrated in Fig. 1, for $f = 5$ GHz. Figure 1 shows that v_φ decreases monotonically with B_0 and increases with E_0.

It should be noted that $c = 7.17\times 10^9$ cm/sec for $\epsilon_l = 17.5$, and the refractive index of the wave $c\beta/\omega = c/v_\varphi$ is in the range of $3\times 10^3 \leqslant c/v_\varphi \leqslant 7\times 10^4$ which implies $|\xi|^2 \gg 1$. Thus, the slow electromagnetic wave is permitted to propagate in the material. It should be noted that if

$$\left|1 - \frac{X^2}{Y^2 + Z^2}\right| \ll 1\,, \tag{28}$$

then the first three terms of the left-hand side of Eq. (27) are of importance, and the propagation constant of the slow waves can be given approximately by solving the following quadratic equation:

$$A_4\xi^2 + A_3\xi + A_2 = 0\,, \tag{29}$$

which can be rearranged into the following form:

$$\zeta^2 - (g + ih)\zeta + (r + is) = 0\,, \tag{30}$$

where

$$g = 2\,, \quad h = -\frac{\gamma_0}{1+\eta_0^2}(1-\delta^2)\,,$$

$$r = 1 - \delta_z^2\left(1 - \frac{\gamma_0^2}{1+\eta_0^2}\right),$$

$$s = -\frac{\gamma_0}{1+\eta_0^2}(1 - \delta^2 - 2\delta_z^2)\,,$$

with

$$\zeta \equiv \xi\delta_z = kv_{0z}/\omega\,, \quad \delta^2 \equiv \delta_x^2 + \delta_z^2\,,$$

$$\eta_0 \equiv Y/Z = -\mu_s B_0\,, \quad \gamma_0 \equiv X/Z = \omega_R/\omega\,.$$

$\omega_R \equiv \sigma_s/\epsilon$ denotes the dielectric relaxation frequency and $\sigma_s = \mu_s q_s n_s$ is the conductivity of the intrinsic material under consideration.

Wave amplification

In order to determine whether or not the wave can be amplified, we set the wave angular frequency ω to be real and investigate the behavior of the complex propagation constant $\tilde{k} = \beta + i\alpha$. Amplification of the wave arises when $\beta\alpha > 0$, and the threshold condition is given by $\alpha = 0$ and $\beta \neq 0$.

Substituting $\zeta = \beta v_{0z}/\omega + i(\alpha v_{0z}/\omega)$ in Eq. (30) yields a set of two real algebraic equations relating β and α, from which we obtain the following:

$$\beta v_{0z}/\omega = \tfrac{1}{2}g \pm \{\tfrac{1}{2}[(p^2+q^2)^{1/2} + p]\}^{1/2}\,, \tag{31a}$$

$$\alpha v_{0z}/\omega = \tfrac{1}{2}h \mp \{\tfrac{1}{2}[(p^2+q^2)^{1/2} - p]\}^{1/2}\,, \tag{31b}$$

where

$$p = \tfrac{1}{4}(g^2 - h^2) - r\,, \tag{31c}$$

$$q = (s - \tfrac{1}{2}gh)\,. \tag{31d}$$

Taking v_{0z} to be positive, since $h < 0$, for $\alpha > 0$ only the lower sign in Eqs. (31a) and (31b) need be considered.

The threshold condition for wave amplification is given by $\alpha = 0$, or

$$2s/gh = 1 \pm (1 - 4r/g^2)^{1/2}\,, \tag{32}$$

which can be expressed as

$$\frac{2\delta_z}{1-\delta^2} = \left[1 - \frac{\gamma_0^2}{1+\eta_0^2}\right]^{1/2}, \tag{33}$$

provided that

$$\gamma_0^2 < 1 + \eta_0^2\,.$$

Under the condition (28), $|q^2/p^2| \ll 1$, so that Eqs.

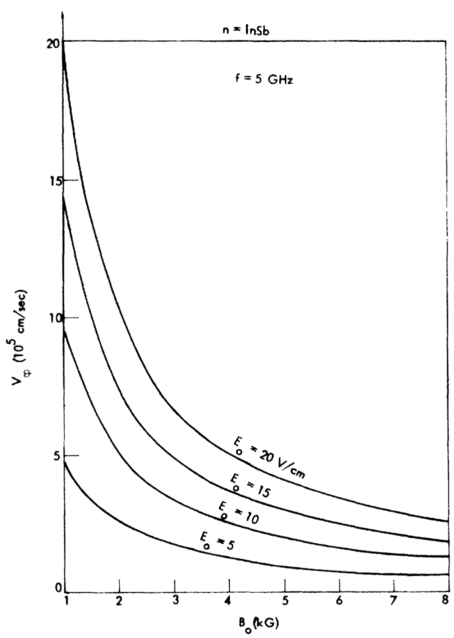

FIG. 1. Variation of phase velocity v_φ with the magnetic field strength B_0 for different values of applied static electric field E_0 at $f = 5$ gHz, in n-type InSb at $T = 77$ °K.

(31a) and (31b) become, respectively,

$$\frac{\beta v_{0z}}{\omega} = 1 - \frac{2\delta_z^2}{1-\delta^2} \quad (34a)$$

and

$$\left(\frac{\alpha v_{0z}}{\omega}\right) = \frac{1+\eta_0^2}{\gamma_0}\left(\frac{2\delta_z}{1-\delta^2}\right)^3 \delta_z . \quad (34b)$$

If δ^2 is much smaller than unity, the phase velocity of the wave $v_\varphi = \omega/\beta$, and the amplitude constant α, are given by Eqs. (34a) and (34b), respectively, as follows:

$$v_\varphi = v_{0z} \quad (35a)$$

and

$$\alpha = \frac{\omega}{c}\frac{1+\eta_0^2}{\gamma_0}(2\delta_z)^3 , \quad (35b)$$

which suggests that when the phase velocity of wave is approximately synchronous with the static carrier velocity in the direction of wave propagation, the amplification of the wave can take place, and the rate of spatial growth $\alpha > 0$, is proportional to E_0^3 and increases with E_0.

INSTABILITY OF SLOW ELECTROMAGNETIC WAVES

In order to determine the instability of the wave, we set the propagation constant k to be real and investigate the behavior of the complex angular frequency $\tilde{\omega} = \omega_r + i\omega_i$. Instability of the wave arises when the imaginary part of the wave angular frequency ω_i becomes negative, and the threshold for the onset of instability will occur when ω_i vanishes.

Threshold conditions

The dispersion relationship of slow electromagnetic waves under consideration, Eq. (29), can be rearranged into the following form:

$$\varphi^2 - (g_0 + ih_0)\varphi + (r_0 + is_0) = 0 , \quad (36)$$

where

$$g_0 = (2K\delta_z/a)(1+\eta_0^2) ,$$

$$h_0 = (1/a)(1-\delta^2-2\delta_z^2) ,$$

$$r_0 = (\delta_z^2/a)[(1+\eta_0^2)K^2+1] ,$$

$$s_0 = (K\delta_z/a)(1-\delta_z^2) ,$$

$$a = (1+\eta_0^2)(1-\delta_z^2) ,$$

in which $K = ck/\omega_R$ and $\varphi = \tilde{\omega}/\omega_R$.

It should be noted that when $\delta_z \to 0$ and $\delta_x \to 0$, then $g_0 \to 0$, $h_0 \to 1/(1+\eta_0^2)$, $r_0 \to 0$, and $s_0 \to 0$, so that $\varphi \to i/(1+\eta_0^2)$. Consequently, $\mathrm{Im}(\varphi) = \omega_i/\omega_R > 0$. This suggests that in the absence of static electric field and density gradient, instability of the wave is not possible.

Equation (36) can be solved analytically for φ which has the form

$$\varphi = \omega_r/\omega_R + i(\omega_i/\omega_R) , \quad (37a)$$

where

$$\omega_r/\omega_R = \tfrac{1}{2}g_0 \pm \{\tfrac{1}{2}[(p_0^2+q_0^2)^{1/2}+p_0]\}^{1/2} \quad (37b)$$

and

$$\omega_i/\omega_R = \tfrac{1}{2}h_0 \mp \{\tfrac{1}{2}[(p_0^2+q_0^2)^{1/2}-p_0]\}^{1/2} , \quad (37c)$$

in which

$$p_0 = \tfrac{1}{4}(g_0^2-h_0^2) - r_0 \quad (37d)$$

and

$$q_0 = s_0 - \tfrac{1}{2}g_0 h_0 . \quad (37e)$$

Since $\omega_R > 0$ and $g_0 > 0$, we take the upper sign in Eqs. (37b) and (37c) so that $\omega_r > 0$. In order to have negative ω_i, it is required that the second term must be greater than the first term on the right-hand side of Eq. (37c), since $h_0 > 0$. Consequently, the threshold condition for the wave instability is given by $\omega_i = 0$. Using the same approach as that used for deriving the threshold condition for wave amplification, with the aid of Eqs. (32) and (36), the desired threshold condition can be obtained in the following form:

$$\frac{\delta_z^2(1+\delta^2)}{1-\delta^2-2\delta_z^2} = \left(\delta_z^2 - \frac{1-\delta_z^2}{K^2(1+\eta_0^2)}\right)^{1/2} . \quad (38)$$

For $\delta^2 \ll 1$, Eq. (38) becomes

$$\delta_z = 1/K(1+\eta_0^2)^{1/2} . \quad (39)$$

On the other hand, the threshold oscillation frequency ω_0 defined as the value of ω_r at which $\omega_i = 0$ in Eq. (37a), can be given as follows:

$$\frac{\omega_0}{\omega_R} = \frac{K\delta_z}{1-\delta_z^2}\left[1 + \left(\delta_z^2 - \frac{1-\delta_z^2}{K^2(1+\eta_0^2)}\right)^{1/2}\right] . \quad (40)$$

If δ^2 is much smaller than unity, then with the aid of Eq. (39), Eq. (40) is reduced to

$$\omega_0/\omega_R = 1/(1+\eta_0^2)^{1/2} . \quad (41)$$

DISCUSSION OF RESULTS

In the absence of the carrier density gradient ($\nabla n_0 = 0$), δ_x and δ_z are given by

$$\delta_x = \frac{-\delta_0}{1+\eta_0^2}, \quad \delta_z = \frac{\eta_0\delta_0}{1+\eta_0^2}, \quad (42)$$

where $\delta_0 = -\mu_s E_0/c$ and $\eta_0 = -\mu_s B_0$. It should be noted that since the carrier drift mobility μ_s takes a negative value for the electron and a positive value for the hole, $\delta_z > 0$ and $\delta_x < 0$ for electron while $\delta_z > 0$ and $\delta_x > 0$ for hole.

For a moderate electric field strength E_0, $\delta_0^2 \ll 1$ so that with the aid of Eq. (42), the threshold condition for wave amplfication, given by Eq. (33), becomes

$$|\delta_0| = \frac{1+\eta_0^2}{2|\eta_0|}\left(1-\frac{\gamma_0^2}{1+\eta_0^2}\right)^{1/2}, \tag{43}$$

and the phase velocity of wave, given by Eq. (35a), and the growth rate, given by Eq. (35b), respectively, become

$$v_\varphi = \frac{c|\eta_0||\delta_0|}{1+\eta_0^2} \tag{44a}$$

and

$$\alpha = \frac{\omega}{c\gamma_0}\,\frac{|2\eta_0\delta_0|^3}{(1+\eta_0^2)^2}. \tag{44b}$$

On the other hand, the threshold condition for wave instability given by Eq. (39) becomes

$$|\delta_0| = (1+\eta_0^2)^{1/2}/K|\eta_0|, \tag{45a}$$

and the threshold oscillation frequency is still given by Eq. (41), which is expressed as

$$f_0 = \frac{1}{2\pi}\,\frac{\omega_R}{(1+\eta_0^2)^{1/2}}. \tag{45b}$$

Equation (43) suggests that the threshold electric field required for wave amplification E_a increases with the applied static magnetic field B_0, and with the wave frequency $f=\omega/2\pi$, whereas it decreases with the carrier concentration n_0. On the other hand, Eq. (44a) suggests that the phase velocity v_φ is proportional to E_0 and it increases with B_0 when $1>\eta_0$, while it decreases with B_0 when $1<\eta_0$. For the case where $1 \ll \eta_0^2$, v_φ is given as E_0/B_0, which is the drift velocity of carrier in the direction of $\vec{E}_0 \times \vec{B}_0$ and also in the direction of wave propagation. This is consistent with Fig. 1. Since $\gamma_0 = \omega_R/\omega$, Eq. (44b) suggests that the rate of wave amplification α is proportional to $f^2E_0^3$ so that it increases with f and E_0. It is also easily seen that α increases with B_0 when $1>|\eta_0|$ while it decreases with B_0 when $1<|\eta_0|$, and α takes its maximum value of $\omega\delta_0^3/c\gamma_0^2$ at $\eta_0 = 1$.

It is of interest to estimate the range of the threshold static electric field strength for wave instability E_{th}, given by Eq. (45a), and the range of the threshold oscillation frequency f_0, given by Eq. (45b). For example, for a n-type InSb at $T=77$ °K, with $\epsilon_l = 17.5$ and $\mu_e = -5\times10^5$ cm^{2}V^{-1} sec^{-1}, if η_0 is in the range of $0.5 \leq \eta_0 \leq 50$, then $|\delta_0|$ is in the range of $2.24\times10^{-3} \geq |\delta_0| \geq 10^{-3}$ for $K=10^3$. This suggests that E_{th} is in the range of $32 \geq E_{th} \geq 14.3$ V cm^{-1} for B_0 in the range of $0.1 \leq B_0 \leq 10$ kG. Thus, E_{th} can be in the range where Ohm's law still holds. On the other hand, if the carrier concentration n_0 is in the range of $10^{13} \leq n_0 \leq 10^{14}$ cm^{-3}, then the dielectric relaxation angular frequency ω_R would be in the range of $5.15\times10^{11} \leq \omega_R \leq 5.15\times10^{12}$ rad. sec^{-1}. Now, if $B_0 = 10$ kG or $\eta_0 = 50$ so that $\omega_0 \simeq \omega_R/\eta_0$, the threshold oscillation frequency f_0 would consequently be in the range of $1.64 \leq f_0 \leq 16.4$ GHz, which is in the microwave-frequency range.

The plot of $|\delta_0|$ vs $|\eta_0|$, given by Eq. (45a), is shown in Fig. 2 for different values of $K = ck/\omega_R$. It should be noted, from Fig. 2, that for the n-type InSb under consideration, $B_0 = 3$ kG or $\eta_0 = 15$, E_{th} lies in the range of $14.3 \leq E_{th} \leq 47$ V cm^{-1}, for the value of K in the range $1000 \geq K \geq 300$. Although there are some experimental results on microwave emission from n-type InSb have been reported,[1,2,5] a direct quantitative comparison with the above theoretical results is not appropriate since the present theory neglects the consideration of the boundary effect. However, it is of interest to note that the plot showing the variation of the observed minimum electric field required for microwave emission,[1] E_{th} vs η_0 has similar shape as that given by Fig. 2 of the present paper.

On the other hand, the variation of the threshold oscillation frequency, f_0, with η_0 (or B_0) given by Eq. (45b), is to be compared with the experimental result reported by Kobyzev and Tager[8] on the study of coherent microwave radiation of n-type InSb. The curve "No. 2" on Fig. 3 of Kobyzev and Tager[8] shows that the radiation frequency decreases monotonically with the transverse magnetic field B_0 from $f=4.8$ GHz at $B_0=2$ kG to $f=4.4$ GHz at $B_0=3$ kG which has similar behavior as that given by Eq. (45b) of this paper.

Although the above qualitative comparisons are not rigorous, the fact that the range of parameters E_0, B_0, and f, under consideration in this paper is quite consistent with those values often encountered in the studies of microwave emission from InSb

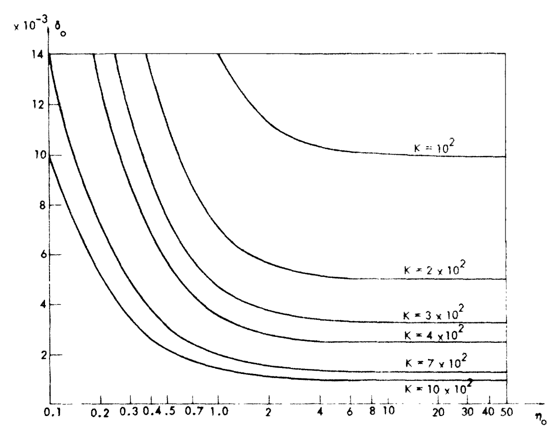

FIG. 2. Plots of δ_0 vs η_0 for different values of K.

tends to suggest that the wave instability in the crossed-fields configuration might be responsible for a certain class of microwave emissions observed experimentally.[2,5,8]

CONCLUDING REMARKS

In the present paper, the propagation characteristics and the possibility of instability for the electromagnetic wave in extrinsic InSb have been studied. The applied static electric field $\vec{E}_0$, the static magnetic field $\vec{B}_0$, and the wave vector $\vec{k}$, are taken to be directed along the x, y, and z directions in the Cartesian coordinate system. The dispersion relationship for the wave was derived under the assumption that the carrier density gradient is not too large so that $|(\nabla n_0)/n_0| \ll k$ is still valid. Under the condition Eqs. (28) and (33) ($\gamma_0^2 \lesssim 1+\eta_0^2$), the detail analysis of the dispersion relationship was made, the threshold condition for wave amplification, Eq. (39), the threshold condition for wave instability, the phase velocity v_φ, Eq. (35a), and the spatial growth rate α, Eq. (35b), as the function of $\delta_z = v_{0z}/c$ are obtained. These equations are applicable both to n-type and p-type materials.

The numerical illustrations given in the preceding sections did not take into account any effect due to the static carrier density gradient which might be present with the material. The quantitative analysis of this carrier density gradient effect requires the knowledge of the spatial distributions of carrier density distribution, $n_0(x, y, z)$, which can only be obtained by solving proper boundary values problems.

For a qualitative analysis, however, the effect of the density gradient on the v_{0z} can be examined with the aid of Eqs. (19a) and (19b). For example in a n-type material since $\mu_s < 0$, v_{0z} will be increased by the presence of density gradient if $\partial n_0/\partial x > 0$ or $\partial n_0/\partial z < 0$. Suppose that the density gradient exists only in the x direction, such that $n_0(x) = N_0 e^{\gamma x}$, then Eqs. (19a) and (19b) yield

$$\delta_x = \frac{u}{c(1+\eta_0^2)}, \quad \delta_z = \frac{-\eta_0 u}{c(1+\eta_0^2)}, \tag{46}$$

where $u = \mu_s E_0 - D_s \gamma_s$, so that it is easily seen that, from Eqs. (35a) and (35b), v_φ and α are increased and α can be increased considerably since it is proportional to v_{0z}. On the other hand, from Eq. (39) the required static electric field for wave instability E_{th} will be reduced in the presence of positive density gradient in the x direction. Thus, the carrier density gradient can have significant effects upon the threshold conditions, phase velocity, and spatial growth rate.

The present study shows that the wave instability is possible under a proper condition. The wave instability is closely related to the phenomena of the excitation of electromagnetic waves and noise emission within the semiconductor material. For example, a random fluctuation of the carrier charge density at any point in the semiconductor may produce a random fluctuation of the carrier current density, which in turn produces random fluctuation of dynamic electromagnetic fields. This random fluctuation of electromagnetic fields may manifest itself as a electromagnetic wave (radiation or noise). Owing to the wave instability, the wave may grow exponentially in time. When the applied static electric fields are well above the required threshold values for the instability, given for example by Eq. (39), the temporal growth rate may be sufficiently large so that the amplitude of dynamic electromagnetic fields would be large enough to propagate out of the material.

The present investigation tends to suggest that once the electromagnetic wave in the microwave-frequency range is excited somewhere within the material, it can propagate and experience the spatial growth if the conditions are proper; i.e., if the condition (33) is satisfied and the growth rate given by Eq. (35b) is sufficiently large. This wave may be coupled in and out of the material, provided that the direction of wave vector is properly oriented with regards to the static electric and magnetic fields.

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