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# Comment on "Coulomb interaction in semiconductor lasers"—A reply

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Points raised in the preceding comment by Teitler, Ngai, and McCombe are considered together with the limitations of the approach taken in this author's original paper.

In the paper[1] (also called A here) to which the preceding comment[2] refers, three main points were made: (i) that the electron-hole excitation in direct-gap semiconductors excited to the lasing regime is characteristically a moderately dense plasma described by values of electron density parameter $r_s$ somewhat greater than unity ($1 \lesssim r_s \lesssim 3$); (ii) that correlation effects as well as exchange are expected to be important in this regime; and (iii) that an approximate treatment using well-known results from theory of metals (many real metals are characterized by $r_s$ values in precisely this range) can account for the gross spectral characteristics of the stimulated optical recombination. It is useful to review these points in the light of more recent theory and experiment.

With regard to the first point, numerous experiments have established surface pumping intensities needed for stimulated emission from semiconductors, and recent results from experiments of the type cited as references for Table I of A continue to yield similar values. Of particular note is a recent experiment of a different type in which the microwave reflectivity of the photoinjected plasma in GaAs (Ref. 3) was used to determine the total electron number. The results are consistent with the estimates made from creation rate and lifetime determinations as tabulated in A. A *direct* measurement of *actual* electron-hole density during strong stimulated optical recombination has not yet been reported, so that the quantitative validity of Table I in A remains only highly probable.

As for the second point, recently Brinkman and Lee[4] have calculated optical gain spectra to be expected for GaAs in zero magnetic field. They have considered potentially important two-electron processes and Coulomb enhancement effects omitted from A. Their results agree with the *shape* as well as magnitude and position of directly measured gain spectra,[5] and certainly support the main tenet of A—that direct optical gain in excited semiconductors indeed arises within a moderately degenerate electron-hole plasma regime.

The third point is that most directly addressed in the preceding comment. Choice of a condensed notation in Sec. IV of A may have contributed to some apparent confusion as to what was actually done therein. The expression for the inverse dielectric function for an electron gas in magnetic field given by Horing[6] was taken as the basis for the numerical calculations described in A. This expression has greater validity to shorter wavelengths than the zero-wave-vector limiting form appropriate for $r_s \ll 1$ apparently considered in the previous comment. At $r_s \sim 1$ one is already into a regime where the distinction between plasmon and quasiparticle modes is not sharp, and the cutoff momentum has no real significance beyond delineating the region of assumed validity for the particular approximation used for the dielectric function. One may well question to what degree extension to states with $k \sim k_F$, as in A, is legitimate for $r_s \gtrsim 1$. Teitler, Ngai, and McCombe[2] do not consider this point, however. In any case the statement in A was that the numerical result, including effects of dispersion, damping, and finite temperature, comes out to be such that the zero-order term in a power-series development of the self-energy is numerically equivalent to that obtained by simply associating the zero-point energy of a zero-wave-vector plasmon with each electron. That Teitler, Ngai, and McCombe[2] do not find such numerical equivalence is not surprising since they do not appear to go beyond the elementary simple-pole zero-wave-vector approximation to the inverse dielectric function.

Specific points raised in the preceding paper require comment. The first twelve equations therein are accurate reproductions of the zero-temperature electron-gas treatment presented by Fetter and Walecka, as cited. However, their Eq. (5) is in error by a factor of 2 for the *plasma* owing to neglect of the hole "bubbles" in the ring diagrams. Other apparent differences between these equations and their counterparts in A are primarily notational, although the specific forms suggested in the preceding comment are patently inappropriate to a finite temperature treatment. Apart from trivial typographical errors,[7] the equations in A appear as intended and the "corrections" Teitler *et al.* suggest are neither necessary nor appropriate.

Similarly, their Eq. (13) is an accurate reproduction of the Hamiltonian form used in Lundqvist's paper. However, the Hamiltonian for a free boson field is

$$H \sim \frac{1}{2} \sum_i (b_i^\dagger b_i + b_i b_i^\dagger)\omega_i = \sum_i (b_i^\dagger b_i + \tfrac{1}{2})\omega_i \ .$$

The zero-point-energy term $\frac{1}{2}$ is usually dropped as of no physical significance, and *either* the form as in A or as in the preceding comment could be used, since either way the electron ground-state energy shift comes from $H_{\text{el-plas}}$ as in Eq. (4.8) of A. The Lundqvist Hamiltonian is an artificial construction that must be interpreted in a manner consistent with the reality that actual electrons constitute *both* the quasielectron *and* the collective modes. Zero-point energy is *not* counted twice in A, contrary to the assertion in the preceding comment.

"Average electrons and holes" were not mentioned in A. $\langle \Sigma(p) \rangle$ was defined analytically as a leading term approximation, and has nothing to do with assertions about "average electrons" or "average holes." It is also a serious oversimplification to argue that the number of plasmon modes is well defined when $r_s \gtrsim 1$. The arguments used by Teitler *et al.* are *only* valid in the high-density limit where $r_s \ll 1$, which is not appropriate to the experiments considered in A.

A complete treatment of optical gain for the intermediate-density interacting electron-hole gas in arbitrary magnetic field or with band anisotropy is exceedingly complex, and two developments will be necessary for real progress—first, experiments as in Ref. 5 in which the actual gain spectrum and shifts thereof in magnetic field are determined, rather than just the over-all shift of total stimulated output; and second, calculations including arbitrary magnetic field comparable in scope to the zero-field work cited in Ref. 4. In the meantime, it remains that the treatment in A is the only suggestion even qualitatively in accord with the experimental observations.[8] The constant offset of peak wavelength shift from $\frac{1}{2}\hbar\omega_c$ ($\omega_c$ = cyclotron frequency) observed clearly at high fields will almost certainly continue to require explanation in terms of correlation effects and Coulomb interaction. The fact that this constant offset appears to be equal to $\frac{1}{2}\hbar\omega_p$ may turn out to be fortuitous, or it may indeed reflect the collective Coulomb interaction. Simplistic repetition of the familiar high-density calculation will not shed any further light on this question.

To close, readers are reminded of the limitations of the approach taken in A, which this author considers adequately enumerated therein. Hopefully a more satisfying analysis will be forthcoming. Unfortunately, the preceding comment offers no positive contribution in that direction. The equations in Sec. IV of A generally appear as intended, and the "corrections" suggested by Teitler, Ngai, and McCombe result at best in a different choice of units and at worst in serious oversimplification leading to consideration of a different problem. Their conclusions follow from a simplified and nonequivalent analysis which makes no attempt whatever to extend the standard high-density treatment to the intermediate density regime as was considered in A, and from their criticism of concepts or hypotheses ("average" electrons) which were neither mentioned *nor* implied in A, and which can only be considered as nascent in their own interpretation.

[1]W. D. Johnston, Jr., Phys. Rev. B 6, 1455 (1972).
[2]S. Teitler, K. L. Ngai, and B. D. McCombe, preceding paper Phys. Rev. B 10, 3715 (1974).
[3]R. F. Leheny, R. E. Nahory, and M. A. Pollack, Phys. Rev. B 8, 620 (1973).
[4]W. F. Brinkman and P. A. Lee, Phys. Rev. Lett. 31, 237 (1973).
[5]K. L. Shaklee, R. E. Nahory, and R. F. Leheny, J. Lumin. 7, 284 (1973).
[6]N. J. Horing, Ann. Phys. (N.Y.) 31, 1 (1965).
[7]There are three printer's errors in Sec. IV of A to be corrected: the limits on all integrations over $\lambda$ in Eq. (4.7) should be 0 and 1; $(q)$ should read $\Pi(q)$ in the third line of Eq. (4.7); $\pi(q)$ should read $\Pi(q)$ wherever it appears in the equation following (4.7). Footnote 21 of Ref. 1 above applies to all equations of Secs. IV and V thereof. Also, in Sec. II of A the condition for Mott transition should have read $n_{\text{Mott}} = (64a_0^3)^{-1}$.
[8]J. L. Shay, W. D. Johnston, Jr., E. Buehler, and J. H. Wernick, Phys. Rev. Lett. 27, 711 (1971); J. L. Shay and W. D. Johnston, Jr., Phys. Rev. B 6, 1605 (1972).