Comment on "Coulomb interaction in semiconductor lasers"

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A recent claim to an explanation of the magnetic field dependence of the frequency of semiconductor lasers in terms of many-body effects is considered. It is shown that the proposed mechanism does not in fact follow from conventional many-body theory. Furthermore, it is shown that even if the mechanism is assumed to be valid for reasons as yet unclear, its proper application leads to quantitative disagreement with experimental results.

Recently there have been several studies investigating the importance of many-body interactions on the optical properties of semiconductors. These have focused on the nature and consequences of the coupling of single-particle excitations to collective excitations. While such interactions have been shown to play a role in certain weakly allowed cyclotron resonance transitions,[1] and in the explanation of cyclotron harmonics,[2] there has also been a claim that they play an essential role in the magnetic field dependence of the frequency of direct-band-gap semiconductor lasers.[3,4] Recently, Johnston[5] has sought to provide a justification for this latter claim. In Johnston's paper[5] (hereafter referred to as A), it is argued that because of many-body interactions, the magnetic field dependence of the frequency of such lasers is determined by the zero-point energy of plasmons propagating perpendicular to the magnetic field. In the present Comment, we clarify the many-body arguments in A and show that they do not, in fact, support such a physical hypothesis.

Our starting point will be the "ring-diagram" contribution to the ground-state energy E. [The latter is related to the thermodynamic potential Ω in the zero-temperature limit by $\Omega(0)=E-\mu N$, where μ is the chemical potential and N is the total number of particles.] As shown, e.g., by Fetter and Walecka,[6] this ring-diagram contribution is

$$E_r=\tfrac{1}{2}i\hbar V\int_0^1 \frac{d\lambda}{\lambda}\int \frac{d^4q}{(2\pi)^4}\,\frac{[\lambda U_0(q)\Pi^0(q)]^2}{1-\lambda U_0(q)\Pi^0(q)}\,. \qquad (1)$$

Here we have used the notation of Fetter and Walecka. The correspondence with the notation in A is as follows. V is the volume, apparently set equal to 1 in A. $\lambda U_0(q)$ is equivalent to $V^\lambda(q)$, where λ is the variable coupling parameter. Furthermore, Π^0 is the equivalent of Π, and here

$$\Pi^0(q)\equiv -2i\int \frac{d^4p}{(2\pi)^4}G^0(p)G^0(p+q)\;. \qquad (2)$$

G^0 is the noninteracting Green's function. Note that an i in the definition for the equivalent of E_r, and a minus i in the definition of Π^0, are omitted in A. Also, the factor λ^{-1} in the integral over λ has been omitted.

We now define the so-called ring-diagram dielectric function[6] in essentially the same way as in A:

$$\epsilon^\lambda(q)=1-\lambda U^0(q)\Pi^0(q)\;. \qquad (3)$$

Then E_r may be written in the form

$$E_r=\frac{i}{2}\hbar V\int_0^1\frac{d\lambda}{\lambda}\int\frac{d^4q}{(2\pi)^4}\left(\frac{1}{\epsilon^\lambda(q)}+\epsilon^\lambda(q)-2\right). \qquad (4)$$

Since E_r is real, we find

$$E_r=-\frac{\hbar}{2}V\int_0^1\frac{d\lambda}{\lambda}\int\frac{d^4q}{(2\pi)^4}\left[\mathrm{Im}\left(\frac{1}{\epsilon^\lambda(q)}\right)+\mathrm{Im}\,\epsilon^\lambda(q)\right]\;. \qquad (5)$$

This is the correct form for the equivalent of Eq. (4.8) of A. Interest in A was confined to the plasmon pole contribution to E_r. Then Eq. (5) becomes

$$E_r=-\frac{\hbar}{2}V\int_0^1\frac{d\lambda}{\lambda}\int\frac{d^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\,\mathrm{Im}\left(\frac{1}{\epsilon^\lambda(q,\omega)}\right), \qquad (6)$$

where $\omega\equiv q^4$ is the fourth component of the momentum. We define $E_r(q)$ as follows:

$$E_r\equiv\int\frac{d^3q}{(2\pi)^3}E_r(q)\;. \qquad (7)$$

Then in the long-wavelength limit, we obtain

$$E_r(q)=\hbar V\int_0^1\frac{d\lambda}{\lambda}\,\frac{\omega_p^\lambda(q)}{4},\quad q<q_c \qquad (8)$$

where q_c is the cutoff momentum for the plasmons, and[7] $\omega_p^2(0)=4\pi Ne^2/m^*V$ so that $\omega_p^\lambda=\omega_p\lambda^{1/2}$. It follows that

$$E_r(q)=\tfrac{1}{2}\hbar V\omega_p(q)\,,\quad q<q_c\;. \qquad (9)$$

Attention in A was confined to these long-wavelength contributions in evaluating the thermodynamic potential. Before commenting further on the use of this result in A, we wish to continue the many-body discussion to encompass the equation equivalent to Eq. (4.9) of A.

In fact, Eq. (4.9) of A is just the plasmon pole contribution to a corrected Eq. (4.7) of A before the convolution integral over d^4p has been carried out to obtain $\Pi^0(q)$ in the remaining integrand. [See our Eq. (2).] The correct equivalent to Eq. (4.7) of A is

$$E=-\frac{iV}{2}\int_0^1\frac{d\lambda}{\lambda}\int\frac{d^4p}{(2\pi)^4}\Sigma^\lambda(p)G^0(p)\ , \qquad (10)$$

where

$$\Sigma^\lambda(p)=i\hbar\int\frac{d^4q}{(2\pi)^4}\frac{\lambda U^0(q)}{\epsilon^\lambda(q)}G^0(p-q)\ . \qquad (11)$$

The plasmon pole[8] contribution to Eq. (10) is

$$E_r=-\frac{i}{2}\int_0^1\frac{d\lambda}{\lambda}\int\frac{d^4p}{(2\pi)^4}[-2i\hbar]\int\frac{d^4q}{(2\pi)^4}|g_q^\lambda|^2 \times\frac{2\omega_q^\lambda\Theta(q_c-|\vec{q}|)}{\omega^2-(\omega_q-i\delta)^2}G^0(p-q)G^0(p)\ . \qquad (12)$$

Here $|g_q^\lambda|^2\equiv\lambda U_0(q)[\partial\epsilon^\lambda(q)/\partial\omega]^{-1}$, $\Theta(x)$ is the unit step function, and the factor of 2 in the square brackets occurs because of the sum over spin. Thus with appropriate corrections, Eq. (4.9) of A is just the plasmon pole contribution to the thermodynamic potential, and only that same contribution to Eq. (4.8) was considered in A.

At this point in A, there was an attempt to interpret Eq. (4.9) in terms of the Lundqvist Hamiltonian, supposedly given by Eq. (4.10). In fact, the Lundqvist Hamiltonian is given by[9]

$$H_{\text{eff}}=\sum_k\epsilon(k)C_k^\dagger C_k+\sum_q\hbar\,\omega_q(b_q^\dagger b_q)+\sum_{k,q}V^{-1/2}g_kC_{k-q}^\dagger C_k(b_q+b_{-q}^\dagger)=H_{\text{el}}^0+H_{\text{plas}}^0+H_{\text{el-plas}}\ . \qquad (13)$$

The last term on the right has been termed the plasmaron coupling.[10] Note there is *no* plasmon zero-point energy in this Hamiltonian.[11] The Lundqvist Hamiltonian is a construct arising from Lundqvist's observation that the plasmon pole contribution to the self-energy of quasielectrons is (in the random-phase approximation) of the same form as the electron self-energy caused by the Fröhlich interaction with phonons. As follows from our Eqs. (10)–(12), the plasmaron-coupling-induced self-energy contribution to the ground-state energy provides the plasmon zero-point energy. If one uses the form given in A or Ref. 10 and includes the plasmon zero-point energy in H_{eff}, one is counting this energy twice in the calculation of the ground-state energy.

Thus as far as ground-state energy calculations are concerned, one must either include the plasmon zero-point energy and leave out the plasmaron coupling, or include the plasmaron coupling and leave out the plasmon zero-point energy. This is just the point that seems to be missed in A in the discussion following Eq. (4.10). Since $H_{\text{el-plas}}$ and the zero-point energy of the plasmons are not coexistent as far as ground-state energy calculations are concerned, there is no meaning to the idea that $H_{\text{el-plas}}$ serves to couple single-particle modes to the zero-point energy of the plasmons.

In summary of this point, the many-body considerations in A lead to nothing more than the usual result[12] that there is a contribution to the many-electron ground-state energy which corresponds to the zero-point energy of the plasmons.

The whole thrust of A thus reduces to the *ad hoc* assumption that[13] "experimental shifts of stimulated light output... should shift in accordance with $\langle\Sigma(p)\rangle$." $\langle\Sigma(p)\rangle$ as *defined* in A is an "average" self-energy correction. Since it is particular electrons and holes which are radiatively recombining and not "average" electrons and holes, this hypothesis seems dubious. However, let us explore its consequences. As will be shown, when consistently applied, this hypothesis is untenable.

Following Eq. (4.6) of A, it is indicated that frequency shifts are only caused by exchange and ring-diagram contributions to $\Sigma(p)$. However, in the later development only the ring-diagram contributions to $\Sigma(p)$ are considered. Thus an unnumbered equation after Eq. (4.10) of A has the form

$$\langle\Sigma(p)\rangle_R=\left(\frac{1}{2n}\right)\sum_{q<q_c}(\hbar\omega_q)\ , \qquad (14)$$

where q_c is the cutoff momentum for the plasmons and n is the electron concentration. Since the number of plasmon modes included in the sum over $q<q_c$ is much less than the number of electrons,[14] $\langle\Sigma(p)\rangle_R$ is actually much less than the zero point energy of a plasmon. For example, for the $CdSnP_2$ material considered in A, the number of plasmon modes is an order of magnitude smaller than the number of electrons. Thus a single electron's "share" of this contribution to the self-energy is *much less* than the zero-point energy of a plasmon.

Next, the effect of a magnetic field is considered in A. However, this consideration does not include the effect of the magnetic field on all terms in Σ, so, e.g., Landau quantization effects on single-particle modes do not enter into the discussion. Rather, attention is confined to $\langle\Sigma(p)\rangle_R$, and a claim is made that

$$\langle\Sigma(p)\rangle_R=\hbar(\omega_p^2+\omega_c^2)^{1/2}K\ . \qquad (15)$$

Here K is supposed to be approximately $\frac{1}{2}$, and $(\omega_p^2+\omega_c^2)^{1/2}$ is the frequency of a plasmon propagating in the direction perpendicular to the magnetic field. However, the number of plasmons propagating within any solid angle encompassing the direction perpendicular to the magnetic field is clearly less than the total number of plasmons.[15] It follows that K must be even smaller than its corresponding value in Eq. (14), i.e., $K \ll \frac{1}{2}$. Since agreement of the discussion in A with experiment is predicated on K being of the order of $\frac{1}{2}$, it follows that the hypothesis in A is untenable.

In conclusion, we have investigated a claim to an explanation for the magnetic field dependence of the frequency shift of direct-band-gap semiconductor lasers. This claim was based on a hypothesis that an "average" electron possesses the zero-point energy of a plasmon because of many-body interactions. We have shown that this hypothesis has no foundation in many-body theory. Furthermore, we have shown that even if one assumes laser action involves such average electrons and average holes, the disparity between the number of electrons and number of plasmons requires that the "share" of each electron of the total plasmon zero-point energy must be much less than that of a single plasmon. Hence we conclude that the proposed explanation is incorrect.

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[1]B. D. McCombe, R. J. Wagner, S. Teitler, and J. J. Quinn, Phys. Rev. Lett. 28, 37 (1972).

[2]K. W. Chiu, K. L. Ngai, and J. J. Quinn, Solid State Commun. 10, 1251 (1972).

[3]J. L. Shay, W. D. Johnston, Jr., E. Buehler, and J. H. Wernick, Phys. Rev. Lett. 27, 711 (1971).

[4]J. L. Shay and W. D. Johnston, Jr., Phys. Rev. B 6, 1605 (1972).

[5]W. D. Johnston, Jr., Phys. Rev. B 6, 1455 (1972).

[6]A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971), p. 156, Eq. (12.23).

[7]It may be noted that in A the screening caused by the background semiconductor medium dielectric function is not explicitly included in the definition of $V(q) \equiv U^0(q)$ and hence ω_p^2. However, the static dielectric function is included in the evaluation of Bohr radii for use in determining r_s values in A.

[8]B. I. Lundqvist, Phys. Kondens. Mater. 6, 193 (1967). For other details of the effects of electron-plasmon interactions, see, e.g., B. I. Lundqvist, Phys. Kondens. Mater. 6, 206 (1967), 7, 117 (1968).

[9]B. I. Lundqvist, Phys. Kondens. Mater. 6, 193 (1967), Eq. (17).

[10]S. Teitler, B. D. McCombe, and R. J. Wagner, in *Proceedings of the Tenth International Conference on the Physics of Semiconductors*, edited by S. P. Keller, J. C. Hensel, and F. Stern (U.S. AEC, Oak Ridge, Tenn., 1970), p. 177.

[11]Such a term was also erroneously included in Ref. 10 but played no role in the discussion therein since only the microscopic effects of plasmaron coupling were considered.

[12]D. Pines and P. Nozières, *The Theory of Quantum Liquids, Vol. I: Normal Fermi Liquids* (Benjamin, New York, 1966), p. 289, Eq. (5.97).

[13]Reference 5, last paragraph on p. 1458.

[14]D. Pines, *Elementary Excitations in Solids*, (Benjamin, New York, 1963), p. 104.

[15]The contention is made in A (p. 1460) that the plasmon "momentum sum will emphasize plasmons propagating perpendicular rather than parallel to any magnetic field." This contention contradicts the results in the quoted reference, i.e., N. D. Mermin and E. Canel, Ann. Phys. (N. Y.) 26, 247 (1964), in which the residue of the plasmon poles are given. For $\omega_p^2 > \omega_c^2$, the plasmons with non-negligible residue vary from a frequency ω_p in the direction parallel to the magnetic field to a frequency $(\omega_p^2 + \omega_c^2)^{1/2}$ in the direction perpendicular to the magnetic field. However, there is no singularity in the residue, and therefore plasmons propagating perpendicular to the magnetic field are not essentially different in number from those propagating in other directions. As an aside, it should be noted that discussion of the plasmon dispersion relation by Mermin and Canel is only applicable when retardation effects are negligible, and hence not appropriate in the extreme long-wavelength limit.