Acoustoelectric absorption in semiconductors with dominant acoustic-phonon scattering*

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(Received 22 May 1973)

It has been shown that the well-known Cohen-Harrison-Harrison (CHH) model of the collision term for electrons in a semiconductor with an acoustic wave is invalid for energy-dependent relaxation times. A modified collision term has been used to calculate the absorption coefficient of an acoustic wave. Our calculations show that for the case of acoustic-phonon scattering and an intermediate range of frequencies, i.e., $v_s/v_0 < ql < v_0/v_s$ is (v_0 is average thermal velocity and v_s is the sound velocity), the values of the absorption coefficients are appreciably different from those obtained using the CHH model; in the two extreme limits, i.e., $ql < v_s/v_0$ and $ql > v_0/v_s$, we obtain some results similar to those using CHH model.

I. INTRODUCTION

In the last decade the phenomenon of acoustoelectric absorption has been extensively investigated both theoretically and experimentally.[1] Most of the theoretical calculations have been made using a constant-relaxation-time model. Recently however, Jacoboni and Prohofsky[2] calculated the absorption coefficient of an acoustic wave in a nondegenerate semiconductor taking into account the relevant energy dependence of the relaxation time. Most recently, one of us calculated[3] the absorption coefficient in the presence of a dc electric field assuming dominant acoustic-phonon scattering. However, in both these papers the Cohen-Harrison-Harrison (CHH) model for the collision term for electrons was used. This collision term (as we shall show) is invalid for an energy-dependent relaxation time for the obvious reason that its volume integral in velocity space does not vanish—a condition which any number-conserving collision term must satisfy. In an investigation of the damping of electron plasma waves in a weakly ionized gas, Bogdanor and Willett[4] recently faced a similar problem with the usual Bhatnagar-Gross-Krook collision term. They suggested a modified collision term for electrons which conserves the number of particles, even for an energy-dependent relaxation time. In this note we use this model instead of the CHH collision model and calculate the absorption coefficient of an acoustic wave for dominant acoustic-phonon scattering, for the four regions of frequencies, i.e., (i) $ql < v_s/v_0 < 1$, (ii) $v_s/v_0 < ql < 1$, (iii) $1 < ql < v_0/v_s$, and (iv) $1 < v_0/v_s < ql$. Our calculations indicate that while in the first and fourth regions we obtain the same results as those obtained by using the CHH model, the calculations for the regions (ii) and (iii) are found to be significantly different from the corresponding calculations[2] obtained by using the CHH model. Our theory predicts that the limiting behavior of absorption coefficient in the high-frequency limit is obtained for $ql \gg 1$ in contrast to Jacoboni and Prohofsky's condition that $\omega\tau \gg v_0/v_s$.

II. MODEL FOR THE COLLISION TERM

The collision term due to Cohen, Harrison, and Harrison[5] is given by

$$\left(\frac{\partial f}{\partial t}\right)_{\text{coll}} = -\frac{f^1}{\tau} + \frac{N_1}{N_0}\frac{f^0}{\tau} + \frac{mvu}{k_B T}\frac{f^0}{\tau} \quad , \tag{1}$$

where f^1 is the part of the electron distribution function which varies as $\exp[-i(\omega t - qr)]$, m and v are the electronic mass and velocity, ω and q are the frequency and wave-vector magnitude of the acoustic wave. N_1 is the fluctuating part of electron density, N_0 is the time-independent electron density, u is the velocity vector of lattice displacement, $\tau = \tau_0(mv^2/2k_BT)^n$, n is a constant which depends on scattering mechanism (for acoustic phonon scattering $n = -\frac{1}{2}$), and $f^0 \propto \exp(-mv^2/2k_BT)$ is the isotropic part of the distribution function. The first term on right-hand side of Eq. (1) is the usual rate of change in distribution function due to electron-phonon collisions. The second term accounts for diffusion of electrons which arises because there is an electron density gradient set up due to the presence of an acoustic wave. The third term arises due to the fact that when impurity or phonon scattering is dominant the distribution of electrons relaxes to the rest frame of moving lattice and this would lead to an effective field of magnitude $(-mu/e\tau)$ acting on electrons in addition to the self-consistent electric field. However, this term is negligible when deformation potential coupling is the dominant coupling mechanism. Thus the relevant collision term for our case becomes

$$\left(\frac{\partial f}{\partial t}\right)_{\text{coll}} = -\frac{f^1}{\tau} + \frac{N_1}{N_0}\frac{f^0}{\tau} \quad . \tag{1'}$$

On taking the volume integral of Eq. (1') in velocity space, we have

$$\int\left(\frac{\partial f}{\partial t}\right)_{\text{coll}} d^3v = -\int \frac{f'}{\tau}\, d^3v + \frac{N_1}{N_0}\int \frac{f^0}{\tau}\, d^3v \, . \tag{2}$$

An expression for f', the part of the distribution function which varies as $\exp[-i(\omega t - qz)]$ was obtained by Jacoboni and Prohofsky.[2] Rewriting this in a more elegant way, we have

$$f^1 = \left(E - \frac{Ciq^2u}{e\omega}\right) f_s^1 + \frac{N_1}{N_0} f_N^1 \, , \tag{3}$$

where E is the self-consistent electric field associated with the acoustic wave, C is the deformation potential, f_s^1 and f_N^1 are given by

$$f_s^1 = \frac{-iev_s v_z f^0}{\omega k_B T(v_s + i/q\tau - v_z)} \, , \tag{4a}$$

$$f_N^1 = \frac{iv_s f^0}{\omega\tau(v_s + i/q\tau - v_z)} \quad , \tag{4b}$$

and the rest of the symbols have their usual meaning. Substituting for f^1 from (3) and (4) and using the relation $N_1 = \int f^1 d^3v$, we get

$$\int\left(\frac{\partial f}{\partial t}\right)_{\text{coll}} d^3v = \left(E - \frac{iCq^2u}{e\omega}\right)\left(N_0 - \int f_N^1\, d^3v\right)^{-1} \times\left[\left(\int f_s^1 d^3v \int \frac{f^0}{\tau}\, d^3v - N_0\int \frac{f_s^1}{\tau}\, d^3v\right) + \left(\int \frac{f_s^1}{\tau}\, d^3v \int f_N^1\, d^3v - \int f_s^1 d^3v \int \frac{f_N^1}{\tau} d^3v\right)\right] . \tag{5}$$

It can be seen from Eq. (5) that when τ is a function of energy, the volume integral of the CHH collision term does not vanish. Thus for energy-dependent relaxation times, the CHH model is not valid. For constant τ, of course, the right-hand side of Eq. (5) is identically zero.

III. THE NEW COLLISION MODEL AND BOLTZMANN'S EQUATION

For a description of electron-neutral collisions in a weakly ionized gas, Bogdanor and Willett[4] have proposed a collision term of the form

$$\left(\frac{\partial f}{\partial t}\right)_{\text{coll}} = -\frac{f - \langle f\rangle}{\tau} \, , \tag{6}$$

where $\langle\cdots\rangle$ is the average over $d\Omega_v$ (i.e., the solid angle in velocity space). This collision term does conserve the particle number even when τ is a function of electron energy, since

$$\int\left(\frac{\partial f}{\partial t}\right)_{\text{coll}} d^3v = -\int \frac{f - \langle f\rangle}{\tau}\, d\Omega_v\, v^2\, dv = 0 \, . \tag{7}$$

For small perturbations, we can expand the distribution function as

$$f = f^0 + f^1 \, . \tag{8}$$

Using the above expansion in Eq. (6), we get

$$\left(\frac{\partial f}{\partial t}\right)_{\text{coll}} = -\frac{f^1 - \langle f^1\rangle}{\tau} \quad . \tag{9}$$

We now use Eq. (9) in place of the CHH model [Eq. (1)] to describe acoustic-phonon scattering of carriers in a nondegenerate semiconductor in the presence of an acoustic wave.

The Boltzmann equation for electrons in the presence of an acoustic wave may be now written as

$$\frac{\partial f^1}{\partial t} + v_z\frac{\partial f}{\partial z} - \frac{eE_s}{m}\frac{\partial f^0}{\partial v_z} = -\frac{f^1 - \langle f^1\rangle}{\tau} \, , \tag{10}$$

where

$$E_s = E - iCq^2u/e\omega \, . \tag{11}$$

$\tau = \tau_0(mv^2/2k_BT)^{-1/2}$; i.e., acoustic-phonon scattering has been assumed to be the dominant scattering mechanism. Equation (10) can be readily solved for f^1, which gives

$$f^1 = \frac{-eE_s}{[1 - i\tau(\omega - qv_z)]}\left(\frac{v_z\tau}{k_BT} f^0 - \frac{\langle v_z\tau(\partial f^0/\partial v)/mv[1 - i(\omega - qv_z)\tau]\rangle}{\{1 - \langle 1/[1 - i(\omega - qv_z)\tau]\rangle\}}\right) . \tag{12}$$

Now the current density associated with the acoustic wave [i.e., the component varying as $e^{-i(\omega t - qz)}$] can be obtained from the expression

$$J_1 = -e\int v_z f^1\, d^3v \, . \tag{13}$$

Substituting (12) into (13) and integrating over angles, one obtains

$$J_1 = -\frac{K_n\pi e^2 i v_s^2 v_0^3 E_s}{\omega k_B T} \times\int_0^\infty \frac{x^{1/2}e^{-x}\{2 + B\ln[(B-1)/(B+1)]\}}{\{1 + (iv_s/2v\omega\tau)\ln[(B-1)/(B+1)]\}}\, dx \, , \tag{14}$$

where

$$B=\frac{v_s}{v}+\frac{i}{qv\tau}, \quad v_0=\left(\frac{2k_BT}{m}\right)^{1/2}, \quad x=\frac{mv^2}{2k_BT},$$

and K_n is the normalization constant of the distribution function which is given by

$$K_n=\frac{N_0}{4\pi\int_0^\infty \exp(-mv^2/2k_BT)v^2\,dv}. \tag{15}$$

Comparing Eq. (14) with $J_1=\sigma^1\sigma_0 E_s$ ($\sigma_0=4N_0e^2\tau_0/3\pi^{1/2}m$), one obtains the following expression for dimensionless conductivity:

$$\sigma^1=-3i\frac{v_s}{v_0}\frac{1}{ql} \times\int_0^\infty \frac{\{2+B\ln[(B-1)/(B+1)]\}x^{1/2}e^{-x}}{\{2+(i/ql)\ln[(B-1)/(B+1)]\}}\,dx. \tag{16}$$

Now the absorption coefficient of an acoustic wave is directly related to σ^1 through the relation[6]

$$\frac{\alpha}{(2N_0m/\tau_0\rho_d v_s)(C/mv_s^2)^2(\omega^2/\omega p^2)^2}=\alpha_0=\mathrm{Re}[\sigma^1], \tag{17}$$

where ρ_d is the density of the semiconductor, and $\omega_p=(4\pi N_0e^2/m)^{1/2}$ is the plasma frequency. The symbol Re stands for the real part, and α_0 is a dimensionless measure of the absorption coefficient of sound wave. The integral in Eq. (16) cannot be exactly evaluated. We shall therefore evaluate it approximately for the following regions of frequencies, i.e., (i) $ql<s<1$, (ii) $s<ql<1$, (iii) $1/s>ql>1$, and (iv) $ql>1/s>1$ (where $s=v_s/v_0$). Thus from (16) and (17) one obtains the following expressions for the absorption coefficients:

(i) $ql<s<1$:

$$\alpha_0\simeq 1, \tag{18}$$

(ii) $s<ql<1$:

$$\alpha_0=\frac{9s^2}{q^2l^2}\left[1-\frac{9s^2}{q^2l^2}\,e^{9s^2/q^2l^2}E_1\left(\frac{9s^2}{q^2l^2}\right)\right], \tag{19}$$

(iii) $1/s>ql>1$:

$$\alpha_0=\frac{3\pi s^2}{2ql}\frac{(1-2/\pi ql)}{(1-\pi/2ql)}, \tag{20}$$

(iv) $ql>1/s>1$:

$$\alpha_0\simeq 3\pi s^2/2ql, \tag{21}$$

where $E_1(z)=\int_z^\infty (e^{-x}/x)\,dx$ is a well-known tabulated integral.[7]

In obtaining Eqs. (18)–(21) we have neglected the terms an order of magnitude smaller than those retained in the analysis.

IV. DISCUSSION

It may be interesting to compare our results with those obtained by Jacoboni and Prohofsky using the CHH model for the collision term. It is seen that for $ql<s<1$ and $ql>1/s>1$ both collision terms give the same results. However, for $s<ql<1$ and $1/s>ql>1$ the ratios of the two calculations are given by

$$\frac{\alpha}{\alpha_{JP}}=\frac{9s^2}{q^2l^2}\left[1-\frac{9s^2}{q^2l^2}\,e^{9s^2/q^2l^2}E_1\left(\frac{9s^2}{q^2l^2}\right)\right] \quad (\text{for } s<ql<1), \tag{22}$$

$$\frac{\alpha}{\alpha_{JP}}=\frac{(1-2/\pi ql)}{(1+2/\pi ql)}\frac{(1+1/s^2q^2l^2)}{(1+4/\pi s^2q^2l^2)} \quad (\text{for } 1/s>ql>1), \tag{23}$$

where α_{JP} in Eqs. (22) and (23) refer to the corresponding values of absorption coefficient obtained by Jacoboni and Prohofsky. It is seen from Eqs. (22) and (23) that for the intermediate region of frequencies, i.e., $s<ql<1/s$, the ratio of the two calculations is significantly different from unity. To have a numerical appreciation of the relative magnitudes of the absorption coefficient, we shall consider the following two cases:

Case A: $ql=\frac{1}{20}$, $s=\frac{1}{30}$,

$\alpha/\alpha_{JP}=0.74$;

Case B: $ql=20$, $s=\frac{1}{30}$,

$\alpha/\alpha_{JP}=0.79$.

Thus we see that taking into account the appropriate collision term results in decreasing the absorption coefficient by as much as 25% or more in the intermediate range of frequencies. Another important consequence of using a more appropriate collision term is that at the high frequencies, the limiting behavior is obtained when $ql\gg 1$. Jacoboni and Prohofsky, however, conclude [see their Eqs. (47) and (48)] that the high-frequency limit is reached for $\omega\tau_0\gg 1/s$, which looks unphysical. The reason for this difference lies in the fact that Jacoboni and Prohofsky use the CHH model, which is not valid for energy-dependent collision frequencies.

An important conclusion of the present investigation is that for frequencies for which $s<ql<1/s$, one must take into account the modified collision term in the calculation of absorption coefficients.

ACKNOWLEDGMENT

One of us (S. S.) is grateful to Professor M. S. Sodha for encouragement and discussions.

*Work partially supported by NSF (USA).

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