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Noise of hot carriers in single-injection solid-state diodes with traps lying below the Fermi level

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An exact and an approximate solution for the current-voltage characteristics of the hot-carrier single-injection solid-state diodes with a single set of traps lying well below the thermal-equilibrium Fermi level (operating in the insulator regime) have been calculated. In addition, the low frequency and thermal noise are calculated at different critical currents. The approximate solution is calculated with the help of the regional approximation. An expression for the critical currents and voltages shows that there is a large change in current occuring over a restricted change in voltage. It appears from the study of the electrical-noise behavior that there is a noise-suppression effect at higher currents.

I. INTRODUCTION

In this paper a study has been made of the electron-conduction and electrical-noise behavior of a hot-carrier single-injection solid-state diode with a single set of traps lying below the Fermi level. A similar approach to the various types of diodes has been presented by Sharma.[1] The expression for the low-frequency and thermal noise in a low electric field (mobility independent of field strength) operating in an insulator regime has been calculated by Sharma[2] and Sharma and Srivastava,[3] respectively, based on the method given by van der Ziel.[4,5]

The method of regional approximation[6–14] is used to calculate the current-voltage characteristics. This method was first used by Lampert[13] (Lampert and Schilling[8] for insulator problems). Shockley and Prim[14] discussed the space-charge limited-current punch-through problem for semiconductors. The corrections to the current-voltage characteristic produced by inclusion of the diffusion current in the problem has been studied by Lampert and Edelman[15] with the aid of a high-speed digital computer. These corrections are of practical interest in that they establish the magnitude of error in the determination of trap density.

van der Ziel[4] and Sergiescu[16] have given an alternate theory of noise in space-charge-limited solid-state diodes, which gives a smaller value for the noise than do Webb and Wright.[17] The experimental results[18,19] are in agreement with Klaassen[20] and van der Ziel,[5] but not with van der Ziel's earlier paper.[4] Sergiescu and Friedman[21] gave a computed solution in a semiconductor based on the pattern given by Van Vliet and Fassett.[22] Noise in single-injection solid-state diodes operating in the hot-carrier regime is calculated by Gisolf and Zijlstra[9] and Sharma.[1] In their paper, Gisolf and Zijlstra[9] have shown that the field-dependent mobility can be applicable throughout the diode. The expression for high-field mobility is

$$\mu(E)=\mu_0(E_c/E(x))^{1/2}, \tag{1}$$

where μ_0 is the low electric-field mobility, E_c is the critical electric field defined in Ref. 9, and $E(x)$ is the electric field at distance x. We have used Eq. (1) in all sections of our paper.

In Secs. II and III the exact and an approximate solution are calculated. In Sec. IV we calculated the low-frequency noise at different critical currents and noted that the mobility is fluctuating with the fluctuation of the electric field. In Sec. V the thermal noise at different critical currents is calculated. In Sec. VI the validity of the approximation is discussed.

II. EXACT ANALYTIC SOLUTION

The equations defining the current density and Poisson's law in the insulator with traps lying below the Fermi level are

$$J=e\mu_0(E_c)^{1/2}n[E(x)]^{1/2}, \tag{2}$$

subject to the boundary condition,

$$E(0)=0 \tag{3}$$

and

$$\begin{aligned}\left(\frac{\epsilon}{e}\right)\frac{dE}{dx}&=(n-N_0)+(p_{t,0}-p_t)\\&=\left(n-\frac{N_tN}{gn}\right)+\left(\frac{N_tN}{gN_0}-N_0\right).\end{aligned} \tag{4}$$

In Eq. (4), we have used the deep-trap relation from Refs. 6 and 10. To calculate the current-voltage characteristics the following dimensionless variables are used:

$$\begin{aligned}u&=\frac{N_0}{n(x)}=\frac{N_0e\mu_0\sqrt{E_c}(E(x))^{1/2}}{J},\\w&=\frac{e^2N_0^2\mu_0(E_c)^{1/2}x}{\epsilon J[E(x)]^{1/2}},\\v&=\frac{e^3N_0^3\mu_0^2V(x)E_c}{\epsilon J^2E(x)}.\end{aligned} \tag{5}$$

Equation (4) and the dimensionless variable u gives

$$\left(\frac{\epsilon}{eN_0}\right)\frac{dE}{dx}=\left(\frac{1}{u}-Bu\right)+(B-1)\,, \tag{6}$$

where

$$B=N_tN/gN_0^2=p_{t,0}/N_0\,. \tag{7}$$

By using dimensionless variables (5) and Eq. (2), Eq. (6) can be written

$$u\,dw+w\,du=(2u^2du)/[1-Bu^2+(B-1)u]\,, \tag{8}$$

giving the solution

$$uw=-\frac{2}{B}\left(u+\frac{(2B-1)\ln(Bu+1)}{B(1+B)}+\frac{(B-2)\ln(u-1)}{B+1}\right), \tag{9}$$

which satisfys the usual boundary condition [Eq. (3)] $w=0$ at $u=0$. In the dimensionless notation the voltage integral is

$$\begin{aligned} v&=\frac{1}{u^2}\int_0^u u^2(u\,dw+w\,du) \\ &=\left(-\frac{2}{Bu^2}\right)\int_0^u\frac{u^4\,du}{u^2-[(B-1)/B]u-1/B}\,. \end{aligned} \tag{10}$$

The voltage integral (10) can be obtained from Eqs. (2), (5), and (8). Making partial fractions [Eq. (10)] and integration gives

$$\begin{aligned} v=-\frac{2}{Bu^2}\Bigg\{&\frac{u^3}{3}-\frac{(B-1)u^2}{2B^2}+\frac{(B^2-B+1)u}{B^2} \\ &+\frac{(B^2-B+1)}{B^3(1+B)}\left[\ln\left(\frac{B}{Bu+1}\right)+B^2\ln(u-1)\right]\Bigg\}. \end{aligned} \tag{11}$$

In terms of dimensionless variables, the current density J and the voltage V can be written

$$J^2=\frac{e^3N_0^3\mu_0^2E_cL}{\epsilon}\,\frac{1}{u_aw_a}\,,\quad V=\frac{eN_0L^2}{\epsilon}\,\frac{v_a}{w_a^2}\,. \tag{12}$$

In Eq. (12) the subscript a corresponds to $x=L$ in Eq. (5). Equation (12) gives the current-voltage characteristics for space-charge-limited hot-carrier injection into an insulator with traps lying below the Fermi level.

III. REGIONAL APPROXIMATION

We obtain a solution to this problem with the help of the method of regional approximation. The method of calculation is the same as with low field-mobility cases. To calculate an accurate approximation the insulator is divided into three regions. In the insulator, the injected free-carrier concentration n_i is a decreasing function of x. At not-too-high currents there will be a transition plane $x_2(J)$ where the concentration of injected carriers (n_i) is equal to the thermally generated free carrier (N_0). To the right of plane x_2, where $N_0>n_i$ we neglect n_i; to the left of plane, where $n_i>N_0$ we neglect N_0. Thus the plane $x_2(J)$ separates the insulator in space-charge and Ohmic regions. Similarly in low-field-mobility cases the space-charge region can be separated into two regions, due to the additional term $(p_{t,0}-p_t)$ in Poisson's equation (4). Thus there will be a plane x_1, where $n_i(x_1)=n(x_1)=p_{t,0}$. To the left of plane x_1, $n>p_{t,0}$ and $p_{t,0}$ is neglected; to the right of plane x_1, up to plane x_2, where $p_{t,0}-p_t>n-N_0$, $n-N_0$ as well as p_t can be neglected. The current density and Poisson's equation in different regions follow.

Region I. $(0\leqslant x\leqslant x_1)$,

$$J=e\mu_0n(x)(E_c)^{1/2}[E(x)]^{1/2}\,, \tag{13}$$

$$\frac{\epsilon\epsilon_0}{e}\,\frac{dE}{dx}=n\,, \tag{14}$$

$$n(x_1)=p_{t,0}\,. \tag{15}$$

Region II. $(x_1\leqslant x\leqslant x_2)$,

$$J=e\mu_0n(x)(E_c)^{1/2}[E(x)]^{1/2}\,, \tag{16}$$

$$\frac{\epsilon\epsilon_0}{e}\,\frac{dE}{dx}=p_{t,0}\,, \tag{17}$$

$$n(x_2)=N_0\,. \tag{18}$$

Region III. $(x_2\leqslant x\leqslant L)$,

$$J=e\mu_0N_0(E_c)^{1/2}[E(x)]^{1/2}\,, \tag{19}$$

$$\frac{\epsilon\epsilon_0}{e}\,\frac{dE}{dx}=0\,. \tag{20}$$

In addition to Eq. (3), the boundary conditions of the problem are the electric-field continuity equations

$$E(x_1^-)=E(x_1^+)\,,\quad E(x_2^-)=E(x_2^+)\,. \tag{21}$$

In terms of dimensionless variables (5), the Poisson equations in regions I, II, and III become

Region I:

$$d(uw)=2u^2\,du\,, \tag{22}$$

Region II:

$$d(uw)=(2u/B)\,du\,, \tag{23}$$

Region III:

$$\frac{du}{dw}=0\,. \tag{24}$$

From Eqs. (5), (15), and (18), the transition planes $x_1(w_1)$ and $x_2(w_2)$ correspond to

$$u_1=u(w_1)=1/B\,,\quad u_2=u(w_2)=1\,. \tag{25}$$

Integrating Eq. (22) gives

Region I:

$$w=\tfrac{2}{3}u^2 \tag{26}$$

satisfying the boundary condition $u=0$ at $w=0$. Equations (25) and (26) give

$$w_1=2/3B^2\,. \tag{27}$$

The integral of Eq. (23) is, using Eqs. (25) and (27),

$$w=(1/B)u+5/3B^2\ . \tag{28}$$

From Eqs. (25) and (28), we derive

$$w_2=1/B+5/3B^2\ . \tag{29}$$

The integral of Eq. (24) is

Region III:

$$u=u_2=1\ , \tag{30}$$

which satisfys the boundary condition (21).

From Eqs. (5), (27), and (30) we obtain

$$x_1=\frac{2\epsilon J^2}{3B^2e^3N_0^3\mu_0^2E_c}\ ,\quad x_2\simeq\frac{\epsilon J^2}{Be^3N_0^3\mu_0^2E_c}\ . \tag{31}$$

Corresponding to the transition planes x_2 and x_1, there are critical currents of $J_{\text{cr},1}$, $J_{\text{cr},2}$, respectively, defined by $x_2(J_{\text{cr},1})=L$ and $x_1(J_{\text{cr},2})=L$. Then from (31) we derive

$$J^2_{\text{cr},1}=\frac{Be^3N_0^3\mu_0^2E_cL}{\epsilon}\ ,\quad J^2_{\text{cr},2}=\frac{3B^2e^3N_0^3\mu_0^2E_cL}{2\epsilon}\ . \tag{32}$$

There are three separate regions in the current-voltage characteristics,

$J<J_{\text{cr},1}$ (Ohm's law regime),

$J_{\text{cr},1}\leqslant J\leqslant J_{\text{cr},2}$ (trap-filled limited regime),

$J_{\text{cr},2}<J$ (trap-free square-law regime).

The potentials in the separate regions are

Region I:

$$v=\tfrac{3}{5}\,uw=\tfrac{2}{5}\,u^3\ . \tag{33}$$

Using Eq. (25) we derive

$$v_1=v(w_1)=2/5B^3\ . \tag{34}$$

Region II:

$$v=v_1+\frac{1}{u^2}\int_{u_1}^{u}\int_{w_1}^{w}u^2(u\,dw+w\,du)\ . \tag{35}$$

From Eqs. (28) and (35), we derive

$$\begin{aligned}v&=v_1+\frac{1}{u^2}\int_{u_1}^{u}\left(\frac{2u^3}{3}+\frac{5u^2}{3B^2}\right)du\\&=-\frac{2}{5B^3}+\frac{u^2}{2B}+\frac{5u}{9B^2}-\frac{19}{18}\,\frac{1}{B^5u^2}\ .\end{aligned} \tag{36}$$

Equations (36) and (25) give

$$v_2=v(w_2)=\frac{1}{2B}+\frac{5}{9B^2}+\frac{2}{5B^3}-\frac{19}{18B^5}\ . \tag{37}$$

Region III:

$$v=v_2+(w-w_2)=w-\frac{1}{2B}-\frac{10}{9B^2}+\frac{2}{5B^3}-\frac{19}{18B^5}\ . \tag{38}$$

For full applied voltage, $w=w_a$, we derive

$$v_a=w_a-\frac{1}{2B}-\frac{10}{9B^2}+\frac{2}{5B^3}-\frac{19}{18B^5}\ . \tag{39}$$

For $J<J_{\text{cr},1}$, the dimensionless current-voltage relationship is

$$\begin{aligned}\frac{v_a}{w_a^2}&=\frac{1}{w_a}-\frac{1}{B}\left(\frac{1}{2}+\frac{10}{9B}-\frac{2}{5B^2}+\frac{19}{18B^4}\right)\left(\frac{1}{w_a^2}\right)\\&\simeq\frac{1}{w_a}-\frac{1}{2B}\left(\frac{1}{w_a}\right)^2\ .\end{aligned} \tag{40}$$

When $J<J_{\text{cr},1}$ all regions (I, II, and III) are present in the insulator. The contribution of region I is negligibly small, since $x_1(J)/x_2(J)\simeq 1/2B\ll 1$ [Eq. (31)]. The left-hand boundary of region II becomes $w=0$, and with the current-voltage characteristics at $J<J_{\text{cr},1}$,

$$v_a/w_a^2\simeq 1/w_a-(1/2B)\,(1/w_a)^2\ . \tag{41}$$

For a trap-filled limited regime the dimensionless current-voltage relation is

$$\frac{v_a}{w_a^2}\simeq\frac{B}{2}-\frac{10}{9B}\left(\frac{1}{w_a}\right). \tag{42}$$

In a trap-free square-law regime only region I is present in the insulator.

The Ohm's-law regime terminates in a trap-filled limited regime at $J=J_{\text{cr},1}$ [from Eqs. (29) and (39) $v_{a,\text{cr},1}\simeq 1/2B$] and at $J=J_{\text{cr},2}$ [from Eq. (34) $v_{a,\text{cr},2}=2/5B^3$]. Using dimensionless variables (5) for these values, the critical voltages corresponding to critical currents are

$$V_{\text{cr},1}=\frac{BeN_0L^2}{2\epsilon}=\frac{ep_{t,0}L^2}{2\epsilon}\ ,\quad V_{\text{cr},2}=\frac{9BeN_0L^2}{10\epsilon}. \tag{43}$$

The ratios of critical voltages and currents are

$$J^2_{\text{cr},2}/J^2_{\text{cr},1}=\tfrac{3}{2}B\ ,\quad V_{\text{cr},2}/V_{\text{cr},1}=\tfrac{9}{5}\ . \tag{44}$$

Equation (44) shows that there is a large change in current occurring over a restricted change in voltage.

IV. LOW-FREQUENCY NOISE IN THE DIODE

The equation of current flow in a single-injection diode with a single set of traps lying below the Fermi level operating in the hot-carrier regime is

$$J=q\mu(E)\ E(x)n(x)\ , \tag{45}$$

where $\mu(E)$ is given in Eq. (1). According to regional approximation, the Poisson equation in the space-charge region of the diode is

$$\frac{dE}{dx}=-\frac{q}{\epsilon}\ n(x)\ . \tag{46}$$

The small-signal equations can be obtained by letting

$$E=E_0+\Delta E,\quad n=n_0+\Delta n,\quad J=J_0+\Delta J,\quad \mu=\mu_0+\Delta\mu\ . \tag{47}$$

Equations (45) and (47) give

$$\Delta J = q[nE\Delta\mu(E) + \mu(E)\{n\Delta E + E_0\Delta n\}] . \tag{48}$$

In the above equation the mobility is also a fluctuating quantity. From Eq. (1), we derive

$$\Delta\mu(E) = (-\tfrac{1}{2})\mu(E)\Delta E/E . \tag{49}$$

The study of noise is made by open circuiting the output of the diode. The low-frequency noise in the open-circuited diode is caused by carrier-density fluctuations in the potential minimum and calculating the voltage fluctuation (ΔV) caused by a fluctuation δn_0 in the carrier density n_0. Assuming the cathode to be an injecting contact with a relatively small barrier and the anode as a blocking contact with a relatively large barrier, the carrier density at the anode is seen to be practically zero at all times, i.e., $\Delta J = 0$.

Letting $\Delta J = 0$ and $\Delta n \simeq \delta n_0$, and using Eqs. (46) (49), and (48) gives,

$$E_0\delta n_0 - \frac{\epsilon}{2q}\frac{dE}{dx}\Delta E(x) = 0 . \tag{50}$$

Integrating Eq. (50) gives

$$V(x)\delta n_0 + (\epsilon/2q)E(x)\Delta E = \alpha , \tag{51}$$

since

$$\int E_0\,dx = -V(x) .$$

In the above equation, α is an integration constant. At the potential minimum $E(x) = 0$ and $V(x) = 0$, $\alpha = 0$. The value of the fluctuating electric field is

$$\Delta E(x) = (-2q/\epsilon)\,[V(x)/E(x)]\delta n_0 . \tag{52}$$

Using the boundary condition (3) and Eqs. (1), (45), and (46) gives the differential equation

$$[E(x)]^{1/2}\frac{dE}{dx} = \frac{J}{\epsilon\mu_0(E_c)^{1/2}} . \tag{53}$$

Following integration, we derive

$$E(x) = [3J/2\epsilon\mu_0(E_c)^{1/2}]^{2/3}\,x^{2/3} \tag{54}$$

and

$$V(x) = \tfrac{3}{5}[3J/2\epsilon\mu_0(E_c)^{1/2}]^{2/3}\,x^{5/3} . \tag{55}$$

Substituting Eqs. (54) and (55) into Eq. (52) we derive

$$\Delta E(x) = (6qx/5\epsilon)\delta n_0 . \tag{56}$$

The fluctuation in voltage at distance x_2 can be obtained by integrating Eq. (56),

$$\Delta V(x_2) = \frac{6q\delta n_0}{5\epsilon}\int_0^{x_2} x\,dx = \frac{3qx_2^2}{5\epsilon}\,\delta n_0 , \tag{57}$$

where x_2 is the transition plane separating the space-charge region and the Ohmic region.

The Fourier analysis of Eq. (57) gives the spectral intensity of voltage fluctuations at distance x_2 as

$$S_{V(x_2)}(f) = 4kTR_n = (3qx_2^2/5\epsilon)^2\,S_{n_0}(f) , \tag{58}$$

where R_n is the noise resistance of the space-charge region and $S_{n_0}(f)$ is the spectral intensity of the carrier-density fluctuations in the potential minimum.

The value of the spectral intensity of the carrier fluctuations can be given (from Ref. 23) as

$$S_{n_0}(f) = 2\bar{n}_0 , \tag{59}$$

where n_0 follows a Poisson distribution. Equation (59) is obtained by applying Carson's theorem (Ref. 23). If, in a diode, each particle carries a charge q then the Poisson average is given by

$$\bar{n}_0 = I_a/q = AJ/q . \tag{60}$$

Substituting the value obtained in (59) into (58), we derive the value of the noise resistance (when the current is less than the $J_{\text{cr},1}$) as

$$R_n = \left(\frac{3qx_2^2}{5\epsilon}\right)^2\frac{2\bar{n}_0}{4kT} = \left(\frac{3\epsilon J^4}{5B^2q^5N_0^6\mu_0^4E_c^2}\right)^2\frac{\bar{n}_0}{2kT} , \tag{61}$$

where the value of x_2 is obtained from Eq. (31) and $\bar{n}_0$ [from Eq. (60)] corresponds to $I_a = AJ$.

A. Low-frequency noise at first critical current ($J = J_{\text{cr},1}$)

There is a trap-filled limited regime in the diode at the first critical current condition. The value of the first critical electric field is given [from Eq. (54)] as

$$x = L ,\ E_{\text{cr},1} = -[3J_{\text{cr},1}/2\epsilon\mu_0(E_c)^{1/2}]^{2/3}\,L^{2/3} . \tag{62}$$

In Eq. (52), substituting the values of $E_{\text{cr},1}$ and $V_{\text{cr},1}$ [from Eqs. (62) and (43), respectively], we get the value of the fluctuating first critical field as

$$\Delta E_{\text{cr},1} = \frac{Bq^2N_0L^2}{\epsilon^2}\left(\frac{2\epsilon\mu_0(E_c)^{1/2}}{3LJ_{\text{cr},1}}\right)^{2/3}\delta n_{\text{cr},1} . \tag{63}$$

Making a Fourier analysis of Eq. (63), the first critical resistance (R_1) is as follows:

$$R_1 = \frac{B^2q^4N_0^2L^4}{\epsilon^4}\left(\frac{2\epsilon\mu_0(E_c)^{1/2}}{3LJ_{\text{cr},1}}\right)^{4/3}\frac{\bar{n}_{\text{cr},1}}{2kT} , \tag{64}$$

where $\bar{n}_{\text{cr},1}$ follows from Eq. (60) for $\bar{n}_0 = \bar{n}_{\text{cr},1}$ and $J = J_{\text{cr},1}$.

B. Low-frequency noise at second critical current ($J = J_{\text{cr},2}$)

For $J = J_{\text{cr},2}$, only region I is present in the insulator. The second critical electric field is

$$E_{\text{cr},2} = -[3J_{\text{cr},2}/2\epsilon\mu_0(E_c)^{1/2}]^{2/3}\,L^{2/3} . \tag{65}$$

The fluctuating electric field ($E_{\text{cr},2}$) is

$$\Delta E_{\text{cr},2} = (9Bq^2N_0L^2/5\epsilon^2)[2\epsilon\mu_0(E_c)^{1/2}/3LJ_{\text{cr},2}]^{2/3}\,\delta n_0 , \tag{66}$$

where the values of $V_{\text{cr},2}$ and $E_{\text{cr},2}$ are substituted into Eq. (56) from Eqs. (43) and (65).

Following the integration of Eq. (68) (within the limits of 0 to L) and making a Fourier analysis, we get the noise resistance,

$$R_2=\frac{81B^2q^4N_0^2L^4}{25\epsilon^4}\left(\frac{2\epsilon\mu_0(E_c)^{1/2}}{3LJ_{\mathrm{cr},2}}\right)^{4/3}\frac{\overline{n}_{\mathrm{cr},2}}{2kT}. \qquad (67)$$

The ratio of the noise resistance of a trap-filled limited regime and a trap-free square-law regime is

$$\frac{R_2}{R_1}=\frac{81}{25}\left(\frac{J_{\mathrm{cr},1}}{J_{\mathrm{cr},2}}\right)^{4/3}\frac{\overline{n}_{\mathrm{cr},1}}{\overline{n}_{\mathrm{cr},2}},\quad \frac{\overline{n}_{\mathrm{cr},1}}{\overline{n}_{\mathrm{cr},2}}=\frac{J_{\mathrm{cr},1}}{J_{\mathrm{cr},2}}, \qquad (68)$$

where the first relation follows from Eq. (64) and (67) and the second corresponds to Eq. (60) for different critical currents. From Eqs. (44) and (68), we derive

$$\frac{R_2}{R_1}=\frac{81}{25}\left(\frac{J_{\mathrm{cr},1}}{J_{\mathrm{cr},2}}\right)^{7/3}=\frac{54}{25B}\left(\frac{2}{3B}\right)^{1/6}. \qquad (69)$$

The low-frequency noise is much reduced in a trap-free square-law regime as compared with a trap-filled limited regime.

The results here obtained are based upon fluctuations in the carrier density at the potential minimum. Though this noise mechanism is always present, it is usually masked by the thermal noise mechanism discussed in Sec. V.

V. THERMAL NOISE IN THE DIODE

If the diode is open circuited for ac signals, then in the frequency interval Δf the thermal noise can be represented by a current generator $(i^2)^{1/2}$ in parallel with the diode,

$$i^2=4kTR_{n'}\Delta fg^2=\sum 4kT\Delta R\Delta fg^2, \qquad (70)$$

where ΔR is the resistance of secion Δx and a summation is carried out over all sections Δx.

In the space-charge limited single-injection hot-carrier current flow the electric-field strength is given by Eq. (54). The concentration of the hot carriers in a space-charge region can be obtained from Eqs. (46) and (54),

$$n=(2\epsilon/3q)[3I_a/2\epsilon\mu_0(E_c)^{1/2}A]^{2/3}x^{-1/3}. \qquad (71)$$

According to van der Ziel[5,24] the noise resistance (ΔR) of a section Δx is

$$\Delta R=\frac{\Delta x}{q\mu_0nA[E_c/E(x)]^{1/2}}=\left(\frac{3}{2\mu_0(E_c)^{1/2}A\epsilon}\right)^{2/3}\frac{x^{2/3}\Delta x}{I_a^{1/3}} \qquad (72)$$

A. Thermal noise when current is less than the critical value ($J<J_{\mathrm{cr},1}$)

Since the current is less than the first critical value, all regions are present in the insulator. Integration of Eq. (72) (within the limits 0 to x_2) gives the noise resistance (R_{n_1}) in the space-charge region of the diode as

$$R_{n_1}=\sum\Delta R=\left(\frac{3}{2\mu_0(E_c)^{1/2}A\epsilon}\right)^{2/3}\frac{3x_2^{5/3}}{5I_a^{1/3}}=\frac{V(x_2)}{I_a}, \qquad (73)$$

where $V(x_2)$ is the voltage corresponding to $x=x_2$ in Eq. (55). The differential conductance (g) for this region is

$$g=\frac{dI_a}{dV(x_2)}=\frac{3I_a}{2V(x_2)}. \qquad (74)$$

From Eq. (73) and (74), we derive

$$R_{n_1}=3/2g. \qquad (75)$$

To calculate the noise resistance (R_{n_2}) of region III, the value of the thermally generated electron (N_0) is substituted (instead of n) in Eq. (72) and we derive

$$\Delta R=\frac{\Delta x}{q\mu_0N_0A[E_c/E(x)]^{1/2}}=\frac{1}{q\mu_0(E_c)^{1/2}N_0A}\times\left(\frac{3J}{2\epsilon\mu_0(E_c)^{1/2}}\right)^{1/3}x^{1/3}\Delta x. \qquad (76)$$

The noise resistance (R_{n_2}) of region III can be obtained by the integration of Eq. (76) (within the limits x_2 to L)

$$R_{n_2}=\frac{3}{4q\mu_0(E_c)^{1/2}N_0A}\left(\frac{3J}{2\epsilon\mu_0(E_c)^{1/2}}\right)^{1/3}(L^{4/3}-x_2^{4/3}). \qquad (77)$$

We represent the noise in a frequency interval Δf by a current generator $\sqrt{i^2}$ in parallel with the diode,

$$i^2=4kTR_{n'}\Delta fg^2, \qquad (78)$$

where

$$R_{n'}=R_{n_1}+R_{n_2}. \qquad (79)$$

Equation (78) gives the estimate of thermal noise in a hot-carrier single-injection diode with traps lying below the Fermi level when current is less than the critical value.

B. Thermal noise at first critical current ($J=J_{\mathrm{cr},1}$)

For $J=J_{\mathrm{cr},1}$, the trap-filled limited regime is present in the insulator. The noise resistance ($R_{\mathrm{cr},1}$) is obtained by integrating Eq. (72) (within the limits 0 to L),

$$R_{\mathrm{cr},1}=\sum\Delta R=\left(\frac{3}{2\epsilon\mu_0\sqrt{E_c}A}\right)^{2/3}\frac{3L^{5/3}}{5I_{\mathrm{cr},1}^{1/3}}=\frac{V_{\mathrm{cr},1}}{I_{\mathrm{cr},1}}, \qquad (80)$$

where the value of $V_{\mathrm{cr},1}$ corresponds to $x=L$ and $I_a=I_{\mathrm{cr},1}$ in Eq. (55).

The critical differential conductance ($g_{\mathrm{cr},1}$) of the diode is

$$g_{\mathrm{cr},1}=\frac{dI_{\mathrm{cr},1}}{dV_{\mathrm{cr},1}}=\frac{3I_{\mathrm{cr},1}}{2V_{\mathrm{cr},1}}. \qquad (81)$$

From Eqs. (80) and (81), we derive

$$R_{\mathrm{cr},1}=\frac{3}{2g_{\mathrm{cr},1}}\,. \tag{82}$$

If the noise at the first critical current is represented by a current generator $\sqrt{\overline{i^2_{\mathrm{cr},1}}}$ in parallel with the diode, then

$$i^2_{\mathrm{cr},1}=4kTR_{\mathrm{cr},1}\Delta f g^2_{\mathrm{cr},1}=6kT\Delta f g_{\mathrm{cr},1}\,. \tag{83}$$

In the above equation the value of $R_{\mathrm{cr},1}$ is obtained from (82). Equations (32), (43), and (81) give

$$g_{\mathrm{cr},1}=3A\mu_0(\epsilon qN_0E_c/BL^3)^{1/2}\,. \tag{84}$$

C. Thermal noise at the second critical current ($J=J_{\mathrm{cr},2}$)

Only region I is in the insulator. This is a perfect trap-free insulator region. The noise resistance ($R_{\mathrm{cr},2}$) is

$$R_{\mathrm{cr},2}=\sum\Delta R=\left(\frac{3}{2\mu_0\sqrt{E_c}\,A\epsilon}\right)^{2/3}\frac{3L^{5/3}}{5I^{1/3}_{\mathrm{cr},2}}=\frac{V_{\mathrm{cr},2}}{I_{\mathrm{cr},2}}\,. \tag{85}$$

The second critical conductance ($g_{\mathrm{cr},2}$) of the diode is

$$g_{\mathrm{cr},2}=\frac{dI_{\mathrm{cr},2}}{dV_{\mathrm{cr},2}}=\frac{3I_{\mathrm{cr},2}}{2V_{\mathrm{cr},2}}\,. \tag{86}$$

Equations (85) and (86) give

$$R_{\mathrm{cr},2}=\frac{3}{2g_{\mathrm{cr},2}}\,. \tag{87}$$

The estimate of the thermal noise in a hot-carrier injection diode under the second critical condition is

$$i^2_{\mathrm{cr},2}=4kTR_{\mathrm{cr},2}\Delta f g^2_{\mathrm{cr},2}=6kT\Delta f g_{\mathrm{cr},2}\,, \tag{88}$$

where

$$g_{\mathrm{cr},2}=5A\mu_0(\epsilon qN_0E_c/6L^3)^{1/2}\,. \tag{89}$$

The value of $g_{\mathrm{cr},2}$ is obtained by substituting the values $V_{\mathrm{cr},2}$ and $I_{\mathrm{cr},2}$ from Eqs. (32) and (43), respectively, into (86). The ratio of thermal-noise resistances is

$$\frac{R_{\mathrm{cr},2}}{R_{\mathrm{cr},1}}=\frac{V_{\mathrm{cr},2}I_{\mathrm{cr},1}}{V_{\mathrm{cr},1}I_{\mathrm{cr},2}}=\frac{9}{5}\left(\frac{2}{3B}\right)^{1/2}\,. \tag{90}$$

Equations (73), (80), (85), and (90) show that the open-circuited thermal noise emf corresponds to the dc resistance (V_a/I_a) and the equivalent noise resistance (R_n) of the device equals to (V_a/I_a), which is the statement given in Ref. 25 for the low-field mobility in the case of the space-charge-limited diode. The ratio of conductances at different critical currents is

$$g_{\mathrm{cr},2}/g_{\mathrm{cr},1}=\tfrac{5}{3}\left(\tfrac{1}{6}B\right)^{1/2}\,. \tag{91}$$

The differential conductance of the diode is greater at low currents and decreases very rapidly as the trap-filled limited regime merges into the trap-free insulated regime. This is due to the fact that all the traps are filled at trap-filled limited voltage followed by a very steep current rise.

VI. DISCUSSION

This paper shows that for a high-field-injection regime the change in current in relation to voltage [Eq. (44)] is not so large as in the low field-mobility case. In the low field-injection case the ratios of critical currents to voltages are

$$J_{\mathrm{cr},2}/J_{\mathrm{cr},1}=2B=2p_{t,0}/N_0\,,\quad V_{\mathrm{cr},2}/V_{\mathrm{cr},1}=\tfrac{8}{3}\,. \tag{92}$$

The rapid increase in current in relation to voltage is due to the fact that at a higher voltage all the traps are filled with electrons, the insulator behaves just like a trap-free regime, and all the injected space charges contribute to the current; the traps no longer affect the current flow. The reasons for the approximation made in Sec. III are similar to those of the low field-mobility case[6,8,10] and high-mobility injection case.[9,1] The study of space-charge-limited current behavior is made in Refs. 26–29.

In Secs. IV and V there is a reduction in resistances at higher voltages because the current is inversely proportional to the noise resistance and the value of the current increases very steeply with restricted increase in voltage. As voltage increases the value of transit time decreases and reduces the fluctuation in current. With an increase in current, the Ohm's-law regime ($J<J_{\mathrm{cr},1}$) is merged into trap-filled limited regime ($J_{\mathrm{cr},1}\leq J\leq J_{\mathrm{cr},2}$). Finally in a trap-free regime ($J>J_{\mathrm{cr},2}$) the noise will decrease rapidly.

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[1] Y. K. Sharma, Solid State Electron. (to be published).

[2] Y. K. Sharma, Can. J. Phys. (to be published).

[3] Y. K. Sharma and B. B. Srivastava J. Appl. Phys. (to be published).

[4] A. van der Ziel, Solid-State Electron. 9, 123 (1966).

[5] A. van der Ziel, Solid-State Electron. 9, 899 (1966).

[6] R. K. Willardson and A. C. Beer, *Semiconductors and Semimetals* (Academic, New York, 1970), Vol. 6.

[7] L. Patrick, J. Appl. Phys. 28, 765 (1957).

[8] M. A. Lampert and R. B. Schilling, Phys. Rev. Lett. 18, 493 (1967).

[9] A. Gisolf and R. J. J. Zijlstra, Solid-State Electron. 16, 571 (1973).

[10] M. A. Lampert and P. Mark, *Current Injection in Solids* (Academic, New York, 1970).

[11] A. Rose, Phys. Rev. 97, 1538 (1955).

[12] A. Rose, RCA Rev. 12, 362 (1951).

[13] M. A. Lampert, Phys. Rev. 103, 1648 (1956).

[14] W. Shockley and R. C. Prim, Phys. Rev. 90, 753 (1953).

[15] M. A. Lampert and F. Edelman, J. Appl. Phys. 35, 2971 (1964).

[16] V. Sergiescu, Brit. J. Appl. Phys. 16, 1435 (1965).

[17] P. W. Webb and G. T. Wright, J. Brit. Inst. Radio

Eng. **23**, 111 (1962).

[18] M. A. Nicolet, Appl. Phys. Lett. **9**, 434 (1966).

[19] S. T. Liu, Appl. Phys. Lett. **10**, 308 (1967).

[20] F. M. Klaassen, Solid-State Electron. **11**, 377 (1968).

[21] V. Sergiescu and A. Friedman, Brit. J. Appl. Phys. **17**, 1408 (1966).

[22] K. M. Van Vliet and J. R. Fassett, *Fluctuation Phenomena in Solids*, edited by R. E. Burgess (Academic, New York, 1966).

[23] A. van der Ziel, *Fluctuation Phenomena in Semiconductors* (Butterworth, London, 1959), Chap. 3.

[24] A. van der Ziel, *Noise, Sources, Characterization, Measurement* (Prentice-Hall, Englewood Cliffs, N. J., 1970).

[25] S. T. Hsu, A. van der Ziel, and E. R. Chenette, Solid-State Electron. **10**, 129 (1967).

[26] W. Shockley, Bell Syst. Tech. J. **30**, 990 (1951).

[27] A. Shumka, Solid-State Electron. **14**, 367 (1971).

[28] G. T. Wright, Proc. IEEE **51**, 1642 (1963).

[29] S. T. Liu, Solid-State Electron. **10**, 253 (1967).