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Magnetic phase transition at a surface: Mean-field theory

Pradeep Kumar
Laboratory for Surface Studies and Department of Physics, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53201
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Within mean-field theory, it is possible to analytically calculate the spatial and temperature dependence of the magnetization near a surface and near a phase transition. These algebraic forms are then used in the Landau-Ginzburg free energy to obtain the surface specific heat and a relaxation time for the surface spins driven to a weak nonequilibrium. These physical quantities have been found to have a temperature dependence that varies continuously with the change in the surface-plane exchange constant. Such a behavior is interpreted in terms of an interplay between the correlation length ξ and the extrapolation length λ where λ effectively determines the range of the surface effects.

I. INTRODUCTION

This paper extends some work, initiated by Mills[1] on the analysis of the effect of a surface on the magnetic properties of simple antiferromagnets and ferromagnets. The basic model considered is analogous to the mean-field-theory (MFT) approach of Ginzburg and Landau.[2] While the Ginzburg-Landau (GL) approach is phenomenological, in that the coefficients of the expansion of the free energy are essentially determined by symmetry properties, Mills has been able to show that similar equations may be derived by considering the Brillouin function for the magnetization. This permits us to consider the GL equation for the free energy with known coefficients; namely, those derived from the Brillouin function.

The subject matter, from the point of view of scaling exponents and scaling relations, has been exhaustively reviewed by Binder and Hohenberg.[3] It is well known that the scaling exponents derived from MFT for systems with infinite extent are unsatisfactory. The exponents derived from rigorous considerations of the Ising model yield more accurate results. However, the mean-field theory permits an easy physical interpretation and the relationship of the Ising model with reality is less than transparent. The only known investigation of purely surface magnetic properties, namely, the magnetic low-energy-electron-diffraction (LEED) scattering,[4] favors the mean-field theory.

In the following we have analyzed the MFT in some detail. Analytic expressions are obtained for the temperature and spatial variation of magnetization near or at the surface within the framework of the Heisenberg model. In the ferromagnetic phase, we study the magnetization in two separate limits, i.e. when the exchange constant at the surface is larger or smaller than the bulk-exchange constant. It is found, in agreement with some observations within the Ising model[3] that the temperature dependence of the magnetization varies continuously with the relative stiffening or softening of the surface-exchange constant. This might appear in apparent contradiction with the scaling hypothesis, i.e., near the transition temperature, the scaling exponents are independent of the strength of interaction. We would like to point out that scaling hypothesis, which depends crucially on the presence of a length scale (the correlation length ξ), has to be modified to include two scaling lengths. The extra length has been described by Binder and Hohenberg as the extrapolation length λ which depends upon the range and the magnitude of the surface effects. We have analyzed all of our results in the two cases when either one of them is long compared to the other. Only when $\lambda \ll \xi$ do we find significant change in the exponents associated with surface properties.

The emphasis is on exact results within a somewhat approximate though physically revealing formalism, and the form of the paper is as follows. In Sec. II we analyze the magnetization with several possible values of λ in both the ferromagnetic and paramagnetic phases. The consequences of the interplay of λ and ξ are studied in detail. In Sec. III we consider the GL free energy, with the coefficients that are commensurate with the differential equation in Sec. I, and derive the thermodynamic properties of the system. Section IV is devoted to a brief and simple analysis of relaxation time for the surface magnetization driven to a weak nonequilibrium. The summary in Sec. V gives a brief survey of the essential results obtained in this paper. The Appendix briefly discusses the response to a magnetic field of a paramagnetic phase.

II. MAGNETIZATION AT THE SURFACE

The differential equation that describes the spatial and temperature dependence of the magnetization has been derived by Mills.[1] Taking the external field to be zero and ignoring variations in the plane parallel to the surface, he finds for a simple cubic ferromagnet

$$\frac{a_0^2}{6}\frac{\partial^2\eta}{\partial z^2}+(1-\tau)\eta(z)-\frac{\beta}{\tau^2}\eta^3(z)=0\ , \tag{1}$$

where $\eta(z)=\langle s^z(z)\rangle/s$, a_0 is the lattice parameter, z is the direction perpendicular to the (100) surface, $\tau=T/T_c$, and β is a numerical constant approximately equal to 1 for $s=\frac{1}{2}$. The equation has been derived under the assumptions that (a) the magnetization is small and (b) its variation over length of the order of lattice parameter is small. Except in one case, the final results are consistent with these assumptions. The latter case will be discussed later in this section. If the surface magnetization is required to feel the same molecular field as in the bulk (except for the missing bond and different coupling constant in the surface plane $J_s=J(1-\Delta)$ where J is the bulk-coupling constant), then one has the boundary condition[1]

$$\left.\frac{\partial\eta}{\partial z}\right|_0=\frac{1}{\lambda}\eta(z=0) \tag{2}$$

where $\lambda=a_0/(1+4\Delta)$. Note that λ may be positive or negative depending on whether Δ is greater or less than $-\frac{1}{4}$. Furthermore, it can be large before changing signs and it is this regime that produces some of the surprising results.

Following Mills, we shall rewrite Eqs. (1) and (2) in terms of $f(y)=\eta(y)/\eta_\infty$, where $\eta_\infty=\tau(1-\tau)^{1/2}/\beta^{1/2}$ is the bulk magnetization, and a reduced length $y=z/\xi$, where $\xi=a_0/[6(1-\tau)]^{1/2}$; i.e., the correlation length. Thus

$$\frac{\partial^2 f}{\partial y^2}+f(y)-f^3(y)=0 \tag{3}$$

and

$$f(0)=\frac{\lambda}{\xi}\left.\frac{\partial f}{\partial y}\right|_{y=0}\ . \tag{4}$$

The solutions are obtained by multiplying Eq. (3) by df/dy and integrating with respect to y from y to ∞:

$$\left(\frac{df}{dy}\right)^2+f^2(y)-\tfrac{1}{2}f^4(y)=\tfrac{1}{2} \tag{5}$$

or

$$\left(\frac{df}{dy}\right)^2=\tfrac{1}{2}[1-f^2(y)]^2\ . \tag{6}$$

We have used the boundary conditions at $y=\infty$; i.e., $f(y=\infty)=1$ and $\partial f/\partial y|_\infty=0$. In taking the square root, caution is needed. In particular, we shall see later that the solution with change in the sign of derivative corresponds to a higher free energy.

Thus the stable low-free-energy solution must correspond to a monotonic function of y. If $f'(y)$ at the surface is positive (i.e., $\lambda>0$), we must have $f(0)$ smaller than in the bulk. If, on the other hand, $\lambda<0$ $[f'(0)<0]$, the surface magnetization must be larger. The latter corresponds to the surface magnetic moment[5] which survives above the Curie temperature T_c, and vanishes at a temperature larger than T_c.

Hence for $\lambda>0$, we have

$$\frac{df}{dy}=\frac{1}{\sqrt{2}}[1-f^2(y)]\ . \tag{7}$$

Equation (7) can be further integrated [using the boundary condition, Eq. (4)] and we find for $\lambda>0$,

$$f(y)=\frac{(2\xi^2+4\lambda^2)^{1/2}-\xi\sqrt{2}+2\lambda\tanh y/\sqrt{2}}{2\lambda+[(2\xi^2+4\lambda^2)^{1/2}-\xi(2)^{1/2}]\tanh y/\sqrt{2}} \tag{8}$$

and

$$f(0)=\frac{(2\xi^2+4\lambda^2)^{1/2}-\xi\sqrt{2}}{2\lambda}\ . \tag{9}$$

These expressions are not as formidable as they look. The extreme behavior of $f(0)$ especially can be seen as

$$f(0)=1\quad\text{for }\lambda\gg\xi\ , \tag{10a}$$

and

$$f(0)\simeq\lambda\sqrt{2}/\xi,\quad\text{for }\lambda\ll\xi\ . \tag{10b}$$

It is the latter behavior $[\eta\propto(T_c-T)]$ that has been widely quoted as a surface property. This is also the temperature dependence that has been observed in the magnetic LEED scattering. The above is the first illustration of the interplay between the extrapolation length λ and the correlation length ξ.[6]

Similarly if $\lambda<0$, Eq. (7) correctly describes the behavior of $f(y)$. Now, however, the spatial dependence of magnetization is given by a different function[7(a),(b)] since $f(y)\geq1$. In particular,

$$f(y)=\coth(y/\sqrt{2}+A)\ , \tag{11}$$

$$f(0)=\coth A=\frac{(2\xi^2+4\lambda^2)^{1/2}+\xi\sqrt{2}}{2|\lambda|}\ . \tag{12}$$

The limiting temperature dependences are

$$f(0)=\xi/|\lambda|\sqrt{2}\quad\text{for }\xi\gg|\lambda|\ , \tag{13a}$$

and

$$f(0)=1\quad\text{for }\xi\ll|\lambda|\ . \tag{13b}$$

Thus in the region close to $\Delta\approx-\frac{1}{4}$, the surface magnetization has the same temperature dependence as the bulk magnetization. If Δ is positive and large we have the "surface" behavior and for $\Delta \ll -\frac{1}{4}$, the surface magnetization is actually temperature independent. This is the aforementioned surface magnetic moment. Figure 1 describes the different regions in the (ξ, λ) plane. Region I corresponds to the bulk temperature dependence. Region II represents the behavior according to surface temperature dependence, and region III where Δ is negative and less than $-\frac{1}{4}$ corresponds to the surface magnetic moment that is temperature independent for $\tau < 1$. The arrow around the λ axis represents the variation in λ as Δ varies from large-positive values to large negative ones.

The behavior of this surface moment for $\tau > 1$ may be calculated easily. Now Eq. (1) has to be written in terms of $\eta(z) = \alpha f(z)$ where $\alpha^2 = \tau^2(\tau - 1)/\beta$ and $y = z/\xi$, $\xi = a_0/[6(\tau-1)]^{1/2}$. With the usual integration [as for Eq. (5)], using the boundary conditions that now are $f = f' = 0$ for $y = \infty$; we find

$$\eta(y) = \sqrt{2}\,\alpha/\sinh(A + y)\,, \tag{14}$$

where

$$A = \coth^{-1}\xi/|\lambda|\,.$$

Also,

$$\eta(0) = \frac{\sqrt{2}\,\tau(\tau-1)^{1/2}}{\beta^{1/2}}\left(\frac{\xi^2}{\lambda^2} - 1\right)^{1/2}$$
$$\simeq (\tau_c^s - \tau)^{1/2} \quad \tau_c^s = \frac{T_c^s}{T_c} = 1 + \frac{(1+4\Delta)^2}{6}\,, \tag{15}$$

a reasonably well-known result. The y dependence, however, is new (in its analytic form). Figure 2 shows the spatial dependence of the magnetization in several cases considered. The transition from $\lambda > 0$ to $\lambda < 0$ passes through an infinity in λ (as a function of Δ).[7(b)] Near this transition λ can be larger than ξ. We find $f(y)$ to be independent of y in that regime.[6] The case of a small and negative λ corresponds to a temperature-independent magnetization as has been discussed above.

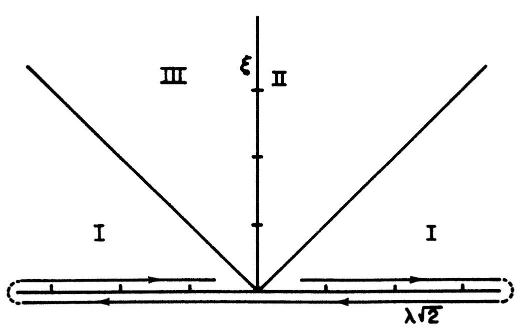

FIG. 1. Different regions in (ξ, λ) plane for the temperature dependence of magnetization.

III. THERMODYNAMICS

The thermodynamic quantities can all be derived from the free energy. The free energy that we use in this section is given by Landau[2]

$$F(\eta, T) = F_0(T) + \int_0^\infty dz \left[A\eta^2(z) + B\eta^4(z) + C\left(\frac{\partial \eta}{\partial z}\right)^2\right]$$
$$+ \frac{C}{\lambda}\eta^2(0)\,. \tag{16}$$

Here $F_0(T)$ is the free energy for the paramagnetic phase. The last term introduced by Kaganov and Omel'yanchuk[8] represents extra contribution due to surface magnetization. In equilibrium, F is a minimum as a function of η. The functional differential of Eq. (16) yields an equation similar to (1) if we identify

$$A = (\tau - 1), \quad B = \beta/2\tau^2, \quad C = \tfrac{1}{6}a_0^2\,.$$

The differentiation with respect to $\eta(0)$ yields the boundary condition (2).

For $\tau > 1$, $\eta(z) = 0$ for $\lambda > 0$, and the magnetization does not contribute to the free energy. For $\tau < 1$, we shall write the magnetic part of the free energy in terms of y and $f(y)$:

$$F^M(\eta, T) = F(\eta, T) - F_0(T)$$
$$= \xi\,\frac{\tau^2(1-\tau)^2}{\beta}\int_0^\infty dy\left[-f^2(y) + \tfrac{1}{2}f^4(y) + \left(\frac{\partial f}{\partial y}\right)^2\right]$$
$$+ \xi\,\frac{\tau^2(1-\tau)^2}{\beta}\,\frac{\xi}{\lambda}f^2(0)\,. \tag{17}$$

If we substitute the solution obtained earlier

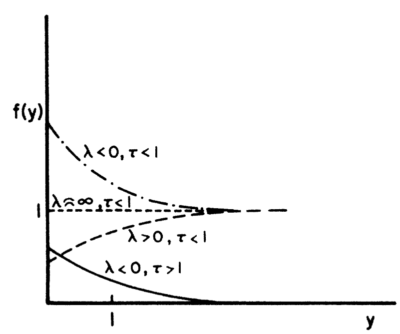

FIG. 2. Spatial dependence of the reduced magnetization $f(y)$. ____, paramagnetic phase $\lambda < 0$; –·–·–, ferromagnetic phase $\lambda < 0$; ------, ferromagnetic phase but very large λ and – – – –, ferromagnetic bulk with $\lambda > 0$.

for $\tau < 1$, we find (the integration can be carried out easily) for the part that is independent of the thickness of the sample

$$F_s^M(T) = \xi \frac{\tau^2(1-\tau)^2}{\beta}\left(\tfrac{2}{3} - f(0) + \tfrac{1}{3} f^3(0) + \frac{\xi}{\lambda} f^2(0)\right); \tag{18}$$

for $\lambda \ll \xi$;

$$F_s^M(T) = \xi \frac{\tau^2(1-\tau)^2}{\beta}\left[\frac{2}{3} + \frac{1}{3}\left(\frac{\lambda}{\xi}\right)^3\right]$$

for $\lambda \gg \xi$, $f(0) \approx 1$ and

$$F_s^M(T) = \frac{\tau^2(1-\tau)^2}{\beta}\frac{\xi^2}{\lambda} .$$

The surface specific heat is defined as

$$C_s = -T\frac{\partial^2 F_s^M}{\partial T^2}$$

and we find

$$C_s \propto (1-\tau)^{-1/2} \text{ for } \lambda \ll \xi$$

and

$$C_s \simeq \text{(nonsingular) for } \lambda \gg \xi .$$

Thus we see that Eq. (18) for the free energy (an exact expression within the present framework) yields the exponents for both the free energy and the specific heat that depend upon the interplay between λ and ξ or, in other words, depend continuously on the variation in J_s. It is also interesting to note that for the case $\lambda \gg \xi$ the magnetic surface free energy behaves exactly as the bulk case; namely, the specific heat has a discontinuity and is nonsingular at the transition temperature. A similar situation holds for $\lambda < 0$. For $\lambda \gg \xi$, the physical quantities are identical to $\lambda > 0$; for $\lambda \ll \xi$, we find that again, the specific heat is nonsingular at the transition temperature. The leading term in the surface free energy F_s^M is, in fact, temperature independent. It acquires weak temperature dependence close to the surface transition temperature T_C^s discussed in Sec. II.

For the paramagnetic phase, if $\lambda > 0$, no surface magnetic moment occurs. Therefore there is no contribution to the surface free energy. For $\lambda < 0$, however, the surface moment is finite. The contribution of this surface moment to the free energy can be calculated by substituting the appropriate solution [Eq. (14)] obtained in Sec. II into Eq. (17). We find

$$F_s^M = 2\xi \frac{\tau^2(\tau-1)^2}{\beta}\left(\tfrac{2}{3}\{1 - [1 + \tfrac{1}{2} f^2(0)]^{3/2}\} - \frac{\xi}{2|\lambda|} f^2(0)\right). \tag{19}$$

The specific heat again is discontinuous at the transition temperature T_C^s (not T_c) but has no singularity. This might be construed as the expected two-dimensional-like behavior.

Thus aside from the $\lambda \ll \xi$ and $\lambda > 0$ case where the specific heat has a singularity, its behavior is quite conventional: i.e., it is discontinuous at the appropriate transition temperature. To ascribe that to a two-dimensional characteristic, at least from the mean-field theory point of view, is difficult. Even in three dimensions, in MFT, the specific heat has the discontinuity. Ising-model calculations[3] seem to indicate that the behavior of the surface moment for $\tau > 1$ is two dimensional rather than three dimensional, as expected.

IV. RELAXATION TIMES

A large number of transport properties can be described in terms of the relaxation time of the weakly perturbed (from equilibrium) order parameter. If an external perturbation is imposed upon the system, which couples only to the surface magnetization, then the energy dissipation would be governed by the relaxation (to equilibrium) of the driven surface moment. The appropriate transport property would be determined by this relaxation time. Such would be the case, for example, for the surface conductivity if the predominant resistive mechanism was spin scattering or for the attenuation of sound waves if phonons were scattered by the magnetic moment. For the latter case it would be necessary to consider short-wavelength-surface sound waves, so as to minimize the penetration depth into the bulk of the sample. This is necessary since the relaxation behavior of surface spins is likely to be different from the bulk spins. In this section we shall concentrate mainly on the relaxation of surface spins.

In particular we shall calculate the temperature dependence of the relaxation time near the transition temperature. Landau and Khalatnikov[9] have calculated the relaxation time for bulk spins within the framework of MFT and we shall adopt the same procedure. For completeness, we shall outline the procedure. If $\eta(0)$ represents the surface moment in nonequilibrium, then

$$\frac{\partial \eta(0)}{\partial t} = \gamma \frac{\partial F_s^M}{\partial \eta(0)} = -\frac{\delta\eta(0)}{\tau_s} , \tag{20}$$

where $\eta(0) = \eta_0(0) + \delta\eta(0)$, with $\eta_0(0)$ being the order parameter in equilibrium, and γ is a constant. Note that the relaxation of $\eta(0)$ is governed only by the surface free energy since the bulk free energy is independent of the surface moment. In the Landau-Khalatnikov procedure, the $\partial F_s^M/\partial\eta(0)$ is expanded around equilibrium and for τ_s^{-1} we find

$$\tau_s^{-1} = -\gamma \frac{\partial^2 F_s^M}{\partial\eta(0)^2}\bigg|_{\eta(0)=\eta_0(0)} \qquad (21)$$

We can now separate the problem into two parts. If the perturbing field is uniform, the mode driven from equilibrium would also be uniform in the plane parallel to the surface; then we can calculate the temperature dependence of its relaxation due to surface effects. This involves simply using the expressions for free energy obtained in Sec. III. In the ferromagnetic phase, we find for $\lambda \ll \xi$,

$$1/\tau_s = (2\gamma\lambda/T_c)[T_c^s - T] , \qquad (22)$$

where T_c^s is the surface transition temperature mentioned previously. The bulk behavior has the same exponent but the singular temperature is T_c. The behavior of τ is nonsingular for both $\lambda \gg \xi$ and also in the $\lambda < 0$ regimes.

On the other hand, the paramagnetic phase with the finite surface moment offers an interesting situation. One might expect the relaxation time to be singular at the transition temperature T_c^s. The temperature range of interest here is $T_c < T < T_c^s$. We find that

$$\frac{1}{\tau_s} = \frac{\gamma\lambda}{T_c}(T_c^0 - T) , \qquad (23)$$

where $T_c^0 = T_c[1+\frac{1}{2}(1+4\Delta)^2]$, namely $T_c^0 > T_c^s$. Thus the relaxation time is never singular.

This is indeed not the complete story. It is easy (within MFT) to consider the behavior (relaxation) of a single mode characterized by $\eta_k(z)e^{iK_\parallel R_\parallel}$ where $K_\parallel$ and $R_\parallel$ are the projections of momentum and coordinate parallel to the surface plane. We have ignored here the mode-mode coupling that arises out of $|\eta|^4$ term. In the presence of this surface inhomogeneity, it is necessary to include a term proportional to $|\nabla_\parallel \eta(z=0)|^2$ in the surface part of Eq. (16). The free energy can now be written as

$$F(\eta, T) = F_0(T) + \int_0^\infty dz\left[(A + CK_\parallel^2)\eta^2(z) + B\eta^4(z) + C\left(\frac{\partial\eta}{\partial z}\right)^2\right] + \frac{C}{\lambda}\eta^2(0) + C\beta K_\parallel^2\eta^2(0) . \qquad (24)$$

Using the Mills equation (now rewritten to include the surface-plane inhomogeneity) we find that the effects of inhomogeneous η can be easily accounted for by shifts in the transition temperature and the scaling length, i.e.,

$$(1-\tau) \to (1 - \tfrac{1}{6}a_0^2K_\parallel^2 - \tau) ,$$

and

$$\lambda = \frac{a_0}{1+4\Delta} \to \frac{a_0}{1+4\Delta - \Delta a_0^2 K_\parallel^2} .$$

These shifts may bring the singularity temperatures closer to the transition temperatures T_c or T_c^s as the case may be for large values of $K_\parallel$, for small $K_\parallel$ the effect is negligible. Similar changes occur in the regimes of Fig. 2, i.e., λ is no longer singular at $\Delta = -\frac{1}{4}$ but at $-1/(4 - a_0^2K_\parallel^2)$, again these effects would be observable only for very large $K_\parallel$.

The role of fluctuations is another important aspect heretofore ignored in the present calculations. The dynamics have been shown, at least for the bulk systems to be strongly dependent on the fluctuations.[10] The treatment of static fluctuations[11] in the semi-infinite case has been found to be exceedingly difficult in other model calculations. If the MFT is found to be satisfactory for the description of surface phenomena (by more experimental measurements) it will be easy to incorporate dynamic fluctuations.[12] This would naturally have to wait until the basic validity of MFT is decided upon by experiments.

V. CONCLUSIONS

To conclude, we would like to summarize the results obtained in this paper.

(a) Exact solutions are obtained for the magnetization as a function of both the temperature and distance from the surface. The scale of rise and the magnitude at the surface of the magnetization is determined by the interplay between the correlation length ξ and the extrapolation length λ. The length λ depends on the change in the surface-plane exchange constant. Even though it has been claimed that the extrapolation length is an artifact of the MFT, we believe, based on the fact that the surface-related properties have been found to vary almost continuously with the change in surface exchange in other model calculations, that such an effect is model independent and that λ may be a useful quantity to invoke to understand these effects.

(b) Surface free energy is evaluated using the solutions for magnetization obtained earlier. Temperature-dependent properties of the surface free energy and its derivative (namely, the specific heat), are analyzed near the bulk critical temperature and have been found to be singular only when $\lambda \ll \xi$, $\lambda > 0$. All other cases have discontinuities at the appropriate transition temperature (at T_c for $\lambda > 0$ and at T_c^s for $\lambda < 0$).

(c) The relaxation time τ for the magnetization driven weakly from equilibrium is studied and we find it is nonsingular in all cases. In the ferromagnetic case $\tau \propto 1/(T_c^s - T)$, where $T_c^s > T_c$, and in the intermediate regime ($T_c < T < T_c^s$) when the bulk is paramagnetic but a finite surface moment exists, $\tau \propto 1/(T_c^0 - T)$ where $T_c^0 > T_c^s$. There is no singularity associated with surface spin relaxation if $\lambda >> \xi$.

These calculations offer an insight into a variety of phenomenon as yet unexplored. The calculation of relaxation time will of necessity have to be done with fluctuations. However, at the present moment when no experiments exist to test the validity of any theory, the complexity of advanced calculations is unwarranted.

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APPENDIX

To further exemplify the interplay between λ and ξ, we will calculate the response to an external, static, and uniform magnetic field. The corresponding magnetization for weak fields is small and if we restrict ourselves to the linear response, we find for the magnetization

$$\frac{a_0^2}{6}\frac{\partial^2\eta}{\partial z^2} - (\tau - 1)\eta(z) = -\tfrac{1}{3}(s+1)h_0 \tag{A1}$$

Here h_0 is measured in reduced units namely $h_0 = g\mu_B H/k_B T_c$ where H is the applied field. Deep in the bulk $\eta(z) = \frac{1}{3}(s+1)[h_0/(\tau - 1)] = \eta_\infty$ the usual behavior. If we solve (A1) now we find

$$\eta(z) = \frac{s+1}{3}\frac{h_0}{\tau - 1}\left(1 - \frac{\xi}{\xi+\lambda}e^{-z/\xi}\right). \tag{A2}$$

Since h_0 is uniform, $\eta(z) = \sum_{z'}\chi(z,z')h_0 = h_0\chi(z)$ where $\chi(z) = \sum_{z'}\chi(z,z')$; $\chi(z,z')$ being the response function:

$$\chi(z) = \frac{s+1}{3(\tau-1)}\left(1 - \frac{\xi}{\xi+\lambda}e^{-z/\xi}\right) \tag{A3a}$$

$$\cong \frac{s+1}{3(\tau-1)}\left(1 - \frac{\xi}{\lambda}e^{-z/\xi}\right) \quad (\lambda >> \xi) \tag{A3b}$$

$$\approx \frac{s+1}{3(\tau-1)}\left(1 - e^{-z/\xi}\right) \quad (\lambda << \xi)\,. \tag{A3c}$$

Similarly, the temperature dependence of the impressed magnetization at the surface is

$$\eta(z=0) = \frac{s+1}{3(\tau-1)}\frac{\lambda}{\xi+\lambda}h_0$$

$$\cong \frac{(s+1)h_0}{3(\tau-1)} \quad \text{for } \lambda >> \xi$$

$$\cong (s+1)h_0/[3(\tau-1)^{1/2}(1+4\Delta)] \quad \text{for } \lambda \ll \xi\,.$$

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[6]The temperature dependence of $\eta(y)$ depends on y. This is to be expected since the exponents, different at the surface from deep in the bulk, should not change discontinuously. For a discussion of such position-dependent temperature dependence see Pradeep Kumar, Phys. Lett. 47A, 187 (1974).

[7](a) In relation to an earlier comment, we would like to point out that Eq. (7) admits solutions that behave like $\coth[y/(2)^{1/2} - A]$. These solutions suffer from two setbacks: (i) they are divergent at $y = \sqrt{2}A$; perhaps this can be overcome by including higher-order terms of the Brillouin function; and (ii) the corresponding free energy is higher. Hence the solutions should not be considered as an equilibrium solution. (b) This is also the regime where approximation (1) in the derivation of Eq. (1) breaks down, i.e., $f(0) \geq 1$.

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