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# Spectrum of superconducting films with quantized resistances

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The lowest-lying quasiparticle states in a superconducting-normal-superconducting layer structure are calculated in the presence of a ground-state current flow parallel to the phase boundaries and an applied magnetic field $H$ parallel to the film surface. Particle-hole scattering at the phase boundaries as well as ordinary reflection processes at the outer film surfaces and at the pair-potential walls are taken into account. Quasiparticle energy and momentum normal to the phase boundaries are obtained as linear functions of a field-dependent parameter $\alpha(H)$, $0 < \alpha(H) < 1/2$, which has been found previously in the experiments on quantized resistances. The relevance of the results with respect to the hypothesis that the bound quasiparticle states are involved in the formation of the observed quantized resistances is discussed.

## I. INTRODUCTION

Recently, Chen, Hayler, and Kim (CHK) detected linear current branches corresponding to quantized resistances in the current-voltage characteristics of current-driven films with a laminar superconducting-normal-superconducting (SNS) layer structure.[1] This structure was either produced by an external magnetic field parallel to the surface of a 4000-Å-thick lead film[1] or by sandwiching a normal silver layer (4000 Å) between two superconducting lead films (each 1500 Å thick).[2] The authors suggest that the quantized resistances are due to current conduction through individual phase-coherent bound quasiparticle states in the normal region and show that good agreement with experiment is obtained, if one assumes that the resistances of the current branches are proportional to the quantized parts of quasiparticle momenta normal to the SNS phase boundaries.

An attempt has been made to explain why the two superconducting sheaths bordering the normal region do not act as shorts between the voltage-biased film ends. It was pointed out[3] that the quasiparticle countercurrent which is being produced by quasiparticle relaxation with the lattice,[4] above a certain critical value may induce a space and time variation in the phase of the superconducting order parameter. Consequently, a voltage proportional to the time variation of phase difference between the film ends[5] may develop in the presence of a dc supercurrent. Voltage (or resistance) quantization would then be a consequence of the single-valuedness of the complex order parameter in the presence of superfluid vorticity created by momentum transfer from the bound quasiparticles decaying in the superconducting regions.

These considerations, however, are not very convincing because one has not yet been able to draw the complementary picture of how the quantized resistances originate in the normal region. Besides, step-like structures in the current-voltage characteristics of superconducting microbridges have been observed by several authors.[6–8] Although their experimental conditions and the results obtained differ in a number of important details from those of CHK, one can not yet rule out with certainty Huebener's vortex channel mechanism[6] and Tinkham's phase-slip centers[8] as alternative explanations of the observations of Chen, Hayler, and Kim.

Therefore, it is important to obtain a thorough understanding of the bound quasiparticle states in the systems investigated by CHK in order to see if they indeed can be responsible for the quantized resistances. The well known Andreev states,[9,10] upon which CHK base the interpretation of their experiments, do not describe very well the excitation spectrum of the films under consideration. They are adequate for normal layers bordered by superconducting regions of a thickness several times larger than the BCS coherence length $\xi_0$. Only in such systems the bound excitations consist of a linear combination of electrons and holes of nearly equal momentum which completely decay in the superconducting regions via transformation into Cooper pairs.[11] If, on the other hand, the bordering superconducting regions are narrow with a thickness less than $\xi_0$, and this seems to be the case in the samples with the most clearly pronounced quantized resistances,[1,2] the quasiparticles with a finite probability will penetrate the S region and suffer reflection from the outer film surfaces. The excitations will consist of a superposition of electron waves with opposite momenta and corresponding hole waves. Furthermore, the energetically lowest bound quasiparticle states with momenta nearly parallel to the phase boundaries are not Andreev states at all. The Andreev approximation of matching only the wave amplitudes at the phase boundaries and not also the derivatives,[9] or, equivalently, the WKBJ approximation of neglecting second-order derivatives of the solutions of the Bogoliubov equations[10] is valid only when the quasiparticle Fermi momentum normal to the phase boundary, $k_{zF}$, is much larger than $(2m\Delta)^{1/2}$; $\Delta$ is the maximum value of the superconducting pair potential (order parameter), and $m$ is the electron mass. If this condition is not satisfied, the small pair potential $\Delta$ is able to invert the small perpendicular quasiparticle momentum as an ordinary potential wall does. Then, with a finite probability, electrons are reflected as electrons and holes as holes at the phase boundaries, instead of being scattered into holes and electrons, respectively. The lowest-lying states with $k_{zF} \leq (2m\Delta)^{1/2}$ are expected to be the first to play a role in transport phenomena parallel to the phase boundaries.

In Sec. II we investigate the spectrum of a vacuum-SNS-vacuum system for states with $k_{zF} \gg (2m\Delta)^{1/2}$, and in Sec. III we look into the states with $k_{zF} \leq (2m\Delta)^{1/2}$. We will discuss in Sec. IV to what extent the characteristic features of the bound quasiparticles' spectrum support the view that these states are responsible for the quantized resistances.

## II. SPECTRUM OF STATES WITH $k_{zF} \gg (2m\Delta)^{1/2}$

Let us look into a vacuum-SNS-vacuum system, realized in a pure film of thickness $2T$ as shown in Fig. 1. The film has macroscopic dimensions in the $x$ and $y$ directions. A magnetic field is applied in the $y$ direction parallel to the film surface. It penetrates the film in the central normal region $-a \leq z \leq +a$, where it has the value $H$. For the corresponding vector potential $\vec{A}$ we make the same approximations as in the first paper of Ref. 10 by assuming the form $\vec{A}(z) = \vec{e}_x Hz\theta(a - |z|)$ and keeping only linear terms in $\vec{A}$. More accurate treatments of the vector potential show that no essential corrections to these approximations are to be expected as long as one can neglect all effects due to Landau orbit quantization.[12] We consider a situation when the film carries a uniform ground-state flow in $y$ direction with an average momentum per electron $\vec{e}_y q$.

The Bogoliubov equations for the quasiparticle excitations[13] of this system are

$$Eu(\vec{r}) = \left[\frac{1}{2m}\left(\frac{\vec{\nabla}}{i} - \frac{e}{c}\vec{A}(z)\right)^2 - \epsilon_F\right] u(\vec{r}) + \Delta(z)\, e^{i2qy} v(\vec{r}) , \tag{2.1a}$$

$$Ev(\vec{r}) = -\left[\frac{1}{2m}\left(\frac{\vec{\nabla}}{i} + \frac{e}{c}\vec{A}(z)\right)^2 - \epsilon_F\right] v(\vec{r}) + \Delta(z)\, e^{-i2qy} u(\vec{r}) . \tag{2.1b}$$

$\Delta(z)$ is real and has the form shown in Fig. 1; $\epsilon_F \equiv k_F^2/2m$ is the Fermi energy and $E$ the quasiparticle energy; $\hbar = 1$.

These equations have four linearly independent

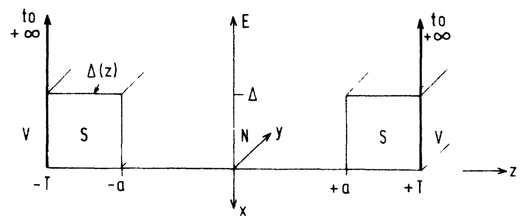


FIG. 1. Model for the spatial variation of surface and pair potentials across a superconducting film with central normal layer.

solutions. In an S region,

$$\begin{bmatrix} u(\vec{r}) \\ v(\vec{r}) \end{bmatrix} = \text{const} \times e^{i(k_x x + k_y y)} \times \begin{bmatrix} e^{i(qy \pm \eta_0/2)} \\ e^{-i(qy \pm \eta_0/2)} \end{bmatrix} e^{\pm i k_{zF} (1+i\delta)^{1/2} z} ; \tag{2.2}$$

in the normal region,

$$\begin{bmatrix} u(\vec{r}) \\ v(\vec{r}) \end{bmatrix} = e^{i(k_x x + k_y y)} \left( a_\pm \begin{bmatrix} e^{iqy} \\ 0 \end{bmatrix} e^{\pm i[k_{zF}(1+\epsilon)^{1/2} + pz]z} + b_\pm \begin{bmatrix} 0 \\ e^{iqy} \end{bmatrix} e^{\pm i[k_{zF}(1-\epsilon)^{1/2} - pz]z} \right) , \tag{2.3}$$

where $a_\pm$ and $b_\pm$ are constants. The sign of

$$\eta_0 \equiv \arccos(E_q/\Delta) \tag{2.4}$$

is coupled to the sign of

$$\delta \equiv (2m/k_{zF}^2)(\Delta^2 - E_q^2)^{1/2} , \tag{2.5}$$

whereas the sign of

$$k_{zF} \equiv (k_F^2 - k_x^2 - k_y^2)^{1/2} \tag{2.6}$$

varies independently from that of $\eta_0$ as one goes from one solution (2.2) to the other. For the treatment of the magnetic field we have assumed that

$$\epsilon \equiv (2m/k_{zF}^2)\, E_q \ll 1 ,$$

where

$$E_q \equiv E - k_y q/m \tag{2.7}$$

and that

$$p \equiv \frac{e}{2c}\,\frac{k_x}{k_{zF}}\, H \ll \frac{k_{zF}}{2a} . \tag{2.8}$$

In Eq. (2.7) a term $q^2/2m$ has been neglected. The last inequality is well satisfied in most of the states for magnetic fields up to some kilogauss and normal region thickness $2a$ of some thousand angstroms.

When the superconducting regions on each side of the normal layer have a thickness of several coherence lengths, the quasiparticle wave functions are simply the solutions with positive *or* negative sign in the exponents of Eq. (2.3) and the exponentially decreasing waves with $+k_{zF}$ *or* $-k_{zF}$ in Eq. (2.2).[10] However, if the superconducting walls are thin so that particle-particle scattering at the film surface is possible (besides particle-hole scattering at the phase boundaries), all four possible solutions are mixed together in each of the three regions and we have twelve unknown integration constants. The matching conditions of the wave functions and their derivatives at the phase boundaries at $z=\pm a$, and the requirement that $u(\vec{r})$ and $v(\vec{r})$ vanish at the outer film surfaces at $z=\pm T$ provide us with 12 independent linear homogeneous equations; a 13th condition is that of normalization. The energy eigenvalues of the system are determined from the usual condition, that the determinant of the coefficients in the equations must vanish. Evaluation of this $12\times 12$ determinant means straightforward but boring work. It has to be done anyway and is simplified considerably, by assuming that

$$\frac{k_{zF}^2}{2m} \gg \begin{cases} \Delta \\ E_q \end{cases} . \tag{2.9}$$

Then, all derivatives produce only a factor $\pm k_{zF}$ in front of the wave functions, and the determinant with originally 80 null elements can be brought into a form with 106 zeroes. While our wave functions (2.2) and (2.3) are exact except for the not very restrictive approximation (2.8), we are now limited to states with large momenta $k_{zF}$ perpendicular to the phase and film boundaries. The determinant for the bound states with $\Delta > E_q$ is found to be

$$\begin{aligned} D = 16\sin^2\eta_0 \Bigg[ & \exp\left(\frac{4m}{k_{zF}}(\Delta^2-E_q^2)^{1/2}(T-a)\right) \\ & \times \sin^2\left(\frac{2m}{k_{zF}}E_q a - \eta_0\right) + \exp\left(-\frac{4m}{k_{zF}}(\Delta^2-E_q^2)^{1/2}(T-a)\right) \\ & \times \sin^2\left(\frac{2m}{k_{zF}}E_q a + \eta_0\right) - 2\sin^2\left(\frac{2m}{k_{zF}}E_q a\right) \\ & - 2\sin^2\eta_0 \cos 4k_{zF}T \Bigg] . \end{aligned} \tag{2.10}$$

The eigenvalue equation is $D=0$. For large thickness of the superconducting regions, $(T-a)\gg \xi_0 = k_F/\pi m\Delta$, the first term in the square brackets dominates, and the requirement that it vanishes reproduces the well-known Andreev spectrum[9,10] from

$$\arccos(E_q/\Delta) = (2m/k_{zF})E_q a - n\pi, \quad n=\text{integer} . \tag{2.11}$$

When $(T-a)\lesssim \xi_0$, analytical solutions of $D=0$ can be found for the low-lying states with $E_q \ll \Delta$ and $\eta_0 \approx \pi/2$. Their eigenvalue equation is

$$\sin^2\left(\frac{2m}{k_{zF}}E_q a\right) = \frac{\cosh[(4m/k_{zF})\Delta(T-a)] - \cos 4k_{zF}T}{\cosh[(4m/k_{zF})\Delta(T-a)]+1} . \tag{2.12}$$

It has the solution

$$E_q = \frac{k_{zF}}{m}\frac{\pi}{2a}(n-\alpha) , \tag{2.13}$$

where

$$\alpha = \alpha(k_{zF}, a) \equiv \frac{1}{\pi}\left| \arcsin \left(\frac{\cosh[(4m/k_{zF})\Delta(T-a)] - \cos 4k_{zF}T}{\cosh[(4m/k_{zF})\Delta(T-a)]+1}\right)^{1/2} \right| , \tag{2.14}$$

$n\geq 1$ is a positive integer and $0\leq\alpha\leq\frac{1}{2}$.

The spectrum of Eq. (2.13) is very similar to the Andreev spectrum one obtains from Eq. (2.11) in the limit $E_q\ll\Delta$ with $n=0, 1, 2, \ldots$. Applying the labeling system of CHK,[1] used in Eq. (2.13), to the Andreev states, their energy eigenvalues can be written as $E_q = k_{zF}\pi(n-\frac{1}{2})/2ma$, $n=1, 2, 3, \ldots$. We see that ordinary reflection from the film surfaces raises the energy of a given state with respect to the corresponding Andreev level by changing the term $\frac{1}{2}$ to $\alpha<\frac{1}{2}$. The physical explanation of this effect is quite simple. When infinite potential walls back the superconducting layers at a distance $(T-a)$ from the phase boundaries that is less than the penetration depth of a bound quasiparticle in S regions of an extension much larger than the coherence length (Andreev case), then the quasiparticle wave functions spread over a distance that is smaller than in the Andreev case and consequently, the energy levels are shifted upward. One has the same situation in an ordinary potential well backed by infinitely high potential walls. [There is a second set of unphysical negative-energy eigenvalues proportional to $(n+\alpha)$ with $n=-1, -2, -3, \ldots$ which always show up in the solutions of the Bogoliubov equations.[14]] Note that the $p$ terms of Eq. (2.8), which explicitly depend upon the magnetic field, have exactly canceled in Eq. (2.10). The physical reason for this was discussed in Ref. 10.

For larger values of $E_q/\Delta$ the Andreev spectrum remains the same except for a change of the normal thickness $2a$ into a somewhat larger effective thickness $2a^* = 2[a+k_{zF}/(2m\Delta)]^4$; from Eq. (2.10) we do not expect that the higher energies will be substantially different from those of Eq. (2.13).

In the limit $(4m/k_{zF})\Delta(T-a)\ll 1$ the solution of Eq. (2.12) is

$$E_q = \pm\left(\frac{k_{zF}}{m}\frac{n'\pi}{2a} - \frac{k_{zF}^2}{m}\frac{T}{a}\right), \tag{2.15a}$$

and for $a\to T$,

$$E_q \to \pm\left(\frac{k_{zF}}{m}\frac{n'\pi}{2T} - \frac{k_{zF}^2}{m}\right) \approx \pm\frac{1}{2m}\left[\left(\frac{n'\pi}{2T}\right)^2 + k_x^2 + k_y^2 - k_F^2\right], \tag{2.15b}$$

where $n'$ is a positive integer. As was to be expected, the excitation spectrum becomes that of an ordinary square well potential with infinitely high walls in the limit of vanishingly thin superconducting layers. The last, approximate equality in Eq. (2.15b) is valid because $k_{zF} \approx n'\pi/2T$ for small $E_q$. The plus sign holds for excitations above and the minus sign for excitations below the Fermi surface.

## III. SPECTRUM OF STATES WITH $k_{zF} \lesssim (2m\Delta)^{1/2}$

The states with the lowest energies should have a small $k_{zF} \lesssim (2m\Delta)^{1/2}$. Because of approximation (2.9), they are not part of the energy spectrum (2.13). According to Eq. (2.5), $\delta \gtrsim 1$ in these states and the damped wave functions of Eq. (2.2) decrease as $e^{-\kappa z}$, where $\kappa$ is essentially $(2m\Delta)^{1/2}$. For typical values of $\Delta$ this quantity is of the order of magnitude of $(100\ \text{Å})^{-1}$. Therefore, in superconducting layers not thinner than some hundred angstroms, the portion of a quasiparticle with small $k_{zF}$, which penetrates the phase boundaries, will have decayed with high probability before reaching the outer film surfaces. For this reason we may neglect reflection from the film surfaces against reflection from the phase boundaries. The appropriate wave functions in the normal region consist of a superposition of all possible solutions given in Eq. (2.3), whereas in each superconducting region we take only the two exponentially decaying solutions from Eq. (2.2). We match these solutions and their derivatives at the two phase boundaries and ignore the outer film surfaces. We also disregard the magnetic field. From Eqs. (2.13) and (2.14) we see that the energy eigenvalues depend upon the magnetic field only via the thickness $2a$ of the normal region.[10] However, the approximation made for the states with $k_{zF} \gg (2m\Delta)^{1/2}$ is no longer valid, because in kG fields Eq. (2.8) fails for a number of states with $k_{zF} \lesssim (2m\Delta)^{1/2}$. Nevertheless, since we are mainly interested in momentum and energy quantization and their possible relation to the quantized resistances, which have also been found in SNS sandwiches [where the magnetic field is of secondary importance only, serving mainly for suppression of the proximity effect],[2] one should be justified in avoiding the more elaborate mathematics which an adequate treatment of the magnetic field in the present case would require.

The matching conditions provide eight linear homogeneous equations for as many free constants. The equation for the energy eigenvalues which make the determinant of the coefficients vanish, in its full generality is too long to be written here. Again, limitation to $E_q/\Delta \ll 1$ ($\eta_0 \approx \pi/2$) and $\epsilon \ll 1$ simplifies the affair considerably and is justified for the low-lying states in which we are interested.

We obtain the energy eigenvalues,

$$E_q = (k_{zF}/m)(\pi/2a)(n-\alpha'), \tag{3.1}$$

where $n = 1, 2, 3, \ldots$ and

$$\alpha' = \alpha'(k_{zF}, a)$$

$$\equiv \frac{1}{\pi}\arccos\left(\frac{\{\delta^2(1+\cos 4k_{zF}a) - 2\sqrt{2}[(1+\delta^2)^{1/2}-1]^{3/2}\sin 4k_{zF}a - 4[(1+\delta^2)^{1/2}-1]\cos 4k_{zF}a\}^{1/2}}{\{2[\delta^2 + 2(1+\delta^2)^{1/2}+2]\}^{1/2}}\right), \tag{3.2}$$

$0 \le \alpha' \le \frac{1}{2}$; $\alpha'$ is always real, because the minimum value of the function under the square root is zero, as may be seen from differentiation with respect to $4k_{zF}a$.

In the limit $\delta = (2m/k_{zF}^2)\Delta \gg 1$, $\pi\alpha'$ becomes $\pm(2k_{zF}a - r\pi)$, where the positive integer $r$ has to be chosen so that $0 \le \pm(2k_{zF}a - r\pi) \le \pi/2$. Again, as in Eq. (2.15b) we obtain an ordinary potential-well spectrum with energies above and below the Fermi surface given by

$$E_q = \pm(1/2m)[(l\pi/2a)^2 + k_x^2 + k_y^2 - k_F^2], \quad l \text{ integer}. \tag{3.3}$$

As discussed in the Introduction, this result was to be expected for small $k_{zF}$.

## IV. DISCUSSION

The quasiparticle spectrum of a vacuum-SNS-vacuum system has the following characteristic features. Quasiparticles which practically move parallel to the phase boundaries in the $x$ and $y$ directions form ordinary standing waves between the pair-potential walls with energy eigenvalues like those of Eq. (3.3), which are typical for a truly normal region. However, the number of these ordinary potential-well states (OPWS) with $k_{zF} \ll (2m\Delta)^{1/2}$ is very small. As soon as $\delta = 2m\Delta/k_{zF}^2$ is no longer large enough so that terms with $\delta^2$ dominate the other ones in Eq. (3.2), the spectrum is that of Eq. (3.1) for $k_{zF} \lesssim (2m\Delta)^{1/2}$, and with further increasing $k_{zF}$ to values larger than $(2m\Delta)^{1/2}$

the energies are given by Eq. (2.13). The only change in a transition from spectrum (3.1) to spectrum (2.13) occurs in the definition of the parameters $\alpha$ and $\alpha'$, which also make the only difference from a true Andreev spectrum. We may say that these states form a generalized Andreev spectrum (GAS). The remarkable fact is that in the investigated complex situation where, in addition to particle-hole scattering at the pair-potential walls, ordinary reflections from the outer film surfaces and the phase boundaries are important, the structure of the spectrum still has the simple form of the pure Andreev case with only particle-hole scattering present. The cooperation of the two different scattering processes only produces the field dependent terms $\alpha$ and $\alpha'$ in the energy spectrum. This *a posteriori* justifies the consideration of Andreev states by CHK[1] in situations where the Andreev approximations are not valid.

The right-hand sides of Eqs. (2.13), (3.1), and (3.3) give the quasiparticle energies $E_q$ in a reference frame which moves with the velocity $q/m$ of the ground-state flow. The energies $E$ in the resting frame of the lattice ions are $E=E_q+k_y q/m$, according to Eq. (2.7). In a stationary situation, the quasiparticles are in equilibrium with the lattice, and the occupation probability of the quasiparticle states is given by the Fermi function $f(E)=(1+e^{E/kT})^{-1}$. Therefore, at temperature $T=0\,^\circ\mathrm{K}$ all states with $k_y<0$ and $E<0$ are occupied. They are defined by the inequalities

$$-k_y q \geq (k_F^2-k_x^2-k_y^2)^{1/2}(\pi/2a)(n-\alpha)\geq 0 \qquad (4.1)$$

in a GAS, where $\alpha$ for $k_{zF}\gg(2m\Delta)^{1/2}$ must be replaced by $\alpha'$, if $k_{zF}\lesssim(2m\Delta)^{1/2}$; for OPWS the condition for occupation is

$$-k_y q \geq \tfrac{1}{2}\left[(l\pi/2a)^2+k_x^2+k_y^2-k_F^2\right]\geq 0 \ . \qquad (4.2)$$

If condition (4.2) were valid for all values of $k_x$ and $k_y$ instead of being limited to $k_{zF}\ll(2m\Delta)^{1/2}$, it would in momentum space define a volume between the ground-state Fermi sphere and the Fermi sphere whose center has been shifted to $k_y=-q$. The negative current density carried by the excited quasiparticles in these states with $k_y<0$ would exactly cancel the ground-state flow in the $N$ region, which is just another way of saying that in a truly normal region no current flows without voltage. However, in our SNS-layer structure, condition (4.2) applies to a tiny fraction of states only. For the overwhelming majority of states, relation (4.1) defines the number of excited quasiparticles with negative $k_y$, and this number is far less than it would be in a truly normal metal. Therefore, the relaxation countercurrent of excited quasiparticles is not sufficient to cancel the ground-state flow in the $N$ region and a current flow without applied voltage is possible. Consequently, the observed zero voltage current $I_0$ in the returning section of the current-voltage characteristic of CHK[1] may be a ground-state flow which extends throughout the entire SNS system. (Discussing the thermodynamical properties of SNS junctions, Ishii[15] has already pointed out that "the electronic state of the 'normal' metal... is more like a gapless superconducting state than the usual normal state.")

Based on the calculated SNS quasiparticle spectrum and the excitation conditions (4.1) and (4.2), the following comments on some aspects of CHK's current-voltage characteristic[1] may sketch the lines along which further theoretical work on the electrodynamics of the system can be oriented. For sufficiently large values of the homogeneous ground-state current density $\sim q$, a number of states of the GAS are populated according to Eq. (4.1). The quasiparticles of the GAS spectrum suffer particle → hole (and hole → particle) scattering at the phase boundaries. Formation (and destruction) of a Cooper pair in an S region via transfer of two electrons from the N to an S region (and an S to the N region) is associated with these processes.[11,16] In general, a net electrical quasiparticle current incident on an N-S interface changes into a supercurrent when the quasiparticles decay and merge into the condensate, transferring their momentum to the Cooper pairs and shifting the phase of the order parameter.[11] Based on this effect Bardeen and Johnson recently explained Josephson current flow through SNS junctions.[4] A study of charge and momentum exchange between the N and the S regions of the system dealt with in this paper, and of the related possible voltage between the film ends, is presently being carried out by the author.

The bound quasiparticle states excited by the ground-state flow in the normal region carry a current opposite to the ground-state current. This aspect differs markedly from CHK's hypothesis about conduction of the current itself through the bound states. Nevertheless, the basic idea that the bound quasiparticles are in some way responsible for the observed current-voltage characteristic deserves further consideration; first, because of the experimental field dependence of the quantized resistances, and secondly, for the theoretical reason that the total quasiparticle momentum normal to the phase boundaries according to Eqs. (2.3), (2.7), (2.13), and (3.1) is

$$k_\perp^{\pm}=k_{zF}\pm(\pi/2a)(n-\alpha^{(\prime)}) \ , \quad p \text{ term neglected.} \qquad (4.3)$$

$\alpha$ and $\alpha'$ as given by Eqs. (2.14) and (3.2) depend upon $k_{zF}$ and the field-dependent normal-region thickness $2a$. This field dependent $\alpha$ is not present in a true Andreev spectrum. It was, however, found in the quantized resistances $r_n = r_0(n - \alpha)$ of CHK.[1] The range of values $0 \leq \alpha$, $\alpha' \leq \frac{1}{2}$ of the calculated $\alpha$, $\alpha'$ compares reasonably well with the range $\frac{1}{5} < \alpha < \frac{1}{2}$ of the experimentally found $\alpha$.

Any quasiparticle-based attempt to explain CHK's observations, which identifies $n - \alpha^{(\prime)}$ of Eq. (4.3) with $n - \alpha$ in $r_n$, leads to the conclusion that only states within a narrow range of $k_{zF}$ should participate in the events that produce the quantized resistances in order to have an $\alpha$ independent of current. As mentioned above, when current-excited quasiparticles with $E_q < \Delta$ decay in the S regions, they transfer their momentum to the condensate and induce space and time variations in the phase of the order parameter with the associated finite voltage between the film ends. The probability that this occurs for a quasiparticle in a given state is proportional to the probability of particle-hole scattering in the S regions.[3,11] It has been shown[17] for the case of an electron incident from a normal region onto a superconducting layer backed by an infinite normal surface potential, that the particle-hole scattering probability has a relatively narrow maximum around a $k_{zF}$ value of the order of magnitude of the inverse thickness of the superconducting layer. The physical reason for this is obvious. If $k_{zF}$ is too small, we have OPWS with particle-particle scattering at the pair-potential walls. If $k_{zF}$ is too large, the electron (hole) penetrates the S region, gets reflected at the outer surface, and returns into the normal region (without having been scattered into its "antiparticle") with an appreciable probability. Only for an intermediate range of $k_{zF}$ can the electron (hole) wave be damped nearly completely along the path (phase-boundary surface phase-boundary) so that particle-hole scattering and all the related processes occur with high probability.

It is hoped that the results obtained in this paper provide a basis from which further investigation might determine if and in which way the quasiparticles are involved in the formation of the quantized resistances. The agreement between the experimental $\alpha$ and the theoretical $\alpha$, $\alpha'$ may be taken as a hint that the quantized quasiparticle states and their momenta $k_\perp^\pm$ normal to the film surface, play a role in this phenomenon. Nevertheless, the quantized resistances still remain a puzzling problem.

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