

## Temperature dependence of the maximum (dc) Josephson current\*

E. P. Balsamo and G. Paternò

*Laboratori Nazionali del Comitato Nazionale per l'Energia Nucleare, Frascati, Italy*

A. Barone, P. Rissman,<sup>†</sup> and M. Russo

*Laboratorio di Cibernetica del Consiglio Nazionale delle Ricerche, Arco Felice, Napoli, Italy*

(Received 5 December 1973)

Experimental investigation on the temperature dependence of the maximum Josephson current (dc) has been performed on both symmetrical and asymmetrical junctions. Agreement with the Ambegaokar and Baratoff calculations has been found for both Sn-Sn<sub>x</sub>O<sub>y</sub>-Sn and Sn-Sn<sub>x</sub>O<sub>y</sub>-Pb junctions. Experimental results on Sn-Sn<sub>x</sub>O<sub>y</sub>-In structures have also been obtained.

### I. INTRODUCTION

Experimental results on the temperature dependence of the maximum Josephson current were first reported by Fiske<sup>1</sup> to verify the theoretical predictions that Ambegaokar and Baratoff<sup>2</sup> derived within the BCS quasiparticle approximation.

In this framework, the relationship between zero-voltage Josephson current and temperature is given—in the case of a junction with electrodes of the same superconductor (symmetrical junction)—by the expression

$$\frac{J_s(T)}{J_s(0)} = \frac{\Delta(T)}{\Delta(0)} \tanh\left(\frac{1}{2} \frac{\Delta(T)}{k_B T}\right). \quad (1)$$

This can be written in terms of the universal function of temperature  $\Delta(T)/\Delta(0)$  as

$$\frac{J_s(T)}{J_s(0)} = \frac{\Delta(T)}{\Delta(0)} \tanh\left[\frac{1}{2} \frac{\Delta(T)}{\Delta(0)} \left(\frac{3.5}{2}\right) \frac{T_c}{T}\right].$$

For a junction made by two different superconductors (asymmetrical junction), the dependence of the Josephson current upon the temperature is given by

$$\begin{aligned} \frac{J_s(T)}{J_s(0)} = & \pi k_B T \frac{\Delta_1(T)}{\Delta_1(0)} \frac{\Delta_2(T)}{K\{1 - [\Delta_1(0)/\Delta_2(0)]^2\}^{1/2}} \\ & \times \sum_{i=0, \pm 1, \dots} \{[\omega_i^2 + \Delta_1^2(T)][\omega_i^2 + \Delta_2^2(T)]\}^{-1/2}, \end{aligned} \quad (2)$$

where  $K$  indicates the complete elliptic integral of the first kind,  $\Delta_1$  and  $\Delta_2$  indicate the gaps of the two superconductors, and  $\omega_l = \pi(2l+1)/k_B T$ .

Fiske's results on Sn-Sn and Sn-Pb junctions agree with this calculation. Such agreement, however, is confined to the range of temperature near  $T_c$ . Careful measurements on symmetrical Josephson junctions (Pb-Pb<sub>x</sub>O<sub>y</sub>-Pb) have been performed by Lim *et al.*<sup>3</sup> These authors have also developed numerical calculations on the basis of strong-coupling superconductivity theory, a development which provides a good agreement with their experimental results.

In the present work, results concerning experimental investigations on both symmetrical and

asymmetrical junctions are reported. Sn-Sn<sub>x</sub>O<sub>y</sub>-Sn and Sn-Sn<sub>x</sub>O<sub>y</sub>-Pb Josephson tunneling structures have been considered in order to improve Fiske's original results; we have used samples made in better experimental conditions and a more sophisticated measurement technique.<sup>3</sup> In addition, measurements on Sn-In samples have been performed. The experimental results are compared with the numerically computed dependence  $I_J(T)$  given by Ambegaokar and Baratoff.

### II. EXPERIMENTAL PROCEDURES

#### A. Sample preparation

The junctions were fabricated in an ultra-high-vacuum system (Ultek ion pump) by thin-film evaporation from molybdenum canoe boats. In order to select the evaporation material and the film geometry without breaking vacuum, a remotely controlled mechanical system was employed to change masks and substrate positions. Tin, lead, and indium having an initial purity of 99.999% were evaporated to a thickness of several thousand angstroms in a time of about 1 min. Before deposition of the first superconductor layer, system pressures were in the range of  $10^{-7}$ – $10^{-8}$  Torr. The substrates were previously cleaned by ordinary techniques<sup>4</sup> and ion cleaned by high-voltage discharge in air atmosphere at a pressure of 20–40 mTorr. Size and pattern definition of the samples were obtained using stainless-steel masks prepared by photore-sist techniques.<sup>5</sup>

The dielectric barrier was obtained by oxidizing the first superconductor layer with a glow-discharge oxidation procedure.<sup>6</sup> After oxidation, the system was pumped down again and the second superconductor layer deposited.

#### B. Test apparatus

The method adopted to measure variations of the critical Josephson current as function of the temperature was basically that used by Lim *et al.*<sup>3</sup> In the present work, solid-states devices (field-effect transistor) for chopping reference voltage have

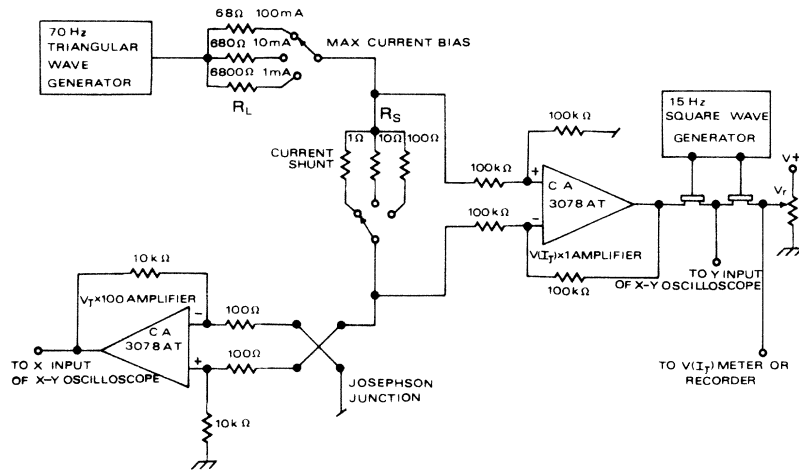


FIG. 1. Schematic of the circuit used to measure junction  $V$ - $I$  characteristic and Josephson current variation.

been employed instead of reed relays. In the scheme of Fig. 1, a triangular-wave voltage generator feeds current into the Josephson junction through the limiting resistor  $R_L$  and the shunt resistor  $R_S$ . The voltage information, proportional to the current in the sample, is picked up by the shunt resistor. The value of  $R_S$  can be chosen by commutation such that the range, or maximum voltage value, of the signal is maintained constant at 100 mV. Both voltage and current signals are amplified by two low-noise low-drift differential amplifiers in order to obtain an optimum decoupling of the junction from the external instrumentation.

The voltage across the junction was continuously applied to the  $x$  input of an  $x$ - $y$  oscilloscope (Tektronix 5103N). On the  $y$  input, the voltage across  $R_S$  proportional to the junction current and a reference voltage  $V_r$ , which can be manually varied by a standard potentiometer, are alternatively displayed. The measurement of the Josephson current was performed by making the  $V_r$  line coincide with the top of the vertical trace (Josephson current).  $V_r$  was measured either with a four-digit

voltmeter or by using an  $x$ - $y$  plotter. The maximum error in the normalized current measurement is less than 0.005.

The junctions, magnetically shielded by a superconducting container,<sup>7</sup> were tested in a  $\text{He}^4$  cryostat connected to rotary and booster pumps. The temperature was regulated by pumping through a manostat in order to keep the helium vapor pressure at a constant value. Each reading was taken after several minutes to allow the system to reach thermal equilibrium. The temperature was measured using a germanium resistance thermometer in close contact with the junction. The voltage across the germanium resistor was measured by a five-digit voltmeter. To avoid spurious thermal emf due to contacts, two distinct voltage readings were taken, inverting the current direction in the thermometer for the second measurement and using the mean value of the two readings. The overall accuracy of the temperature measurement was  $\pm 0.01$  K.

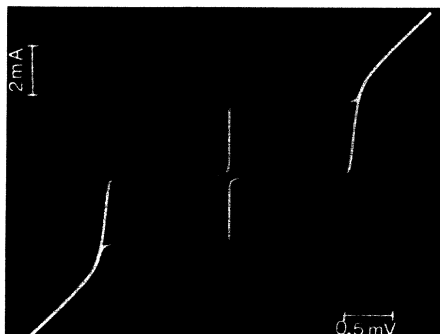


FIG. 2. Typical  $V$ - $I$  characteristic for a  $\text{Sn-Sn}_x\text{O}_y\text{-Sn}$  Josephson junction at  $T=1.52$  K. Horiz. scale: 0.5 mV/div.; vert.: 2 mA/div.

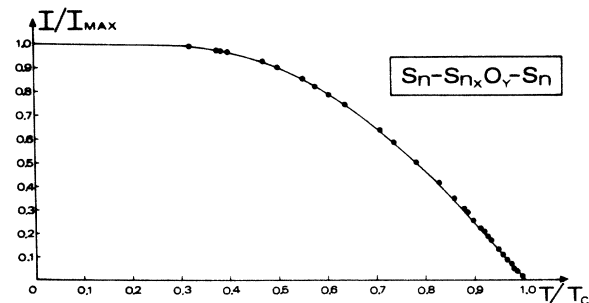


FIG. 3. Temperature dependence of the maximum (dc) Josephson current for  $\text{Sn-Sn}_x\text{O}_y\text{-Sn}$  junction. The experimental data (solid circles) are compared with the theoretical curve (solid line) calculated using the Ambegaokar and Baratoff results. The maximum experimental error in this graph and following ones is smaller than the dimensions of the solid circles.

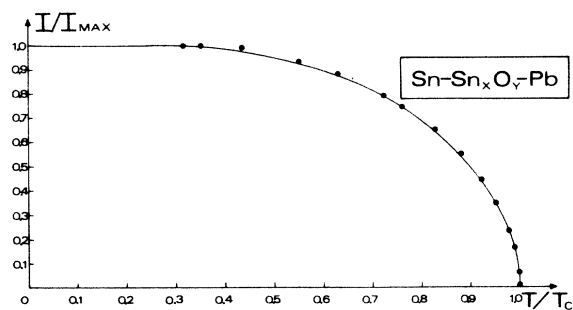


FIG. 4. Temperature dependence of the maximum (dc) Josephson current for Sn-Sn<sub>x</sub>O<sub>y</sub>-Pb junction. Experimental data: solid circles; theoretical curve using Ambegaokar and Baratoff calculations: solid line.

### III. RESULTS

Following the procedures outlined in Sec. II, the dependence of the maximum zero-voltage Josephson current has been measured for Sn-Sn<sub>x</sub>O<sub>y</sub>-Sn, Sn-Sn<sub>x</sub>O<sub>y</sub>-Pb, and Sn-Sn<sub>x</sub>O<sub>y</sub>-In junctions. The experimental results are compared with the theoretical expressions (1) and (2) which are valid in the weak-coupling limit for symmetrical and asymmetrical junctions, respectively. Such theoretical expressions have been previously calculated by computer (IBM 360/44) using the ratio  $\Delta(T)/\Delta(0)$  obtained by polynomial interpolation of the numerical results given by Mühlischlegel.<sup>8</sup> Several values of the ratio  $\Delta(T)/\Delta(0)$  have been considered.<sup>9</sup>

#### A. Sn-Sn<sub>x</sub>O<sub>y</sub>-Sn junctions

Typical values of the ratio  $L/\lambda_J$  for these junctions ( $L$  is the largest junction dimension and  $\lambda_J$  the Josephson penetration depth) ranged between 0.2 and 1.0. The normal tunneling resistance  $R_{NN}$  was  $\approx 0.1 \Omega$ , and the ratio between the observed maximum Josephson current  $I_J$  and its theoretical value  $I_T$  was  $I_J/I_T > 0.8$ . A typical  $V$ - $I$  characteristic is shown in Fig. 2.



FIG. 5.  $V$ - $I$  characteristic for a Sn-Sn<sub>x</sub>O<sub>y</sub>-In Josephson junction at  $T=1.21$  K. Horiz. scale: 0.5 mV/div.; vert.: 1 mA/div.

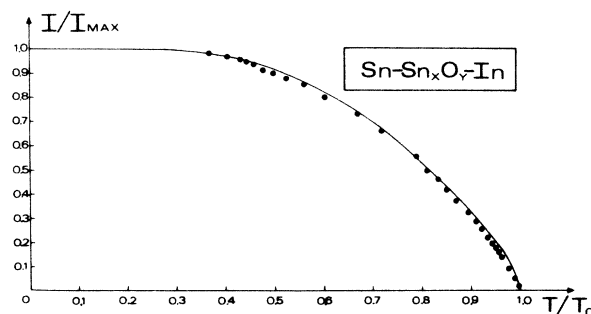


FIG. 6. Temperature dependence of the maximum (dc) Josephson current for Sn-Sn<sub>x</sub>O<sub>y</sub>-In junction. Experimental data: solid circles; theoretical curve using Ambegaokar and Baratoff calculations: solid line.

The measured temperature dependence of the maximum Josephson current is compared with the calculated theoretical curve in Fig. 3. The data refer to a sample having a critical temperature  $T_c = 3.73 \pm 0.01$  K and an energy-gap value  $\Delta_{Sn} = 0.565 \pm 0.005$  meV. Experimental current values were normalized to the maximum current observed and fitted to the theoretical curve by matching the corresponding point in temperature. The agreement with the Ambegaokar and Baratoff calculations is excellent over the entire temperature range.

#### B. Sn-Sn<sub>x</sub>O<sub>y</sub>-Pb junctions

These samples also exhibit values of the ratio  $L/\lambda_J$  of less than one. The normal tunneling resistances were typically  $\approx 0.3 \Omega$ , and the measured maximum Josephson current was about 70% of the theoretical value.

In Fig. 4 the experimental data for a tin-lead junction are reported together with the theoretical curve. For this particular junction the measured critical temperature for tin was  $3.73 \pm 0.01$  K. The lead energy gap was obtained from the measured value  $\Delta_{Pb} + \Delta_{Sn} = 1.950 \pm 0.005$  meV by subtracting the previously measured value of the tin gap ( $\Delta_{Sn} = 0.565$  meV). The resulting value is  $\Delta_{Pb} = 1.38 \pm 0.01$  meV, in agreement with that measured on Pb-Pb junctions by Lim *et al.*<sup>3</sup> In the computation of the theoretical curve, the value  $T_c = 7.19$  K for lead was used. Since the critical temperatures of the two superconductors are markedly different, slight changes in their value do not significantly affect the theoretical curve. Also, for such structures very good agreement with Ambegaokar and Baratoff predictions has been found. No deviation due to strong-coupling effects is present, since in the experimental temperature range the lead was always far below its critical temperature  $T_c$ .

C. Sn-Sn<sub>x</sub>O<sub>y</sub>-In junctions

As for the previously discussed junctions, these samples also exhibit a value of the ratio  $L/\lambda_J$  of less than one. The normal resistance values were about  $0.4 \Omega$  and the ratio  $I_J/I_T$  was of the order of 70%. In Fig. 5, a typical  $V-I$  characteristic of a Sn-Sn<sub>x</sub>O<sub>y</sub>-In junction is shown. The peculiar shape of the curve, which exhibits an almost linear branch from  $V=0.5$  mV to  $V=(1/e)(\Delta_1+\Delta_2)$  mV, was observed in all junctions measured. This effect is probably connected with multiparticle tunneling.<sup>10</sup> Following the same procedure reported in Sec. III B, the value of the indium energy gap  $\Delta_{In}=0.535 \pm 0.01$  meV has been determined. The critical temperatures of the tin and the indium were measured to be, respectively,  $3.69 \pm 0.01$  K and  $3.40 \pm 0.01$  K.

In Fig. 6 the experimental data are compared to the calculated curve. The slight discrepancy can be explained by a lack of means for an independent measurement of the two energy gaps for the asymmetrical junction. Small variations of the ratio of the gaps, particularly when gaps and critical temperatures are similar, can in fact produce sig-

nificant changes in the computation of the theoretical curve. Further extension of these measurements will be the subject of future publication.

## IV. CONCLUSIONS

Reliable Josephson junctions made by ultra-high-vacuum deposition of thin films and with a plasma-oxidized tunneling barrier have been investigated in order to measure the temperature dependence of the maximum Josephson current. Both symmetrical (Sn-Sn<sub>x</sub>O<sub>y</sub>-Sn) and asymmetrical (Sn-Sn<sub>x</sub>O<sub>y</sub>-Pb, Sn-Sn<sub>x</sub>O<sub>y</sub>-In) junctions have been considered. For tin-tin-oxide-tin and tin-tin-oxide-lead junctions, excellent agreement with the Ambegaokar and Baratoff calculations has been found. For tin-tin-oxide-indium junctions, although reasonable agreement has already been obtained, it would be interesting to perform further measurements.

## ACKNOWLEDGMENTS

The authors would like to express their appreciation for the technical assistance of G. Della Volpe, M. Greco, C. Salinas, and C. Salvia.

†Present address: Department of Electrical and Computer Engineering, University of Wisconsin, Madison, WI. Partially supported by the Consiglio Nazionale delle Ricerche and the NSF under the US-Italy Program for Cooperative Research in Science.

†On research leave from Department of Electrical Engineering, University of Wisconsin, Madison, WI.

\*Partially supported by the Consiglio Nazionale delle Ricerche and the NSF under the US-Italy Program for Cooperative Research in Science.

<sup>1</sup>M. D. Fiske, *Rev. Mod. Phys.* **36**, 221 (1964).

<sup>2</sup>V. Ambegaokar and A. Baratoff, *Phys. Rev. Lett.* **10**, 486 (1963); **11**, 104(E) (1964).

<sup>3</sup>C. S. Lim, J. D. Leslie, H. J. T. Smith, P. Vashista, and J. P. Carbotte, *Phys. Rev. B* **2**, 1651 (1970).

<sup>4</sup>P. Rissman and T. Palholmen, *Solid State Electron.* **17**, 611 (1974).

<sup>5</sup>G. Della Volpe, Lab. di Cibernetica CNR, Internal Report RI/35, 1973 (unpublished).

<sup>6</sup>W. J. Johnson and A. Barone, *J. Appl. Phys.* **41**, 2958 (1970).

<sup>7</sup>B. Cabrera and W. O. Hamilton, in *Science and Technology of Superconductivity*, edited by W. D. Gregory, W. N. Mathews, Jr., and E. A. Edelsack (Plenum, New York, 1973), Vol. II, p. 587.

<sup>8</sup>B. Mühlischlegel, *Z. Phys.* **155**, 313 (1959).

<sup>9</sup>E. P. Balsamo, K. Baker, A. Barone, and G. Paternò, Laboratori Nazionali di Frascati internal report (unpublished).

<sup>10</sup>C. J. Adkins, *Rev. Mod. Phys.* **36**, 211 (1964).

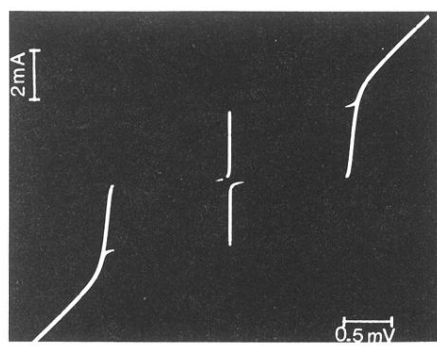


FIG. 2. Typical  $V$ - $I$  characteristic for a Sn-Sn<sub>x</sub>O<sub>y</sub>-Sn Josephson junction at  $T=1.52$  K. Horiz. scale: 0.5 mV/div.; vert.: 2 mA/div.

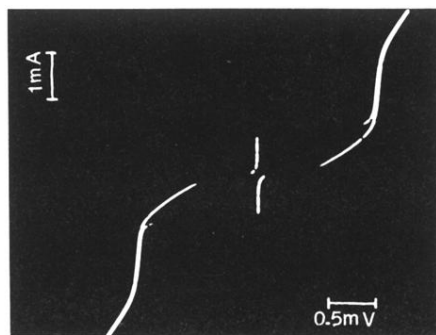


FIG. 5.  $V$ - $I$  characteristic for a Sn-Sn <sub>$x$</sub> O <sub>$y$</sub> -In Josephson junction at  $T=1.21$  K. Horiz. scale: 0.5 mV/div.; vert.: 1 mA/div.