Generalized transport equation*

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A transport equation for the density operator of the Landau quasiparticle in the presence of a constant magnetic field is derived using the generalized self-consistent field (GSCF) method. An appropriate matrix representation in $\vec{k}$ space is introduced. The resulting transport equation for the Landau one-quasiparticle density matrix is gauge invariant and not restricted to the long-wavelength limit. In the long-wavelength limit the GSCF transport equation becomes Silin's phenomenological transport equation. The role of exchange and correlation is discussed. It is shown that to a good degree of approximation, the quasiparticle velocity can be replaced by the bare particle velocity, and the Lorentz-type term involving the fluctuation of the self-consistent-field potential can be neglected.

I. INTRODUCTION

The transport equation for a system of interacting electrons has long been a subject of interest and controversy. The Boltzmann equation of electrical (and thermal) transport has been widely used to calculate, in the free-electron approximation, transport coefficients in metals and semiconductors.[1] Quantum-mechanical derivations of the Boltzmann equation have appeared in the literature.[2] Transport equations for the interacting electrons in solids, treated within the framework of Landau's theory of the Fermi liquid,[3] were introduced by several authors. Silin[4] proposed a semiclassical equation which he used to study oscillations of the degenerate electron fluid.[5] He predicted the existence of paramagnetic spin waves, later on observed by Schultz and Dunifer in alkaline metals.[6] Abrikosov and Dzialoshinskii extended these results to ferromagnetic metals.[7] Extension of the theory in order to be applied to antiferromagnetic materials or to materials with a more complex magnetic ordering, was proposed by de Graaf and Luzzi,[8] and elaborated in detail by Cohen,[9] being from then on identified under the name of the generalized self-consistent field (GSCF) method. This method was applied to the study of equilibrium and nonequilibrium processes.[8–11] The GSCF method can also be a convenient one to describe, in a unified way, light scattering from a Fermi liquid since the scattering cross section can be related to the imaginary part of a transport coefficient (or generalized susceptibility) through the fluctuation-dissipation theorem.[12]

A most interesting case appears when the electron fluid is embedded in a constant magnetic field. Quantum-mechanical derivations of transport equations in a uniform magnetic field have recently been presented by van Zandt[13] and Weisenthal and de Graaf.[14] These authors obtained an equation for the transverse spin magnetization of a paramagnetic electron gas. They compare their equations with the phenomenological equation of Platzman and Wolff[15] used for the interpretation of the experiments of Ref. 6. Conflicting results are reported concerning the character of the velocity entering the Lorentz-force-type term and on the role of the (self-consistent) exchange energy. The time-dependent Hartree-Fock calculation of Weisenthal and de Graaf[14] reproduces the results of Platzmann and Wolff[15] when the range of the exchange interaction is smaller than the cyclotron radius.

Two main problems have generally plagued a quantum-mechanics derivation of a transport equation in a magnetic field, namely, (a) the restriction to long-wavelength perturbations and (b) gauge dependence. Here we present a derivation of a transport equation for the Landau quasiparticle density operator within the framework of the GSCF method. The treatment by the GSCF method is inherently valid for short-wavelength excitations thus removing restriction (a) and, furthermore, the use of an appropriate representation makes the Liouville-Landau equation gauge independent, and there is the additional advantage of using Bloch band one-electron wave functions.

The GSCF-Landau-Liouville equation is derived in Sec. II, and their principal characteristics are discussed and next summarized in Sec. III. In Appendix B the collision operator is briefly considered.

II. GSCF TRANSPORT EQUATION

Let us consider a charged Fermi fluid embedded in a uniform constant magnetic field $\vec{H}_0 = H_0\hat{z}$, in the presence of a random distribution of impurities, and subjected to an external perturbation characterized by a potential $V(\vec{r}, t)$. We assume that the charged fermion system is described by a self-consistent single quasiparticle density operator $\hat{\rho}$. The GSCF single-quasiparticle Hamiltonian $\hat{\epsilon}[\rho]$, i.e., the first variational derivative of the average field energy of the whole many-body system with respect to $\hat{\rho}$, is a functional of ρ, which in turn depends on $\hat{\epsilon}$ itself, thus stablishing the self-consistency condition.[8,9] The time dependence of $\hat{\rho}$ in nonequilibrium situations is given by Liouville equation[8,9,16]

$$i\frac{\partial}{\partial t}\langle\vec{r}s|\hat{\rho}(t)|\vec{r}'s'\rangle=\langle\vec{r}s|[\hat{\epsilon}[\hat{\rho}]+\hat{V},\hat{\rho}]|\vec{r}'s'\rangle, \quad (1)$$

where $\vec{r}$ and s are the spatial and spin coordinates ($\hbar=1$ throughout this paper).

We write $\hat{\rho}=\hat{\rho}_0+\hat{\rho}_1$, where ρ_0 is the unperturbed ($V=0$) density operator and $\hat{\epsilon}[\hat{\rho}]=\hat{\epsilon}_0+\delta\hat{\epsilon}+\hat{w}$, where $\hat{\epsilon}_0=\hat{\epsilon}[\hat{\rho}_0]$, $\delta\hat{\epsilon}$ is the variation of the self-consistent potential induced by the external perturbation (polarization effect), and $\hat{w}$ is the potential due to the impurities. The linearized Liouville equation is then

$$i\frac{\partial}{\partial t}\langle\vec{r}s|\hat{\rho}_1|\vec{r}'s'\rangle-\langle\vec{r}s|[\hat{\epsilon}_0,\hat{\rho}_1]|\vec{r}'s'\rangle-\langle\vec{r}s|[\delta\hat{\epsilon},\hat{\rho}_0]|\vec{r}'s'\rangle=\langle\vec{r}s|[\hat{V},\hat{\rho}_0]|\vec{r}'s'\rangle+\langle\vec{r}s|[\hat{w},\hat{\rho}_1]|\vec{r}'s'\rangle . \quad (2)$$

The last term in Eq. (2), due to scattering by the impurities, is separately studied in Appendix B.

The uniform constant magnetic field in the z direction can be described in the most general way by the vector potential $\vec{A}(\vec{r})=\vec{A}_0(\vec{r})+\nabla g(\vec{r})$, where $\vec{A}_0(\vec{r})=\frac{1}{2}(\vec{H}_0\times\vec{r})$ and $g(\vec{r})$ is any twice differentiable function. Because of the presence of the magnetic field we find convenient to introduce the following representation for an operator $\langle\vec{r}s|\hat{A}|\vec{r}s'\rangle$:

$$\{\vec{k}n\sigma|\hat{A}|\vec{k}'n'\sigma'\}=\int d^3r\,d^3r'\langle\vec{r}s|\hat{A}|\vec{r}'s'\rangle\times\exp[i\Lambda(\vec{r},\vec{r}')]\phi_{\vec{k}n\sigma}(\vec{r})\phi_{\vec{k}'n'\sigma'}(\vec{r}') , \quad (3)$$

where the ϕ_s' are one-quasiparticle SCF Bloch functions characterized by the wave number $\vec{k}$, the band index n, and spin index σ. The phase function Λ is defined as

$$\Lambda(\vec{r}\vec{r}')=2(e/c)\vec{\xi}\cdot\vec{A}_0(\vec{R})+(e/c)[g(\vec{r})-g^*(\vec{r}')] , \quad (4)$$

with $\vec{R}=\frac{1}{2}(\mathbf{r}+\mathbf{r}')$ and $\vec{\xi}=\vec{r}-\vec{r}'$. Similar transformations to that of Eq. (3) have been proposed by several authors to study different transport problems.[2,17,18]

The choice of Eq. (4) renders Eq. (2) gauge invariant, as will be shown below (cf. also Appendix A). Furthermore, the band Hamiltonian is diagonal in the representation defined by Eq. (3), i.e.,

$$\{\vec{k},n,\sigma|\hat{\epsilon}_0|\vec{k}',n',\sigma'\}=\epsilon(\vec{k},n,\sigma)\delta_{\vec{k},\vec{k}'}\delta_{nn'}\delta_{\sigma\sigma'} , \quad (5)$$

where $\epsilon(\vec{k},n,\sigma)$ are the single-quasiparticle self-consistent-field (SCF) Bloch band energies. In what follows we will omit the band index and, everywhere, $\vec{k}$ will represent the pair of indexes $\vec{k}n$.

Taking matrix elements, as defined by Eq. (3), in Eq. (2) and introducing the notation

$$\{\vec{k}\alpha|\hat{A}|\vec{k}'\alpha'\}=A(\vec{k}\alpha,\vec{k}'\alpha') ,$$

we obtain

$$i\frac{\partial}{\partial t}\{\vec{k}\alpha|\hat{\rho}_1|\vec{k}'\alpha'\}=i\frac{\partial}{\partial t}\rho_1(\vec{k}\alpha,\vec{k}'\alpha') \quad (6)$$

for the first term in Eq. (2).

The second term becomes

$$\{\vec{k}\alpha|[\hat{\epsilon}_0,\hat{\rho}_1]|\vec{k}'\alpha'\}=\{\epsilon(\vec{k}\alpha)-\epsilon(\vec{k}'\alpha')-(ie/2c)[\vec{H}_0\times(\vec{u}_{\vec{k}\alpha}+\vec{u}_{\vec{k}'\alpha'})]\cdot(\nabla_{\vec{k}}+\nabla_{\vec{k}'})\}\rho_1(\vec{k}\alpha,\vec{k}'\alpha'), \quad (7)$$

where $\vec{u}_{\vec{k}\alpha}$ is the *quasiparticle* (SCF) *velocity*.

We next turn to the third term in Eq. (2). According to the GSCF method one has

$$\langle\vec{r}s|\delta\hat{\epsilon}|\vec{r}'s'\rangle=\sum_{\sigma\sigma'}\int d^3R\,d^3R'\langle\vec{R}\sigma|\hat{\rho}_1|\vec{R}'\sigma'\rangle\times[\langle\vec{R}'\sigma',\vec{r}s|\hat{\Phi}|\vec{r}'s',\vec{R}\sigma\rangle-\langle\vec{R}'\sigma',\vec{r}s|\hat{\Psi}|\vec{R}\sigma,\vec{r}'s'\rangle], \quad (8)$$

where $\hat{\Phi}$ and $\hat{\Psi}$ are the direct and exchange contributions to the Landau interaction function,[3,9] i.e., the second variational derivative of the whole SCF energy of the many-particle system with respect to the one-quasiparticle density operator. Whereas these quantities can be written in terms of a two-quasiparticle potential, that is to say,

$$\hat{\Phi}=v_c(\vec{r}-\vec{R}')\delta(\vec{R}'-\vec{R})\delta(\vec{r}-\vec{r}')\delta_{\sigma\sigma'}\delta_{ss'} , \quad (8a)$$

$$\hat{\Psi}=v_x(\vec{r}-\vec{R}')\delta(\vec{R}'-\vec{r}')\delta(\vec{R}-\vec{r})\delta_{\sigma's'}\delta_{\sigma s} , \quad (8b)$$

one gets

$$\{\vec{k}\alpha|[\delta\hat{\epsilon},\rho_0]|\vec{k}'\alpha'\}=\delta_{\alpha\alpha'}(N_{\vec{k}'\alpha'}-N_{\vec{k}\alpha})\sum_{\vec{p}\vec{p}'\beta}\langle\vec{p},\vec{k}|v_c|\vec{k}'\vec{p}'\rangle\rho_1(\vec{p}'\beta,\vec{p}\beta)+\sum_{\vec{k}_1\vec{k}_2\vec{k}_3}\int d^3R\,v_x(\vec{r}-\vec{R})\times e^{i\lambda(\vec{r},\vec{r}',\vec{R})}\phi^*_{\vec{k}_1\alpha}(\vec{r})\phi^*_{\vec{k}_1\alpha'}(\vec{r}')\phi^*_{\vec{k}\alpha}(\vec{r})\phi_{\vec{k}_2\sigma}(\vec{R})\phi_{\vec{k}_3\alpha'}(\vec{r}')\times\phi_{\vec{k}'\alpha'}(\vec{r}')N_{\vec{k}_1\alpha}\rho_1(\vec{k}_1\alpha,\vec{k}_3\alpha')+\text{same term with }\vec{r}\leftrightarrow\vec{r}' , \quad (9)$$

where

$$\lambda(\vec{r},\vec{r}',\vec{R})=(e/2c)\vec{H}_0[(\vec{r}'-\vec{R})\times(\vec{r}-\vec{R})] . \quad (10)$$

Use was made of the fact that

$$\rho_0(\vec{p}\beta, \vec{p}'\beta') = N_{\vec{p}\beta}\delta_{\vec{p}\vec{p}'}\delta_{\beta\beta'} , \tag{11}$$

$N_{\vec{p}\beta}$ being the Fermi-Dirac distribution function.

Finally, the term involving the driving potential V becomes

$$\{\vec{k}\alpha|[\hat{V}, \hat{\rho}_0]|\vec{k}'\alpha'\} = (N_{\vec{k}'\alpha'} - N_{\vec{k}\alpha})\langle\vec{k}\alpha|\hat{V}|\vec{k}'\alpha'\rangle . \tag{12}$$

Collecting the contributions of Eqs. (6), (7), (9), and (12) and the scattering operator of Appendix B one obtains the GSCF transport equation for the Landau single-quasiparticle density matrix. We omit writing the cumbersome resulting equation, and give it later [cf. Eq. (13)] in a more convenient way. We first observe that the GSCF transport equation just derived is gauge independent and not restricted, since $\vec{k}$ and $\vec{k}'$ could be any vector in the first Brillouin zone, to the long-wavelength limit.

Neglecting spin, putting $\delta\epsilon = 0$ and $\epsilon(\vec{k}\alpha) = k^2/2m$, and introducing the new variables $\vec{k} = \vec{k} + \frac{1}{2}\vec{q}$, $\vec{k}' = \vec{k} - \frac{1}{2}\vec{q}$, the GSCF transport equation becomes, in the small q limit, the Boltzmann equation, once the identification

$$\{\vec{k} + \tfrac{1}{2}\vec{q}|\rho_1|\vec{k} - \tfrac{1}{2}\vec{q}\} = \int d^3R\, e^{i\vec{q}\cdot\vec{R}} f(\vec{k}, \vec{R}, t)$$

is made, and where $f(\vec{k}, \vec{R}, t)$ is the Boltzmann distribution function.

An important point already stressed by Weisenthal and de Graaf[14] should be brought to note. Because of correlation effects the potential v_x is screened.[19] If J_s is the inverse of the range of this potential, and λ_c is the magnetic length $|\hbar c/eH_0|^{1/2}$ one can observe that when $\lambda_c J_s \ll 1$, the contribution from the exchange term in Eq. (9) will tend to cancel out because of the rapidly varying phase factor. On the other hand, if $\lambda_c J_s \gg 1$ what implies either small fields or short-range interaction, the expansion $e^{i\lambda} \simeq 1 + i\lambda$ can be made. Under such conditions the GSCF transport equation becomes

$$\begin{aligned}
&\left(i\frac{\partial}{\partial t} - \epsilon(\vec{k}\alpha) + \epsilon(\vec{k}'\alpha') + \frac{ie}{2c}[\vec{H}_0\times(\vec{u}_{\vec{k}\alpha} + \vec{u}_{\vec{k}'\alpha'})]\cdot(\nabla_{\vec{k}} + \nabla_{\vec{k}'})\right)\rho_1(\vec{k}\alpha, \vec{k}'\alpha') + (N_{\vec{k}\alpha} - N_{\vec{k}'\alpha'})\\
&\quad\times\sum_{\vec{p}\beta,\vec{p}'\beta'}\langle\vec{p}\beta, \vec{k}\alpha|v_c|\vec{k}'\alpha', \vec{p} - \vec{k}' + \vec{k}\beta'\rangle\delta_{\alpha\alpha'}\delta_{\beta\beta'}\rho_1(\vec{p}\beta, \vec{p} - \vec{k}' + \vec{k}\beta')\\
&\quad+\left[(N_{\vec{k}\alpha} - N_{\vec{k}'\alpha'}) + \frac{ie}{2c}[\vec{H}_0\times(\vec{u}_{\vec{k}\alpha} + \vec{u}_{\vec{k}'\alpha'})]\left(\frac{\partial N_{\vec{k}\alpha}}{\partial\epsilon_{\vec{k}\alpha}}\nabla_k + \frac{\partial N_{\vec{k}'\alpha'}}{\partial\epsilon_{\vec{k}'\alpha'}}\nabla_{k'}\right)\right]\\
&\quad\times\sum_{\vec{p}\beta,\vec{p}'\beta'}\langle\vec{k}\alpha, \vec{p}\beta|v_x|\vec{k}'\alpha', \vec{p} - \vec{k}' + \vec{k}\beta'\rangle\delta_{\alpha'\beta}\delta_{\alpha\beta'}\rho_1(\vec{p}\beta, \vec{p} - \vec{k}' + \vec{k}\beta')\\
&\quad= (N_{\vec{k}'\alpha'} - N_{\vec{k}\alpha})\langle\vec{k}\alpha|\hat{V}|\vec{k}'\alpha'\rangle + J_{\text{coll}}\{\rho\} .
\end{aligned} \tag{13}$$

When $\alpha = \uparrow$, $\alpha' = \downarrow$ and $\vec{k}' = \vec{k} + \frac{1}{2}\vec{q}$, $\vec{k} = \vec{k} - \frac{1}{2}\vec{q}$ and in the long-wavelength limit, Eq. (13) becomes the Platzman-Wolff[15] equation for the transverse magnetization of an electron gas. In case of a contact exchange interaction, i.e., $v_x(r - r') = U\delta(r - r')$, clearly, $\lambda = 0$ and no Lorentz-type term involving $\delta\epsilon$ appears, and for $\vec{q}\times\vec{H}_0 = 0$ one recovers the results of Ref. 11.

Equation (13) is then the basic equation to obtain response functions (or generalized susceptibilities) of the Fermi fluid. Let us recall that Eq. (13) corresponds to the case of short-range interaction between quasiparticles. Let us write

$$\epsilon(\vec{k}\alpha) = \epsilon^0(\vec{k}\alpha) + v(\vec{k}\alpha)$$

for the quasiparticle energy, where ϵ^0 is the (bare) one-electron Bloch band energy and $v(\vec{k}\alpha)$ the effective exchange and correlation potential for quasiparticle states. In the Hartree-Fock approximation $\nabla_{\vec{k}} v(\vec{k}\alpha)$ presents a logarithmic singularity at $k = k_F$. However, Hedin[19] has shown, by means of a new method for calculating the one-electron Green's function at metallic densities, that correlation effects could make $v(\vec{k}\alpha)$ almost k independent and therefore it represents a very short-range potential in direct space. This implies that $\vec{u}_{\vec{k}\alpha} = \nabla_{\vec{k}}\epsilon(\vec{k}\alpha) \simeq \nabla_{\vec{k}}\epsilon^0(\vec{k}\alpha)$ or $u_{\vec{k}\alpha} \simeq \vec{k}/m$ in the plane-wave representation. Hence, the quasiparticle velocity in the transport equation can be replaced by the bare particle velocity. Furthermore, the Lorentz-type term involving $\delta\epsilon$ is also small and can be neglected in first approximation.

III. CONCLUSIONS

Within the framework of the generalized Landau quasiparticle picture[9] we have derived a transport equation for the Landau quasiparticle density operator in the presence of a constant uniform magnetic field. An appropriate representation for the operators entering the Landau-Liouville equation was introduced. This representation has the advantages of making diagonal the SCF-Bloch Hamiltonian and of allowing to work in the $\vec{k}$ space. The transport equation obtained in this way is gauge invariant and not restricted to the long-wavelength limit. The Landau one-quasiparticle density matrix solution of the transport equation provides all necessary information to determine, in the linear approximation, any space, spin and time-dependent generalized susceptibility of the interacting Fermi fluid. This generalized susceptibility can, through the fluctuation-dissipation theorem, be related to a correlation function thus allowing comparison with measurements.[20]

APPENDIX A: CHOICE OF THE PHASE FACTOR

Pursuing an analysis similar to that used on studying the Schrödinger equation for a particle in a magnetic field described by a vector potential $\vec{A}$,[21] we write

$$\langle\vec{r}|\hat{\rho}|\vec{r}'\rangle=\langle\vec{r}|\hat{R}|\vec{r}'\rangle e^{-i\Lambda(\vec{r},\vec{r}')}, \tag{A1}$$

where Λ is an arbitrary twice differentiable function of $\vec{r}$ and $\vec{r}'$. Substituting Eq. (A1) into Eq. (2), at V and w zero, produces

$$i\frac{\partial}{\partial t}\langle\vec{r}|\hat{R}|\vec{r}'\rangle=\langle\vec{r}|[\hat{\epsilon}(0),\hat{R}]|\vec{r}'\rangle-\hat{L}(\vec{r},\vec{r}')\langle\vec{r}|\hat{R}|\vec{r}'\rangle, \tag{A2}$$

where $\hat{\epsilon}(0)$ is the single-quasiparticle Hamiltonian with $\vec{A}=0$ and, after a convenient rearrangement of terms

$$\begin{aligned}\hat{L}(\vec{r},\vec{r}')=&\{[\nabla_{\vec{r}}-\nabla_{\vec{r}}]\Lambda-(e/c)[\vec{A}(\vec{r})+\vec{A}^*(\vec{r}')]\}\cdot(\nabla_{\vec{r}}+\nabla_{\vec{r}'})\\&+[\nabla_{\vec{r}'}\Lambda+(e/c)\vec{A}^*(r')]\cdot\nabla_{\vec{r}}-[\nabla_{\vec{r}}\Lambda\\&-(e/c)\vec{A}(\vec{r})]\cdot\nabla_{\vec{r}'}\ .\end{aligned} \tag{A3}$$

Requiring that the first term on the right-hand side of Eq. (A3) be zero, one gets the following equation for the correction to the action:

$$\begin{aligned}(\nabla_{\vec{r}}-\nabla_{\vec{r}'})\Lambda-(e/c)[\vec{A}_0(\vec{r})+\vec{A}_0(\vec{r}')]\\+(e/c)[\nabla_{\vec{r}}g(\vec{r})+\nabla_{\vec{r}'}g^*(\vec{r}')]=0\ ;\end{aligned} \tag{A4}$$

the solution to Eq. (A4) is

$$\begin{aligned}\Lambda(\vec{\xi},\vec{R})=&\frac{2e}{c}\int^{\vec{\xi}}d\vec{\xi}'\cdot\vec{A}_0(\vec{R})\\&+\frac{e}{c}\int^{\vec{\xi}}d\vec{\xi}'\cdot\nabla_{\xi'}[g(\vec{R}+\tfrac{1}{2}\vec{\xi}')-g^*(\vec{R}-\tfrac{1}{2}\vec{\xi}')],\end{aligned} \tag{A5}$$

which is just Eq. (4). It is verified that $\Lambda(\vec{r}\vec{r})=0$ as it should, and using solution (A5) the operator L becomes

$$\hat{L}(\vec{r},\vec{r}')=\tfrac{1}{2}[\vec{H}_0\times\vec{\xi}]\cdot\nabla_{\vec{\xi}}\ .$$

This is the gauge independent Lorentz-type term of the transport equation. Let us remark that representation (3) for $\hat{\rho}$ is equivalent to take matrix elements between Bloch states of the operator $\hat{R}$.

APPENDIX B: SCATTERING OPERATOR

In this Appendix we consider the collision operator $[\hat{w},\hat{\rho}_1]$, i.e., the scattering due to the random impurity distribution. Using the representation defined by Eq. (3) one gets

$$\begin{aligned}J_{\text{coll}}\{\rho\}=&\{\vec{k}\alpha|[\hat{w},\hat{\rho}_1]|\vec{k}'\alpha'\}\\=&\sum_{\vec{p}\beta}[\langle\vec{k}\alpha|\hat{w}|\vec{p}\beta\rangle\rho_1(\vec{p}\beta,\vec{k}\alpha')\\&-\langle\vec{p}\beta|\hat{w}|\vec{k}'\alpha'\rangle\rho_1(\vec{k}\alpha,\vec{p}\beta)].\end{aligned} \tag{B1}$$

Assuming a time dependence of ρ_1 as $e^{i\omega t}$, and using a method of calculation similar to the method used previously,[11,12] one obtains

$$\begin{aligned}J_{\text{coll}}\{\rho\}=&-n\sum_{\vec{p}}\frac{|\langle\vec{k}|\hat{u}|\vec{p}\rangle|^2}{\omega-\epsilon(\vec{p}\alpha)+\epsilon(\vec{k}'\alpha')+is}\\&\times[\rho_1(\vec{k}\alpha,\vec{k}'\alpha')-\rho_1(\vec{p}\alpha,\vec{k}'-\vec{k}+\vec{p}\alpha')]\\&-n\sum_{\vec{p}}\frac{|\langle\vec{p}|\hat{u}|\vec{k}'\rangle|^2}{\omega-\epsilon(\vec{k}\alpha)+\epsilon(\vec{p}'\alpha')+is}\\&\times[\rho_1(\vec{k}\alpha,\vec{k}'\alpha')-\rho_1(\vec{k}-\vec{k}'+\vec{p}\alpha,\vec{p}\alpha')],\end{aligned} \tag{B2}$$

where $\hat{u}$ is the single-impurity potential and n the impurity density.

In deriving Eq. (B2) it was supposed that $|\vec{k}-\vec{k}'|l\ll 1$, where l is the electron mean free path, what implies that ρ_1 is in direct space essentially constant over regions much larger than the volume per impurity. This allows the average to be taken over all possible arrangement of impurities (Ref. 18, p. 329). The real part of J_{coll} can be thought of as a renormalization of the single-quasiparticle energies. The imaginary part, under isotropic conditions, can be rewritten in the form of the Boltzmann collision operator in the relaxation time approximation,[1] used for example in Ref. 15 (cf. also Refs. 11 and 22).

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[1]E. G. A. H. Wilson, *The Theory of Metals* (Cambridge, U.P., England, 1965); J. M. Ziman, *Electrons and Phonons* (Oxford U.P., Oxford, 1960).

[2]L. Weisenthal and A. M. de Graaf, Am. J. Phys. 40, 1469 (1972), and references therein.

[3]L. D. Landau, Zh. Eksp. Teor. Fiz. 30, 1058 (1956) [Sov. Phys.-JETP 3, 920 (1956)].

[4]V. P. Silin, Zh. Eksp. Teor. Fiz. 33, 495 (1957) [Sov. Phys.-JETP 6, 387 (1958)].

[5]V. P. Silin, Zh. Eksp. Teor. Fiz. 35, 1243 (1958) [Sov.

Phys.-JETP 8, 870 (1959)].
[6]S. Schultz and G. Dunifer, Phys. Rev. Lett. 18, 283 (1967).
[7]A. A. Abrikosov and I. E. Dzialoshinskii, Zh. Eksp. Teor. Fiz. 35, 771 (1958) [Sov. Phys.-JETP 8, 535 (1959)].
[8]A. M. de Graaf and R. Luzzi, Nuovo Cimento 38, 285 (1964).
[9]M. H. Cohen, *Proceedings of the International School of Physics "Enrico Fermi", Course XXXVII*, edited by W. Marshall (Academic, New York, 1967).
[10]D. R. Penn and M. H. Cohen, Phys. Rev. 155, 468 (1967); A. M. de Graaf and R. Luzzi, Phys. Lett. 16, 256 (1965); R. Luzzi, R. Lobo and A. M. de Graaf, Nuovo Cimento B 61, 185 (1969); R. Luzzi and M. C. Terrile, *ibid*. 16, 293 (1973).
16, 293 (1973).
[11]R. Luzzi and A. M. de Graaf, Phys. Rev. 182, 597 (1969).
[12]A. R. Vasconcellos and R. Luzzi, Nuovo Cimento (to be published); R. Luzzi. Prog. Theor. Phys. 47, 1 (1972).
[13]L. L. van Zandt, Phys. Rev. B 1, 3217 (1970); 1, 3223 (1970); 1, 3235 (1970); 4, 1641 (1971).
[14]L. Weisenthal and A. M. de Graaf, Phys. Rev. B 6, 2900 (1972).
[15]P. M. Platzman and P. A. Wolff, Phys. Rev. Lett. 18, 280 (1967).
[16]R. Tolman, *Statistical Physics* (Oxford U. P., Oxford, 1959).
[17]R. B. Stinchcombe, Proc. Phys. Soc. Lond. 78, 275 (1961).
[18]A. A. Abrikosov, L. P. Gorgov, and I. E. Dzialoshinskii, *Methods of Quantum Field Theory in Statistical Physics* (Prentice-Hall, Englewood Cliffs, N. J., 1964), p. 321.
[19]L. Hedin and S. Lundquist, *Solid State Physics Series* (Academic, New York, 1969), Vol. 23, p. 89.
[20]P. C. Martin, *Many Body Physics*, edited by C. de Witt and B. Balian (Gordon and Breach, New York, 1969).
[21]J. J. Sakurai, *Advanced Quantum Mechanics* (Addison-Wesley, Reading, Mass., 1967), p. 16.
[22]T. Sasada and H. Hasegawa, Prog. Theor. Phys. 45, 1072 (1971).