Radiation damping in magnetic resonance. II. Continuous-wave antiferromagnetic-resonance experiments

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It is shown that magnetic-dipolar radiation damping can be a primary source of the broadening of the uniform mode in antiferromagnetic resonance (AFMR) as well as ferromagnetic resonance. From a detailed study of the dependence of the AFMR linewidth ΔH in MnF_2 on the volume of the sample and its position relative to the termination of a shorted waveguide, a quantitative comparison between theory and experiment is obtained. An empirical procedure is outlined for separating the intrinsic (magnon scattering) contributions to ΔH from the radiation-induced broadening. Uniform-mode linewidths are now measured which not only agree with the dipolar pit-scattering theory of Loudon and Pincus, but also resolve the dilemma of Kotthaus and Jaccarino, who found the linewidth of the uniform mode to be much larger than those of the magnetostatic modes with which it is nearly degenerate.

INTRODUCTION

For more than two decades the phenomena of ferromagnetic resonance (FMR) and antiferromagnetic (AFMR) have been the subject of numerous investigations.[1,2] Many of these studies include the problem of the resonance linewidth ΔH—its dependence on the magnetization, frequency, sample size and shape, impurity content, surface preparation, and temperature. A rather remarkable and detailed understanding now exists of nearly all the linewidth and relaxation processes; in the main these are imperfection scattering and thermal broadening.

It is rather surprising to find that so little attention[3-5] has been given to the broadening that might result from classical magnetic-dipolar radiation damping, especially in view of the extremely narrow widths (less than 0.05 Oe) that have been found in some FMR experiments. In the following we will show that radiation broadening is an observable phenomenon in cw AFMR experiments. These results complement the recent parallel studies made on radiation broadening in cw FMR experiments.[6] (We will refer to the latter work as I hereafter.) Moreover, it will become clear from our studies that radiation damping has been a primary source of the residual broadening of the *uniform* ($k=0$) mode resonance in recent AFMR studies in MnF_2.[7]

We begin by calculating the radiation damping of an extended, precessing magnetic dipole in free space and obtain the widths for FMR, AFMR, and paramagnetic resonance due to this process. Then we relate the free space widths to the transient-radiation-decay times obtained by LeGall *et al.*[4] for an FMR sample placed in a shorted-waveguide structure. Finally, we compare the theoretical predictions for the radiation broadening of the AFMR with the observed dependence of the MnF_2 AFMR uniform-mode width on sample volume and sample position relative to the short in a terminated waveguide.

FREE-SPACE RADIATION DAMPING

One knows from classical electromagnetic theory[8] that a point magnetic dipole $\vec{\mu}$, precessing at a frequency $\omega = kc$, radiates energy at a rate

$$dE/dt = P_r = \tfrac{2}{3} ck^4 \mu_\perp^2 . \qquad (1)$$

Here $\mu_\perp$ is the instantaneous component of $\vec{\mu}$ in the plane perpendicular to the axis of precession. For a finite or extended magnetization distribution, characterized by a largest linear dimension d, one would expect Eq. (1) to apply equally well provided that

$$d \ll k^{-1} . \qquad (2)$$

If (2) were not satisfied (e.g., if $d \sim k^{-1}$) then a significant portion of the power radiated would have multipole character of higher order and in general this would impede the radiative loss of energy. However, in the absence of compensating external driving forces, the perpendicular component of the magnetization will decay with time whenever (2) holds.

The relaxation rate η_r associated with the magnetic-dipolar radiation process is obtainable from the ratio of the power radiated [Eq. (1)] to the stored magnetic energy (Zeeman, exchange, and anisotropy). In the Appendix, η_r is derived from this point of view for the FMR and AFMR cases. Alternatively, we may use the small signal theory of magnetic resonance to obtain η_r. In the linear response regime the power absorbed P_a by a magnetic spin system in an rf field H_1, as a function of frequency ω, is[9]

$$P_a = \tfrac{1}{2}\,\omega\chi''\,VH_1^2, \tag{3}$$

where χ'' is the imaginary part of the frequency-dependent transverse (volume) susceptibility and V is the sample volume. If the processes which determine the dynamic response of the system may be characterized by an exponential decay time τ, then $\chi''(\omega)$ will have the familiar Lorentzian profile[9]

$$\chi''(\omega) = \frac{\frac{1}{2}\gamma^2 M[\frac{1}{2}(\Delta H)]}{(\omega-\omega_0)^2+\gamma^2[\frac{1}{2}(\Delta H)]^2}, \tag{4}$$

where ω_0 is the Larmor frequency, γ is the gyromagnetic ratio, and M is the magnetization per unit volume. τ, η_r, and ΔH are related by $\tau^{-1} = 2\eta_r = \gamma(\Delta H)$, where τ is the *energy* relaxation time, η_r is the *magnetization* relaxation rate due to magnetic-dipolar radiation processes, and ΔH is the *full* linewidth between half-power points. In terms of $\chi''(\omega)$ the total *transverse* magnetization developed at a frequency ω in the rf field H_1 is just $\chi''(\omega)VH_1$. Hence if equilibrium is to be established between the incident rf field and the radiating spin system—and radiation damping were the *sole* cause of the line broadening—it would require that

$$P_a = P_r = \tfrac{2}{3}\,ck^4(\chi''\,VH_1)^2 \tag{5}$$

from Eq. (1). But we see from Eqs. (3) and (5) that on resonance, (i.e., $\omega=\omega_0$) where $\chi''(\omega) = M/\Delta H$, this would result in a free-space (fs) linewidth

$$(\Delta H)_r^{\text{fs}} = \tfrac{4}{3}k^3 M\,V. \tag{6}$$

For the ferromagnet (or paramagnet) $M\,V$ would represent the total magnetic moment at a temperature T_0 and dc field H_0; that is $M\,V \to M_0(T_0, H_0)V$. For the easy axis antiferromagnet, such as MnF_2, where the precessing magnetization is smaller than the sublattice magnetization M_s by the ratio H_A/H_C we must make the identification $M\,V \to M_s(T_0)(H_A/H_C)V$ in applying Eq. (6) to AFMR. The quantities H_A and H_C are the anisotropy and critical (spin-flop) fields, respectively.[10] Thus for the two cases of most interest, we have

$$\text{FMR:}\quad (\Delta H)_r^{\text{fs}} = \tfrac{4}{3}M_0(T_0, H_0)\,V(\omega/c)^3 = [\tfrac{1}{3}(8\pi^2)](4\pi M_0)V\lambda^{-3}, \tag{7}$$

$$\text{AFMR:}\quad (\Delta H)_r^{\text{fs}} = \tfrac{4}{3}M_s(T_0)(H_A/H_C)\,V(\omega/c)^3 = [\tfrac{1}{3}(8\pi^2)](4\pi M_s)\,V(H_A/H_C)\lambda^{-3}. \tag{8}$$

The ω^3 dependence of $(\Delta H)_r^{\text{fs}}$ suggests that radiation damping could become a measurable source of broadening at high frequencies, subject to the restriction of the inequality (2). Indeed it was shown in I for yttrium iron garnet (YIG) spheres of 4-mm diameter, in a free-space configuration at 9 GHz, that the broadening caused by radiation damping, $(\Delta H)_r^{\text{fs}} \simeq 40$ Oe, is almost two orders of magnitude larger than that which would be attributable to intrinsic processes.

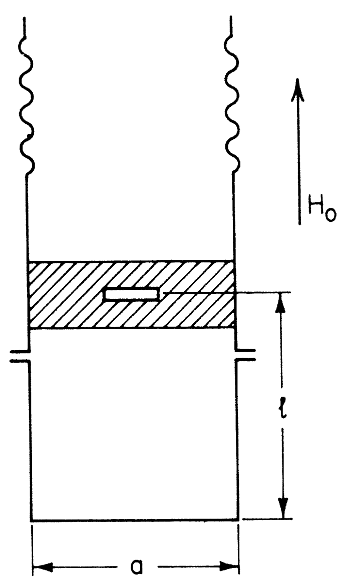

FIG. 1. Shown here is the sample, mounted in polystyrene and located a distance l from the reflecting short. A short piece of flexible waveguide allows precise orientation of the sample with respect to H_0. Adjustment of l can be made without disturbing the orientation by exchanging a series of shorted waveguide sections.

RADIATION DAMPING IN A SHORTED WAVEGUIDE

It is inconvenient to study AFMR in a "free-space" configuration because of the need to carefully align the crystal easy axis parallel to the external field H_0 at low temperatures. Instead, in the studies on MnF_2 we have placed the sample at a distance l from the reflecting short in a rectangular wave guide of cross section ab in which only the fundamental (TE_{10}) mode is propagated at the frequencies employed (see Fig. 1). Using the results of LeGall *et al.*[4] for the transient radiation decay behavior in a shorted waveguide, we find that the counterpart of the basic free-space radiation damping width [Eq. (6)] would, in the waveguide, be

$$(\Delta H)_r^{\text{wg}} = 16\pi^2(M\,V/ab\lambda_g)\cos^2(k_g l), \tag{9}$$

where k_g and λ_g are the guide wave vector and wavelength, respectively.

At first glance, one might believe that the $\cos^2 k_g l$ dependence of Eq. (9) reflects the standing-wave structure of the incident microwave radiation. However, Eqs. (3) and (5) still hold when the sample is placed in a waveguide, except that Eq. (5) should include a term which expresses the modified ability of the sample to radiate when confined by the waveguide. Therefore, $(\Delta H)_r^{wg}$ will be independent of H_1, but will be influenced by interference between radiation emitted directly toward the detector (in the direction of $\vec{H}_0$ in Fig. 1) and radiation emitted toward the short and reflected back toward the detector. This interference causes a $\cos k_g l$ dependence in the field, and a $\cos^2 k_g l$ dependence in the net power radiated by the sample and, therefore, in $(\Delta H)_r^{wg}$ as well. Of course the coefficients in Eq. (9) must be determined by a detailed consideration[4] of all the boundary conditions imposed by the waveguide.

It is instructive to express Eq. (9) in terms of the free-space widths. For both FMR and AFMR one finds immediately that

$$(\Delta H)_r^{wg} = \left(\frac{3}{2\pi}\frac{\lambda^3}{ab\lambda_g}\cos^2 k_g l\right)(\Delta H)_r^{fs} . \qquad (10)$$

Note that for $k_g l = n\pi$, and operating at a frequency away from the "cutoff" condition ($\nu = c/2a$ for TE_{10}), so that $ab\lambda_g \approx \lambda^3$, the free-space and shorted-waveguide widths are of the same magnitude. In the spirit of a golden rule, transition-probability approach, this implies that the final-state density is similar in the two cases and that the $\cos^2 k_g l$ behavior in the waveguide is a superimposed spectral modulation of the coupling to the electromagnetic field. As we shall presently show, the oscillatory dependence of $(\Delta H)_r^{wg}$ on $k_g l$ is a useful check on the relative contribution that radiation damping makes to the observed widths.

EXPERIMENTAL PROCEDURE AND RESULTS

With the definitive results on cw radiation broadening of the FMR obtained in I and the qualitative suggestion implicit in Eqs. (8) and (10) that similar effects might be important in the AFMR linewidth in MnF_2,[7] we set out to make a quantitative study of radiation damping in the latter system. The experimental arrangement used is virtually identical to that described in Ref. 7—a 23-GHz reflection spectrometer which has the sample arm of the magic-T bridge terminated in a shorted waveguide section (see Fig. 1) in which microwave propagation is parallel to the surrounding superconducting solenoid. Measurements were made at 4.2 K in a field in the vicinity of 90 KOe.

The disk-shaped MnF_2 single-crystal samples—with the c axis normal to the disk plane—were mounted in a thin slab of polystyrene and then inserted into the waveguide. Each was centered and carefully oriented with respect to the field using the AFMR absorption. The sole innovation introduced was a provision for adjustment of l without disturbing the sample orientation. This was achieved by exchanging a series of precisely cut, shorted, waveguide sections, as shown in Fig. 1.

We were fortunate also to have access to some of the samples used in the original AFMR experiments.[7] In particular, a study was made of the sample described in Fig. 1 of Ref. 7. This sample has a volume $V_1 = 1.77$ mm^3 from which a predicted maximum radiation induced width $(\Delta H)_r^{wg} \simeq 19$ Oe is obtained using Eqs. (8) and (10) and the values of $M_s = 600$ Oe, $H_A/H_C = 0.090$, and $\cos^2 k_g l = 1$. On the other hand, the contribution to ΔH expected from two magnon dipolar pit scattering[2]—the only other mechanism of any importance at this low temperature—was less than 2 Oe for the surface polish conditions obtained.[7] Thus as $\cos^2 k_g l$ varied from 0 to 1 (as the sample position was changed along the waveguide), we might expect a tenfold change in linewidth, assuming the total width $(\Delta H)_T$ is simply[11]

$$(\Delta H)_T = (\Delta H)_r^{wg} + (\Delta H)_{int} , \qquad (11)$$

with $(\Delta H)_{int}$ representing all nonradiative intrinsic contributions to ΔH.

The observed dependence of ΔH on $k_g l$ is shown in Fig. 2. The minimum value of $\Delta H \simeq 1.5$ Oe at $k_g l = 2.5\pi$ quantitatively agrees with the Loudon-Pincus theory[2] for the parameters given in Ref. 7. It is also satisfying that the periodic dependence of ΔH is in accord with the predictions of Eqs. (9) and (10) and the assumption of additivity contained in Eq. (11); i.e., $\Delta H = C\cos^2 k_g l + 1.5$ Oe. However the quantity C is nearly a factor of $\frac{2}{3}$ smaller than the calculated value, using Eq. (10). This may be due in part to the use of nonellipsoidal sample shapes. Because the magnetization is not constant throughout a nonellipsoidal sample, the material in the "corners" of our disks may be unable to participate in the uniform precession, although it may participate in $k \neq 0$ modes. There is some indication of this in the integrated spectrum of magnetostatic modes taken at $\cos^2 k_g l = 1$. Here the ratio of the integrated power in the uniform mode to the integrated power in the entire spectrum is roughly equal to the ratio C_{obs}/C_{calc}. In addition, one notices that the ratio of the volumes of an ellipsoid and its circumscribing disk is exactly $\frac{2}{3}$, suggesting that only the volume within such an ellipsoid is active in the uniform-mode resonance of a disk-shaped sample. As it stands, this evi-

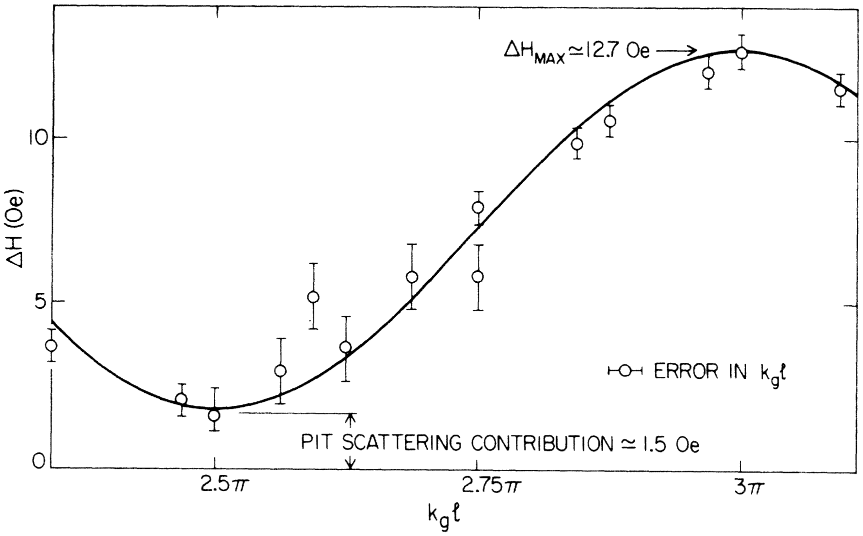

FIG. 2. Uniform-mode linewidth is plotted vs $k_g l$ for a disk-shaped sample of volume $V_1 = 1.77\ \text{mm}^3$ at 23 GHz and 4.2 K. The curve is $C\cos^2 k_g l + 1.5$ Oe, where C is chosen to agree with $\Delta H_{\max} = 12.7$ Oe.

dence is tenuous at best, and an investigation of linewidths in polished spheres of MnF_2 is planned to clear up this aspect of the problem.

To test the explicit volume dependence of the radiation induced broadening, a second smaller sample was prepared with $V_2 = 0.68\ \text{mm}^3$ but with the same surface polish as the specimen considered above. Again, at $\cos^2 k_g l = 0$, a minimum value of $\Delta H \simeq 1.5$ Oe was observed, indicating that the nonradiative contributions to the linewidth are volume independent. After subtracting the latter intrinsic contribution to ΔH for both samples we find that the remaining widths $(\Delta H)'$, at values of $k_g l$ corresponding to $\cos^2 k_g l = 1$, accurately scale with the volumes, i.e., $V_1/V_2 = 2.60$, $(\Delta H_1)'/(\Delta H_2)' = 2.65$. (This relationship remains undisturbed by the assumption that only the material within the inscribed ellipsoid may be active, since the *ratio* of the volumes is unchanged.)

There is an interesting consequence of the $\cos^2 k_g l$ dependence of the linewidth arising from radiation damping as manifested in the total power absorbed. Using Eqs. (3) and (11) the peak power absorption at $\omega = \omega_0$ for the AFMR case is

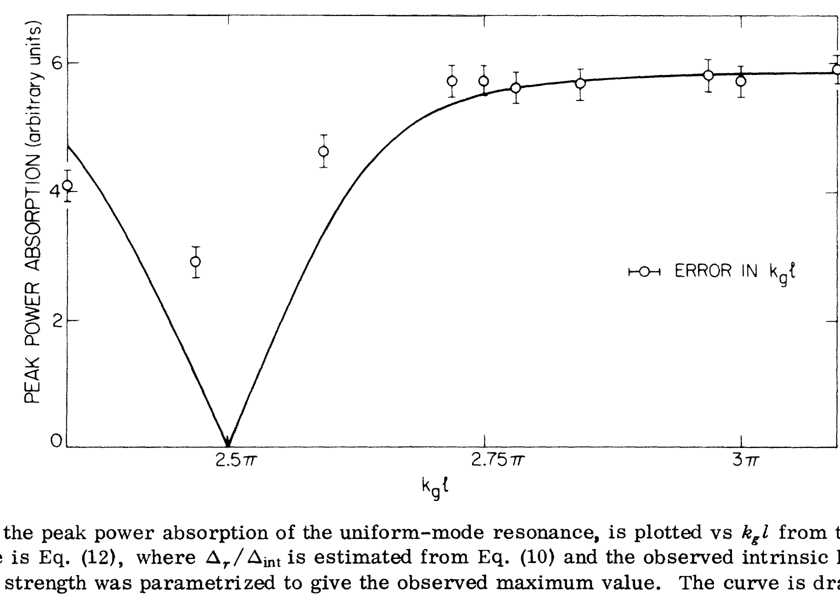

FIG. 3. $P_a(\omega_0)$, the peak power absorption of the uniform-mode resonance, is plotted vs $k_g l$ from the same data as Fig. 2. The curve is Eq. (12), where $\Delta_r/\Delta_{\text{int}}$ is estimated from Eq. (10) and the observed intrinsic linewidth, and where the rf field strength was parametrized to give the observed maximum value. The curve is drawn for a point sample; our data for a finite sample, therefore, agree poorly near the minimum, but show the general behavior of constant peak power absorption for the region in which $\Delta_r >> \Delta_{\text{int}}$.

$$P_a(\omega_0) = \tfrac{1}{2}\,\omega_0 M_s V \frac{H_A}{H_C}\left(\frac{H_1^2}{[(\Delta H)_r + (\Delta H)_{\rm int}]}\right). \quad (12)$$

However, both H_1^2 and $(\Delta H)_r$ are proportional to $\cos^2 k_g l$. Hence, for those values of $k_g l$ for which $(\Delta H)_r \gg (\Delta H)_{\rm int}$, $P_a(\omega_0)$ should be approximately independent of $k_g l$. At that value of $k_g l$ where $(\Delta H)_r = (\Delta H)_{\rm int}$, $P_a(\omega_0)$ will decrease to half of its maximum value. This general behavior is observed, as is seen in Fig. 3 where $P_a(\omega_0)$ vs $k_g l$ is plotted, providing additional support for the existence of radiation damping.

Finally, we remark on an empirical procedure for obtaining the intrinsic linewidth and show how our observations relate to the apparently anomalous (relative to the widths of the magnetostatic modes) behavior of the uniform-mode width found in the earlier study of AFMR in MnF_2.[7]

After we found the periodic behavior of ΔH as a function of $k_g l$ for the original sample, as shown in Fig. 2, we repeated the experiment of Kotthaus and Jaccarino by placing the sample on the terminating short face (i.e., at $l=0$). Instead of the 12.7-Oe maximum width that was observed at all other values of $\cos^2 k_g l = 1$, we found here that $\Delta H = 19.5$ Oe, a value identical with that of Ref. 7. This "wall effect" has been observed in previous FMR[3] studies and is ascribed to the additional Ohmic loss produced by eddy currents induced in the wall by the precessing magnetization. The magnitude of the observed increase in ΔH, over and above the processes considered earlier, is in rough agreement with the FMR results[3] when scaled down by the small precessing moment in the AFMR case. Thus the earlier uniform-mode width measurement[7] was larger than would be obtained if both radiation damping and eddy current, wall-induced broadening had been absent. Indeed, if one takes our value of $\Delta H = 1.5$ Oe for the intrinsic width it is seen to fit extremely well to the curve of linewidth versus mode position relative to the bottom of the spin-wave band (Fig. 2 of Ref. 7) and thus removes a discrepancy between experiment and the two-magnon pit-scattering theory.

In view of the above, if one wished to obtain "intrinsic" linewidths in a terminated waveguide structure, it is essential not to place the sample on the termination plane. Empirically we found it best to start with the sample at a value of $l \neq 0$, but for which $\cos^2 k_g l = 1$. Here the uniform mode is easily identified relative to the magnetostatic modes by its larger intensity and width. This also facilitates the location and alignment of the sample. As one moves away from the $\cos^2 k_g l = 1$ condition the peak power absorption on resonance for the uniform mode will remain essentially constant until the radiative width becomes as small as the intrinsic width. In the region where $\cos^2 k_g l \simeq 0$ the intensity of the uniform mode will decrease and, for the flat-disk geometry, some magnetostatic modes may have larger intensities than does the uniform mode. However, the identification as to which of the observed resonances is the uniform mode is made clear by comparison with the spectrum taken at $k_g l = n\pi$. It should be remarked here that no changes in the linewidths of any of the magnetostatic modes were observed as $k_g l$ was varied. Since only for the uniform mode is the precessing magnetization everywhere in phase throughout the sample, it is perhaps not surprising to find that the radiation damping for the nonuniform modes is much smaller and may in fact be negligible for all practical purposes.

Because Eq. (8) exhibits a temperature dependence for $(\Delta H)_r$, one may be concerned about the possible effect on investigations of thermally induced contributions to the linewidth, $\Delta H_{\rm th}(T)$. MnF_2 has a nearly constant perpendicular susceptibility for temperatures below the Néel temperature, so that $(\Delta H)_r$ is expected to be independent of temperature. This is confirmed by an earlier study,[12] in which an "anomalously broad" (due to radiation damping) uniform-mode linewidth was reported to be temperature independent below 10 K. However, had precautions such as those outlined above been taken to inhibit radiation damping, the small temperature dependence of the remaining linewidth would have been observable down to 5 K, below which the dominant cause of linewidth is imperfection scattering. In the same study the magnetostatic modes (which are not radiation broadened) *did* exhibit a temperature dependence in the region $5 < T < 10$ K. Therefore, in materials which have a nearly temperature-independent perpendicular susceptibility, measurements of $\Delta H_{\rm th}(T)$ will be unaffected by radiation damping, except that inhibiting the radiation damping contribution to the linewidth will extend the range of temperatures for which $\Delta H_{\rm th}(T)$ may be extracted from the uniform-mode linewidth. However, experiments done in other materials which have a strongly temperature-dependent perpendicular susceptibility should be reexamined, since their values of $\Delta H_{\rm th}(T)$ may include temperature-dependent contributions from radiation damping.

CONCLUSIONS

We have shown radiation damping to be an important source of line broadening of the uniform mode in an AFMR experiment in a shorted waveguide. The expected $\cos^2 k_g l$ dependence of the radiation broadening in this electromagnetic configuration has been verified for the condition $d \ll \lambda$, where d is the largest linear dimension of the sample. In a subsequent paper[13] we will reexamine the problem of radiation damping in a cw magnetic-resonance experiment in a *resonant cavity* and show experimentally and theoretically that the conclusion reached by Bloembergen and Pound[14] (i.e., that radiation damping is unobservable in a cw resonant-cavity experiment) is in error.

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APPENDIX

We can also derive Eqs. (7) and (8) by consideration of the stored energy on resonance. In the case of FMR, the stored energy is just the Zeeman energy $VH_0M(1-\cos\varphi)$. Here V is the sample volume, $\vec{H}_0$ is the applied dc magnetic field, $\vec{M}$ is the volume magnetization, and φ is the angle between $\vec{M}$ and $\vec{H}_0$. For small angles we may rewrite the stored energy in terms of $M_\perp$, the component of $\vec{M}$ perpendicular to $\vec{H}_0$:

$$U^{\text{FMR}} = VH_0(M_\perp^2/2M)\,. \tag{A1}$$

From Eq. (1) and the assumption that all losses are due to radiation,

$$P_r = \frac{d}{dt}\,U^{\text{FMR}} = -\tfrac{2}{3}\,ck^4V^2M_\perp^2\,. \tag{A2}$$

Combining Eqs. (A1) and (A2) we have

$$\frac{d}{dt}\,U^{\text{FMR}} = -\left(\frac{4MV}{3H_0}\right)ck^4\,U^{\text{FMR}}, \tag{A3}$$

which has a solution of the form $U^{\text{FMR}} = U_0e^{-t/\tau}$, characterized by a relaxation time τ. We can easily solve Eq. (A3) to obtain

$$\tau = 3H_0/4MVck^4\,. \tag{A4}$$

Notice that τ is actually the ratio of stored energy to radiated power. Using the relation $\tau^{-1} = 2\eta_r = \gamma(\Delta H)$ [see discussion following Eq. (4)] and the resonance condition $H_0 = \omega/\gamma$, we find

$$\text{FMR:}\quad (\Delta H)_r^{\text{fs}} = \tfrac{4}{3}M\,V(\omega/c)^3\,, \tag{A5}$$

which is seen to correspond identically with Eq. (7).

In the case of AFMR, the stored energy is the sum of the Zeeman and anisotropy energies for each sublattice and the exchange energy:

$$U^{\text{AFMR}} = V\{H_0M_s(1-\cos\theta_1) + H_AM_s(1-\cos\theta_1) - H_0M_s(1-\cos\theta_2) + H_AM_s(1-\cos\theta_2) + H_EM_s[1-\cos(\theta_1-\theta_2)]\}\,. \tag{A6}$$

V is the sample volume; H_0, H_A, and H_E are, respectively, the dc field applied parallel to the easy axis of the crystal, the anisotropy field, and the exchange field; M_s is the magnitude of the sublattice magnetization (same for each sublattice); $\theta_1(\theta_2)$ is the angle between the up (down) sublattice magnetization and the easy axis of the crystal. Again, for small angles

$$U^{\text{AFMR}} = \tfrac{1}{2}M_s\,V[H_0(\theta_1^2-\theta_2^2) + H_A(\theta_1^2+\theta_2^2) + H_E(\theta_1-\theta_2)^2]\,. \tag{A7}$$

Substituting $\theta_1/\theta_2 \simeq 1 + H_C/H_E$ gives the solution for the high-frequency mode,[10] while the low-frequency mode is obtained by substituting $\theta_2/\theta_1 \simeq 1 + H_C/H_E$. Making the further approximation $H_C^2 \simeq 2H_AH_E$, and neglecting higher-order terms, we obtain, for the two modes,

$$U^{\text{AFMR}} = \theta_2^2M_s\,V(\pm H_0 + H_C)(H_C/H_E)\,. \tag{A8}$$

The radiation power expression is

$$P_r = \tfrac{2}{3}\,ck^4V^2(\sin\theta_1 - \sin\theta_2)^2\,, \tag{A9}$$

which, by the same assumptions made above, becomes

$$P_r = -\frac{d}{dt}\,U^{\text{AFMR}} = \tfrac{2}{3}\,ck^4V^2\theta_2^2(H_C/H_E)^2\,. \tag{A10}$$

Combining Eqs. (A8) and (A10) again yields an expression for the characteristic relaxation time

$$\tau = \frac{2ck^4V}{3M_s[\pm H_0 + H_C]}\left(\frac{H_E}{H_C}\right). \tag{A11}$$

From this, the resonance condition $\omega/\gamma \simeq \pm H_0 + H_C$, and the relation $H_C/H_E \simeq 2H_A/H_C$, we obtain

$$\text{AFMR:}\quad (\Delta H)_r^{\text{fs}} = \tfrac{4}{3}M_s(H_A/H_C)V(\omega/c)^3\,, \tag{A12}$$

which corresponds identically to Eq. (8).

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