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# Theory of cyclotron-resonance absorption*

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An expression for the cyclotron-resonance power absorption is derived with the help of a projection operator. It is shown that the correct evaluation of this formula requires the summation of an infinite number of terms in order to describe the effects of the scattering interaction to second order. These results are in contradiction with those obtained from some recently proposed theories of this phenomenon. It is then demonstrated that the correct expression for the cyclotron-resonance power absorption is identical to the one obtained through the use of the quantum kinetic equation.

## I. INTRODUCTION

The study of the cyclotron-resonance absorption line shape has been very useful in obtaining information about the scattering mechanisms for the charge carriers in solids. A great many theoretical studies[1–10] have appeared recently on this topic, dealing in particular with the linewidth and the line shift of the absorption line and its dependence on the magnetic field.

Among these, the theories of Kawabata[5] and of Lodder and Fujita[7] appear to be quite general and rigorous. They provide a criticism of other theories[1,2] and they yield explicit expressions for the linewidth and the line shift, which have been evaluated in the second order of the scattering interaction $\lambda$. These expressions have been used in a number of specific applications.[4–8,11]

In this paper we derive a general expression for the cyclotron-resonance power absorption, which provides the basis for a critique of these recent theories.[5,7] In particular, we consider in detail the theory of Kawabata,[5] which we derive in a much more direct and simple way by a new technique we have introduced recently.[12] We point out that his expressions for the linewidth and the line shift evaluated to order $\lambda^2$ cannot be correct in general. Since Lodder and Fujita[7] claim that their theory leads to expressions for the linewidth and the lineshift to order $\lambda^2$ identical to those of Kawabata,[5] it is subject to the same criticism (although their general theory is sufficiently different to warrant a separate study). Without any detailed investigation, however, one may ask the following question, which immediately raises doubts about the general validity of these theories: Since these theories[5,7] are valid for arbitrary strengths of the magnetic field including the value zero, why do they always yield an explicit expression for the (complex) relaxation time for the current (of order $\lambda^2$) while it is well known that for zero magnetic field such a relaxation time does *not* exist in general? In fact, for *any* magnetic field the appropriate distribution functions for the calculation of the current have been shown[13–16] to satisfy integral kinetic equations, which *in general* do *not* possess simple solutions that describe the effects of the scattering interaction to second order in terms of a (complex) relaxation time. This discrepancy among the existing theories has provided the motivation of this investigation.

In Sec. II we derive a general expression for the cyclotron-resonance power absorption of free charge carriers in random impurities, subject to the same approximations that Kawabata[5] has made. The method of derivation is not only a great deal simpler and more direct than that of Kawabata,[5] but it is also of such a form that allows us to consider and sum infinite subsets of terms of higher order in $\lambda$. This is of importance in Sec. III, where we point out that Kawabata's treatment of the effects of the scattering mechanism to order $\lambda^2$ is in error, especially for the case he considers, namely, for frequencies near resonance. In fact, it is shown that the explicit formula[5,7] for the power absorption with the relaxation time to order $\lambda^2$ is nothing more than a perturbation expansion in powers of $\lambda$ of the current followed by an inversion of this series to order $\lambda^2$. The error lies in the fact that in the general formal expression for the relaxation time there are terms of order $\lambda^3$ and higher that diverge at the resonant frequency. Thus, Kawabata's expression for the power absorption should be valid in general only at the wings of the absorption line. However, another approximation made by Kawabata invalidates this expression even in this region. In Sec. IV we sum the dominant divergent terms mentioned above to all orders in $\lambda$. We obtain a different expression for the power absorption, which, although of the same order in $\lambda$, requires the solution of an integral equation, in accord with the quantum transport theories.[15,16] In Sec. V we treat the general problem of cyclotron-resonance absorption by free charge carriers in random impurities without the approximations of Sec. II on the basis of a generalization of the technique introduced in that section. We obtain a formal expression for the power absorption, correct to all orders in $\lambda$, which is shown to be equivalent to the one that can be obtained more simply by the method of kinetic equations.[15,16]

## II. AN EXPRESSION FOR THE CYCLOTRON-RESONANCE POWER ABSORPTION

We present in this section an expression for the conductivity tensor of dynamically independent electrons in a uniform magnetic field that will provide the basis for our discussion of Kawabata's theory for the cyclotron-resonance power absorption. This expression is obtained by a new method that the authors[12] have introduced recently in a discussion of a different problem.

For dynamically independent electrons it suffices[13] to consider from the very start the motion of a single electron. The energy of the electron in the magnetic field $B$, taken in the $z$ direction, is given by the unperturbed Hamiltonian

$$H_0 = \frac{1}{2m}\left(\vec{p} - \frac{e}{c}\vec{A}\right)^2 , \tag{2.1}$$

where $\vec{A} = (0, Bx, 0)$ is a convenient vector potential. The eigenstates $|\alpha\rangle = |n\vec{k}\rangle$ and eigenvalues $\epsilon_\alpha$ of $H_0$ are then specified by $n = 0, 1, 2, \ldots$ and $\vec{k} = (k_y, k_z)$, a two-dimensional wave vector, so that

$$\langle \vec{r}\,|\,\alpha\rangle = \langle \vec{r}\,|n\vec{k}\rangle \propto \varphi_n(x + k_y/m\omega_c)\, e^{i\vec{k}\cdot\vec{r}} , \tag{2.2}$$

$$\epsilon_\alpha = \epsilon_{n\vec{k}} = (n + \tfrac{1}{2})\,\omega_c + k_z^2/2m . \tag{2.3}$$

Here $\varphi_n(x + k_y/m\omega_c)$ are the eigenfunctions of a simple harmonic oscillator of frequency $\omega_c \equiv |e|B/mc \equiv$ the cyclotron frequency, centered at $-k_y/m\omega_c$, and $\hbar$ is taken equal to 1. In the following we shall adopt the notation $|\alpha + 1\rangle$ to indicate the state $|n+1, \vec{k}\rangle$ if $|\alpha\rangle = |n\vec{k}\rangle$. The scattering mechanism is taken to be a set of identical impurities located at the random positions $\{\vec{r}_s\}$. Thus, the scattering potential is

$$V(\vec{r}) = \sum_s v(\vec{r} - \vec{r}_s), \tag{2.4}$$

where $v(\vec{r})$ is the potential of a single impurity located at the origin. Finally, the system is driven by an oscillating electric field of frequency $\omega$, so that, in a scalar gauge, the interaction Hamiltonian is

$$F(t) = -\,e x_i E_i(\omega)\, e^{i\omega t} + \text{H. c.}, \tag{2.5}$$

where the summation convention for the Cartesian components $x_i$, $E_i$ is adopted.

The induced steady-state current density is then $\vec{j}(t) = \vec{j}(\omega)\, e^{i\omega t} + \text{c. c.}$ with

$$j_i(\omega) = \mathrm{Tr}\,\{J_i\, \rho(\omega)\} , \tag{2.6}$$

if we take for convenience the volume of the sample equal to unity. Here $J_i = ei[H_0, x_i]$ is the $i$th component of the current operator, and $\rho(\omega)$ is the steady-state density operator for the electron, linear in the electric field. This satisfies the equation of motion[14]

$$(-\omega^- - L)\,\rho(\omega) = -\,eE_i(\omega)\,[x_i, f(H)] . \tag{2.7}$$

Here $\omega^- \equiv \omega - i\eta$ with $\eta$ a positive infinitesimal $(\eta \to 0^+)$, $f(H)$ stands for the Fermi-Dirac distribution function, and $L$ denotes the Liouville operator corresponding to $H \equiv H_0 + V$, i.e.,

$$LX \equiv [H, X] \tag{2.8}$$

for any operator $X$. The formal solution of (2.7) is

$$\rho(\omega) = G(-\omega^-)\,[-\,e x_i ,\ f(H)]\, E_i(\omega), \tag{2.9}$$

where we have introduced the propagator $G$ corresponding to the Liouville operator $L$, namely,

$$G(z) = (z - L)^{-1} , \tag{2.10}$$

for any complex number $z$. Thus, the conductivity tensor $\sigma_{ij}(\omega)$, as obtained from (2.6) and (2.9), is given by

$$\sigma_{ij}(\omega) = \mathrm{Tr}\,\{J_i G(-\omega^-)\,[f(H),\ e x_j\,]\} . \tag{2.11}$$

In order to be able to make a direct comparison with Kawabata's expression, we note that due to the invariance property of the trace under cyclic permutation this can also be written

$$\sigma_{ij}(\omega) = \mathrm{Tr}\,\{e x_j [\,f(H),\ G(\omega^-) J_i]\} , \tag{2.12}$$

since $Gf(H)J = f(H)GJ$ and $GJf(H) = (GJ)f(H)$.

The average power absorbed by the electrons when driven by an electric field circularly polarized in the plane perpendicular to $\vec{B}$ is proportional[7] to

$$\mathrm{Re}\,\sigma_{+-}(\omega) = \mathrm{Re}[\sigma_{11}(\omega) + i\sigma_{21}(\omega) + \sigma_{22}(\omega) - i\sigma_{12}(\omega)] . \tag{2.13}$$

From (2.12) we have then

$$\sigma_{+-}(\omega) = \mathrm{Tr}\,\{e x^- [\,f(H),\ G(\omega^-) J^+]\} , \tag{2.14}$$

where we have put

$$J^\pm \equiv J_1 \pm iJ_2 , \quad x^\pm \equiv x_1 \pm i x_2 .$$

This is an *exact* expression for $\sigma_{+-}(\omega)$, if we take its average over the random distribution of the impurities.

If we now adopt the approximation that Kawabata[5] makes (which he considers valid for $\omega \cong \omega_c$), we may replace $f(H) = f(H_0 + V)$ by $f(H_0)$ and thus obtain instead of (2.14)

$$\sigma_{+-}(\omega) \cong \mathrm{Tr}\,\{e x^- [\,f(H_0),\ G(\omega^-) J^+]\} . \tag{2.15}$$

Evaluating the trace in the $|\alpha\rangle$ representation as given by (2.2), and making use of the selection rule

$$(ex^-)_{\alpha\alpha'} = (i/\omega_c)(J_\alpha^+)^*\,\delta_{\alpha',\alpha+1}\,, \tag{2.16}$$

we find for (2.15)

$$\sigma_{+-}(\omega) \cong \frac{1}{i\omega_c}\sum_\alpha\,[f(\epsilon_\alpha) - f(\epsilon_\alpha+\omega_c)] \times (J_\alpha^+)^* R(\omega)_\alpha\,. \tag{2.17}$$

Here we have introduced the convenient notation

$$X_\alpha \equiv \langle \alpha+1\,|X|\,\alpha\rangle \tag{2.18}$$

for any operator $X$, and the operator

$$R(\omega) \equiv G(\omega^-)J^+\,. \tag{2.19}$$

We must now evaluate the quantity $R(\omega)_\alpha$. Our method of evaluation proceeds as follows. From the definition (2.19) and Eq. (2.10) we note that the operator $R(\omega)$ obeys the equation

$$(\omega^- - L)R(\omega) = J^+\,. \tag{2.20}$$

We introduce the projection operator $P$ defined by

$$PX \equiv J^+(X_\alpha/J_\alpha^+), \tag{2.21}$$

where $X_\alpha$, $J_\alpha^+$ are defined by (2.18). We note that

$$PJ^+ = J^+, \quad P'J^+ \equiv (1-P)J^+ = 0\,, \tag{2.22}$$

while

$$PR(\omega) = J^+[R(\omega)_\alpha/J_\alpha^+]\,. \tag{2.23}$$

In (2.20) we introduce the splitting $R(\omega) = PR(\omega) + P'R(\omega)$ and then operate with $P$ and $P'$, separately, to obtain, with the use of (2.22),

$$(\omega^- - PL)PR - PLP'R = J^+, \tag{2.24a}$$

$$(\omega^- - P'L)P'R - P'LPR = 0\,. \tag{2.24b}$$

Solving (2.24b) for $P'R$ in terms of $PR$, we get

$$P'R(\omega) = G'(\omega^-)P'LPR(\omega)\,, \tag{2.25}$$

where we have introduced the new propagator

$$G'(z) \equiv (z - P'L)^{-1}\,. \tag{2.26}$$

We now substitute (2.25) into (2.24a) and obtain for $PR(\omega)$ the equation

$$\{\omega^- - PL[1 + G'(\omega^-)P'L]\}PR(\omega) = J^+\,. \tag{2.27}$$

We note that all terms on the left-hand side of (2.27) are, according to (2.21), simple scalar multiples of the operator $J^+$. Thus, with the help of (2.23), we find for the quantity of interest $R(\omega)_\alpha$, the expression

$$R(\omega)_\alpha = J_\alpha^+\{\omega^- - (1/J_\alpha^+)([L + LG'(\omega^-)P'L]J^+)_\alpha\}^{-1}\,. \tag{2.28}$$

Since we shall be interested in a weak scattering potential $V$, it is convenient to introduce the Liouville operators $L_0$ and $L_1$ corresponding to $H_0$ and $V$, respectively, i.e., $L = L_0 + L_1$, with

$$L_0X \equiv [H_0, X], \quad L_1X \equiv [V, X]\,. \tag{2.29}$$

We then note that $L_0J^+ = \omega_cJ^+$, $P'L_0J^+ = 0$, and $(L_0P'X)_\alpha = 0$, as it follows from the definitions of $L_0$ and $P'$. We thus can write (2.28) in the form

$$R(\omega)_\alpha = \frac{iJ_\alpha^+}{i(\omega^- - \omega_c) + \Gamma_\alpha(\omega)}\,, \tag{2.30}$$

where

$$\Gamma_\alpha(\omega) \equiv -\frac{i}{J_\alpha^+}\,([L_1 + L_1G'(\omega^-)P'L_1]J^+)_\alpha\,. \tag{2.31}$$

Using (2.30) in (2.17), in conjunction with (2.19), we have

$$\mathrm{Re}\,\sigma_{+-}(\omega) \cong \frac{1}{\omega_c}\,\mathrm{Re}\sum_\alpha \frac{[f(\epsilon_\alpha) - f(\epsilon_\alpha+\omega_c)]\,|J_\alpha^+|^2}{i(\omega - \omega_c) + \Gamma_\alpha(\omega)}\,. \tag{2.32}$$

Equations (2.30) and (2.31) are exact formal expressions for $\langle\alpha+1\,|\,G(\omega^-)J^+|\,\alpha\rangle$, valid for arbitary scattering potential $V$. Thus (2.32) gives an expression for $\mathrm{Re}\sigma_{+-}(\omega)$, which is exact except for two considerations: (i) in (2.17) use was made of the approximation $f(H) \cong f(H_0)$, which was introduced in (2.15), and (ii) no averaging over the distribution of the impurities has been carried out.

Expression (2.32) will provide the basis for our discussion of Kawabata's[5] theory for the cyclotron-resonance power absorption in Sec. III. Later, in Sec. V, we shall return and discuss the points (i) and (ii) we just mentioned.

## III. CRITIQUE OF OTHER THEORIES

In this section we derive on the basis of (2.31) and (2.32) Kawabata's[5] expression for the power absorption and discuss its shortcomings.

We are interested in obtaining an explicit expression for $\Gamma_\alpha(\omega)$ of (2.31) for a weak scattering potential $V$. If we denote the strength of this interaction by $\lambda$, we can generate a power series in $\lambda$ for $\Gamma_\alpha(\omega)$ by expanding the propagator $G'(\omega^-)$ in powers of $V$. From (2.26) we have

$$G'(\omega^-)P' = (\omega^- - L_0 - P'L_1)^{-1}P' = [G_0(\omega^-) + G_0(\omega^-)P'L_1G_0(\omega^-) + \cdots]P'\,, \tag{3.1}$$

where we have introduced the unperturbed propagator

$$G_0(\omega^-) \equiv (\omega^- - L_0)^{-1}\,. \tag{3.2}$$

The first equality in (3.1) follows from (2.26) and the fact that $PL_0P' = 0$, as can easily be verified. Substituting (3.1) in (2.31) we have the formal expansion in powers of $\lambda$:

$$\Gamma_\alpha(\omega) = -\frac{i}{J_\alpha^+}\left([L_1 + L_1 G_0 P' L_1 + L_1 G_0 P' L_1 G_0 P' L_1 + \cdots]J^+\right)_\alpha . \tag{3.3}$$

More explicitly, we have, if we evaluate the first two terms in the $|\alpha\rangle$ representation

$$\Gamma_\alpha(\omega) = -i(V_{\alpha+1,\alpha+1} - V_{\alpha\alpha}) + {\sum_\beta}' \frac{[V_{\alpha\beta} - V_{\alpha+1,\beta+1}(J_\beta^+/J_\alpha^+)]V_{\beta\alpha}}{i(\omega - \epsilon_{\alpha+1} + \epsilon_\beta) + \eta}$$

$$+ {\sum_\beta}' \frac{V_{\alpha+1,\beta}[V_{\beta,\alpha+1} - V_{\beta-1,\alpha}(J_{\beta-1}^+/J_\alpha^+)]}{i(\omega - \epsilon_\beta + \epsilon_\alpha) + \eta} + \mathcal{O}(\lambda^3), \tag{3.4}$$

where we recall that $\eta \to 0^+$, and the prime on $\sum_\beta$ denotes the deletion of the terms associated with the diagonal matrix elements of $V$, i.e., in the first sum $\beta \neq \alpha$ and in the second sum $\beta \neq \alpha + 1$. We point out that these exclusions arise from the fact that $(P' L_1 J^+)_\alpha = 0$, and that for the excluded states the respective denominators are equal to $\omega - \omega_c$.

Expression (2.32) is identical to Kawabata's[5] expression (3.9) for $\mathrm{Re}\sigma_{+-}(\omega)$, if in our Eq. (3.4) for $\Gamma_\alpha(\omega)$ we ignore the terms of first order in $V$, as Kawabata does, and consider the terms denoted by $\mathcal{O}(\lambda^3)$ as negligible. [There is a minor difference between the exclusions in the second sum $\sum'_\beta$ in our expression (3.4) for $\Gamma_\alpha(\omega)$ and the corresponding sum in Kawabata's expression (3.6) for the same quantity, but this is apparently an oversight on the part of Kawabata.] Furthermore, Kawabata takes the average of the expression for $\Gamma_\alpha(\omega)$ over the random distribution of the impurities, which also justifies the neglect of the first order in $V$ terms in $\Gamma_\alpha(\omega)$. This is, as Kawabata correctly points out, an *ad hoc* procedure, since $\mathrm{Re}\sigma_{+-}(\omega)$ is proportional to the impurity average of $R(\omega)_\alpha$ rather than that of $\Gamma_\alpha(\omega)$. Kawabata identifies $\mathrm{Re}\Gamma_\alpha(\omega)$ and $-\mathrm{Im}\Gamma_\alpha(\omega)$ with the inverse "transport relaxation time" $1/\tau_c(\alpha, \omega)$, or the linewidth, and the line shift $\Delta(\alpha, \omega)$, respectively. He finally approximates these by their values for $\omega = \omega_c$.

We now show that such a procedure is in error. More specifically, we show that the expansion of $\Gamma_\alpha(\omega)$ in powers of $\lambda$, as in (3.3) and (3.4), breaks down for $\omega = \omega_c$, i.e., higher-order terms become infinite for $\omega = \omega_c$, and thus this expansion is only valid for sufficiently large $\omega - \omega_c$, i.e., most probably it is valid only in the wings of the absorption line. In brief, the approximation of $\Gamma_\alpha(\omega)$ by terms of order up to $\lambda^2$ in (3.4), which Kawabata has adopted, cannot describe correctly the shape of the cyclotron-resonance absorption line, especially near the center of the line.

In order to emphasize this point, namely, that Kawabata's expression for $\Gamma_\alpha(\omega)$ is valid only for $(\delta\omega)\tau \gg 1$, where

$$\delta\omega \equiv \omega - \omega_c , \tag{3.5}$$

and $\tau$ is of the order of magnitude of the correct relaxation time near resonance, we observe below that Kawabata's theory can be derived in a straightforward manner by expanding the quantity $R(\omega)_\alpha$ in powers of $\lambda$ up to $\lambda^2$ and then rewriting the expression so obtained as the inverse of another series in $\lambda$ up to second order. Thus, no elaborate projection, or diagrammatic, techniques are necessary for such a theory. To show this we first solve (2.20) for $R(\omega)$ up to second order in $\lambda$ by iteration. We obtain

$$R(\omega) = [G_0(\omega^-) + G_0(\omega^-) L_1 G_0(\omega^-)$$

$$+ G_0(\omega^-) L_1 G_0(\omega^-) L_1 G_0(\omega^-) + \cdots ]J^+ . \tag{3.6}$$

More explicitly for $R(\omega)_\alpha$ we find

$$R(\omega)_\alpha = \frac{J_\alpha^+}{\delta\omega}\,(1 + \lambda n_1 + \lambda^2 n_2 + \cdots), \tag{3.7}$$

where

$$\lambda n_1 = \frac{V_{\alpha+1,\alpha+1} - V_{\alpha\alpha}}{\delta\omega}, \tag{3.7a}$$

$$\lambda^2 n_2 = \frac{1}{\delta\omega}\sum_\beta\left(\frac{[V_{\alpha\beta} - V_{\alpha+1,\beta+1}(J_\beta^+/J_\alpha^+)]V_{\beta\alpha}}{\omega^- - \epsilon_{\alpha+1} + \epsilon_\beta}\right.$$

$$\left.+ \frac{V_{\alpha+1,\beta}[V_{\beta,\alpha+1} - V_{\beta-1,\alpha}(J_{\beta-1}^+/J_\alpha^+)]}{\omega^- - \epsilon_\beta + \epsilon_\alpha}\right). \tag{3.7b}$$

We note that there are no exclusions in the sums $\sum_\beta$ in (3.7b), i.e., some of the terms there diverge for $\delta\omega = 0$. We may now rewrite (3.7) in the form

$$R(\omega)_\alpha = \frac{J_\alpha^+}{\delta\omega}\,(1 + \lambda d_1 + \lambda^2 d_2 + \cdots)^{-1} . \tag{3.8}$$

By comparison we find that

$$d_1 = -n_1 , \tag{3.8a}$$

$$d_2 = n_1^2 - n_2 . \tag{3.8b}$$

Introducing (3.7a) and (3.7b) in (3.8)–(3.8b), we recover expression (2.30) with $\Gamma_\alpha(\omega)$ given by the first three terms of (3.4), i.e., Kawabata's expression for the same quantity. The $n_1^2$ term in (3.8b) serves to bring in the desired exclusions in the sums $\sum'_\beta$ of expression (3.4). (Incidentally, this method of derivation justifies the impurity averaging procedure of Kawabata.)

From this discussion it is evident that the projection technique we introduced in Sec. II gives directly an explicit expression for the *inverse* of $R(\omega)_\alpha$ to all orders in $\lambda$, as shown by (2.30) and (3.3).

We can now demonstrate that in the expression (3.3) of $\Gamma_\alpha(\omega)$ in powers of $\lambda$, the terms of order $\lambda^3$ and higher involve some terms that diverge for $\delta\omega = 0$ (and in the final limit $\eta \to 0^+$). First, we note that every $G_0(\omega^-)$ in (3.3) is followed by $P' = 1 - P$, so that there are always terms that involve $G_0(\omega^-) J^+$. Now, since $L_0 J^+ = \omega_c J^+$, we have $G_0(\omega^-) J^+ = J^+/\delta\omega$. Thus, these terms diverge for $\delta\omega = 0$, except for the terms in (3.3) of order $\lambda^2$, which vanish, because, as we mentioned before, $(P' L_1 J^+)_\alpha = 0$. More generally, however, we note that $G_0(\omega^-)$ operating on the part $X_1$ of any operator $X$ that is defined in the $|\beta\rangle$ representation by the matrix

$$(X_1)_{\beta\beta'} \equiv X_{\beta,\beta-1}\,\delta_{\beta',\beta-1} \qquad (3.9)$$

gives rise to terms that diverge for $\delta\omega = 0$, since

$$\begin{aligned}(G_0(\omega^-) X_1)_{\beta\beta'} &= (\omega^- - \epsilon_\beta + \epsilon_{\beta'})^{-1} (X_1)_{\beta\beta'} \\ &= \frac{(X_1)_{\beta,\beta-1}\,\delta_{\beta',\beta-1}}{\delta\omega}\ . \end{aligned} \qquad (3.10)$$

By contrast, the part $X_2 = X - X_1$ of any operator $X$ gives rise to regular terms when $G_0(\omega^-)$ operates on it, since

$$(G_0(\omega^-) X_2)_{\beta\beta'} = (\omega^- - \epsilon_\beta + \epsilon_{\beta'})^{-1} X_{\beta\beta'} \quad (\beta' \neq \beta - 1) \qquad (3.11)$$

in the summation over the intermediate states becomes (after the thermodynamic limit is taken)

$$[\,i\pi\,\delta(\omega - \epsilon_\beta + \epsilon_{\beta'}) - (\omega - \epsilon_\beta + \epsilon_{\beta'})_p^{-1}\,] X_{\beta\beta'},$$

in the limit $\eta \to 0^+$. Thus, in the expansion (3.3) of $\Gamma_\alpha(\omega)$ in powers of $\lambda$, every factor $G_0(\omega^-) P'$, except the last one that operatres on $L_1 J^+$, brings in terms $\propto \lambda/\delta\omega$, which clearly diverge for $\omega = \omega_c$ (and $\eta \to 0^+$). Therefore, in (3.3) there are terms $\propto \lambda^2(\lambda/\delta\omega)^n$ with $n \geq 1$. It follows that the procedure of keeping terms of order $\lambda^2$ and ignoring the others in the expansion (3.3) of $\Gamma_\alpha(\omega)$ is not valid for frequencies $\omega$ close to $\omega_c$, the very frequencies for which $\Gamma_\alpha(\omega)$ is evaluated.

The theory of Lodder and Fujita[7] is quite different from that of Kawabata in its general formulation, and we shall not discuss it here in any detail. When, however, they treat the effects of the scattering interaction in the lowest order ($\lambda^2$), they arrive at an expression for $\sigma_{+-}(\omega)$ that is identical to the one obtained by Kawabata and discussed above. Thus the criticism mentioned above is applicable to their theory too, although a more detailed study is necessary in order to find out its relation to the general theory presented here and especially in Sec. V.

It must be pointed out, however, that it may prove possible for Kawabata's expression for $\Gamma_\alpha(\omega)$ to be valid for a special scattering interaction. Such a case can arise when the coefficients of all the divergent terms of order $\lambda^3$ or higher vanish for the particular type of impurity interaction.

## IV. CORRECTION OF OTHER THEORIES

In this section we show how the problems inherent in the earlier theories we discussed in Sec. III can be eliminated within the theoretical framework we have introduced. We shall then show that the same correct results can be obtained by a different method in a more direct manner.

In order to remedy the earlier theories, we must sum the infinite subset of terms in the expansion (3.3) of $\Gamma_\alpha(\omega)$ that are of the form $\lambda^2(\lambda/\delta\omega)^n$ ($n \geq 1$). These are the dominant terms for sufficiently small $\lambda$, and will yield an expression for $R(\omega)_\alpha^{-1}$ correct to order $\lambda^2$.

The isolation of the desired terms is facilitated by the introduction of the operator $\Delta$ that projects the part $X_1$ of any operator $X$ that was defined in (3.9), i.e.,

$$(\Delta X)_{\beta\beta'} \equiv X_{\beta,\beta-1}\,\delta_{\beta',\beta-1} = X_{\beta-1}\,\delta_{\beta',\beta-1}, \qquad (4.1)$$

where the last equality follows from the notation introduced in (2.18). The operator that projects the part $X_2 = X - X_1$ of $X$ is then $\Delta' \equiv 1 - \Delta$, with the obvious properties $\Delta'^2 = \Delta'$, $\Delta'\Delta = \Delta\Delta' = 0$. The divergent terms now arise whenever $G_0(\omega^-)$ operates on $\Delta X$, since, as we saw in (3.10),

$$G_0(\omega^-)\Delta = \frac{1}{\delta\omega}\,\Delta\ . \qquad (4.2)$$

By contrast $G_0(\omega^-)\Delta'$ yields regular terms. Thus, it is convenient *not* to use the expansion of $G'(\omega^-)$ in powers of $P'L_1 = L_1 - PL_1$, as we did in (3.1) to obtain (3.3) for $\Gamma_\alpha(\omega)$, but rather to expand $G'(\omega^-)$ in (2.31) for $\Gamma_\alpha(\omega)$ in powers of $\Delta L_1$ and $PL_1$. This is accomplished simply by writing the first equality of (3.1) as

$$G'(\omega^-) = (\omega^- - L_0 - \Delta' L_1 - \Delta L_1 + PL_1)^{-1}\ , \qquad (4.3)$$

since then it follows that

$$\begin{aligned} G'(\omega^-) &= G''(\omega^-) + G''(\omega^-)(\Delta L_1 - PL_1) G''(\omega^-) + \cdots \\ &= G''(\omega^-) + G''(\omega^-)\,\Delta P' L_1 G''(\omega^-) + \cdots\ . \end{aligned} \qquad (4.4)$$

The second equality follows from the fact that $P = \Delta P$, since $\Delta' J^+ = 0$. In (4.4) we have introduced the propagator

$$\begin{aligned} G''(\omega^-) &\equiv (\omega^- - L_0 - \Delta' L_1)^{-1} \\ &= G_0(\omega^-) + G_0(\omega^-)\Delta' L_1 G_0(\omega^-) + \cdots\ . \end{aligned} \qquad (4.5)$$

From the structure (4.5) of $G''(\omega^-)$ and the fact that $G_0(\omega^-)\Delta'$ yields regular terms, we note that $G''(\omega^-)\Delta'$ brings in no divergent terms, whereas in $G''(\omega^-)\Delta$ there are divergent terms with a *single* $1/\delta\omega$ factor. By a simple rearrangement these divergent terms can be made manifest, by noting that

$$G''(\omega^-)\Delta = \frac{1}{\delta\omega}\,[\Delta + G''(\omega^-)\Delta' L_1 \Delta]\ . \tag{4.6}$$

Thus, in order to isolate the divergent terms in $\Gamma_\alpha(\omega)$ as they appear upon introduction of the expansion (4.4) into (2.31), we must separate the $\Delta$ and $\Delta'$ parts of any operator that is acted upon by $G''(\omega^-)$. We thus write $P'$ in (2.31) for $\Gamma_\alpha(\omega)$ as $\Delta P' + \Delta' P'$ and observe that $\Delta' P = P - \Delta P = 0$, so that $P' = \Delta P' + \Delta'$. We now can use (4.4) to write

$$L_1 G' P' L_1 = L_1 G''(\Delta P' + \Delta') L_1 \\ + L_1 G'' \Delta P' L_1 G''(\Delta P' + \Delta') L_1 + \cdots\ . \tag{4.7}$$

If we add $L_1$ to both sides, operate with $\Delta$ from the left, and rearrange the series, we find

$$\Delta[L_1 + L_1 G' P' L_1] = \Delta[L_1 + L_1 G''\Delta' L_1] + \Delta L_1 G''\Delta P'\Delta[L_1 + L_1 G''\Delta' L_1] \\ + \Delta L_1 G''\Delta P'\Delta L_1 G''\Delta P'\Delta[L_1 + L_1 G''\Delta' L_1] + \cdots\ , \tag{4.8}$$

since $\Delta P' = \Delta P'\Delta$. According to (4.6), the dominant divergent terms in (4.8) arise from the "factors"

$$\Delta L_1 G''(\omega^-)\Delta = \frac{1}{\delta\omega}\,\Delta[L_1 + L_1 G''(\omega^-)\Delta' L_1]\Delta, \tag{4.9}$$

while the last "factor" $\Delta[L_1 + L_1 G''(\omega^-)\Delta' L_1]$ can be approximated by $\Delta[L_1 + L_1 G_0(\omega^-)\Delta' L_1]$. Thus, we find

$$\Delta[L_1 + L_1 G' P' L_1] \cong S + \frac{S}{\delta\omega}\,P'S + \frac{S}{\delta\omega}\,P'\,\frac{S}{\delta\omega}\,P'S + \cdots \tag{4.10}$$

where

$$S(\omega) \equiv \Delta[L_1 + L_1 G_0(\omega^-)\Delta' L_1]\ . \tag{4.11}$$

Recalling the definition of $P' = 1 - P$, we can write this as

$$\Delta[L_1 + L_1 G' P' L_1] J^+ \cong (\delta\omega)\left[\frac{S}{\delta\omega} + \frac{S}{\delta\omega}\left(1 - \frac{J^+}{J^+_\alpha}\,\mathrm{Tr}\,|\alpha\rangle\langle\alpha+1|\right)\frac{S}{\delta\omega} + \cdots\right] J^+\ . \tag{4.12}$$

The series (4.12) can be rearranged to read

$$\Delta[L_1 + L_1 G' P' L_1] J^+ \cong \delta\omega\,(\bar{S}J^+)\left[1 - \frac{1}{J^+_\alpha}\,(\bar{S}J^+)_\alpha + \cdots\right] \\ = \delta\omega\,(\bar{S}J^+)\left[1 + \frac{1}{J^+_\alpha}\,(\bar{S}J^+)_\alpha\right]^{-1}, \tag{4.13}$$

where

$$\bar{S} \equiv \frac{S}{\delta\omega} + \left(\frac{S}{\delta\omega}\right)^2 + \cdots = \left(1 - \frac{S}{\delta\omega}\right)^{-1} - 1\ . \tag{4.14}$$

Thus, from (2.31), (4.13), and (4.14) we find

$$\Gamma_\alpha(\omega) \cong -i\delta\omega\,\frac{(\bar{S}J^+)_\alpha}{J^+_\alpha}\left[1 + \frac{(\bar{S}J^+)_\alpha}{J^+_\alpha}\right]^{-1} \\ = -i\{\delta\omega - J^+_\alpha[((\delta\omega - S)^{-1}J^+)_\alpha]^{-1}\}\ . \tag{4.15}$$

Finally, from (2.30) and (4.15) we obtain for the quantity of interest

$$R(\omega)_\alpha \cong ([\omega - \omega_c - S(\omega)]^{-1} J^+)_\alpha\,, \tag{4.16}$$

where $S(\omega)$ is given by (4.11). The meaning of this equation is that the quantity of interest $\Delta R(\omega)$ is determined by the equation

$$\{\omega - \omega_c - \Delta[L_1 + L_1 G_0(\omega^-)\Delta' L_1]\}\Delta R(\omega) = J^+. \tag{4.17}$$

This is clearly a matrix-integral equation for the set of the desired matrix elements

$$\{\langle n+1, \vec{\mathrm{k}}\,|R(\omega)|\,n\vec{\mathrm{k}}\rangle\} \equiv \{R_{n\vec{\mathrm{k}}}\} = \{R_\alpha\},$$

namely:

$$(\omega - \omega_c - V_{\alpha+1,\alpha+1} + V_{\alpha\alpha})\,R_\alpha + (W[R])_\alpha = J^+_\alpha, \tag{4.18}$$

where

$$(W[R])_\alpha = \sum_\beta{}'\left[R_\alpha\left(\frac{|V_{\alpha+1,\beta}|^2}{\omega^- - \epsilon_\beta + \epsilon_\alpha} + \frac{|V_{\alpha\beta}|^2}{\omega^- - \epsilon_{\alpha+1} + \epsilon_\beta}\right) \right. \\ \left. - R_\beta\left(\frac{V_{\alpha+1,\beta+1}V_{\beta\alpha}}{\omega^- - \epsilon_{\alpha+1} + \epsilon_\beta} + \frac{V_{\alpha+1,\beta+1}V_{\beta\alpha}}{\omega^- - \epsilon_{\beta+1} + \epsilon_\alpha}\right)\right], \tag{4.19}$$

and the prime on $\sum$ denotes the deletion of the diagonal matrix elements of $V$.

In fact, Eq. (4.17) for $\Delta R(\omega)$ can be obtained much more directly from its defining equation

(2.20). If we introduce there the splitting $R(\omega) = \Delta R(\omega) + \Delta' R(\omega)$ and then operate on it with $\Delta$ and $\Delta'$, separately, we find

$$(\omega^- - \Delta L)\Delta R - \Delta L \Delta' R = \Delta J^+ = J^+ , \tag{4.20}$$

$$(\omega^- - \Delta' L)\Delta' R - \Delta' L \Delta R = \Delta' J^+ = 0 . \tag{4.21}$$

From (4.21) we find $\Delta' R(\omega) = G''(\omega^-)\Delta' L \Delta R(\omega)$, which when substituted in (4.20) gives

$$[\omega^- - \Delta L - \Delta L G''(\omega^-)\Delta' L]\Delta R(\omega) = J^+ . \tag{4.22}$$

If we now make use of $\Delta L_0 = L_0 \Delta = \omega_c \Delta$, $\Delta' L_0 \Delta = 0$ and approximate $G''(\omega^-)$ by $G_0(\omega^-)$, we obtain an equation for $\Delta R(\omega)$ with coefficients correct up to order $\lambda^2$, which is exactly Eq. (4.17). This method of derivation is the same as that which has been used[15] for the magnetoresistance of the same system. Actually, (4.22) is an *exact* equation for $\Delta R(\omega)$, from which approximations of higher order in $\lambda$, or of a different type, than (4.17) can be obtained.

The integral equation (4.18) for $R(\omega)_\alpha$ we just obtained, should be contrasted with the explicit expression for it, Eqs. (2.30) and (3.4), we obtained before from Kawabata's theory. Thus, if Kawabata's theory were correct, it would have been of extreme practical importance, because it does not require the solution of a complicated integral equation. Unfortunately, however, as we have seen, such an expression for $R(\omega)_\alpha$ is correct only for $(\delta\omega)\tau \gg 1$ and thus cannot be trusted to give the correct shape of the cyclotron-resonance absorption line.

With regard to the last comment of Sec. III, we note that if for a special scattering potential Kawabata's expression for $\Gamma_\alpha(\omega)$ is correct, then for the same potential the integral transport equation (4.18) admits of a simple solution and a relaxation time exists.

## V. GENERAL THEORY

The theories we examined in the previous sections are restricted by the two considerations we mentioned in Sec. II, namely, the neglect of the effects of the scattering potential $V$ arising from the approximation $f(H) \cong f(H_0)$ and the *ad hoc* manner in which the impurity average was taken, and, in addition, by the fact that all other effects of the scattering potential were calculated up to second order in $\lambda$. In this section we show how all these limitations can be lifted, by deriving a formal expression for the cyclotron-resonance power absorption within the appropriately modified technique introduced in Sec. II. We then demonstrate that this expression is equivalent to that obtained by the method of kinetic equations.[16]

We first specify the averaging operation over the random distribution of the impurity positions $\{\vec{r}_s\}$ $(s = 1, \ldots, N_s)$ by the introduction of the operator $P_i$, defined by

$$P_i \Phi(\vec{r}_1, \ldots, \vec{r}_s)$$
$$\equiv \int d^3 r_1 \cdots \int d^3 r_{N_s} \Phi(\vec{r}_1, \ldots, \vec{r}_{N_s}) = \overline{\Phi} . \tag{5.1}$$

Thus, an exact expression for $\sigma_{+-}(\omega)$ for a system of dynamically independent electrons can be obtained from (2.14) after operating on it with $P_i$, if the distribution of the impurities is random. Thus,

$$\sigma_{+-}(\omega) = P_i \operatorname{Tr}\{ex^- G(\omega^-)[f(H), J^+]\}$$
$$= \frac{i}{\omega_c} \sum_\alpha (J_\alpha^+)^* \overline{Q}(\omega)_\alpha , \tag{5.2}$$

where we have introduced the operator

$$Q(\omega) \equiv G(\omega^-) D, \tag{5.3}$$

$$D \equiv [f(H), J^+], \tag{5.4}$$

and made use of (2.16) and the notation (2.18). From (5.3) it is clear that the operator $Q(\omega)$ satisfies the equation

$$(\omega^- - L) Q(\omega) = D . \tag{5.5}$$

The quantity of interest $\overline{Q}(\omega)_\alpha = (P_i Q(\omega))_\alpha$ can be found, as in Sec. II, with the help of a projection operator $P$ defined now by

$$PX \equiv D \frac{1}{d} (P_i X)_\alpha , \tag{5.6}$$

$$d \equiv (P_i D)_\alpha . \tag{5.7}$$

We note that this $P$ includes the impurity averaging operation $P_i$ and it projects all operators onto $D$, rather than $J^+$. It is clear that $PD = D$ and $P'D \equiv (1 - P)D = 0$, while $PQ(\omega) = D\overline{Q}(\omega)_\alpha / d$. Thus, a formal expression for $\overline{Q}(\omega)_\alpha$ can be obtained exactly as in Sec. II, by treating (5.5) as in (2.20)–(2.27). We find

$$\overline{Q}(\omega)_\alpha = \frac{d}{\omega^- - Z_\alpha(\omega)} , \tag{5.8}$$

where

$$Z_\alpha(\omega) \equiv \frac{1}{d} (P_i [L + L G'(\omega^-) P' L] D)_\alpha , \tag{5.9}$$

and

$$G'(\omega^-) \equiv (\omega^- - P'L)^{-1} \tag{5.10}$$

as before, but with $P' = 1 - P$ defined by (5.6). Equations (5.8)–(5.10) in conjunction with (5.2) constitute a formally exact expression for $\sigma_{+-}(\omega)$ for the system under consideration.

One may attempt to evaluate $Z_\alpha(\omega)$ in powers of $\lambda$, by the standard expansion of $f(H) = f(H_0 + V)$, and thus of $D$, in powers of $V$, and by the expansion

$$G'(\omega^-)P' = [G_0(\omega^-) + G_0(\omega^-)P' L_1 G_0(\omega^-) + \cdots ]P'$$

of the propagator $G'(\omega^-)$. As we saw in Sec. III, however, such an expansion is not valid for

$\omega \cong \omega_c$, because whenever $G_0(\omega^-)$ operates on $\Delta X$ a factor $1/(\omega - \omega_c)$ appears. Thus an evaluation of $Z_\alpha(\omega)$ up to $\mathcal{O}(\lambda^2)$ from such an expansion would not give a correct description of the shape of the cyclotron-resonance absorption line. As before, a summation of an infinite set of terms of the form $\lambda^2[\lambda^2/(\omega - \omega_c)]^n$ has to be carried out in order to obtain a value for $Z_\alpha(\omega)$ that would be valid for arbitrary values of $(\omega - \omega_c)\tau$, with $1/\tau$ evaluated to order $\lambda^2$. This can be carried through as in Sec. IV.

In the following we show how $Z_\alpha(\omega)$ and $\overline{Q}(\omega)_\alpha$ can be calculated formally to all orders in $\lambda$, thus generalizing the treatment of the previous section. We shall also show that the result is identical to the one obtained more directly by the method of kinetic equations.[16]

In (5.8) the propagator $G'(\omega^-)$ is given by (5.10) and can be written in the form

$$G'(\omega^-) = (\omega^- - \Lambda'L - \Lambda L + PL)^{-1} , \qquad (5.11)$$

where we have introduced another projection operator

$$\Lambda \equiv \Delta P_i , \qquad (5.12)$$

$\Delta$ being the operator used before and defined by (4.1). From (5.11) we see that $G'(\omega^-)$ satisfies the identity

$$G'(\omega^-) = \overline{G}(\omega^-) + \overline{G}(\omega^-)(\Lambda L - PL)G'(\omega^-), \qquad (5.13)$$

where $\overline{G}(\omega^-)$ is defined by

$$\overline{G}(\omega^-) \equiv (\omega^- - \Lambda'L)^{-1} . \qquad (5.14)$$

From the definitions it follows that $P = P\Lambda$ and, therefore, $P\Lambda' = 0$. Thus, the identity (5.13) can be written in the form

$$G' = \overline{G} + \overline{G}P'\Lambda LG' . \qquad (5.15)$$

We now perform on both sides of (5.15) the operation $\Lambda L \ldots P'LD$ and then add $\Lambda LD$. After a small rearrangement and the use of $P'\Lambda' = \Lambda'$, we obtain an identity for $\Lambda(L + LG'P'L)D$, namely,

$$\Lambda(L + LG'P'L)D = \Lambda(L + L\overline{G}\Lambda'L)D + \Lambda L\overline{G}\Lambda(L + LG'P'L)D - \Lambda L\overline{G}D(1/d)(\Lambda(L + LG'P'L)D)_\alpha . \qquad (5.16)$$

From the definition (5.14) of $\overline{G}(\omega^-)$ it follows that $1 + \overline{G}\Lambda'L = \omega^-\overline{G}$, and thus the first term of (5.16) can be written as $\omega^-\Lambda L\overline{G}D$. We can then solve (5.16) for the quantity of interest to find

$$\Lambda(L + LG'P'L)D = [\omega^- - Z_\alpha(\omega)](1 - \Lambda L\overline{G})^{-1}\Lambda L\overline{G}D. \qquad (5.17)$$

If in the right-hand side of (5.17) we split $D$ into $\Lambda D + \Lambda'D$, we can express it in terms of the quantities

$$S(\omega) \equiv \Lambda L[1 + \overline{G}(\omega^-)\Lambda'L]\Lambda = \omega^-\Lambda L\overline{G}(\omega^-)\Lambda , \qquad (5.18)$$

and

$$C(\omega) \equiv \Lambda L\overline{G}(\omega^-)\Lambda'D . \qquad (5.19)$$

We thus find that (5.17) can be written as

$$\Lambda(L + LG'P'L)D = \{-\Lambda D + \omega^-[\omega^- - S(\omega)]^{-1}(\Lambda D + C)\} \times [\omega^- - Z_\alpha(\omega)] . \qquad (5.20)$$

Taking the $\langle \alpha + 1 | \cdots | \alpha \rangle$ matrix element of the operator equation (5.20) and dividing by $d$, we get the numerical equation for $Z_\alpha(\omega)$

$$Z_\alpha(\omega) = \{-1 + (\omega^-/d)([\omega^- - S(\omega)]^{-1}(\Lambda D + C))_\alpha\} \times [\omega^- - Z_\alpha(\omega)] . \qquad (5.21)$$

Solving this equation for $Z_\alpha(\omega)$ and substituting it in (5.8) we finally obtain for the quantity of interest

$$\overline{Q}(\omega_\alpha) = ([\omega^- - S(\omega)]^{-1}(\Lambda D + C))_\alpha , \qquad (5.22)$$

where $S(\omega)$ is given by (5.18) and $C$ by (5.19). This formal expression gives the set of the desired matrix elements

$$\{\overline{Q}(\omega)_\alpha\} = \Lambda Q(\omega) = \Delta P_i Q(\omega) = \Delta\overline{Q}(\omega)$$

through the matrix-integral equation

$$[\omega^- - S(\omega)]\Delta\overline{Q}(\omega) = \Lambda D + C(\omega), \qquad (5.23)$$

with $S(\omega)$ and $C(\omega)$ given by (5.18) and (5.19), respectively. The coefficients of this equation, i.e., $S(\omega)$, $\Lambda D$, and $C(\omega)$, can be obtained in power series in $\lambda$ from their definitions. In these expansions there are no divergent (for $\omega = \omega_c$) terms, in the thermodynamic limit, in contradistinction to the formal series expansion of $Z_\alpha(\omega)$. The difference between $Z_\alpha(\omega)$ and these new quantities $S(\omega)$ and $C(\omega)$ lies in the fact that the propagator in $Z_\alpha(\omega)$ is

$$G'(\omega^-) = (\omega^- - P'L)^{-1} ,$$

which upon expansion is powers of $\lambda$ presents divergent terms (for $\omega = \omega_c$), while in $S(\omega)$ and $C(\omega)$ the propagator is

$$\overline{G}(\omega^-) = (\omega^- - \Lambda'L)^{-1} ,$$

which by construction does not give rise to divergent terms upon expansion in powers of $\lambda$, in the thermodynamic limit. (For $D$ there is no such difficulty in its power series expansion.)

With regard to the two approximations of the earlier theories we mentioned in Sec. II, we note firstly that the *ad hoc* manner in which the impurity averaging was taken is justified by this method, and secondly that the approximation $f(H) \cong f(H_0)$ amounts to approximating the right-hand side of (5.23) by its lowest order term in $\lambda$, i.e., by

$$\Lambda D^0 = D^0 \equiv [f(H_0), J^+] .$$

This latter approximation amounts to ignoring, apart from the effects of the electron energy shifts due to $V$, also the effects of the electric field on the collisions, which are described by the interference term $C(\omega)$. The errors produced by this approximation can be quite serious. Firstly, it can easily be checked that such an approximation ($C=0$) gives gauge-dependent results; e.g., if the interaction with the electric field is described in a vector gauge, then the expression for $\mathrm{Re}\,\sigma_{+-}(\omega)$ corresponding to (2.17) after this approximation is made differs from it in that the first factor $1/\omega_c$ is replaced by $1/\omega$. Secondly, and more importantly, the effects of the interference term are of the same order of magnitude as the ones kept in the earlier theories, in the region $(\omega-\omega_c)\tau \gg 1$ in which those results are valid. This situation is quite similar to that of dc magnetoresistance for large Hall angles, as has been discussed elsewhere.[15]

Thus, the results of the earlier theories are *not* strictly valid for $(\omega-\omega_c)\tau \lesssim 1$ for the reasons we explained in Secs. III and IV, and are also invalid for $(\omega-\omega_c)\tau \gg 1$ due to the neglect of the effects of the electric field on the collisions.

We now show that the final result (5.23) for $\Delta\bar{Q}(\omega)=\Lambda Q(\omega)$ can be obtained more easily and directly with the method of transport equations.[16] In Eq. (5.5) for $Q(\omega)$, we write $Q=\Delta\bar{Q}+\Lambda' Q$, and then operate, separately, with $\Lambda$ and $\Lambda'$ to obtain

$$(\omega^- - \Lambda L)\Delta\bar{Q} - \Lambda L \Lambda' Q = \Lambda D\,, \tag{5.24}$$

$$(\omega^- - \Lambda' L)\Lambda' Q - \Lambda' L \Delta\bar{Q} = \Lambda' D\,. \tag{5.25}$$

We solve (5.25) for $\Lambda' Q$ to obtain

$$\Lambda' Q = \bar{G}(\omega^-)[\Lambda' L\bar{Q} + \Lambda' D]\,.$$

Upon substitution in (5.24) we obtain

$$[\omega^- - S(\omega)]\Delta\bar{Q}(\omega) = \Lambda D + C(\omega)\,, \tag{5.26}$$

with $S(\omega)$ and $C(\omega)$ given by (5.18) and (5.19), respectively. This is exactly Eq. (5.23). Thus a consistent application of the method of Sec. II yields results identical to those obtained much more directly by the method of kinetic equations.[16]

In the kinetic equation (5.23) for $\Delta\bar{Q}(\omega)$, the coefficients $S(\omega)$, $C(\omega)$ can be evaluated in powers of $\lambda$, by use of the expansion

$$\bar{G}(\omega^-) = G_0(\omega^-) + G_0(\omega^-)\Lambda' L_1 G_0(\omega^-) + \cdots$$

in (5.18) and (5.19), respectively, and the standard expansion of $f(H_0+V)$ for $\Lambda D = \Delta\bar{D}$. These expansions do not bring in divergent terms in the thermodynamic limit, for the same reason we explained in the previous section. A formal expansion in powers of $\lambda$ is known to be valid for both low concentration of impurities and small scattering cross section of each impurity. Other approximations for for the evaluation of $S(\omega)$, $C(\omega)$, and $\Delta\bar{D}$ can, however, be introduced[17] which are valid for low concentrations but for arbitrary scattering cross sections of the impurities.[18]

Finally, it is interesting to note that one can construct a kinetic equation for $\bar{Q}(\omega) = P_i Q(\omega)$ in a way identical to the one just used in the derivation of the kinetic equation for $\Lambda Q(\omega) = \Delta\bar{Q}(\omega)$. One finds

$$[\omega^- - \bar{S}(\omega)]\bar{Q}(\omega) = \bar{D} + \bar{C}(\omega), \tag{5.27}$$

where

$$\bar{S}(\omega) = P_i L[1 + (\omega^- - P_i' L)^{-1} P_i' L] P_i\,, \tag{5.28}$$

$$\bar{C}(\omega) = P_i L(\omega^- - P_i' L)^{-1} P_i' D. \tag{5.29}$$

This equation has been considered elsewhere,[16] where an evaluation of the coefficients $\bar{S}(\omega)$ and $\bar{C}(\omega)$ up to order $\lambda^2$ has been carried out. It is pointed out there that to this order the structure of the operator $\bar{S}(\omega)$, for a large class of scattering potentials $v(\vec{r})$, is such that in $\Delta\bar{S}(\omega)\bar{Q}(\omega)$ only $\Delta\bar{Q}(\omega)$ is involved, i.e.,

$$\Delta\bar{S}(\omega)\,\bar{Q}(\omega) = \Delta\bar{S}(\omega)\,\Delta\bar{Q}(\omega)\,.$$

In such a case an equation for the set of the matrix elements

$$\{\bar{Q}(\omega)_\alpha\} = \Delta\bar{Q}(\omega) = \Lambda Q(\omega),$$

necessary for the calculation of the conductivity $\sigma_{+-}(\omega)$, can be obtained directly by considering the $\langle\alpha+1|\cdots|\alpha\rangle$ matrix elements of (5.27). In other words, for these potentials up to second order, Eq. (5.26) for $\Delta\bar{Q}(\omega)$ can be obtained from Eq. (5.27) for $\bar{Q}(\omega)$ by operating with $\Delta$ on it. In fact, for all scattering potentials and to all orders in $\lambda$, the impurity average operator $P_i$ in $\bar{S}(\omega)$ has the effect of bringing in only the diagonal in $\vec{k}$ matrix elements of $\bar{Q}(\omega)_{n\vec{k},n'\vec{k}'}$ which is part of the operator $\Delta$, defined in (4.1). The additional selection rule for $(\Delta\bar{Q})_{nn'} \propto \delta_{n',n-1}$ is not always automatic in $\Delta\bar{S}\bar{Q}$. Thus, the procedure of using the operator $\Lambda = \Delta P_i$ in deriving quantum kinetic equations, although it is superfluous in some cases, is advantageous from a general point of view as it is sure to avoid any divergences in $S(\omega)$ and $C(\omega)$.

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