[20]A. J. Arko, J. A. Marcus, and W. A. Reed, Phys. Letters 23, 617 (1966).
[21]D. B. McWhan and T. M. Rice, Phys. Rev. Letters 19, 846 (1967).
[22]F. Heiniger, E. Bucher, and J. Muller, Phys. Letters 19, 163 (1965); F. Heiniger, Physik Kondensierten Materie 5, 285 (1966).
[23]D. W. Juenker, L. J. LeBlanc, and C. R. Martin, J. Opt. Soc. Am. 58, 164 (1968).
[24]J. C. Kimball and L. M. Falicov, Phys. Rev. Letters 20, 1164 (1968).
[25]S. H. Liu, Phys. Letters 27A, 493 (1968).
[26]T. M. Rice, A. S. Barker, Jr., B. I. Halperin, and D. B. McWhan, J. Appl. Phys. 40, 1337 (1969).
[27]M. O. Steinitz, L. W. Schwartz, J. A. Marcus, E. Fawcett, and W. A. Reed, Phys. Rev. Letters 23, 979 (1969).
[28]S. Arajs and G. R. Dunmyre, J. Appl. Phys. 37, 1017 (1966).
[29]M. V. Vedernikov, Fiz. Tverd. Tela 9, 3018 (1967) [Soviet Phys. Solid State 9, 2381 (1968)].
[30]J. Als-Nielsen and O.W. Dietrich, Phys. Rev. Letters 22, 290 (1969).
[31]A. Jayaraman, T. M. Rice, and E. Bucher, J. Appl. Phys. (to be published).
[32]L. W. Bos, D. W. Lynch, and J. L. Stanford, Phys. Letters 30A, 17 (1969).
[33]J. L. Stanford (private communication).
[34]T. M. Rice and B. I. Halperin, Phys. Rev. (to be published).
[35]H. W. Verleur, J. Opt. Soc. Am. 58, 1356 (1968).
[36]M. M. Kirillova and M. M. Noskov, Phys. Metals Metallog. USSR 26 (1968).

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Effect of Magnon Drag on Electron Mass and Mobility

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We consider an idealized model of a ferromagnetic metal where the conduction electrons of the s band interact via an exchange interaction with a system of spins represented by a Heisenberg Hamiltonian. Using double-time Green's functions, the effective mass and resistivity are evaluated to second order in the coupling of electrons to the spin system, while the spin magnetization is given by the self-consistent approximation of Bogoliubov and Tyablikov. The resistivity shows a sharp change in slope at the critical temperature that agrees qualitatively with the experimental results for gadolinium.

I. INTRODUCTION

To study the effect of magnon drag on the effective mass and transport properties of electrons in ferromagnetic metals, we consider an idealized model in which the conduction electrons of the s band interact via an exchange interaction with a system of spins of magnitude $\frac{1}{2}$ localized at the sites of a cubic lattice, and represented by a Heisenberg Hamiltonian. The model was previously considered by Methfessel and Mattis[1] in connection with the "magnetic polaron" problem.

If we represent the lattice spins by Pauli operators that commute at different sites but anticommute at the same site

$$S_i^+ = b_i, \quad S_i^- = b_i^+, \quad S_i^z = \tfrac{1}{2}(1 - 2b_i^+ b_i), \tag{1}$$

$$[b_i, b_j] = [b_i, b_j^+] = [b_i^+, b_j^+] = 0 \quad \text{if} \quad i \neq j,$$
$$\{b_i, b_i^+\} = 1, \quad (b_i)^2 = (b_i^+)^2 = 0, \quad n_i = b_i^+ b_i, \tag{2}$$

our system is represented by a Hamiltonian

$$H = H_e + H_S + H_I, \tag{3}$$

$$H_e = \sum_{k,s} \epsilon_k a_{ks}^+ a_{ks}, \tag{4}$$

$$H_S = \tfrac{1}{2} J_0 \sum_j n_j - \tfrac{1}{2} \sum_{i,j} J_{ij} b_i^+ b_j - \tfrac{1}{2} \sum_{i,j} J_{ij} n_i n_j - \tfrac{1}{8} N J_0, \tag{5}$$

$$H_I = \frac{-g}{2N} \sum_{kk'} \sum_j e^{-i(\vec{k}-\vec{k}')\cdot\vec{R}_j} \{ b_j^+ a_{k\uparrow}^+ a_{k'\downarrow} + b_j a_{k\downarrow}^+ a_{k'\uparrow} + (\tfrac{1}{2} - b_j^+ b_j)(a_{k\uparrow}^+ a_{k'\uparrow} - a_{k\downarrow}^+ a_{k'\downarrow}) \}. \tag{6}$$

The a_{ks}^+, a_{ks} are creation and destruction operators for electrons in the conduction band with momentum $\vec{k}$, spin $s = \uparrow$ or $\downarrow$, and satisfy the usual anticommutation rules:

$$\{a_{ks}, a_{k's'}^+\} = \delta_{kk'} \delta_{ss'},$$
$$\{a_{ks}^+, a_{k's'}^+\} = \{a_{ks}, a_{k's'}\} = 0 . \tag{7}$$

The square and curly brackets in Eqs. (2) and (7) mean a commutator and anticommutator, respectively. We have, in Eq. (5),

$$J_{ij} = J, \quad \text{if } i, j \text{ are nearest neighbors}$$
$$= 0, \quad \text{otherwise,} \tag{8}$$

and $J_0=\sum_{\vec{R}_{ij}} J_{ij}$.

We calculate the effective mass of the conduction electrons at the Fermi surface and the resistivity to second order in the coupling constant g, as a function of temperature, using retarded, double-time Green's functions.[2]

In this approximation, we can replace the thermal average of the spin operators by their value when $g=0$. We will consider the magnetization for the Heisenberg ferromagnet as it was evaluated by Bogoliubov and Tyablikov,[2,3] that reproduces the results of spin-wave theory at low temperature, while for $T \simeq T_c$ (T_c = critical temperature) the magnetization coincides with the expression given by molecular field theory. The decoupling of the Green's functions for the spin operators that leads to this solution is consistent with our decoupling of the electron Green's functions to lowest order in g.

In Sec. II, we calculate the electron self-energy, and in Sec. III, an explicit expression for the effective mass at the Fermi surface and the resistivity is given.

II. GREEN'S FUNCTIONS

We indicate with a double bracket $\langle\langle A|B\rangle\rangle_\pm$ the Fourier transform

$$\langle\langle A|B\rangle\rangle_\pm=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i(\omega+i\eta)t}G^{\pm}_{AB}(t)\,dt \tag{9}$$

of the retarded Green's function[2]

$$G^{\pm}_{AB}(t)=-i\,\theta(t)\langle\,[A(t),\,B(0)]_\pm\,\rangle, \tag{10}$$

where A, B are any two operators, $\langle\;\cdots\rangle$ means a thermal average, $[A,\,B]_\pm$ is either the commutator or anticommutator, $\theta(t)$ is the step function, and η is a positive infinitesimal.

From the Hamiltonian (3) and the commutation relations (2) and (7), we derive the equations of motion for the Green's functions:

$$(\omega-\epsilon_{n\uparrow})\langle\langle a_{n\uparrow}|a^+_{n'\uparrow}\rangle\rangle_+ =\frac{\delta n,n'}{2\pi}-\frac{g}{2N}\sum_{\vec{k},j}e^{-i(\vec{n}-\vec{k})\cdot\vec{R}_j}\langle\langle b_j a_{k\downarrow}|a^+_{n'\uparrow}\rangle\rangle_+ -\frac{g}{2N}\sum_{\vec{k}j}e^{-i(\vec{n}-\vec{k})\cdot\vec{R}_j}\langle\langle n_j a_{k\uparrow}|a^+_{n'\uparrow}\rangle\rangle_+, \tag{11}$$

$$(\omega-\epsilon_{n\downarrow})\langle\langle a_{n\downarrow}|a^+_{n'\downarrow}\rangle\rangle_+=\frac{\delta n,n'}{2\pi} -\frac{g}{2N}\sum_{\vec{k},j}e^{-i(\vec{n}-\vec{k})\cdot\vec{R}_j}\langle\langle b^+_j a_{k\uparrow}|a^+_{n'\downarrow}\rangle\rangle_+ +\frac{g}{2N}\sum_{\vec{k},j}e^{-i(\vec{n}-\vec{k})\cdot\vec{R}_j}\langle\langle n_j a_{k\downarrow}|a^+_{n'\downarrow}\rangle\rangle_+, \tag{12}$$

where $\epsilon_{k\uparrow}=\epsilon_k-\frac{1}{4}g,\;\epsilon_{k\downarrow}=\epsilon_k+\frac{1}{4}g.$ (13)

$$(\omega-\tfrac{1}{2}J_0)\langle\langle b_i|b^+_j\rangle\rangle_-=(1-2\bar{n})\frac{\delta_{ij}}{2\pi}-\frac{1}{2}\sum_f J_{if}\langle\langle b_f|b^+_j\rangle\rangle_-+\sum_f J_{if}\langle\langle n_i b_f\,|\,b^+_j\rangle\rangle_- -\sum_f J_{if}\langle\langle b_i n_f|b^+_j\rangle\rangle_- -\frac{g}{2N}\sum_{kk'}e^{-i(\vec{k}-\vec{k}')\cdot\vec{R}_i}\langle\langle(1-2b^+_ib_i)\,a^+_{k\uparrow}a_{k'\downarrow}|b^+_j\rangle\rangle_- +\frac{g}{2N}\sum_{kk'}e^{-i(\vec{k}-\vec{k}')\cdot\vec{R}_i}\langle\langle b_i(a^+_{k\uparrow}a_{k'\uparrow}-a^+_{k\downarrow}a_{k'\downarrow})|b^+_j\rangle\rangle \;. \tag{14}$$

Four new Green's functions appear in the right-hand side of Eqs. (11) and (12). To lowest order, we can decouple $\langle\langle n_j a_{ks}|a^+_{n's}\rangle\rangle_+$, but we should look for the equations of motion of $\langle\langle b_j a_{k\downarrow}|a^+_{n'\uparrow}\rangle\rangle_+$ and $\langle\langle b^+_j a_{k\uparrow}|a_{n'\downarrow}\rangle\rangle_+$. They are

$$(\omega-\epsilon_{k\downarrow}-\tfrac{1}{2}J_0)\langle\langle b_j a_{k\downarrow}|a^+_{n'\uparrow}\rangle\rangle_+=-\frac{g}{2N}\sum_{\vec{k}'f}e^{-i(\vec{k}-\vec{k}')\cdot\vec{R}_f}\langle\langle b_j b^+_f a_{k'\uparrow}|a^+_{n'\uparrow}\rangle\rangle_+ +\frac{g}{2N}\sum_{\vec{k}'f}e^{-i(\vec{k}-\vec{k}')\cdot\vec{R}_f}\langle\langle b_j n_f a_{k'\downarrow}|a^+_{n'\uparrow}\rangle\rangle_+ -\frac{g}{2N}\sum_{\vec{k}',\vec{k}''}e^{-i(\vec{k}'-\vec{k}'')\cdot\vec{R}_j}\langle\langle(1-2b^+_jb_j)a^+_{k'\uparrow}a_{k''\downarrow}a_{k\downarrow}|a^+_{n'\uparrow}\rangle\rangle_+ +\frac{g}{2N}\sum_{\vec{k}'\vec{k}''}e^{-i(\vec{k}'-\vec{k}'')\cdot\vec{R}_j}\langle\langle b_j(a^+_{k'\uparrow}a_{k''\uparrow}-a^+_{k'\downarrow}a_{k''\downarrow})a_{k\downarrow}|a^+_{n'\uparrow}\rangle\rangle_+ -\frac{1}{2}\sum_f J_{jf}\langle\langle b_f a_{k\downarrow}|a^+_{n'\uparrow}\rangle\rangle_+ +\sum_f J_{jf}\langle\langle n_j b_f a_{k\downarrow}|a^+_{n'\uparrow}\rangle\rangle_+ -\sum_f J_{jf}\langle\langle b_j n_f a_{k\downarrow}|a^+_{n'\uparrow}\rangle\rangle_+ . \tag{15}$$

$$(\omega-\epsilon_{k\uparrow}+\tfrac{1}{2}J_0)\langle\langle b^+_j a_{k\uparrow}|a^+_{n'\downarrow}\rangle\rangle_+=-\frac{g}{2N}\sum_{\vec{k}',f}e^{-i(\vec{k}-\vec{k}')\cdot\vec{R}_f}\langle\langle b^+_j b_f a_{k'\downarrow}|a^+_{n'\downarrow}\rangle\rangle_+ -\frac{g}{2N}\sum_{\vec{k}',f}e^{-i(\vec{k}-\vec{k}')\cdot\vec{R}_f}\langle\langle b^+_j n_f a_{k'\uparrow}|a^+_{n'\downarrow}\rangle\rangle_+ +\frac{g}{2N}\sum_{\vec{k}',\vec{k}''}e^{-i(\vec{k}'-\vec{k}'')\cdot\vec{R}_j}$$

$$\langle\langle(1-2b_j^+b_j)a_{k'\downarrow}^+a_{k''\uparrow}a_{k\downarrow}|a_{n'\uparrow}^+\rangle\rangle_+ -\frac{g}{2N}\sum_{\vec{k}',\vec{k}''} e^{-i(\vec{k}'-\vec{k}'')\cdot\vec{R}_j}\langle\langle b_j^+(a_{k'\uparrow}^+a_{k''\uparrow}-a_{k'\downarrow}^+a_{k''\downarrow})a_{k\downarrow}|a_{n'\uparrow}^+\rangle\rangle_+$$

$$+\frac{1}{2}\sum_f J_{jf}\langle\langle b_f^+a_{k\downarrow}|a_{n'\uparrow}^+\rangle\rangle_+ +\sum_f J_{jf}\langle\langle b_j^+n_fa_{k\downarrow}|a_{n'\uparrow}^+\rangle\rangle_+ -\sum_f J_{jf}\langle\langle b_f^+n_ja_{k\downarrow}|a_{k'\uparrow}^+\rangle\rangle_+ . \tag{16}$$

Now, to express the right-hand side of Eqs. (11) and (12) up to order g^2, we need to evaluate Eqs. (15) and (16) up to order g. Therefore, we decouple the Green's functions:

$$\langle\langle b_jb_f^+a_{k'\downarrow}|a_{n'\downarrow}^+\rangle\rangle_+\simeq\langle b_jb_f^+\rangle\langle\langle a_{k'\downarrow}|a_{n'\downarrow}^+\rangle\rangle_+ =\delta_{jf}(1-\bar{n})\langle\langle a_{k'\downarrow}|a_{n'\downarrow}^+\rangle\rangle_+ + O(g^2),$$

$$\langle\langle b_j^+b_fa_{k'\uparrow}|a_{n'\uparrow}^+\rangle\rangle_+\simeq\delta_{jf}\bar{n}\langle\langle a_{k'\uparrow}|a_{n'\uparrow}^+\rangle\rangle_+ + O(g^2),$$

$$\langle\langle(1-2b_j^+b_j)a_{k'-s}^+a_{k''s}a_{k-s}|a_{n's}^+\rangle\rangle_+ \simeq -(1-2\bar{n})\langle a_{k'-s}^+a_{k-s}\rangle\langle\langle a_{k''s}|a_{n's}^+\rangle\rangle_+ + O(g^2),$$

where $\langle b_j^+b_j\rangle=\langle n_j\rangle=\bar{n}$
independent of j owing to translational invariance,

and $\langle a_{k's}^+a_{ks}\rangle=\delta_{kk'}f(\epsilon_{ks})$, (17)

$f(\epsilon)=1/(e^{\epsilon/T}+1)=$ Fermi function,
where $k_B=1$.
We neglect

$$\langle\langle b_j(a_{k's}^+a_{k''s}-a_{k'-s}^+a_{k''-s})a_{ks}|a_{n's}^+\rangle\rangle\simeq O(g^2).$$

The other new Green's functions in Eqs. (15) and (16) involve the product of three b's operators, and they are decoupled together with the Green's functions appearing in Eq. (14) in the following way:

$$\langle\langle n_jb_f\cdots|\cdots\rangle\rangle_\pm\simeq\langle n_j\rangle\langle\langle b_f\cdots|\cdots\rangle\rangle_\pm . \tag{18}$$

We can now solve for the closed system of equations and we obtain

$$\langle\langle a_{ks}|a_{ks}^+\rangle\rangle=\frac{1}{2\pi}\frac{\delta_{kk'}\delta_{ss'}}{\omega-\tilde{\epsilon}_{ks}-\Sigma_{ks}(\omega)}, \tag{19}$$

where
$$\tilde{\epsilon}_{ks}=\epsilon_k-\tfrac{1}{4}g\sigma,\quad s=\uparrow$$
$$=\epsilon_k+\tfrac{1}{4}g\sigma,\quad s=\downarrow \tag{20}$$

and the self-energy is given by

$$\Sigma_{ks}(\omega)=\frac{g^2}{4N}\sum_{\vec{q}}\left\{\frac{\frac{1}{2}(1+\sigma)-\sigma f(\tilde{\epsilon}_{q\uparrow})}{\omega-\tilde{\epsilon}_{q\uparrow}-\sigma\omega(\vec{k}-\vec{q})+i\eta}\,\delta_{s,\downarrow}+\frac{\frac{1}{2}(1-\sigma)+\sigma f(\tilde{\epsilon}_{q\downarrow})}{\omega-\tilde{\epsilon}_{q\downarrow}+\sigma\omega(\vec{k}-\vec{q})+i\eta}\,\delta_{s,\uparrow}\right\}, \tag{21}$$

$$\omega(\vec{k}-\vec{q})=\tfrac{1}{2}[J_0-J_{\vec{k}-\vec{q}}],$$

$$J_{\vec{k}}=\sum_{\vec{r}}e^{-i\vec{k}\cdot\vec{r}}J_{\vec{r}}, \tag{22}$$

$$\sigma=2\langle S_i^z\rangle=1-2\bar{n}. \tag{23}$$

We can see by the definition, Eq. (23), that σ is the relative magnetization and that to lowest order in g we can replace in Eq. (21) its value for the ferromagnet when $g=0$. The self-consistent evaluation of σ from Eq. (14) with the decoupling (18) and $g=0$ was studied in Refs. 2 and 3 with the result, in zero magnetic field,

$$\sigma=1-\sum_{j\geq3}A_j\tau^{j/2},\qquad \tau<\tau_c \tag{24a}$$

$$\sigma=[(3/\tau)(1-\tau/\tau_c)]^{1/2},\quad \tau\lesssim\tau_c(\tau_c-\tau\ll1) \tag{24b}$$

$$\sigma=0,\qquad \tau>\tau_c \tag{24c}$$

where $\tau=T/J_0$,
$$\tau_c=T_c/J_0=\frac{1}{1.516}.$$

From Eq. (22), we can identify $\omega(\vec{k})$ with the excitation energy of a spin wave of momentum $\vec{k}$. A very similar expression for the self-energy of the conduction electrons was previously derived by Kim.[4]

If we look at his Eqs. (3.1a) and (3.1b) we can see that our Eq. (21) can be written

$$\Sigma_{ks}(\omega)=\Sigma_{ks}^{(\mathrm{Kim})}+\frac{g^2}{4N}\sum_{\vec{q}}\left\{\frac{\delta_{s,\downarrow}}{\omega-\tilde{\epsilon}_{q\uparrow}-\sigma\omega(\vec{k}-\vec{q})}+\frac{\delta_{s,\uparrow}}{\omega-\tilde{\epsilon}_{q\downarrow}+\sigma\omega(\vec{k}-\vec{q})}\right\}, \tag{25}$$

the difference being a temperature-independent smooth function of ω that can be considered equal to zero.

When $T=0$ and $\sigma=1$, Eq. (21) coincides with the self-energy calculated by Davis and Liu.[5]

III. EFFECTIVE MASS AND RESISTIVITY

The self-energy $\Sigma_{ks}(\omega)$ as it is expressed in Eq. (21) is a complex function, whose real part renormalizes the single-particle energy, while its imaginary part gives the lifetime of a conduction electron.

The effective mass[5] and resistivity are given by

$$\frac{m^*}{m} = 1 - \left[\mathrm{Re}\,\frac{\partial \Sigma_{ks}}{\partial \tilde{\epsilon}_{ks}}\right]_{\tilde{\epsilon}_{ks}=\epsilon_F}, \tag{26}$$

$$\rho = (m/ne^2)\,(1/\tilde{\tau}), \tag{27}$$

where $\tilde{\tau}$ is the lifetime of a conduction electron at the Fermi surface

$$1/\tilde{\tau} = -[\mathrm{Im}\Sigma_{ks}(\tilde{\epsilon}_{ks})]_{\tilde{\epsilon}_{ks}=\epsilon_F}. \tag{28}$$

The replacement of ω by $\tilde{\epsilon}_{ks}$ in the argument of Σ_{ks} in Eqs. (26)–(28) is consistent with our approximation to lowest order in g.

To evaluate Σ_{ks} we use a Debye model for the spin-wave spectrum, that is,

$$\omega(\vec{k}) = k^2/2M \le \omega_m, \tag{29}$$

where ω_m is some maximum frequency and

$$\epsilon_k = k^2/2m. \tag{30}$$

We express the sum in Eq. (21) $(1/N)\sum_{\vec{q}}$ as an integral $[\Omega_0/(2\pi)^3]\int dq^3$, where Ω_0 is the volume of a unit cell, and introduce a new variable

$$p = |\vec{k} - \vec{q}|, \quad p_0 \le p \le p_m$$

$$p\,dp = -kq\,d(\cos\theta), \quad \cos\theta = \vec{k}\cdot\vec{q}/kq. \tag{31}$$

When $\tilde{\epsilon}_{ks}$ is close to the Fermi surface, the important contributions to the integral in Eq. (21) will come from $\tilde{\epsilon}_{q,-s} \simeq \epsilon_F$.

For these values of $\tilde{\epsilon}_{ks}, \tilde{\epsilon}_{q,-s}$, the minimum value of p would be, from Eqs. (20) and (31),

$$p_0 \simeq (2m)^{1/2}\,|(\epsilon_F - \tfrac{1}{4}g\sigma)^{1/2} - (\epsilon_F + \tfrac{1}{4}g\sigma)^{1/2}| \simeq g\sigma/2v_F, \tag{32}$$

while the maximum value p_m is given by writing

$$\omega_m = p_m^2/2M \tag{33}$$

in Eq. (29).

Hence, approximating the integral in Eq. (21) by the value it takes when $\tilde{\epsilon}_{ks}, \tilde{\epsilon}_{q,-s}$ are both close to the Fermi surface, and introducing the variable p of Eq. (31), we can express $\Sigma_{ks}(\tilde{\epsilon}_{ks})$:

$$\Sigma_{ks}(\tilde{\epsilon}_{ks}) \simeq \frac{g^2}{4}\frac{\Omega_0}{(2\pi)^3}\frac{2\pi}{k_F}\int q\,dq\int_{p_0}^{p_m} p\,dp \times \left\{\frac{\frac{1}{2}(1+\sigma) - \sigma f(\tilde{\epsilon}_{q\uparrow})}{\tilde{\epsilon}_{k\downarrow} - \tilde{\epsilon}_{q\uparrow} - \sigma\omega(p) + i\eta}\,\delta_{s,\downarrow} + \frac{\frac{1}{2}(1-\sigma) + \sigma f(\tilde{\epsilon}_{q\downarrow})}{\tilde{\epsilon}_{k\uparrow} - \tilde{\epsilon}_{q\downarrow} + \sigma\omega(p) + i\eta}\,\delta_{s,\uparrow}\right\}. \tag{34}$$

The integrals are now separable and easy to evaluate. Assuming half-filled bands, $\epsilon_F = 0$ and calling D half the bandwidth, we find for $\tilde{\epsilon}_{ks} \simeq \epsilon_F$,

$$\mathrm{Re}\frac{\partial \Sigma_{ks}}{\partial \tilde{\epsilon}_{ks}} = g^2\frac{\Omega_0}{4\pi^2}\frac{1}{k_F}mM\left\{P\int_{-D}^{D} d\epsilon\, f(\epsilon)\left[\frac{1}{\sigma\omega_m + \tilde{\epsilon}_{k\uparrow} - \epsilon} - \frac{1}{\sigma\omega_0 + \tilde{\epsilon}_{k\uparrow} - \epsilon}\right]\delta_{s,\uparrow} + P\int_{-D}^{D} d\epsilon\, f(\epsilon)\left[\frac{1}{-\sigma\omega_m + \tilde{\epsilon}_{k\downarrow} - \epsilon} - \frac{1}{-\sigma\omega_0 + \tilde{\epsilon}_{k\downarrow} - \epsilon}\right]\delta_{s,\downarrow}\right\}, \tag{35}$$

where $\omega_0 = p_0^2/2M$, p_0 is given in Eq. (32), and k_F is the Fermi momentum.

The integral

$$P\int_{-D}^{D}\frac{f(\epsilon)}{\omega - \epsilon}\,d\epsilon = \frac{1}{2}\ln\frac{(D^2 - \omega^2)}{4\pi^2T^2} - \mathrm{Re}\Psi\left(\frac{1}{2} + \frac{\omega}{2\pi i T}\right), \tag{36}$$

where $\Psi(z)$ is the digamma function,[6] has been evaluated previously in connection with problems in dilute magnetic alloys.[7]

We can approximate, at low temperatures,

$$\Psi(z) \simeq \ln(z), \quad |z| \gg 1 \tag{37}$$

and we obtain for the effective mass in a neighborhood of the Fermi surface

$$\left.\frac{m^*}{m}\right]_{s=\uparrow} = 1 + g^2\frac{\Omega_0}{8\pi^2}\frac{mM}{k_F}\ln\frac{(\sigma\omega_m + \tilde{\epsilon}_{k\uparrow})^2 + \pi^2T^2}{(\sigma\omega_0 + \tilde{\epsilon}_{k\uparrow})^2 + \pi^2T^2}, \tag{38}$$

$$\left.\frac{m^*}{m}\right]_{s=\downarrow} = 1 + g^2\frac{\Omega_0}{8\pi^2}\frac{mM}{k_F}\ln\frac{(\sigma\omega_m - \tilde{\epsilon}_{k\downarrow})^2 + \pi^2T^2}{(\sigma\omega_0 - \tilde{\epsilon}_{k\downarrow})^2 + \pi^2T^2}.$$

At $T = 0$, we recover again the result of Ref. 5, but our expression for m^*/m differs from that obtained in Ref. 4 because Kim used the Einstein approximation for the spin-wave spectrum.

We find that at finite temperature $0 < T \le T_c$, the effective mass (or the density of states) is enhanced at a distance $\pm\sigma(T)\omega_0$ from the Fermi surface for $s = \uparrow$ or $\downarrow$, but for $T \ne 0$ the peaks are finite.[5]

The approximate behavior of m^*/m as a function of $\tilde{\epsilon}_{ks}$ for different temperatures is shown in Fig. 1.

From Eqs. (28) and (34), and using the relationship

$$\frac{1}{\omega - \epsilon + i\eta} = P\frac{1}{\omega - \epsilon} - i\pi\delta(\omega - \epsilon),$$

we obtain for both electrons with spin up and down

$$\frac{1}{\tilde{\tau}} = g^2\frac{\Omega_0}{8\pi^2}\frac{1}{k_F}mM\left\{(\omega_m - \omega_0)(1-\sigma) - 2T\ln\frac{1 + e^{-\sigma\omega_m/T}}{1 + e^{-\sigma\omega_0/T}}\right\}. \tag{39}$$

We can see from Eq. (24a) that the second term

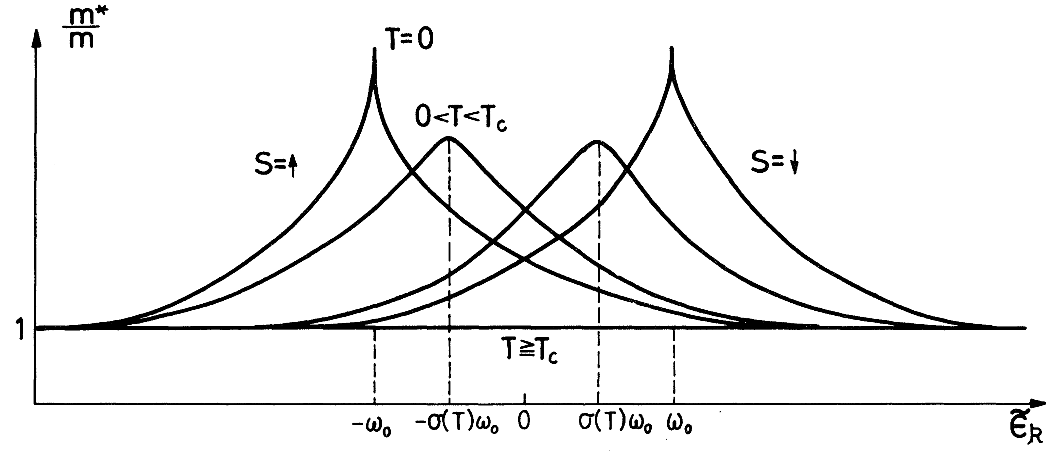

FIG. 1. Relative effective mass m^*/m of conduction electrons near the Fermi surface for different values of T. The effective mass for spins ↓ (↑) is enhanced at a distance $+(-)\sigma(T)\omega_0$ of the Fermi energy $\epsilon_F=0$. $\sigma(T)$ is the relative magnetization and ω_0 is the minimum frequency at which a spin wave can be emitted or absorbed.

in the right-hand side of (39) becomes exponentially small when $T\to 0$, while $1-\sigma(T)\simeq T^{3/2}$, hence we obtain for the resistivity at low temperatures, using Eq. (27),

$$\rho \simeq \text{const}\times T^{3/2}, \quad T \gtrsim 0 . \qquad (40)$$

As $\sigma(T)\to 0$ when $T\to T_c$, for this region of temperature we can expand the second term in the right-hand side of (39) in powers of σ and we find

$$\frac{\rho(T)}{\rho(\infty)} \simeq 1-(\omega_m+\omega_0)\tfrac{1}{4}\sigma^2, \quad T\lesssim T_c \quad (1-T/T_c\ll 1), \qquad (41)$$

where $\sigma(T)$ is given by Eqs. (24b) and (24c).

IV. CONCLUSIONS

We found that, at finite temperature $0<T\leq T_c$, the partial disorder of the lattice modifies the $T=0$ results[5] in two ways: (a) The exchange splitting of the conduction band depends on the magnetization through Eq. (20), and vanishes for $T>T_c$; (b) the magnon frequencies appear in the self-energy multiplied by the magnetization, a result that was also obtained in Ref. 4, therefore, the enhancement in the effective mass for electrons with spin up or down[5] becomes smaller and closer to the Fermi surface, disappearing finally for $T>T_c$, as we show in Fig. 1.

Our expression for the resistivity when $T\simeq T_c$, Eq. (41), shows the same dependence on the magnetization σ that was previously obtained by de Gennes and Friedel,[8] and by van Peski-Tinbergen and Dekker,[9] but the term with σ^2 is multiplied by a different coefficient.

Somewhat different results were obtained by Kim[10] considering the effect of fluctuations of the short-range order of spins near the critical temperature. These considerations are outside the

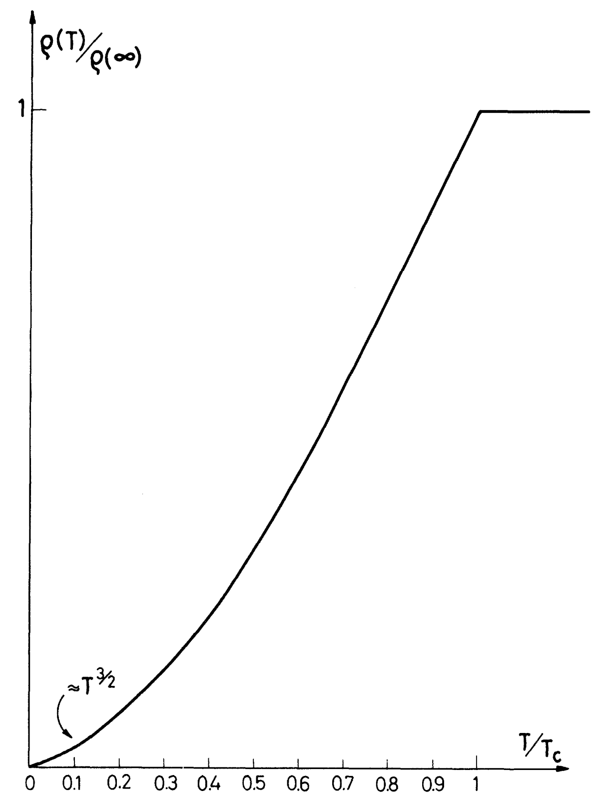

FIG. 2. Relative resistivity, in units of $\rho(T\to\infty)$, versus T/T_c, where T_c is the critical temperature. $\rho(T)/\rho(\infty)\simeq(T/T_c)^{3/2}$ when $T\to 0$ while $\rho(T)/\rho(\infty)\to 1-\text{const}\times(1-T/T_c)$ when $T\to T_c$.

Bogoliubov-Tyablikov approximation for the ferromagnet, but the fact that this solution is consistent with the decoupling of the Green's functions that gives the self-energy of the conduction electrons to second order in g allows us to find an expression for the resistivity, Eq. (39), valid for all temperatures, and, in particular, the low-temperature result of Eq. (40).

A detailed comparison with experiments would require a precise knowledgment of the parameter ω_m, but an approximate plot of the resistivity versus temperature is given in Fig. 2. It shows the qualitative features of the resistivity of gadolinium as measured by Colvin, Legvold, and Spedding.[11] Gadolinium metal has localized spins and approximates best the assumptions of the theory.

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[1]S. Methfessel and D. C. Mattis, in *Handbuch der Physik*, edited by S. Flügge (Springer, Freiburg, 1968), Vol. XVIII/1, Chap. C.
[2]D. N. Zubarev, Usp. Fiz. Nauk 71, 71 (1960) [Soviet Phys. Usp. 3, 320 (1960)].
[3]V. L. Bonch-Bruevich and S. V. Tyablikov, *The Green Function Method in Statistical Mechanics* (North-Holland, Amsterdam, 1962), Chap. VII.
[4]D. J. Kim, Phys. Rev. 167, 545 (1968).
[5]L. C. Davis and S. H. Liu, Phys. Rev. 163, 503 (1967).
[6]*Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1965).
[7]See, for instance, Ph. E. Bloomfield and D. R. Hamann, Phys. Rev. 164, 856 (1967).
[8]P. G. de Gennes and J. Friedel, J. Phys. Chem. Solids 4, 71 (1958).
[9]T. van Peski-Tinbergen and A. J. Dekker, Physica 29, 917 (1963).
[10]D. J. Kim, Progr. Theoret. Phys. (Kyoto) 31, 921 (1964).
[11]R. V. Colvin, S. Legvold, and F. H. Spedding, Phys. Rev. 120, 741 (1960).

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Ising Model with Antiferromagnetic Next-Nearest-Neighbor Coupling: Spin Correlations and Disorder Points*

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The existence of a *disorder point* T_D is established for some one- and two-dimensional Ising lattices with antiferromagnetic next-nearest-neighbor interactions. Within the disordered phase, the decay of axial next-nearest-neighbor pair correlations with increasing spin separation is positive monotonic exponential below T_D and oscillatory with exponential envelope above T_D. A general definition of a disorder point is formulated, and a method of estimating T_D described.

This paper summarizes some exact calculations performed recently to determine the nature of pair correlations between two spins on an axis of an Ising lattice when next-nearest-neighbor interactions are present. The primary result concerns the existence of a disorder point T_D in one- and two-dimensional Ising lattices with ferromagnetic nn[1] and antiferromagnetic nnn[1] exchange. The results also hold when the nn interaction is antiferromagnetic, at least when the nn lattice is loose packed. A definition of T_D is framed and a procedure indicated for its estimation on lattices for which exact solutions are not yet available. Figure 1 illustrates the lattices considered, and Fig. 2 shows the dependence of the disorder point T_D and the critical point T_C (Curie point) on the ratio of nnn to nn interactions for the "union-jack" lattice.[2]

Consider an Ising model with spin variables $\sigma_{\vec{r}} = \pm 1$ at lattice sites $\vec{r}$, and Hamiltonian

$$\mathcal{H} = -J_1 \sum_{\text{nn}} \sigma_{\vec{r}} \sigma_{\vec{r}'} - J_2 \sum_{\text{nnn}} \sigma_{\vec{r}} \sigma_{\vec{r}''}, \quad J_2 < 0 < J_1 \qquad (1)$$