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Flux Conservation in Multiply Connected Type-I Superconductors in Fields Generated in the Voids*

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The behavior of multiply connected type-I superconductors in magnetic fields generated in the voids can be accounted for quantitatively by assuming flux conservation within the total system rather than in the void alone. The properties are similar to the "anomalous" properties observed when hollow cylinders are cooled in static fields and do not appear to involve intermediate-state formation. The results and analysis of the results show that in these configurations the transition from the pure superconducting state to the normal state occurs through a single sharp *S-N* boundary which begins at the inside wall and moves to the outside wall with increasing solenoid current (or internal field). This interface moves reversibly, both with increasing and decreasing field and with increasing and decreasing temperature. Such an interface is not associated with hysteresis in the bulk material and does not lead to field retention in the void when the solenoid current is reduced to zero. Field retention in the void and in the bulk, when the solenoid current is removed, occurs only after sufficient current is initially applied to drive the wall into the intermediate state. The effects also occur in hollow cylinders of unfavorable geometries (short thick walls) and provide a means of making unambiguous measurements of the velocity of the superconducting-normal interface.

The conservation of flux in a multiply connected type-I superconductor has been treated by London.[1] For sufficiently thick walls, London's conditions are $B=0$ in the wall and $\dot{B}=0$ in the void. Various authors[2,3] have used these equations to predict the behavior of long hollow cylinders of type-I superconductors. In this idealized situation, the wall is assumed to be either entirely superconducting or entirely normal. Field in the void when the wall is superconducting is associated with a persistent current on the inside surface. Real samples show many deviations from this simple picture, some of which have been attributed to end effects or to the formation of a complicated intermediate state. One interesting type of behavior which deviates from the idealized predictions is observed in experiments in which hollow cylinders are cooled from above T_c to below T_c in constant external fields. In these experiments, the field in the void does not remain constant but increases as T decreases below T_c.[4-6] Moreover, this behavior is associated with a reversible exchange of flux between the wall and the void.[6] Flux is conserved in the total system of wall plus void, not in the void alone. The purpose of this paper is to show that this behavior is not associated with the intermediate state, but is the result of a single superconducting-normal interface which moves reversibly in the wall. This conclusion can be inferred from a new type experiment in which a solenoid is used to generate a magnetic field within the void of the hollow superconducting cylinder. A schematic of the geometry and the behavior observed for Pb shields of various dimensions (all of which were only slightly longer than the solenoid) is shown in Fig. 1 for measurements made at 4.2°K. The type of probe used and the calibration techniques were described elsewhere.[6] Results obtained with copper solenoids and with superconducting Nb-Ti solenoids were the same. However, the use of superconducting solenoids eliminated heating problems at high currents. The data shown in Fig. 1 are typical. For low currents, the field inside the solenoid varies linearly with current, but is always less than in the absence of the shield. The field inside the solenoid is *reduced* by relatively large amounts when the annular spacing between solenoid and shield is small. The results show that the initial slope is a function of the annular spacing and is independent of shield thickness. We shall call this region A. At a certain characteristic current I_s, there is a knee in the curve followed by another straight-line region. This line is parallel to that for the unshielded coil and is displaced down from it. Measurements made at other temperatures as shown in Fig. 1, and additional measurements made with Sn tubes confirm that this displacement is H_c, the critical field of the shield. We shall call this region B.

By invoking Ampère's law and the assumption of flux conservation *within the outer diameter* of the tube it is possible to account for the observed behavior and to calculate how the fields inside and outside the solenoid vary with current as a function of (a) the critical field (or critical current density) of the shield, (b) the thickness of the

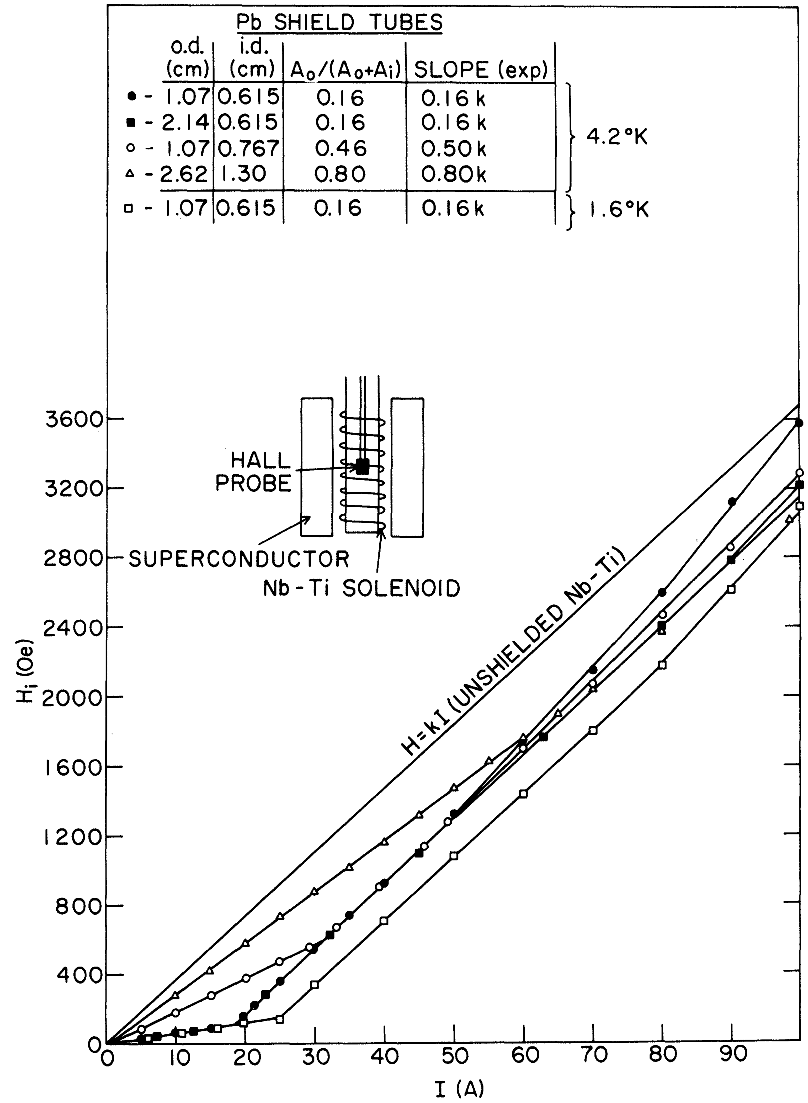

FIG. 1. Internal field variation with current of solenoids inserted in the voids of Pb hollow cylinders of different radial dimensions at two temperatures.

shield, (c) the annular spacing between the shield and the solenoid, and (d) the diameter of the solenoid.

From Ampère's law

$$H_i + H_0 = kI \quad , \tag{1}$$

where H_i and H_0 are the magnitudes of the fields (of opposite sense) inside and outside the solenoid, I is the applied current, and k is the solenoid constant. From flux conservation within the shield,

$$H_i A_i = H_0 A_0 \quad , \tag{2}$$

where A_i is the cross-sectional area of the solenoid and where A_0 is the area of the annular region between solenoid and shield. At first we assume the shield does not break down or enter the intermediate state and A_0 is constant (region A). Combining (1) and (2)

$$H_i = [A_0/(A_0 + A_i)]kI \quad . \tag{3}$$

Equation (3) accurately describes the experimentally determined initial slopes in region A. The calculated and observed values are compared in the legend of Fig. 1. The agreement between Eq. (3) and the observed data supports the assumption that A_0 remains constant and that the shield remains completely superconducting in this region. When H_0 reaches H_c, the critical current density of the shield is exceeded and an inner shell goes normal. This determines the extent of region A. Combining (2) and (3)

$$I_s = (H_c/k)(A_0/A_i + 1) \quad . \tag{4}$$

This is in good agreement with the observed locations of the knees. When I exceeds I_s the effective annular spacing increases and we shall assume that this happens in the following simple way. There is a cylindrical normal-superconducting interface within the shield. Inside this interface (and outside of the solenoid) the field is constant, $H_0 = H_c, A_0$ increasing as I does. From Eq. (1)

$$H_i = kI - H_c \quad . \tag{5}$$

This is an accurate description of region B. Note that in this region the interpretation differs from the conventional treatment referred to previously in which flux conservation is restricted to the actual void.

The data support a model in which a sharp superconducting-normal interface exists in the wall; in other words there is no formation of the intermediate state in regions A and B. The model requires that the sharp superconducting-normal interface (i.e., absence of intermediate state) is associated with the depression of H_i equal to H_c. It is clear from Fig. 1 that at sufficiently high solenoid current, the depression becomes less than H_c and the H_i curve begins to approach the curve obtained in the absence of the tube. We attribute the termination of region B to the onset of intermediate-state formation in the wall. This allows an examination of field retention in the void with two well-defined conditions of flux presence in the wall, viz., in one case the intermediate state is absent and in the other it is presumed to be present.

The experimental data on this point were obtained by a series of "minor" current cycles performed as follows. A current I_1 was applied and removed and the residual field was measured.

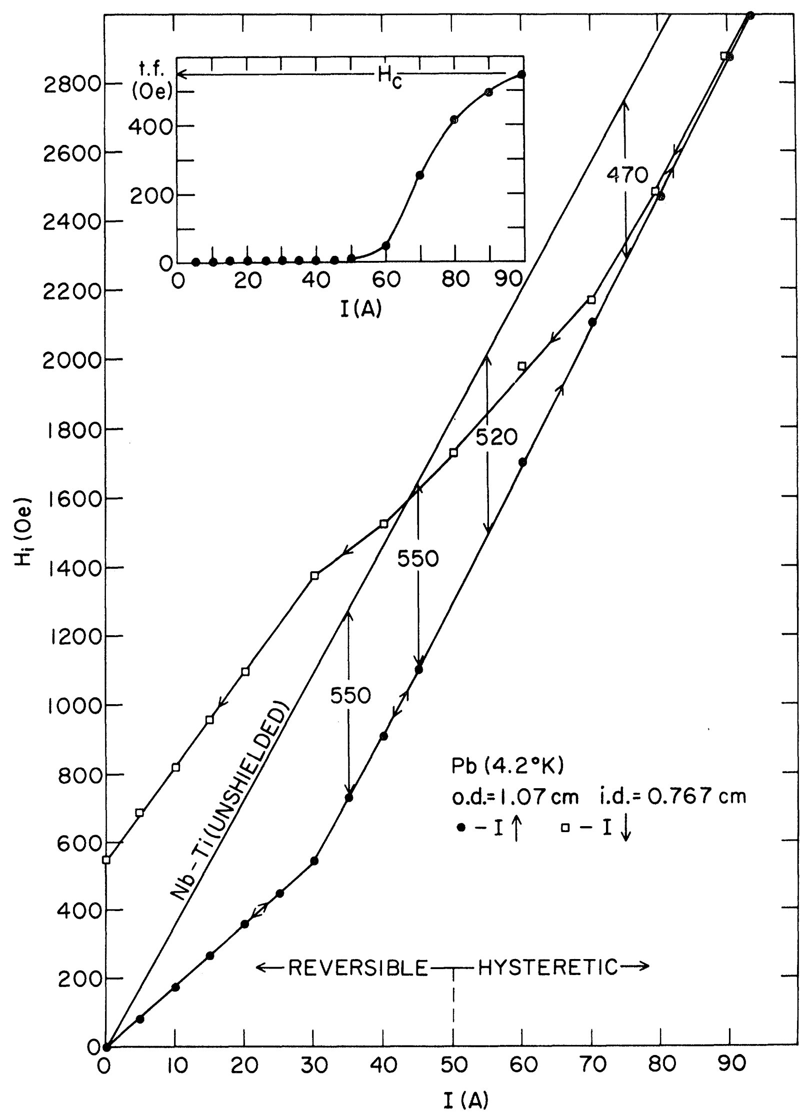

FIG. 2. Field trapping properties of a hollow cylinder of Pb with variations of internally generated fields. Inset shows the field retaining properties with minor cycling of the solenoid current.

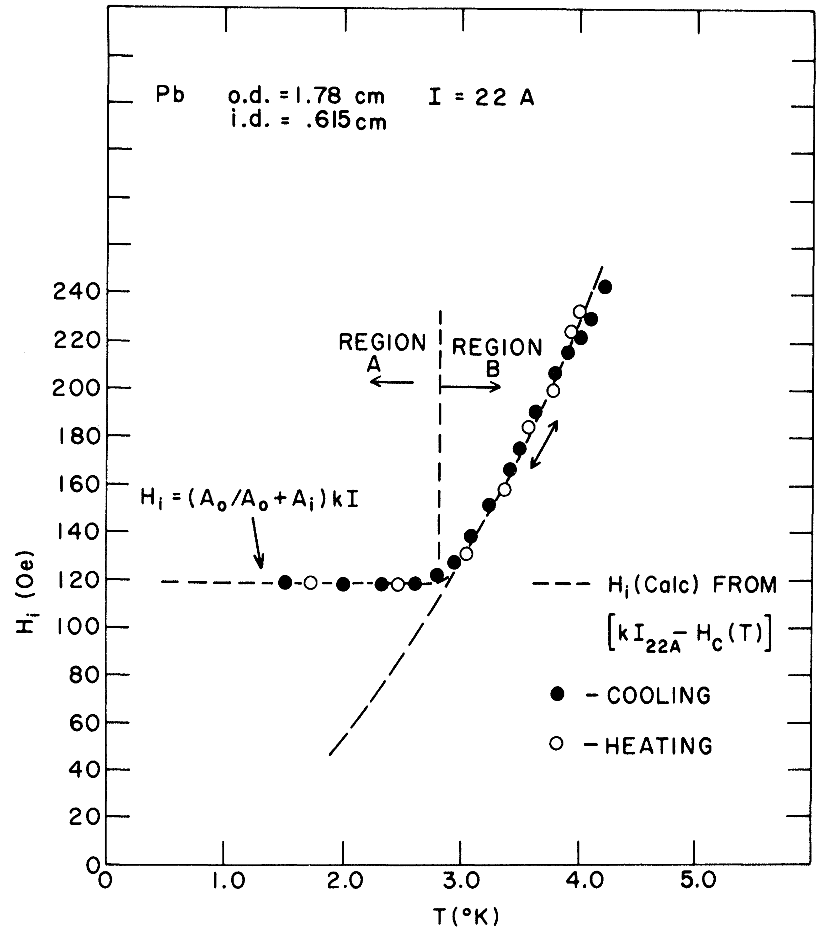

FIG. 3. Reversible exchange of field between the wall and the void at constant solenoid current and varying temperature.

This was repeated for a current I_2 a little bit larger than I_1 and so on until the entire curve of trapped field tf versus I was obtained. Typical data are shown in Fig. 2 for Pb at 4.2 °K. The insert shows the tf-versus-I curve for the series of minor cycles just described. It can be seen that there is no field retention for currents up to 50 A. This current coincides very closely with the point at which the field depression begins decreasing from the value H_c (i.e., the end of region B) as is clearly shown in the main part of the figure. Concomitantly, in the region below 50 A, the H_i-versus-I curve is reversible as indicated in the figure. Also shown are the complete forward and reverse curves for the largest possible solenoid current. This shows that H_c is trapped long before the entire cylinder wall is normal.

The sharp normal-superconducting interface moves reversibly with increasing and decreasing solenoid current in region B so that the field in the normal interior part of the wall does not exceed H_c. The reversibility of interface movement also occurs when the temperature is varied at constant solenoid current. The procedure was to vary the temperature between limits such that for a given solenoid current, the high temperature corresponded to a point in region B ($H_i = kI - H_c$) and the low temperature corresponded to a point in region $A\{H_i = [A_0/(A_0 + A_i)]\, kI\}$. The resulting field variation shown in Fig. 3 is reversible and checks with the model quantitatively.

The above behavior appears to provide an explanation of the spontaneous enhancement effect which occurs when hollow cylinders are cooled in static external fields. The reversible exchange of flux between the wall and the void which is observed in this experiment appears to be due to an interface of the sort discussed above.

The data in Fig. 2 show that field retention in the voids of finite hollow superconducting cylinders occurs when the intermediate state forms in the wall. Moreover, in isothermal magnetization and Meissner experiments (to be described elsewhere) where field is retained in the void we have found there is field retained in the wall. This means that when flux is trapped in the void of a hollow cylinder the situation does not correspond simply to that of a persistent surface current on the inside surface. There must be additional currents flowing either in the bulk or on the outer surface even in the absence of an external field. An interesting consequence of this model shows that it is possible to have a persistent current on the inside surface of the cylinder and no field in the void. This can be seen from Eq. (3) by letting $A_0 \to 0$. In this case there would be no field in the wall except for an infinitesimal layer just outside the surface current.

The observation that this technique allows one to generate a single sharp superconducting interface which can be moved easily and reversibly without intermediate-state complications suggests the possible application to NMR experiments and others in which time-dependent effects are important. In this work we have not determined the velocity of interface motion.

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Light Scattering from Plasmas and Single-Particle Excitations in Cadmium Sulfide near Resonance

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Coupled phonon-plasmon excitations and single-particle excitations in CdS have been observed by means of inelastic light-scattering experiments using argon and krypton ion lasers with emission frequencies near that of the CdS band gap. The coupled-mode frequencies, linewidths, and cross sections have been measured as functions of temperature, laser frequency, and carrier concentrations and have been compared with theoretical predictions. The simultaneous observation of coupled one-LO-phonon excitations and uncoupled two-LO-phonon excitations is explained in terms of plasmon dispersion and damping which uncouple the plasmon and LO phonon at wave vectors near q_{FT}, the Fermi-Thomas wave vector.

Raman scattering from electrons in semiconductors has previously been observed in experiments using infrared lasers.[1–3] The reason semiconductors such as CdS and ZnTe, which have band gaps in the visible region of the spectrum, have not been subject to such experimentation is that they have characteristically larger effective masses and lower mobilities than the III-V's. This results in low-frequency, highly damped plasmons, and, therefore, difficulties in detection. In the present study of high-carrier-concentration CdS, we have found that these anticipated drawbacks are compensated experimentally by resonant enhancement and by the excellent properties of the S-20 phototube; we have been able to obtain very good Raman data on coupled LO phonon-plasmon excitations, using low-power argon and krypton ion lasers.

Data were obtained from right-angle scattering experiments on CdS samples (Harshaw) having nominal room-temperature carrier concentrations (due to deviation from stoichiometry) of 2×10^{19} and 4×10^{18} cm^{-3}. Low-power (< 1W) Ar II (5145 Å) and Kr II (5208 and 5682 Å) cw lasers were used, and detection was by means of a Spex 1400 double monochromator, a cooled S-20 (ITT FW-130) photomultiplier, and a Keithley 610B electrometer. A conventional helium Dewar was used for measurements between 6 and 190°K. Figure 1 presents traces of the spectrum of each sample. The