same experimental results. Even strongly singular functions $V(\mathbf{k},\mathbf{k}')$ may be quite consistent with the experimental results.

If our transport equation is applicable, the experiments represent a measurement of Pauli paramagnetism of enormously improved precision. However, the derivations of the transport equations are uncertain in the range of experiments so far carried out, raising doubts about the significance of the numbers which either theory inexorably produces.

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Impurity Ionization in Germanium in Strong Magnetic Fields*

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Low-temperature electrical breakdown in p-type germanium has been investigated at magnetic-field strengths up to 52 kOe. The electric-field strength required for the onset of breakdown in gallium-doped germanium increased from 4.2 V/cm at zero magnetic field to 22 V/cm in a 50-kOe transverse magnetic field. For a longitudinal magnetic field of the same range, there is a smaller variation of breakdown electric field with a maximum of 7.2 V/cm at 50 kOe. The effect is independent of the polarity of electric field applied to the specimen in all cases. The increase in electric field required for breakdown appears to be in reasonable agreement with recent theories of impact-ionization phenomena, provided that the quantization and shift of energy levels in a strong magnetic field are considered together with the effect of transverse magnetic fields on the rate at which carriers gain energy from an electric field. Measurements of the changes of the far-infrared photoconductivity of the specimen in the magnetic field provide further evidence of the effect of the field on the ionization phenomena.

INTRODUCTION

THE low-temperature electrical-breakdown effect in germanium containing group-III impurities has been investigated, both experimentally and theoretically.[1–4] There is fair qualitative agreement between the experimental results and the theoretical descriptions of the effect in most aspects. The nature of the breakdown phenomena has been experimentally investigated as a function of a variety of experimental parameters by both Sclar and Burstein[1] and by Zavaritskaya.[2] However, the effect of a magnetic field on the impact-ionization process has been investigated only in the low-field regime[1,2,5] where quantization effects are not important. This paper reports measurements of low-temperature electrical breakdown in strong magnetic fields where quantization and shift of the energy levels are significant. The results are interpreted in terms of recent theories of breakdown phenomena taking into account the effect of the magnetic field on the energy-level separation, the mobility and the recombination coefficient. Measurements of the effect of the magnetic field on the far-infrared extrinsic photoconductivity of the same specimens are used to derive supporting evidence as to the nature of the physical phenomena involved.

Low-temperature electric breakdown in impurity semiconductors is caused by impact ionization of neutral impurity centers by hot carriers.[6] As the strength of the applied electric field increases, the average kinetic energy of the charge carriers becomes sufficient to ionize neutral centers. The recombination process which reduces the number of free carriers becomes less effective at high fields since the capture cross section decreases with increasing carrier energy. For electric-field intensities exceeding a certain critical value, the ionization rate exceeds the recombination rate and a nonequilibrium condition is obtained. The critical field is regarded as the breakdown field with the breakdown condition being expressed in terms of the kinetic equation[7]

$$dp/dt = A_T(N_A - N_D - p) + p[A_I(N_A - N_D - p) - B_T(N_D + p)] - p^2 B_I(N_D + p). \quad (1)$$

The increase in the number of free holes p is related to A_T and A_I, which represent the rates of carrier generation by thermal processes and impact ionization, respectively; B_T and B_I describe the thermal and Auger recombination rates, respectively. In a compensated material containing both types of impurities, N_D is the density of donor centers, while N_A is the acceptor density. Under steady-state conditions, if we neglect Auger recombination and thermal-carrier generation

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and assume that both N_A-N_D, $N_D \gg p$, we have

$$A_I(N_A-N_D)=B_T N_D \tag{2}$$

as the simplified breakdown condition.

The coefficients A_I and B_T are dependent on the lattice temperature and carrier distribution as well as other parameters. The thermal recombination of free carriers with a trap can be calculated by assuming carriers are captured by highly excited states of the trapping center.[8] The energy loss of the carrier is through a series of phonons emitted as the carrier cascades down through the excited state of the center. Estimates of the temperature dependence of the recombination coefficient show it to be approximately described by[3]

$$B_T=1.7\times10^{-5}/T_h \text{ cm}^3/\text{sec}, \tag{3}$$

where T_h is the carrier temperature and the numerical coefficient has been determined to be valid for p-type Ge near breakdown at 4.2°K lattice temperature using early experimental data.

The impact-ionization coefficient A_I has been computed by Yamashita, who took the scattering cross section to be independent of wave vector **k**, and also by Cohen and Landsberg, who have performed a more complete **k**-dependent calculation.[4] Comparison of the results of these calculations with the experimental data shows the more complete treatment to give a better description of the effect. The impact-ionization coefficient A_I for a trap state with a depth E_t has been computed as a function of carrier temperature for different values of trap depth based on the latter calculations. These results are illustrated in Fig. 1, and are used in comparing the experimental data with the theoretical model for the behavior observed. The relation which closely describes the carrier temperature for Ge in the 10^{14}-cm^{-3} impurity-concentration range is[4]

$$T_h(°\text{K})=AE_B^{1/2}, \tag{4}$$

where E_B is the breakdown field in V/cm and the coefficient A is an adjustable parameter equal to about 10°K $(\text{cm/V})^{1/2}$ for this case. The results show that a substantially higher carrier temperature is required to maintain the ionization coefficient at the breakdown condition as the impurity ionization energy increases.

EXPERIMENTAL

The experiments were conducted on a gallium-doped germanium crystal with an acceptor concentration of 1×10^{14} cm^{-3} with a compensation ratio of approximately 0.05. Samples with dimensions of $0.15\times0.2\times1$ cm were cut from the crystal in a plane normal to the [111] axis and etched with *CP*4 etch. Indium contacts to the bar were used for electrical measurements; these were found to be Ohmic. A high-impedance direct-current source was used to supply current to the specimen while both specimen voltage and current through a sampling resistor were plotted on an *xy* recorder. The sample was mounted in a stainless-steel light pipe in the liquid-helium Dewar so that measurements of photoconductive properties could be made as well as voltage-current characteristics. The specimen was centered in a 60-kOe superconducting magnet and was alternately placed with its axis in a transverse or longitudinal orientation relative to the magnetic-field direction.

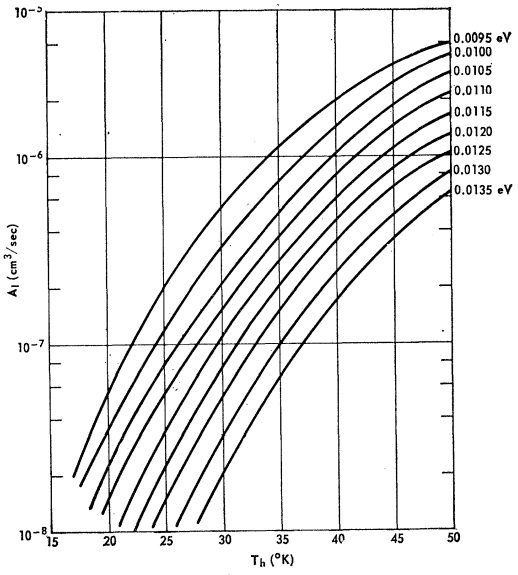

FIG. 1. Impact-ionization coefficient as a function of electron temperature for different impurity-level depths (indicated).

A far-infrared H_2O laser was used to generate emission lines with energies of the same order as the impurity ionization energy of the specimen. The effect of the magnetic field on the photoconductive response was measured on the same specimens on which breakdown measurements were conducted. The experiments were carried out using a gas-laser source which has been described in detail in other publications.[9,10] The laser was pumped by a 1-μF capacitor charged to 4.5 kV, which yielded a 1-μsec current pulse with typical operation at a 27-Hz repetition rate.

The effect of far-infrared radiation was measured by changes in the conductivity of the specimen when operated with a bias current of 2 μA. The modulation in conductivity of the specimen appeared as a 20-μsec voltage pulse which was fed into a gated amplifier and integrator.

The general characteristics of the low-temperature breakdown effect are illustrated in Fig. 2; the breakdown curve for zero magnetic field is characterized by an Ohmic region where the current is proportional to

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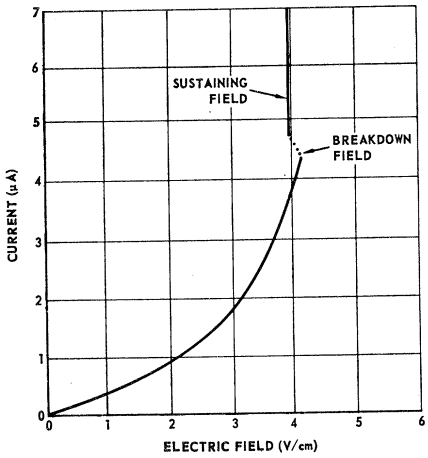

FIG. 2. Current-electric-field characteristic for Ge:Ga at 4.2°K.

applied electric-field strength and a prebreakdown region where the current increases at a much greater rate than the field. The breakdown occurs at an electric-field strength of approximately 4.25 V/cm accompanied by a small negative-resistance effect and a transition to a sustaining field strength of 4.0 V/cm. This small difference between the breakdown and sustaining field strengths is characteristic of specimens which are weakly compensated.

The presence of a transverse magnetic field alters the nature of the current-voltage characteristic in each of the regions as is shown in Fig. 3. In the Ohmic region, the resistivity of the specimen increases as the magnetic-field strength is increased and then reaches a saturated value at high fields. The saturation at high magnetic fields when the impurity ionization energy is increasing may be associated with impurity-band conduction. The prebreakdown region is more strongly influenced by the magnetic field with the prebreakdown resistivity increasing at high magnetic-field strengths to values approaching the resistivity of the Ohmic region. This effect is also a saturating one, with only small changes in resistivity occurring at fields above 20 kOe.

The electric-field strength required for breakdown is seen to increase in proportion to the increase in magnetic-field strength. The relationship between breakdown-field and magnetic-field strength, illustrated in Fig. 4, appears to be composed of two approximately linear regions with a discernible break occurring at about 10 kOe. The continuing increase in the breakdown electric field is in sharp contrast to the strongly saturated behavior of the resistivity of the specimen in the Ohmic and prebreakdown regions of the characteristics. The results shown were found to be completely independent of the polarity of the electric field for all magnetic-field values.

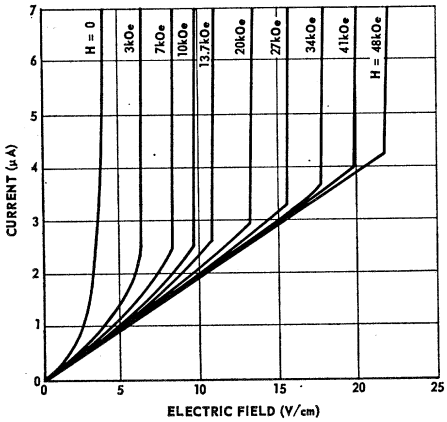

FIG. 3. Low-temperature current-electric-field characteristics in a magnetic field. Small negative resistance effects which occur are not shown.

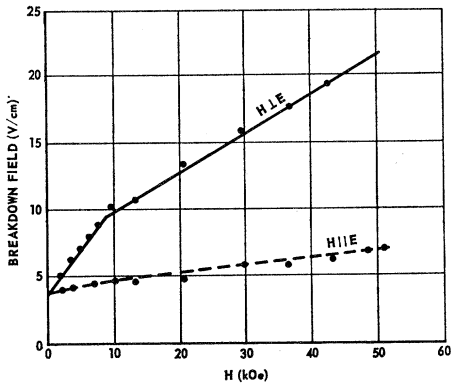

FIG. 4. Electric-breakdown field strength in transverse and longitudinal magnetic fields.

The specimen, on which measurements were made in a transverse magnetic field, was reoriented so that the electric field was parallel to the magnetic-field direction. There is an increase in the electric-field strength required for breakdown in a high longitudinal field, but the variation is only a small fraction of the change observed in a transverse magnetic field. The effect of the longitudinal magnetic field on the characteristic curves in the prebreakdown region is also not as pronounced. The variation of breakdown field strength with longitudinal field is plotted on Fig. 4 for comparison with the transverse-magnetic-field data.

To further investigate the effect of a magnetic field on the ionization processes, measurements were made of the changes in photoconductivity of the specimen induced by the 119-μm H_2O laser line in the presence of an external field. The close agreement between the 10.8-meV impurity ionization energy and the 10.4-meV photon energy of the laser line gives rise to a large extrinsic photoconductivity in gallium-doped germanium. The observed photoconductive signal was

expected to be sensitive to any shift in the impurity ionization energy due to the magnetic field. As illustrated in Fig. 5, the transverse magnetic field causes a large reduction in the photoconductivity up to fields of 30 kOe while above this value there is a residual photoconductive signal which decreases slowly with increasing field. The photoconductive signal for the longitudinal magnetic-field configuration decreases more slowly than the transverse-field case as is also illustrated in Fig. 5.

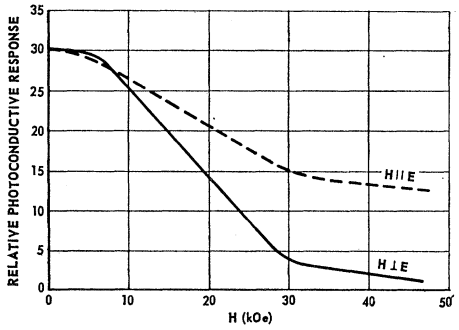

FIG. 5. Photoconductive response of gallium-doped germanium at 119 μm in transverse and longitudinal magnetic fields.

DISCUSSION

Both the threshold for impact ionization and the far-infrared photoconductivity were found to be sensitive to a magnetic field parallel to the direction of current flow through the gallium-doped germanium specimen. The same processes were even more sensitive to the presence of a transverse magnetic field of equivalent strength. The modification of these ionization effects by the magnetic field can be attributed to two factors. First, the magnetic field will cause the ionization energy of the impurity to increase. Second, the condition for breakdown should depend on the magnetic field through the dependence of electron temperature on magnetic field. The magnetic field will influence the electric field required to produce breakdown by reducing the rate at which the carrier distribution gains energy from the electric field. This can result from both a reduction of the drift velocity of the distribution along the electric-field direction, and changing the distribution function with a reduction in the proportion of hot carriers. These changes should appear as a change in the mobility or conductivity of the specimen as a magnetic field is applied, and should be the predominant effect at low magnetic fields.

Measurements of the resistance of the specimens in a transverse magnetic field show a large magnetoresistance effect in the Ohmic and prebreakdown regions at low fields reaching a resistance value almost independent of field above 10 kOe for most bias currents (Fig. 6). Measurements in a longitudinal magnetic field show only small changes in the low-magnetic-field range. On the basis of these observations it will be assumed that the increase in the impact-ionization threshold in the case of longitudinal magnetic fields depends mainly on the increase in impurity ionization energy. In the case of transverse magnetic fields, both the increase in impurity ionization energy and the reduction of electron temperature by the magnetic field will be considered.

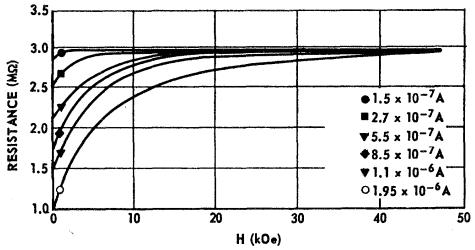

FIG. 6. Variation of resistance of Ge:Ga specimen in a transverse magnetic field at fixed bias currents.

The effect of the magnetic field on the impurity ionization energy has been calculated by several investigators.[11–13] A substantial increase in the binding energy of the impurity is expected in a magnetic field. The results of calculation of this effect are best expressed in terms of the parameter

$$\gamma = \hbar^2\epsilon^2(\hbar\omega_c)/m^*e^4 = \hbar\omega_c/2R,$$

where ω_c is the cyclotron frequency of a free carrier in a magnetic field, m^* is the mass at the band extremum, ϵ the static dielectric constant, and R is the effective Rydberg constant. None of the calculations which are performed for relatively simple band models can be expected to yield accurate quantitative results for the case of the valence bands in p-type Ge, particularly in the case of low γ. For germanium with a heavy hole mass of $0.3m_0$ and a dielectric constant $\epsilon=16$, γ will reach a value of 0.26 at a field strength of 60 kOe. This would lead to an increase in the impurity ionization energy to $1.23E_t$, where E_t is the zero-magnetic-field ionization energy.[12] The 10.8-meV zero-field ionization energy associated with gallium-doped germanium would then be increased to approximately 13.0 meV in such a magnetic field.

The increase in impurity ionization energy in a longitudinal magnetic field based on the above considerations will then require a higher carrier temperature (and electric-field strength) to reach the threshold for impact ionization. From the relationship between the breakdown electric-field strength and carrier temperature given by Eq. (4) we can plot the experimental

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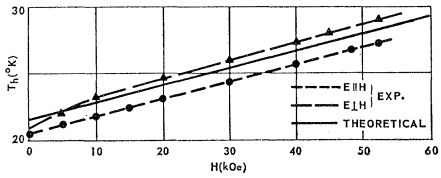

FIG. 7. Effective carrier temperature required for breakdown as a function of magnetic field.

carrier temperature at breakdown as a function of longitudinal magnetic field (Fig. 7). The carrier temperature required for ionization can also be calculated theoretically by combining the breakdown equation [Eq. (2)] and the equation for the recombination coefficient [Eq. (3)], yielding

$$A_I=1.7\times10^{-5}N_D/(N_A-N_D)T_h=8.5\times10^{-7}/T_h, \quad (5)$$

where a compensation ratio of 0.05 is used in the last expression. By referring to Fig. 1, which uniquely relates A_I and T_h for a trap of a given depth, it can be seen that for a trap depth of 10.8-meV breakdown should occur at a temperature $T_h=21.5$°K with $A_I=4\times10^{-8}$ cm³/sec. For an applied longitudinal magnetic field, the equation relating the ionization coefficient and the carrier temperature [Eq. (5)] will still be valid, but it must be satisfied for a greater trap depth as the magnetic field increases the ionization energy. For example, at a field of 60 kOe the trap depth will be approximately 0.013 eV,[12] so that the solution of Eq. (5) for this trap depth from Fig. 1 will yield values of $T_h=29$°K and $A_I=3\times10^{-8}$ cm³/sec. Results derived from the model in this fashion are also shown in Fig. 7 for the longitudinal magnetic-field case, and appear to be in good agreement with the experimental data.

The effect of a transverse magnetic field is more complex since both the impurity ionization energy and the rate at which the carriers are heated are changed by the magnetic field. Since the increased impurity ionization energy in a magnetic field is isotropic with respect to orientation of the magnetic field, the theoretical curve of Fig. 7 will also be valid for this case. The relation between carrier temperature and the applied electric field [Eq. (4)] must be modified to reflect the reduced heating effect of the electric field in the transverse magnetic field. The carrier temperature can be related to the applied electric field E by the expression

$$T_h=A(\mu_B/\mu_0E)^{1/2}, \quad (6)$$

where μ_B/μ_0 is the ratio of carrier mobilities with and without a magnetic field, respectively. Under constant-carrier concentration condition, this can be expressed as

$$T_h=A\left(\frac{\rho_0}{\rho_B}E\right)^{1/2}, \quad (7)$$

where ρ_0/ρ_B is the corresponding ratio of resistivity without and with a magnetic field, respectively. The effective carrier temperature in a transverse magnetic field can be derived by multiplying the breakdown electric field by either the mobility ratio or resistivity ratio as given in Eqs. (5) or (6), respectively. The carrier temperatures derived in this fashion are shown in Fig. 7, and are seen to agree reasonably well with the theoretical curve and the results in a longitudinal field. These results appear reasonable since approximately the same ionization energy and hence carrier temperature for breakdown should be required in either magnetic-field orientation while a strong anisotropy exists in the heating of carriers in the two magnetic-field orientations.

The effect of the longitudinal magnetic field on the photoconductive response is qualitatively consistent with an increase in impurity ionization energy in the field. Measurements of the infrared absorption spectra of impurities in germanium by Fan[14] have shown the absorption to drop rapidly for photoenergies below the impurity ionization energy followed by a slowly decreasing tail at long wavelengths. Recent measurements by Jeffers[15] of the spectral response of a Ge:Ga photoconductor have shown a similar sharp edge beyond the impurity ionization energy followed by a slowly changing tail. The response of the photoconductor with a fixed photon energy in a varying magnetic field then appears consistent with the response of a photoconductor with a fixed impurity ionization energy and a changing input spectrum. The larger change in photoconductive response observed in a transverse magnetic field is consistent with a reduction in mobility in this magnetic-field orientation.

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