Electronic Properties of the Superconductor in the Transition Region*

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The calculations of Abrahams, Redi, and Woo which give the electronic properties above the superconducting transition have been extended into the "critical region" of the transition.

IN a recent work, Abrahams, Redi, and Woo[1] have calculated the correction to the single-electron Green's function due to the interaction with (fluctuation) Cooper pairs in the "classical region" of the superconductor above the transition point. Using this result, they have proceeded to the calculation of the electronic density of states and the thermal conductivity. However, the results are not valid close to the critical point, because the linearized, "classical" form of the Cooper-pair propagator was used.

Recently, we have proposed a method[2] which takes into account the interaction between the fluctuations in the first order of perturbation theory. Using this method we have been able to extend the theory from the classical region to describe a number of properties at the superconducting transition. The recent measurements of the resistive transition on thin films[2] and whiskers[3] are in good agreement with the theoretical predictions. The microscopic form of this model has been presented by Schmid.[4]

As the transition temperature is approached, the interaction between the (fluctuation) Cooper pairs can no longer be neglected. This interaction is represented in Fig. 1(a). The bare vertex part of the interaction of two fluctuations has essentially a constant value[5]

$$\Gamma_0=\tfrac{7}{8}\xi(3)/(\pi T_c)^2\,N_0. \tag{1}$$

To first order, the self-energy contribution in the fluctuation propagator is represented in Fig. 1(b). The largest contribution comes from the $\omega=0$ term in the sum

$$\Sigma(\omega_n=0)=\Gamma_0 T\int\frac{d^sq}{(2\pi)^s}D(\mathbf{q},\,\omega_n=0). \tag{2}$$

$D_0(\mathbf{q},\omega_\nu)=N_0^{-1}[\tau+(\pi/8T_c)\omega_\nu+\xi^2(0)q^2]^{-1}$ is the propagator of the fluctuation field, $\tau=(T-T_c)/T_c$ and $\xi(0)$ is the Ginzburg-Landau coherence length at $T=0$. Following Schmid,[4] we introduce the quantity $\tilde{\tau}$ defined by

$$\tilde{\tau}=\tau+\Gamma_0 T\int\frac{d^sq}{(2\pi)^s}[\tilde{\tau}+\xi^2(0)q^2]^{-1}, \tag{3}$$

and the resulting expression for the fluctuation propagator becomes

$$D(\mathbf{q},\omega_\nu)=N_0^{-1}[\tilde{\tau}+(\pi/8T_c)\omega_\nu+\xi^2(0)\mathbf{q}^2]^{-1}. \tag{4}$$

The correction to D has been evaluated self-consistently, i.e., the wavy line in Fig. 1(a) represents D and not D_0. The procedure of Schmid is different in the sense that the correction to D is calculated from the corrections to the electron propagator, and not from the direct interaction between the fluctuations. The only difference in the result, however, is an extra factor of 2 in Eq. (2). We believe that our procedure is correct since, as it will be shown, it reduces to the BCS results for lower temperatures.[2]

The first-order correction to the single-electron Green's function is easily found to be

$$\Sigma(\mathbf{p},\omega_\nu)=-T\sum_{\omega_n}\int\frac{d^sq}{(2\pi)^s}D(\mathbf{q},\omega_n)\mathcal{G}_0(\mathbf{q}-\mathbf{p},\,\omega_n-\omega_\nu). \tag{5}$$

For lower temperatures the previous equation can be written[3]

$$\Sigma(\mathbf{p},\omega_\nu)=-A^2\mathcal{G}_0(-\mathbf{p},\,-\omega_\nu), \tag{6}$$

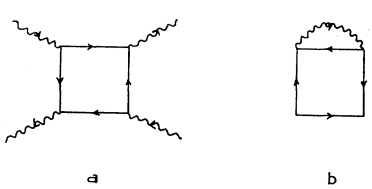

FIG. 1. (a) Interaction of fluctuation Cooper pairs; (b) first-order diagram for the self-energy of a Cooper pair.

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[1] E. Abrahams, M. Redi, and J. W. F. Woo, Phys. Rev. (to be published).

[2] W. E. Masker, S. Marčelja, and R. D. Parks, Phys. Rev. Letters **22**, 124 (1969); Phys. Rev. **188**, 745 (1969).

[3] R. J. Warburton, B. R. Patton, W. W. Webb, and J. W. Wilkins, Proceedings of the Conference on the Science of Superconductivity, Stanford University, 1969 (unpublished).

[4] A. Schmid, Z. Physik (to be published).

[5] K. Maki, Progr. Theoret. Phys. (Kyoto) **40**, 193 (1968).

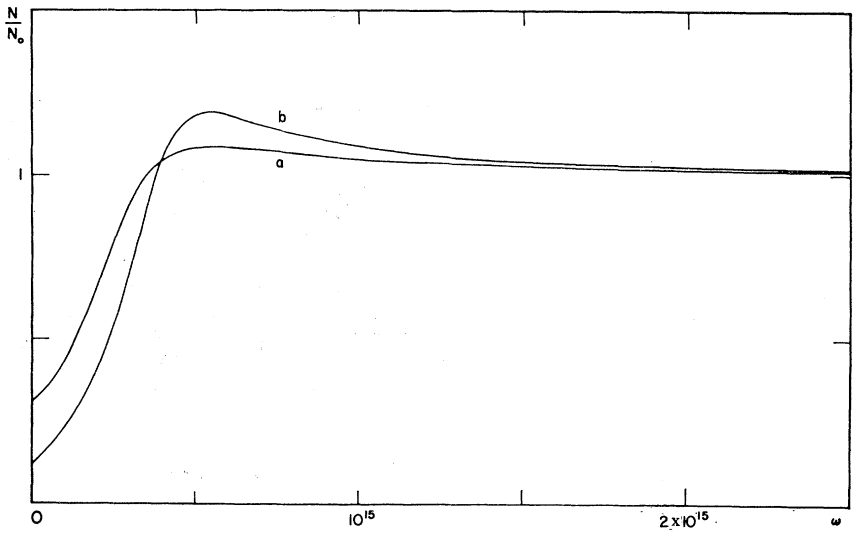

FIG. 2. Density of states for the film with $R_\square = 100\ \Omega/\square$ and $T_c = 2°K$. (a) $\tau = 0.01$; (b) $\tau = 0.0$.

where

$$A^2 = \frac{T}{N_0} \int \frac{d^s q}{(2\pi)^s} [\tilde{\tau} + \xi^2(0) q^2]^{-1} \approx \Delta_0^2 \tag{7}$$

and Δ_0 is BCS value for the energy gap. The electron Green's function reduces to the Gor'kov form:

$$\mathcal{G}^{-1} = \mathcal{G}_0^{-1} - \Sigma = -(\omega^2 + \xi^2 + \Delta_0^2)/(i\omega + \xi). \tag{8}$$

The calculation of the electron Green's function and the density of states proceeds exactly as in Ref. 1, with the only change that τ is replaced by $\tilde{\tau}$, given in Eq. (3). The interesting results are those for thin films, where experiments are most easily performed.[6] With the notation $\tau_c = 2T^2\tau/(\pi^2 N_0 d D \Delta_0^2)$, the relation between τ and $\tilde{\tau}$ is (in the dirty limit)

$$\tau = \tilde{\tau} - \tau_c \ln((\tilde{\tau}+1)/\tilde{\tau}). \tag{9}$$

The Abrahams, Redi, and Woo expression for the density of states is still given by

$$N(\omega) = N_0 \operatorname{Re}[1 + S(i\omega_n \to \omega + i0)]^{-1/2}, \tag{10}$$

where $S(\omega)$ is now

$$S(\omega) = \frac{4\Delta_0^2 \tau_c}{\Gamma_s^2 \tilde{\tau}} \left\{ \frac{1}{(2\omega/\Gamma_s - i)^2} \times \left[\ln \frac{2\omega/\Gamma_s + i}{2i} - \frac{2\omega/\Gamma_s - i}{2\omega/\Gamma_s + i} \right] \right\} \tag{11}$$

and $\Gamma_s = (8T_c/\pi)\tilde{\tau}$. The resulting density of states has the form of Abrahams, Redi, and Woo in the classical region above T_c; as the temperature is lowered it approaches gradually to the BCS form. As an illustration of this behavior, the plot of Eq. (10) for the

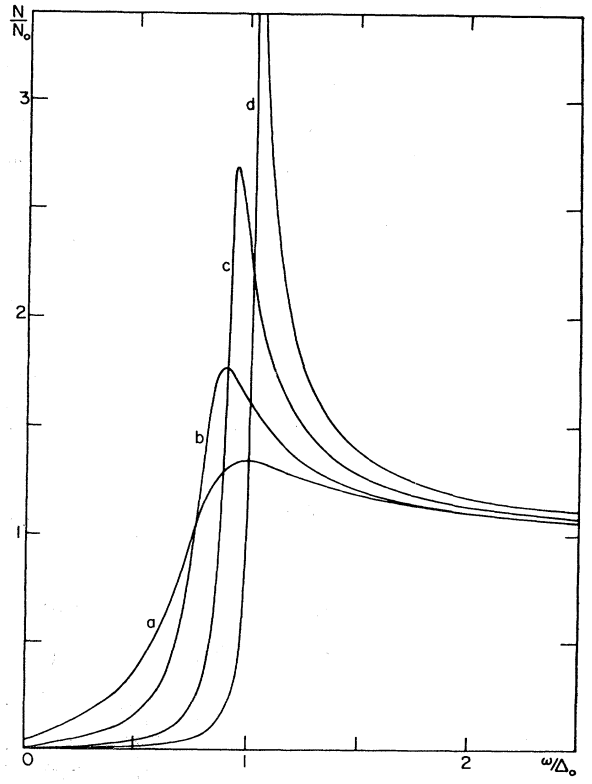

FIG. 3. Density of states below T_{c0}: (a) $\tau = -0.005$; (b) $\tau = -0.01$; (c) $\tau = -0.02$; (d) $\tau = -0.03$. The film parameters are same as in Fig. 2.

[6] R. W. Cohen, B. Abeles, and C. R. Fuselier, Phys. Rev. Letters **23**, 377 (1969).

different values of temperature is presented in Figs. 2 and 3. The sharpening of the gap edge as the temperature is lowered has been observed by Cohen[7] *et al.* and is in qualitative agreement with the present result.

In conclusion, we have extended the calculations of Ref. 1 into the transition region. The present model reduces to the Gor'kov form of the BCS model as the temperature is gradually lowered. However, as discussed in Ref. 2, it is not expected to be valid in detail for high Cooper-pair densities.

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[7] R. W. Cohen (private communication through A. Schmid).