Comments and Addenda

Generation-Recombination Noise of a Three-Level Semiconductor

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The generation-recombination spectral noise density of a three-level semiconductor in thermodynamic equilibrium is shown to decrease monotonically with increasing frequency, independent of the drift mobilities of the free carriers.

IT has been demonstrated by Wessels and Kruizinga[1] that, under special conditions, the generation-recombination noise spectrum of a three-level semiconductor out of thermodynamic equilibrium can show a local maximum. Colligan and Van Vliet[2] state that the spectrum can be shown to exhibit a local maximum, even in thermodynamic equilibrium. In contradiction with the last statement we shall prove that, in thermodynamic equilibrium, the spectral densities of the occupancies and the current noise spectrum never shows a local maximum, with due observance of the following conditions: (a) The semiconductor should be homogeneous, (b) space-charge neutrality will be in force, (c) only one-electron transitions are considered, and (d) transport noise will be neglected.

Consider a semiconductor with three energy levels between which electron transitions take place, while μ_1, μ_2, and μ_3 are the drift mobilities of free carriers and n_1, n_2, n_3 are the average numbers of free carriers in these levels, respectively. The assumption of space-charge neutrality means that $\sum_i \Delta(-1)^{k_i} n_i = 0$, with $k_i = 0$ or 1 for electron or hole conduction in the i level, respectively. Since one variable (i.e., n_3) can be eliminated, the current fluctuations of the equivalent Norton generator has a spectral density represented by[2]

$$G_I(\omega) = (I/\sigma)^2 [\mu_1 - (-1)^{k_1+k_3}\mu_3]^2$$
$$\times \{G_{11}(\omega) + b[G_{21}(\omega) + G_{12}(\omega)] + b^2 G_{22}(\omega)\}, \quad (1)$$

where

$$\sigma = \sum_i n_i\mu_i, \quad b = [\mu_2 - (-1)^{k_2+k_3}\mu_3]/[\mu_1 - (-1)^{k_1+k_3}\mu_3],$$

G_{11} and G_{22} are the spectral densities of charge carriers

in levels 1 and 2; G_{12} and G_{21} are the cross correlation functions. The quantities G_{ij} are equal to[3]

$$G_{11} = \Phi(\tau_1,\tau_2)[P\tau_1^2(1+\omega^2\tau_1^2)^{-1} + U\tau_2^2(1+\omega^2\tau_2^2)^{-1}], \quad (2a)$$

$$G_{22} = \Phi(\tau_1,\tau_2)[Q\tau_1^2(1+\omega^2\tau_1^2)^{-1} + X\tau_2^2(1+\omega^2\tau_2^2)^{-1}], \quad (2b)$$

$$G_{21} = \Phi(\tau_1,\tau_2)[R\tau_1^2(1+\omega^2\tau_1^2)^{-1} + Y\tau_2^2(1+\omega^2\tau_2^2)^{-1}], \quad (2c)$$

$$G_{12} = \Phi(\tau_1,\tau_2)[T\tau_1^2(1+\omega^2\tau_1^2)^{-1} + Z\tau_2^2(1+\omega^2\tau_2^2)^{-1}], \quad (2d)$$

where

$$P = p_{12}(\tau_1^{-1} - M_{22} - M_{12}) + p_{13}(\tau_1^{-1} - M_{22}),$$
$$U = p_{12}(-\tau_2^{-1} + M_{22} + M_{12}) + p_{13}(-\tau_2^{-1} + M_{22}),$$
$$Q = p_{12}(\tau_1^{-1} - M_{11} - M_{21}) + p_{23}(\tau_1^{-1} - M_{11}),$$
$$X = p_{12}(-\tau_2^{-1} + M_{11} + M_{21}) + p_{23}(-\tau_2^{-1} + M_{11}),$$
$$R = p_{12}(-\tau_1^{-1} + M_{11} + M_{21}) + p_{13}M_{21},$$
$$Y = p_{12}(\tau_2^{-1} - M_{11} - M_{21}) - p_{13}M_{21},$$
$$T = p_{12}(-\tau_1^{-1} + M_{22} + M_{12}) + p_{23}M_{12},$$
$$Z = p_{12}(\tau_2^{-1} - M_{22} - M_{12}) - p_{23}M_{12},$$
$$\Phi = (\tau_1,\tau_2) = 4\tau_1\tau_2(\tau_2 - \tau_1)^{-1}.$$

p_{ij} is equal to the mean transition rate of carriers between level i and j.

In thermodynamic equilibrium, the terms G_{ij} are real, $G_{ij} = G_{ji}$ (thus, $R = T$ and $Y = Z$), $p_{ij} = p_{ji}$ and τ_1, τ_2 are real.[3] M_{ij} are the elements of the real two-dimensional relaxation matrix $\mathbf{M}$ defined by $d\mathbf{n}/dt + \mathbf{M}\cdot\mathbf{n} = 0$, and τ_1, τ_2 are the reciprocal eigenvalues

[1] A. C. E. Wessels and S. Kruizinga, Phys. Letters 20, 243 (1966).

[2] M. B. Colligan and K. M. Van Vliet, Phys. Rev. 171, 881 (1968) and (private communication).

[3] K. M. Van Vliet and J. R. Fasset, in *Fluctuation Phenomena in Solids*, edited by R. E. Burgess (Academic Press Inc., New York, 1965) pp. 268-359.

of **M**:

$$\tau_1^{-1} = \tfrac{1}{2}[S + (S^2 - 4D)^{1/2}],$$
$$\tau_2^{-1} = \tfrac{1}{2}[S - (S^2 - 4D)^{1/2}], \tag{3}$$

with $S = \text{Tr}(\mathbf{M})$ and $D = \text{Det}(\mathbf{M})$, while $\tau_2 > \tau_1$.

To prove that $G_I(\omega)$ shows no local maximum it is sufficient to do this for $G_{11} + 2bG_{21} + b^2 G_{22}$. Therefore, [see Eq. (2)] if we prove that $P + 2bR + b^2 Q$ and $U + 2bY + b^2 X$ are always non-negative, then $G_I(\omega)$ will decrease monotonously with increasing frequency or $G_I(\omega)$ will be equal to zero.

First, it will be proved that P as well as U are non-negative. This means that the spectral density of carriers in level 1 can not show a local maximum. To prove this, it is necessary and sufficient to demonstrate that

$$\tau_1^{-1} \geq M_{22} + [p_{12}/(p_{12} + p_{13})]M_{12} \geq \tau_2^{-1} \tag{4}$$

will always hold. With the aid of Eq. (3), this inequality can be written as

$$-(S^2 - 4D)^{1/2} \leq M_{22} - M_{11}$$
$$+ [2p_{12}/(p_{12} + p_{13})]M_{12} \leq (S^2 - 4D)^{1/2}, \tag{5}$$

and so

$$|M_{22} - M_{11} + [2p_{12}/(p_{12} + p_{13})]M_{12}|$$
$$\leq [(M_{11} - M_{22})^2 + 4M_{12}M_{21}]^{1/2}. \tag{6}$$

Inequality (6) may be squared and after rearranging some terms, this yields

$$p_{12}(M_{22} + M_{12} - M_{11} - M_{21}) \cdot (p_{12} + p_{13})M_{12}$$
$$- p_{12}p_{13}M_{12}{}^2 - p_{13}(p_{12} + p_{13})M_{12}M_{21} \leq 0. \tag{7}$$

With the aid of the equality $G_{12} = G_{21}$, it follows that [see Eq. (2)]

$$p_{12}(M_{22} + M_{12} - M_{11} - M_{21}) = p_{13}M_{21} - p_{23}M_{12} \tag{8}$$

and adding this to inequality (7) one gets instead

$$-(p_{12}p_{13} + p_{12}p_{23} + p_{13}p_{23})M_{12}{}^2 \leq 0. \tag{9}$$

Inequality (9) is always true, and it is easy to see that inequality (4) holds also, therefore P and U are always non-negative.

A proof, similar to that of P and U, shows that Q and X are also non-negative. Indeed, because of permutation considerations, it follows that, if G_{11} has no local maximum, G_{22} consequently has no local maximum.

Now it will be proved that for every value of b, it holds that:

$$f_1(b) \equiv P + 2bR + b^2 Q \geq 0$$

and

$$f_2(b) \equiv U + 2bY + b^2 X \geq 0.$$

Since P, Q, U, and X are non-negative, f_1 and f_2 are parabolic functions of b (provided that $Q \neq 0$ and $X \neq 0$), with a minimum at $b = -R/Q$ and $b = -Y/X$, respectively. The minimum value of f_1 is $(PQ - R^2)/Q$ and that of f_2 is $(UX - Y^2)/X$. From the equality $G_{12} = G_{21}$, and Eq. (3), and after some calculations, it can be proved that

$$PQ - R^2 = PQ - RT = 0 \quad \text{and} \quad UX - Y^2 = UX - YZ = 0.$$

So f_1 and f_2 are non-negative for any b. If Q or X is equal to zero, then R or Y is also equal to zero, and as a result, $f_1 = P$ or $f_2 = U$.

In the limiting case, where $\tau_2 = \tau_1 = \tau$, one can prove that

$$G_{11} + 2bG_{21} + b^2 G_{22}$$
$$= 4\tau^2[(1-b)^2 p_{12} + b^2 p_{23} + p_{13}](1 + \omega^2 \tau^2)^{-1}. \tag{10}$$

Thus, $G_I(\omega)$ is either a monotonously decreasing function of frequency or $G_I(\omega)$ is equal to zero.

This proof is based on the property that the minimum of f_1 and f_2 is exactly equal to zero. Approximations of the noise formulas 1 and 2 could erroneously lead to a negative value for f_1 or f_2. This again would lead to the wrong conclusion that a local maximum can appear in the spectrum at thermodynamic equilibrium.[2]