Temperature Dependence of the Cyclotron Effective Mass in Zinc*

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(Received 8 September 1969)

The temperature dependence of the cyclotron effective masses of three orbits in zinc has been measured in the temperature range of 4.2–11°K, and the masses were found to increase as the absolute temperature squared. The experimental results presented are in reasonable agreement with the theoretical calculations of Allen and Cohen reported in the following paper for the temperature dependence of the electron-phonon mass enhancement. The experiments were performed in the usual Azbel'-Kaner geometry. The zinc specimens used had optically flat surfaces and had $\omega_{rf}\tau$ as large as 45.

I. INTRODUCTION

THE purpose of this paper is to present the results of a set of experiments which measured the temperature dependence of the cyclotron effective mass in zinc. These experiments were made possible by a technique that had previously been developed[1] to produce oriented single crystals of zinc with optically flat surfaces and with $\omega_{rf}\tau$ as large as 50. A theoretical calculation by Grimvall[2] has shown that the interaction between lattice vibrations and conduction electrons in a metal gives rise to a temperature-dependent mass enhancement for the cyclotron effective mass. In the following paper,[3] Allen and Cohen have extended this calculation to specific metals including zinc and they have compared our results with their calculations.

To provide a meaningful comparison to the theoretical calculations which assume a spherical Fermi surface, three orbits on that piece of the Fermi surface of zinc called the lens were studied. Previous studies of Azbel'-Kaner cyclotron resonance in zinc[1,4,5] have provided a thorough knowledge of orbits on the lens and have shown that the lens has the very nearly free-electron shape of two spherical caps. The temperature dependencies of the mass enhancements for these orbits agrees in order of magnitude with the results of Allen and Cohen.

The experiments were performed in the Azbel'-Kaner geometry, that is, with the static magnetic field **H** parallel to the surface of the sample. The effective masses were extracted from the experimental resonance curves by plotting reciprocal resonance fields against the harmonic number.[6]

II. EXPERIMENTAL DETAILS

The experiments reported here were performed at a microwave frequency of 32 Gc/sec and at temperatures between 4.2 and 11.2°K using a standard microwave reflection spectrometer. The zinc sample formed one wall of a cylindrical cavity resonating in the TE_{111} mode. The cavity was split, rotatable, and had a design similar to the one described by Spong and Kip.[7]

The dc magnetic field was modulated at an audio frequency and the klystron frequency was stabilized to the resonant frequency of the cavity. Thus, when the power absorbed by the cavity was a function of the magnetic field, the amplitude of the audio component of the power reflected from the cavity was proportional to the derivative of the microwave absorption with respect to the dc magnetic field, i.e., proportional to dR/dH, where R is the real part of the surface impedance of the sample. In these experiments the modulation was typically 1–4 Oe at 43 Hz. The dc magnetic field was swept linearly with time and was measured by a Rawson rotating-coil magnetic-field probe which was calibrated against a nuclear-magnetic-resonance probe. The accuracy of the field measurements was estimated to be 0.01% over the range of 100 Oe to 10 kOe.

The cavity assembly was isolated from the liquid-helium bath by enclosing it in a vacuum-tight can which was sealed wlth Wood's metal solder. Thermal isolation of the cavity assembly was achieved by using a 1-in. piece of stainless steel waveguide between the cavity and the top of the helium exclusion can. In addition, #42 manganin leads were used for the resistance thermometer and for the heater. Thus, the main source of thermal contact between the sample and the helium bath was the helium exchange gas in the exclusion can. The temperature of the sample and cavity was changed by varying the heater current and the pressure of the exchange gas in the can surrounding the cavity assembly. With an exchange gas pressure of $10\,\mu$, approximately 8 mW of heater power was needed to raise the sample temperature to 11°K.

The temperature of the sample was measured by a carbon-resistance thermometer. The insulation and part of the carbon were ground off one side of a $\frac{1}{10}$-W 470-Ω resistor. The resistor was then glued to a small copper plate with GE 7031 varnish. A piece of cigarette paper was inserted between the resistor and the copper plate before varnishing to provide electrical insulation. The

* Work supported in part by Grant No. AF-68-1352.
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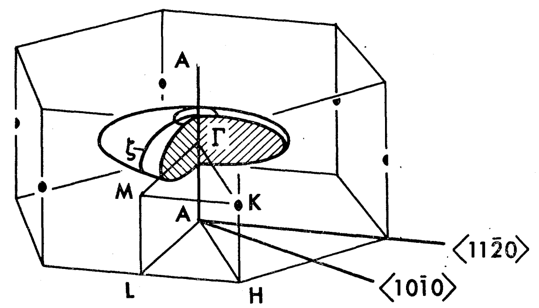

FIG. 1. Third-band electron surface (lens) of zinc.

copper plate had a 5-mm No. 30 copper wire lead which was then soldered to the zinc sample with indium. The resistor was calibrated at three temperatures, the boiling points of liquid hydrogen (20.4°K), liquid helium (4.2°K), and at 3°K. The three-constant carbon-resistor equation of Clement and Quinnell[8] was used to calculate temperature versus resistance between the calibration points. The resistance at 4.2°K was checked each time the system was cooled and was found to be reproducible to within 0.5%. The estimated error in the temperature measurement was at worst ±0.1°K. The resistance measurements were made with a dc bridge having equal-ratio arms and a three-lead connection to the thermometer to cancel lead resistance.

The heater consisted of 24 in. of No. 45 nichrome wire wound on a $\frac{1}{8}$-in. copper slug. The slug had four #30 copper wires soldered to it. Two of these leads were soldered to the cavity and two were soldered with indium to the zinc sample. The temperature was controlled by a feedback circuit which amplified the output of the resistance bridge and changed the amount of heater current to keep the bridge output at zero.

The zinc samples used in these experiments were ones used for effective-mass study, and their preparation is discussed in detail in a previous paper.[1] Two samples were used and both had (10$\bar{1}$0) surfaces. The surfaces were optically flat and both samples exhibited an $\omega_{\text{rf}}\tau$ of approximately 40 at 4.2°K for the resonances studied.

III. EXPERIMENTAL RESULTS

In this section, we report the temperature dependence of the cyclotron effective masses for three orbits in zinc. These orbits lie on that part of the Fermi surface called the lens which is an electron surface in the third band of zinc, as shown in Fig. 1. The three orbits that were studied as a function of temperature were the limit point on the top and bottom of the lens and the two central orbits on the lens that are observed when the magnetic field is perpendicular and parallel to the ⟨0001⟩ axis.

The theory by Grimvall[1] of the temperature dependence of the effective mass and the calculations of Allen and Cohen in the following paper use a spherical Fermi surface. For this reason, orbits on the lens were used in the study presented here, since the lens has a very nearly-free-electron shape. The studies by Shaw *et al.*[5,9] of the effective mass anisotropy of the central orbit on the lens leads to the conclusion that the shape of the lens is very nearly that of two spherical caps with the sharp edges rounded off at the zone boundary. The results of a study by this author[1] of the mass anisotropy of the central orbit on the lens are presented in Fig. 2. Curve B in the figure is the free-electron mass for the central orbit on the lens given by the expression[9]

$$\frac{m^*}{m_0}=\frac{2}{\pi}\cos^{-1}\frac{\sin\theta}{[(k_F/b_3)^2-\cos^2\theta]^{1/2}}\,, \qquad (1)$$

where m^* is the cyclotron effective mass, m_0 is the free-electron mass, b_3 is the reciprocal-lattice vector along ⟨0001⟩, and θ is the angle between **H** and ⟨0001⟩. The solid curve A is this same expression, but it is normalized to the observed effective mass when **H** is perpendicular to ⟨0001⟩. As can be seen from Fig. 2, the observed effective masses fit Eq. (1) for two spherical caps except when **H** is closer than 12° from ⟨0001⟩. This suggests that the temperature dependence of orbits on the lens can be used to provide a meaningful comparison to the calculation of Allen and Cohen. The data in Fig. 2 will be used to extract needed band masses later in this discussion.

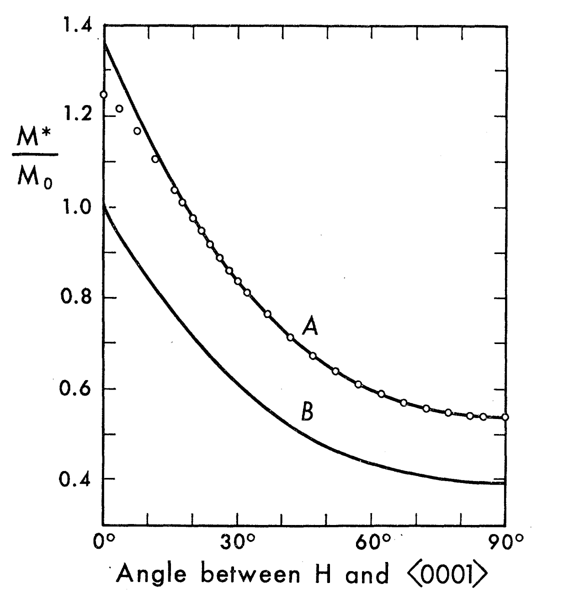

FIG. 2. Mass anisotropy of the central orbit on the lens. The points are measured effective masses. Curve B gives the effective masses for the free-electron model of two spherical caps. Curve A is Curve B normalized to the observed mass for **H** perpendicular to ⟨0001⟩.

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The temperature dependences of the cyclotron effective masses for three orbits are shown in Fig. 3 along with a theoretical curve of Allen and Cohen. In Fig. 4 of the following paper, three theoretical calculations are compared with the experimental points presented in this paper. Each data point is the ratio of the effective-mass enhancement due to all effects other than band structure, that is, we have plotted

$$\Delta m(T)/\Delta m(0)=[m^*(T)-m_b]/[m^*(0)-m_b], \quad (2)$$

where $m^*(T)$ is the measured cyclotron effective mass at the temperature T, m_b is the band mass for the orbit, and $m^*(0)$ is an extrapolated zero-temperature effective mass. We have presented our data in this form because the calculations in the following paper yield a quantity $\lambda(T)/\lambda(0)$ which, if we knew $m^*(0)$ and m_b exactly, would be the same as our $\Delta m(T)/\Delta m(0)$. When the measured $m^*(T)$ was plotted against T^2, a straight line was obtained and $m^*(0)$ was chosen by extrapolating this line to 0°K.

Several temperature effects that can change the cyclotron effective mass are now considered. For the zinc samples used in this study, $\omega_{rf}\tau$ begins to decrease rapidly above 6°K. Moore[10] has shown that a finite $\omega_{rf}\tau$ causes an error in the measured cyclotron effective mass and that this error is a function of the relaxation time τ. His calculations were based on the Azbel'-Kaner equation[11,12] describing cyclotron resonance in a metal with a quadratic Fermi surface. As a typical example, for $\omega_{rf}\tau$ of 40 at 4.2°K and 10 at 11°K, the increase in the measured mass would be

$$m^*(11°\text{K})-m^*(4.2°\text{K})=0.0013m_0^*,$$

where m_0^* is the effective mass for an infinite relaxation time. These very small corrections were applied to the data in Fig. 3, and they caused about a 5% change in the value of $\Delta m(T)/\Delta m(0)$ above 9°K. To apply these corrections, $\omega_{rf}\tau$ was estimated from the number of observable subharmonics in the resonance.

Another possible cause of a mass increase for an orbit on the lens is lattice parameter changes as a function of temperature. Using the spherical-caps model for the lens described by Eq. (1) and the low-temperature expansion data of White[13] for zinc, we estimate that the change in the effective mass of the central orbit on the lens due to lattice expansion is less than $5\times10^{-5}m_0$ between 0 and 11°K, so we can neglect corrections caused by the effects of lattice expansion in this temperature range.

The experimental results for each orbit are discussed below. The temperature dependence of the limit point mass on the lens is given by curve 1 in Fig. 3. The

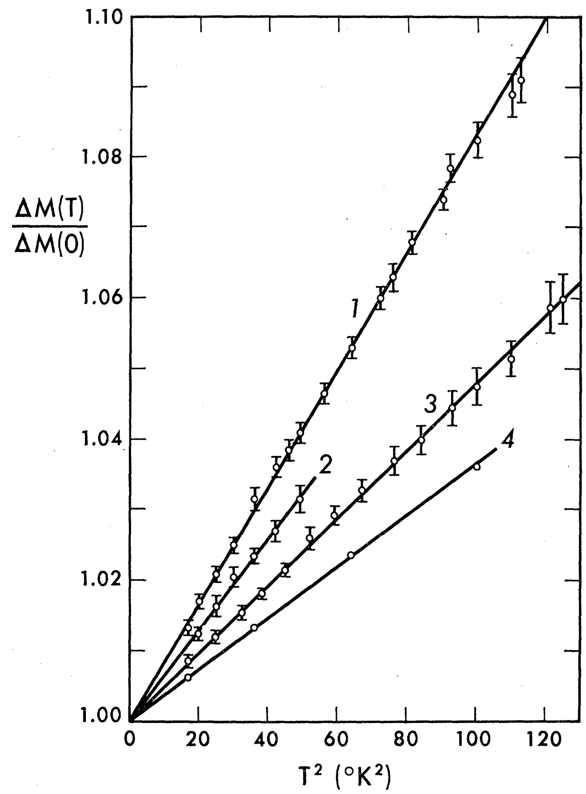

FIG. 3. Effective-mass enhancement as a function of T^2. One is the limit point orbit. Two and three are the central orbits on the lens with H parallel and perpendicular to ⟨0001⟩, respectively. Four is a theoretical curve of Allen and Cohen.

measured mass of this orbit increased from m^* of $1.3902m_0$ at 4.2°K to $1.4204m_0$ at 10.6°K. The extrapolated zero-temperature mass was determined to be $1.3852m_0$. Since the top and bottom of the lens are well away from the zone boundary, it is expected that the band mass would be m_0, and this value was used in Eq. (2). However, calculations in the following paper and elsewhere[14] give a zero-temperature mass enhancement of $0.42m_0$. This value then leads to a band mass of $0.965m_0$ and, if this value were used in Eq. (2), then the magnitude of the temperature increase of the limit point mass would be lowered. This lower band mass would then produce better agreement with the theoretical curve in Fig. 3. The temperature dependencies of the masses of the central orbits on the lens are given by curves 2 and 3 in Fig. 3. The orbit on the lens when **H** is perpendicular to the c axis exhibited the smallest temperature dependence. The resonance from this orbit exhibited 45 subharmonics at 4.2°K, which allowed a very accurate determination of the effective mass. For this orbit, the measured mass changed from m^* of $0.5374m_0$ at 4.2°K to $0.5450m_0$ at 11.2°K. The extrapolated mass at 0°K was taken as $0.5362m_0$. To extract the band mass, we used the value from the two-spherical-

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caps model (curve B of Fig. 2) of $0.3930m_0$. This value is probably too small because of the rounding off of the lens where it contacts the zone boundary. If a larger band mass were used in Eq. (2), then the magnitude of the temperature increase of mass of the orbit when **H** is perpendicular to the c axis would be increased. The second central orbit measured was with **H** parallel to the c axis and the results are presented by curve 2 in Fig. 3. The resonance from this orbit could only be followed to 7.0°K because of $\omega_{rf}\tau$ effects caused by the orbit lying on the zone boundary. The nearly-free-electron model predicts a band mass of m_0, but this model neglects the rounding of the lens at the zone boundary. As seen in Fig. 2, the measured mass value of $1.25m_0$ is lower than the predicted value of $1.37m_0$. We subtracted the defect of $0.12m_0$ from the free-electron value to account for the rounding of the lens and used a band mass of $0.88m_0$. The measured mass for this orbit changed from $1.2545m_0$ at 4.5°K to $1.2617m_0$ at 7.0°K and the extrapolated mass at 0°K was taken as $1.2500m_0$. We cannot attach too much significance to the relative ordering of the temperature increases of the masses for the two central orbits on the lens since selection of a proper band mass is rather arbitrary. Curve 4 in Fig. 3 is characteristic of the theoretical calculations in the following paper. Considering our uncertainties in determining a band mass and a zero temperature mass for each orbit the experimental results presented here are in good agreement with the theoretical calculations.

IV. CONCLUSIONS AND SUMMARY

The temperature dependence of the cyclotron effective mass has been measured for three orbits in zinc at temperatures up to 11°K. The origin of the temperature dependence of the effective mass was not $\omega_{rf}\tau$ or lattice effects and is shown in the following paper to be the electron-phonon interaction. Theoretical calculations by Allen and Cohen for the temperature dependence of the effective mass of a spherical Fermi surface are in fair agreement with the observed temperature dependence of the effective mass of an orbit. No attempt has been made to do a detailed calculation for orbits on the lens in zinc. The selection of a band mass was shown to be important in determining the absolute magnitude of the temperature increase in the mass enhancement but does not effect the experimental result that for temperatures up to 11°K the cyclotron effective mass increases as the square of the temperature.

ACKNOWLEDGMENTS

The author wishes to thank Professor A. F. Kip for many helpful discussions, W. Kolbe for calibrating the thermometer, and Professor M. L. Cohen for originally suggesting this problem. He is also grateful to P. B. Allen for many helpful discussions on the physical origin of the electron-phonon temperature dependence of the effective mass and for his collaboration in preparing this work.