Inelastic Light Scattering from Semiconductor Plasmas in a Magnetic Field*

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The cross section for the inelastic scattering of light from mobile carriers in semiconductors immersed in a dc magnetic field $\mathbf{B}_0$ is calculated approximately in a manner sufficiently general to include directly particle-particle Coulomb interactions (in the random-phase approximation) and energy-band structure of an arbitrary nature. The effect of Coulomb interactions on the momentum matrix elements occurring in the calculation is neglected, but is kept in the evaluation of the correlation function of a generalized electron-pair operator. The results encompass scattering from the various longitudinal magnetoplasma collective modes, and single-particle excitations between Landau levels and spin states. Resonant enhancement factors are automatically included, as are spin-orbit-induced effects such as scattering from spin-density fluctuations and spin waves. Low-temperature electrons in semiconductors of the indium antimonide type are used as a specific example to illustrate general features of the scattering for the two major geometries: $\mathbf{q}\perp\mathbf{B}_0$ and $\mathbf{q}\parallel\mathbf{B}_0$, where $\mathbf{q}$ is the scattering wave vector. For $\mathbf{q}\perp\mathbf{B}_0$, inter-Landau-level scattering is shown to suffer significant screening due to Coulomb interactions. Also, in this geometry it is shown that the strength of the scattering from the Bernstein modes is of the order of the strength for the associated inter-Landau-level excitation, contrary to the conclusions of previous authors.

I. INTRODUCTION

CONSIDERABLE interest has developed in recent years concerning the use of the inelastic scattering of light as a tool for studying the properties of mobile carriers in semiconductor crystals.[1] In particular, one class of measurements involves studies of crystals immersed in a magnetic field, the carriers in some unfilled energy band constituting a magnetoplasma whose properties one wishes to examine.[2–10] Theoretical calculations of cross sections for scattering from excitations in such semiconductor magnetoplasmas have fallen largely into two categories. First, there are calculations which begin with an effective-mass Hamiltonian and include particle-particle Coulomb interactions (which are treated in a self-consistent-field approximation).[4,5] The cross section resulting from such a calculation is generally expressed in terms of a response function which is proportional to the imaginary part of the reciprocal of an effective dielectric constant. These theories have the virtue of treating collective excitation phenomena (plasmons), but are inaccurate for photon energies of the order of the band-gap energy, and under conditions where effects such as energy-band nonparabolicity and spin-orbit coupling become important.[3] Indeed, effective-mass calculations overlook totally some important coupling mechanisms.[2,3] In a second category fall calculations which neglect Coulomb interactions altogether, but use a single-electron Hamiltonian which properly accounts for complex energy band structure and spin-orbit effects.[2,3] Band-structure effects can play a crucial and essential role, giving rise to scattering from excitations which would not be experimentally observable in a free electron gas.[2–10]

It is desirable to bridge the gap between the two categories described above, formulating a many-electron theory which retains important solid-state effects. The complexities of scattering from real semiconductor magnetoplasmas can only be dealt with in a consistent and realistic manner by using results based on such a formulation. The purpose of this article is to complement and/or extend the efforts made toward this end by Wolff,[6] and McWhorter and Argyres.[7] The cross section for the inelastic scattering of light from mobile carriers in semiconductors immersed in a magnetic field $\mathbf{B}_0$ is calculated approximately in a manner sufficiently general to include Coulomb interactions and arbitrary energy band structure. Sample calculations yield results which differ considerably from conclusions based on previous treatments and which are important for the interpretation of experimental data. For a low-temperature Fermi gas, the calculated cross section encompasses scattering from the various longitudinal magnetoplasma collective waves (including spin waves) and single-particle excitations between Landau levels and spin states.

McWhorter and Argyres[7] have discussed scattering from magnetoplasma waves in semiconductors with arbitrary band structure. However, they did not

* Work sponsored by the Department of the Air Force.

[1] See, for example, *Light Scattering Spectra of Solids*, edited by G. B. Wright (Springer-Verlag, New York, 1969).

[2] P. A. Wolff, Phys. Rev. Letters **16**, 225 (1966).

[3] Y. Yafet, Phys. Rev. **152**, 858 (1966); G. B. Wright, P. L. Kelley, and S. H. Groves, in Ref. 1, p. 335; V. P. Makarov, Zh. Eksperim. i Teor. Fiz. **55**, 704 (1968) [English transl.: Soviet Phys.—JETP **28**, 366 (1969)].

[4] A. L. McWhorter, in *Proceedings of the International Conference on Quantum Electronics, Puerto Rico, 1965* (McGraw-Hill Book Co., New York, 1965), p. 111; P. M. Platzman, Phys. Rev. **139**, A379 (1965).

[5] P. M. Platzman, P. A. Wolff, and N. Tzoar, Phys. Rev. **174**, 489 (1968).

[6] P. A. Wolff, Phys. Rev. **171**, 436 (1968).

[7] A. L. McWhorter and P. N. Argyres, in Ref. 1, p. 325.

[8] R. E. Slusher, C. K. N. Patel, and P. A. Fleury, Phys. Rev. Letters **18**, 77 (1967); C. K. N. Patel and R. E. Slusher, Phys. Rev. **167**, 413 (1968); C. K. N. Patel and R. E. Slusher, Phys. Rev. Letters **21**, 1563 (1968); C. K. N. Patel and R. E. Slusher, Phys. Rev. **177**, 1200 (1969); C. K. N. Patel, *Modern Optics* (Polytechnic Press, Brooklyn, N. Y., 1967), Vol. XVII, p. 19.

[9] S. S. Jha, Phys. Rev. **179**, 764 (1969); **182**, 815 (1969).

[10] B. S. Wherrett and P. G. Harper, Phys. Rev. (to be published).

consider scattering from single-particle excitations, a subject of considerable interest in view of experimental data.[8] Wolff[6] also has made one step toward the treatment of real semiconductors by performing a one-band effective Hamiltonian calculation which includes energy-band nonparabolicity effects and Coulomb interactions. But Wolff's treatment is only valid for weak magnetic fields and for incident photon energies small compared to the band-gap energy of the semiconductor. This limit is not appropriate for common experimental situations and some important phenomena are discarded in taking it. For example, in zero magnetic field, Wolff's theory fails to predict the quasielastic scattering from spin-density fluctuations as observed by Mooradian[11] and explained by Hamilton and McWhorter.[12] The calculations of this paper extend the treatment of magnetoplasma light scattering by adaptation of the Hamilton-McWhorter approach. Following Hamilton and McWhorter,[12] we neglect the effect of Coulomb interactions on the momentum matrix elements occurring in the calculation. Making this approximation, electro-optic scattering mechanisms are neglected. Concentrating on mobile carriers, we generally ignore the effect of phonons. However, a brief discussion of the coupling of longitudinal phonons and magnetoplasma waves in polar semiconductors is given.

In Sec. II of this paper a general formulation of the cross section calculation is given. Generality is maintained by expressing all results as sums, over appropriate carrier quantum numbers, of factors involving matrix elements, energies, and statistical occupation numbers for single-particle eigenstates. The quantum numbers include energy band index, orbital and spin labels in a completely general way. One merely need use the correct wave function from a one-electron band-theory calculation. The complexity of the wave functions for carriers in real semiconductors in a magnetic field necessitates such a general approach. Within the approximations made, the resulting differential cross section formula is valid both with and without a magnetic field, and reduces, in general, to the results of previous authors. Resonant enhancement factors are automatically included, as are spin-orbit-induced effects such as scattering from spin-density fluctuations and spin waves. Section III discusses briefly scattering from carriers in a simple, idealized semiconductor. The semiconductor is assumed to have mirror symmetry, parabolic energy bands, and a direct energy gap. Spin-orbit coupling is neglected. This simple semiconductor model is clearly an oversimplification, considering the calculations given in Sec. IV for semiconductors with a complex energy-band structure. However, the discussion of the simple case gives a valuable perspective on the treatment of real semiconductors. In Sec. IV semiconductors of the indium antimonide type are used as specific examples to illustrate general features for the two major scattering geometries, $\mathbf{q}\perp\mathbf{B}_0$ and $\mathbf{q}\|\mathbf{B}_0$, where $\mathbf{q}$ is the scattering wave vector. For $\mathbf{q}\perp\mathbf{B}_0$ inter-Landau-level scattering in a low-temperature Fermi plasma is shown to suffer significant screening due to Coulomb interactions. This screening is particularly dramatic for high magnetic fields. Also, in this geometry it is shown that the strength of the scattering from the Bernstein modes is of the order of the strength for the associated inter-Landau-level excitation, contrary to the conclusions of previous authors. The number and variety of excitations of the low-temperature Fermi gas, which may be observed by light scattering and which are encompassed by the theoretical treatment given here, is noteworthy. The excitations include: quasi-elastic single-electron excitations, acoustic plasmons, optical plasmons (including Bernstein modes), inter-Landau-level excitations, spin-flip excitations, and spin waves.

II. GENERAL FORMULATION

In this section we derive a general expression for the cross section for inelastic scattering of light by many electrons[13] in thermal equilibrium in a crystalline semiconductor with the Hamiltonian,

$$H=\sum_i H_0+\tfrac{1}{2}\sum_{i\neq j}\frac{e^2}{r_{ij}}, \tag{1}$$

which includes Coulomb interactions, and where

$$H_0=\left(\mathbf{p}-\frac{e}{c}\mathbf{A}_0\right)^2\Big/2m+V(\mathbf{r})$$
$$+\left[\hbar\left(\mathbf{p}-\frac{e}{c}\mathbf{A}_0\right)\cdot(\boldsymbol{\sigma}\times\boldsymbol{\nabla}\mathbf{V})\right]\Big/4m^2c^2+\tfrac{1}{2}g\beta\boldsymbol{\sigma}\cdot\mathbf{B}_0 \tag{2}$$

is the Hamiltonian for one mobile electron in the semiconductor. Preserving complete generality H_0 includes the effect of the crystal periodic potential through the electrostatic energy $V(\mathbf{r})$ and the spin-orbit coupling term. The static, uniform magnetic field $\mathbf{B}_0$ appears in H_0 through the associated vector potential $\mathbf{A}_0$ and the magnetic dipole energy term, where g is the free-electron g factor, β is the Bohr magneton, and $\boldsymbol{\sigma}$ is the Pauli-spin operator. One of the virtues of our treatment is that we will deal with the problem at hand without making specific approximation concerning the single-particle eigenstates $|\alpha\rangle$: $H_0|\alpha\rangle=E_\alpha|\alpha\rangle$. The symbol α will merely be taken to denote all appropriate quantum labels for the single-particle Hamiltonian H_0, including band label, spin, and orbital quantum numbers. In this way, we can use directly the complex wave functions for electrons in a semiconductor in a magnetic field, without resorting to special approximations. This

[11] A. Mooradian, Phys. Rev. Letters **20**, 1102 (1968).
[12] D. C. Hamilton and A. L. McWhorter, in Ref. 1, p. 309.

[13] Throughout the text we refer to electrons as the mobile carriers, having in mind doped n-type semiconductors. The analysis easily generalizes to the case of holes in p-type materials.

approach is in direct contrast to effective-mass[4,5] and nonparabolicity[6] calculations which replace Eq. (2) by an effective one-band Hamiltonian. The general procedures for deriving the results from perturbation theory follow closely the treatments of previous authors. What is new and useful in this treatment is the general form of the expressions for the cross section and their ramifications for the semiconductor magnetoplasma problem. Resonance-enhancement effects and one-electron effects such as nonparabolicity and spin-orbit coupling are automatically included. The expressions reduce in appropriate limits to the less general ones derived previously.[4–6]

In order to calculate the scattering cross section we must consider the coupling of the electromagnetic radiation field to the electron system. We will ignore the small magnetic dipole coupling and replace the momentum $\mathbf{p}$ in Eq. (2) by $[\mathbf{p}-(e/c)\mathbf{A}_\omega]$, where $\mathbf{A}_\omega$ is the vector potential of the electromagnetic field at the point $\mathbf{r}$. We take the semiconductor to be transparent to the incident and scattered radiation. We ignore, for the time being, coupling to phonons. This matter will be discussed at the end of this section. Treating $\mathbf{A}_\omega$ as a perturbation, the transition probability for the scattering of a photon from state $(\omega_I,\mathbf{k}_I,\boldsymbol{\varepsilon}_I)$ to state $(\omega_F,\mathbf{k}_F,\boldsymbol{\varepsilon}_F)$ and the concomitant transition of the many-electron system from the state $|I\rangle$ to the state $|F\rangle$ is given by matrix elements of operators of the form $\mathbf{A}_\omega^2$ and $\boldsymbol{\pi}\cdot\mathbf{A}_\omega$, where $\boldsymbol{\pi}=\mathbf{p}+(\hbar\boldsymbol{\sigma}\times\boldsymbol{\Delta}\mathbf{V})/(4mc^2)$ is the momentum including spin-orbit contribution. Here, the symbols $(\omega,\mathbf{k},\boldsymbol{\varepsilon})$ denote, respectively, the frequency, wave vector, and polarization of a photon. Treatment of the A_ω^2 term by first order perturbation theory is straightforward and "exact," leading to cross sections expressed in terms of a dynamic form factor[4,14] or dielectric constant. It is the handling of the $(\boldsymbol{\pi}\cdot\mathbf{A}_\omega)$ term in second order which is approximate in what follows. The general expression for the differential scattering cross section[15] is given by[12]

$$\frac{d^2\sigma}{d\Omega d\omega}=\hbar\frac{\omega_F}{\omega_I}\left\langle\sum_F|M_{FI}|^2\delta(E_F-E_I-\hbar\omega)\right\rangle, \tag{3}$$

where the angular brackets denotes a thermal equilibrium average over initial states and $\omega=(\omega_I-\omega_F)$. This is the total cross section for scattering from all the electrons in the volume illuminated by the incident beam, not the cross section per particle. For simplicity, this scattering volume is taken to be unity throughout. The matrix element M_{FI} is given by

$$M_{FI}=\frac{e^2}{mc^2}\Bigg[\boldsymbol{\varepsilon}_I\cdot\boldsymbol{\varepsilon}_F\sum_{\alpha,\alpha'}m_{\alpha\alpha'}\langle F|C_\alpha^\dagger C_{\alpha'}|I\rangle + \frac{1}{m}\sum_i\sum_{\substack{\alpha,\alpha'\\ \beta,\beta'}}\langle\alpha|j_I|\alpha'\rangle\langle\beta|j_F|\beta'\rangle \times\left(\frac{\langle F|C_\alpha^\dagger C_{\alpha'}|i\rangle\langle i|C_\beta^\dagger C_{\beta'}|I\rangle}{E_I-E_i+\hbar\omega_I} + \frac{\langle F|C_\beta^\dagger C_{\beta'}|i\rangle\langle i|C_\alpha^\dagger C_{\alpha'}|I\rangle}{E_I-E-\hbar\omega_F}\right)\Bigg], \tag{4}$$

where

$$\begin{Bmatrix}j_I\\ j_F\end{Bmatrix}=\begin{Bmatrix}\boldsymbol{\varepsilon}_I\\ \boldsymbol{\varepsilon}_F\end{Bmatrix}\cdot\left(\boldsymbol{\pi}-\frac{e}{c}\mathbf{A}_0\right)\exp\left[i\begin{Bmatrix}\mathbf{k}_I\\ -\mathbf{k}_F\end{Bmatrix}\cdot\mathbf{r}\right]$$

and

$$m_{\alpha\alpha'}=\langle\alpha|e^{i\mathbf{q}\cdot\mathbf{r}}|\alpha'\rangle.$$

The states $|I\rangle$, $|F\rangle$, and $|i\rangle$ are many electron eigenstates of H, $C_\alpha^\dagger$, and C_β are single-particle creation and destruction operations for the single-particle state $|\alpha\rangle$, and $\mathbf{q}$ is the difference between the incident and scattered wave vectors $\mathbf{q}\equiv(\mathbf{k}_I-\mathbf{k}_F)$. The quantity $m_{\alpha\beta}$ is the single-particle matrix element associated with density fluctuations. The first part (proportional to $\boldsymbol{\varepsilon}_I\cdot\boldsymbol{\varepsilon}_F$) of M_{FI} derives from the A_ω^2 term treated in first order, while the second part derives from the $\boldsymbol{\pi}\cdot\mathbf{A}_\omega$ term in second order. Equations (3) and (4) are exact as they stand within the framework of perturbation theory. However, further calculation as things stand is a formidable problem.

Following Hamilton and McWhorter[12] we approximate M_{FI}. At this point we concentrate our attention on scattering transitions such that the states $|I\rangle$ and $|F\rangle$ differ only by excitation of electrons within a given band of the semiconductor, e.g., the conduction band. The effect of Coulomb interactions on the momentum matrix elements and energy denominators appearing in Eq. (4) will be neglected. It is assumed that the intermediate many-particle state differs from the initial and final states only by a single particle excitation. Making these approximations, we discard electro-optic contributions to the scattering cross section.[7,12,16] Considering first scattering from one-band single-particle excitations, one finds[17]

$$M_{FI}=\sum_{\alpha,\beta}\gamma_{\alpha\beta}\langle F|C_\alpha^\dagger C_\beta|I\rangle, \tag{5}$$

where

$$\gamma_{\alpha\beta}=\frac{e^2}{mc^2}\Bigg[m_{\alpha\beta}\boldsymbol{\varepsilon}_I\cdot\boldsymbol{\varepsilon}_F+\frac{1}{m}\sum_{\beta'}\left(\frac{\langle\alpha|j_F|\beta'\rangle\langle\beta'|j_I|\beta\rangle}{E_\beta-E_{\beta'}+\hbar\omega_I} + \frac{\langle\alpha|j_I|\beta'\rangle\langle\beta'|j_F|\beta\rangle}{E_\alpha-E_{\beta'}-\hbar\omega_I}\right)\Bigg]. \tag{6}$$

[14] See, for example, D. Pines and P. Nozieres, *The Theory of Quantum Liquids, I* (W. A. Benjamin, Inc., New York, 1966), p. 266.

[15] This cross section multiplied by the incident photon flux (photons per second per cm^2) yields the number of photons scattered per second into the solid-angle increment $[\Omega,\Omega+d\Omega]$ and frequency range $[\omega,\omega+d\omega]$. The power scattering cross section is obtained by multiplying the right-hand side of Eq. (3) by (ω_F/ω_I).

[16] A. Mooradian and A. L. McWhorter, in Ref. 1, p. 297.

[17] It is assumed that there is only one partially filled energy band.

Using closure, we have eliminated the intermediate many-particle state from the expressions. The matrix element is now much simpler in form and involves the electron pair operator $C_\alpha^\dagger C_\beta$. Note that this result is exact for noninteracting particles. We keep the particle interactions only through evaluation of a correlation function for the pair operator.

Considering next scattering from one-band plasma collective modes, we find Eq. (5) for M_{FI} subject, however, to the following restrictions: We replace the energy denominators in Eq. (6) by $(E_g \pm \hbar\omega_I)$ for interband intermediate-state contributions to γ and neglect the intraband intermediate-state contributions to γ which are usually small[3,12] compared to interband terms. These replacements and the resulting expressions are appropriate only if the characteristic single-particle and collective-mode energies are small compared to $(E_g - \hbar\omega_I)$, and only if $|\omega| \ll \omega_I$. Thus, we use the expression

$$\gamma_{\alpha\beta} = \frac{e^2}{mc^2}\left[m_{\alpha\beta}\boldsymbol{\varepsilon}_I \cdot \boldsymbol{\varepsilon}_F + \frac{1}{m}\sum_{\beta'}\left(\frac{\langle\alpha| j_F |\beta'\rangle\langle\beta'| j_I |\beta\rangle}{E_g + \hbar\omega_I} + \frac{\langle\alpha| j_I |\beta'\rangle\langle\beta'| j_F |\beta\rangle}{E_g - \hbar\omega_I} \right) \right] \quad (7)$$

for evaluation of scattering from collective modes, where the sum over β' includes only states outside the band of interest. Note that the single-particle excitation form of γ [Eq. (6)] becomes that given in Eq. (7) if we apply the approximations made for the collective-mode scattering. When the two forms of scattering are mixed (the collective modes are not well defined), the latter form for γ [Eq. (7)] should be used. The fact that there are two forms for γ does not affect the rest of the calculation which proceeds using γ merely as a parameter. It is interesting to note that γ is exactly the matrix element which is calculated in single-electron scattering theories.[2,3,9,10] It is particularly significant that this one-electron matrix element appears as a parameter in the results given here; the results of the previous calculations which include complicated band-structure effects can be incorporated directly.

Proceeding with the calculation, we express the cross section in terms of the Fourier transform of a correlation function for the generalized pair operator N, $(N \equiv \sum_{\alpha,\beta} \gamma_{\alpha\beta} C_\alpha^\dagger C_\beta)$,

$$\frac{d^2\sigma}{d\Omega d\omega} = \hbar\frac{\omega_F}{\omega_I}\int_{-\infty}^{\infty} \tfrac{1}{2} dt\, e^{i\omega t}\langle N^\dagger(t) N(0)\rangle. \quad (8)$$

Assuming the electrons are in thermal equilibrium, we use the fluctuation dissipation theorem[18] to find

$$\frac{d^2\sigma}{d\Omega d\omega} = \hbar\frac{\omega_F}{\omega_I}\frac{(n_\omega+1)}{\pi}\,\mathrm{Im}\int_{-\infty}^{\infty} dt\, e^{i\omega t} \left[-i\theta(t)\langle [N^\dagger(t), N(0)]\rangle \right], \quad (9)$$

where $n_\omega = [\exp(\hbar\omega/kT) - 1]^{-1}$ is the Einstein-Bose occupation factor. Defining $G(t) \equiv -i\theta(t)\langle [N^\dagger(t), N(0)]\rangle$ and $g_{\alpha\beta}$ by

$$G \equiv \sum_{\alpha,\beta} \gamma_{\alpha\beta}^* g_{\alpha\beta},$$

then writing the equation of motion for $g_{\alpha\beta}$ in the random-phase approximation (RPA) yields

$$g_{\alpha\beta} = \gamma_{\alpha\beta}\left(\frac{n_\beta - n_\alpha}{\hbar\omega + E_\beta - E_\alpha} \right) + \frac{4\pi e^2}{q^2}\left(\frac{n_\beta - n_\alpha}{\hbar\omega + E_\beta - E_\alpha} \right) m_{\alpha\beta} \sum_{i,j} m_{ij}^* g_{ij}, \quad (10)$$

where n_α is the thermal-equilibrium occupation number for the state $|\alpha\rangle$. We neglect relaxation effects. A phenomenological collision time can be included when necessary. Solving the integral equation for $g_{\alpha\beta}$ gives[19]

$$\frac{d^2\sigma}{d\Omega d\omega} = \hbar\frac{\omega_F}{\omega_I}\frac{(n_\omega+1)}{\pi}\,\mathrm{Im}\left(L_2 + \frac{4\pi e^2}{q^2}\frac{L_1 \tilde{L}_1}{\epsilon} \right), \quad (11)$$

where

$$L_2 = \sum_{\alpha,\beta} |\gamma_{\alpha\beta}|^2 \left(\frac{n_\beta - n_\alpha}{\hbar\omega + E_\beta - E_\alpha} \right), \quad (12)$$

$$L_1 = \sum_{\alpha,\beta} \gamma_{\alpha\beta} m_{\alpha\beta}^* \left(\frac{n_\beta - n_\alpha}{\hbar\omega + E_\beta - E_\alpha} \right), \quad (13)$$

$$\tilde{L}_1 = \sum_{\alpha,\beta} \gamma_{\alpha\beta}^* m_{\alpha\beta} \left(\frac{n_\beta - n_\alpha}{\hbar\omega + E_\beta - E_\alpha} \right), \quad (14)$$

and

$$\epsilon(\omega, \mathbf{q}) = 1 - \frac{4\pi e^2}{q^2}\sum_{\alpha,\beta} |m_{\alpha\beta}|^2 \left(\frac{n_\beta - n_\alpha}{\hbar\omega + E_\beta - E_\alpha} \right). \quad (15)$$

In the above equations, $\epsilon(\omega, \mathbf{q})$ is the longitudinal dielectric constant of the valence electron gas, including *interband* contributions to the sum over (α,β). These interband terms occur naturally in the solution of Eq. (10) and are important, since they yield the optical dielectric constant ϵ_∞ of the semiconductor.[20]

Although detailed evaluation of the cross section as given by Eqs. (11)–(15) can be rather involved and complex in practice, there are several rather general and important features of these equations. If we neglect Coulomb interactions $(4\pi e^2/q^2) \to 0$, we find

$$\frac{d^2\sigma}{d\Omega d\omega} = \hbar\frac{\omega_F}{\omega_I}\frac{(n_\omega+1)}{\pi}\,\mathrm{Im}\left[\sum_{\alpha,\beta} |\gamma_{\alpha\beta}|^2 \left(\frac{n_\beta - n_\alpha}{\hbar\omega + E_\beta - E_\alpha} \right) \right],$$

with $\gamma_{\alpha\beta}$ given by Eq. (6). This equation is a completely general and correct expression for noninter-

[18] D. N. Zubarev, Usp. Fiz. Nauk **71**, 71 (1960) [English transl.: Soviet Phys.—Usp. **3**, 320 (1960)].

[19] The derivation of the cross section given here neglects coupling to transverse current fluctuations (see Ref. 4) and therefore treats the collective modes of the plasma in the longitudinal wave approximation.

[20] D. Pines, *Elementary Excitations in Solids* (W. A. Benjamin, Inc., New York, 1964), p. 174.

acting electrons, including a simple generalization to include *interband* scattering processes. The second term in the brackets in Eq. (11) results from Coulomb interactions and gives rise to two significant effects. First, scattering from certain types of single-particle excitations as embodied in $\mathrm{Im}(L_2)$, are screened by the Coulomb term. Second, the Coulomb term introduces scattering from collective modes which occur at the zeros of the dielectric constant $\epsilon(\omega,\mathbf{q})=0$. Equations (11)–(15) have formal similarity to those given originally by Wolff[6] in considering nonparabolicity induced scattering phenomena. Their general form is noteworthy, and it is particularly significant that matrix elements involving the exact eigenstates of the general one-electron Hamiltonian appear in these equations.

The general expressions given above reduce to the less general ones derived by previous authors in appropriate limits. For plane-wave Bloch states (zero magnetic field), Eqs. (11)–(15) reduce to those given by Hamilton and McWhorter.[12] Taking the limit of zero incident and scattered light frequencies (photon energies small compared to the band-gap energy), and considering states near the band edge of a semiconductor, one finds the result of calculations which use a one-band effective-mass Hamiltonian.[4,5] For weakly nonparabolic bands the cross-section expressions reduce to those derived by Wolff.[6] For finite light frequency ($\hbar\omega_I \lesssim E_g$) and small frequency shifts ($\omega \ll \omega_I$), the interband terms in γ have enhancement[2] factors of the approximate form $[E_g/(E_g-\hbar\omega_I)]$, where E_g is the semiconductor band-gap energy. The cross sections for all scattering processes (single particle and collective mode) tend to show this resonant enhancement as $\hbar\omega_I$ approaches E_g.

Finally, we mention briefly the effect of lattice vibrations on the spectra of scattered light. For many common semiconductors one expects first-order inelastic scattering from zone-center phonon modes. Infrared-active longitudinal-optical phonons will be coupled to the longitudinal plasma waves. This coupling to longitudinal plasma waves ($B_0=0$) has been observed and studied in detail in GaAs.[16,21] Being consistent with our neglect of electro-optic-induced scattering phenomena, one can approximately account[22] for the effect of the phonons in polar semiconductors by adding to Eq. (15) for ϵ,

$$\epsilon_\infty\left(\frac{\omega_l^2-\omega_t^2}{\omega_t^2-\omega^2}\right),$$

where $\omega_l(\omega_t)$ is the zone-center longitudinal (transverse) optical-phonon frequency. This replacement is appropriate for the III-V compound semiconductors at which much of the discussion is aimed. Using this procedure, we neglect polaron effects[23] on the one-electron energies, i.e., we assume $\omega_c,\ \omega_s \neq \omega_l$. The result for the problem being considered here will be coupled phonon-magnetoplasma modes.[24] However, we will generally neglect this coupling in what follows since it can be included in a straightforward manner when necessary. It must be emphasized that this procedure is sufficient only for scattering geometries for which electro-optic mechanisms are negligible. Direct phonon scattering processes are not included; all scattering strengths go to zero as the mobile electron density goes to zero.

III. SIMPLE SEMICONDUCTORS

Before giving detailed examples of the usefulness for complex semiconductors of the expressions derived in Sec. II, we consider briefly scattering from electrons in a simple idealized semiconductor. We define a simple semiconductor to be one in which the energy bands are parabolic and spherically symmetric, and in which spin-orbit coupling is unimportant. Only two energy bands are considered of importance (a conduction band and a valence band). These two bands are assumed to have a direct energy gap E_g at the zone center and effective masses of magnitude m^* [we assume $(E_F/E_g) \ll 1$]. For spherical bands, this model is one step beyond the one-band effective-mass calculations.[4,5] The matters discussed here give good contrast to the results for more complex semiconductors discussed in later sections. Some important properties not previously pointed out will be discussed.

For small incident photon energies ($\hbar\omega_I < E_g$), small frequency shifts ($|\omega| \ll \omega_I$), and small effective mass $[(m^*/m) \ll 1]$, the matrix element $\gamma_{\alpha\beta}$ can be shown to be given by

$$\gamma_{\alpha\beta} \approx \left(\frac{E_g^2}{E_g^2-(\hbar\omega_I)^2}\right)\left(\frac{e^2}{m^*c^2}\right)\boldsymbol{\varepsilon}_I\cdot\boldsymbol{\varepsilon}_F m_{\alpha\beta}.$$

The scattering cross section [Eq. (11)] then becomes

$$\frac{d^2\sigma}{d\Omega d\omega} \approx -\frac{\omega_F}{\omega_I}\left(\frac{E_g^2}{E_g^2-(\hbar\omega_I)^2}\right)^2\frac{(n_\omega+1)}{\pi}(\boldsymbol{\varepsilon}_I\cdot\boldsymbol{\varepsilon}_F)^2 \times\left(\frac{e^2}{m^*c^2}\right)^2\left(\frac{\hbar q^2}{4\pi e^2}\right)\mathrm{Im}\left(\frac{1}{\epsilon}\right), \quad (16)$$

and shows the resonant enhancement mentioned in Sec. II. Equation (16) is general and valid or without a magnetic field. Apart from the resonant enhancement factor, Eq. (16) has the form expected for a free-electron gas with a mass m^*. Note that all scattering processes have the polarization selection rule $\boldsymbol{\varepsilon}_I\cdot\boldsymbol{\varepsilon}_F$ and involve charge density fluctuations. The effect of the magnetic field is buried in the longitudinal dielectric

[21] A. Mooradian and G. B. Wright, Phys. Rev. Letters **16**, 999 (1966); A. Mooradian and A. L. McWhorter, *ibid.* **19**, 849 (1967).
[22] B. B. Varga, Phys. Rev. **137**, A1896 (1965).
[23] See, for example, E. J. Johnson and D. M. Larsen, Phys. Rev. Letters **16**, 655 (1966); D. M. Larsen and E. J. Johnson, J. Phys. Soc. Japan Suppl. **21**, 433 (1966).
[24] See, for example, S. Iwasa, in *Physics of Solids in Intense Magnetic Fields*, edited by E. D. Haidemenakis (Plenum Press, Inc., New York, 1969), p. 126.

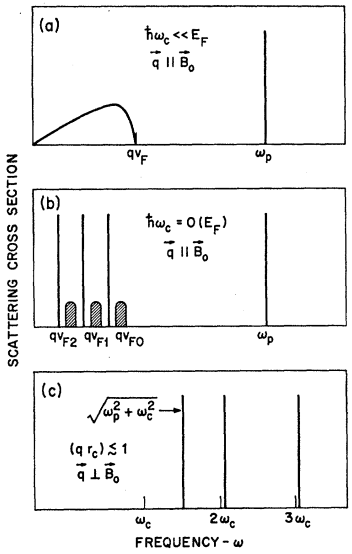

FIG. 1. Schematic display showing qualitative features of the long-wavelength scattering from a low-temperature Fermi plasma in the model semiconductor discussed in Sec. III. For (a) and (b), $\omega_p > qv_F$. For (c), $\omega_p \lesssim \omega_c$.

constant which has been calculated for both Fermi[25] and Maxwell[26] gases. The two vectors $\mathbf{q}$ and $\mathbf{B}_0$ lead to two characteristic observational geometries: (a) $\mathbf{q} \| \mathbf{B}_0$ and (b) $\mathbf{q} \perp \mathbf{B}_0$.

For $\mathbf{q} \| \mathbf{B}_0$, the cross section will exhibit [see Figs. 1(a) and 1(b)] scattering peaks in the range $0 < \omega \lesssim q_{\|} v_F$, where v_F is the Fermi velocity (thermal velocity for a Maxwellian electron plasma) corresponding to a Fermi energy E_F, and at the electron plasma frequency ω_p.[4,6,11,12] Here, the long-wavelength limit $(q_{\|} v_F/\omega_p) \ll 1$ is assumed. These two characteristic peaks have been observed $(B_0 = 0)$.[11,16] For $\hbar\omega_c \ll E_F, \kappa T$ the so-called quasielastic scattering in the range $0 < \lesssim q_{\|} v_F$ is a broad smooth peak covering this entire range. [The symbol ω_c denotes the electron cyclotron frequency (eB_0/m^*c).] It results from the scattering of single electrons from inside to outside the Fermi surface. For a simple semiconductor plasma the quasielastic scattering strength is very small.[4,6] Complex band-structure effects can increase this strength to an observable level.[6,11,12] For a Fermi plasma with a well-defined Fermi surface $(E_F \gg \kappa T)$ and with $\hbar\omega_c = 0(E_F)$, the quasielastic scattering spectrum will begin to split into a series of separated peaks as shown in Fig. 1(b). There is one peak for each distinct energy level lying below the Fermi surface. Assuming that $q \ll k_F$, where $k_F = (2m^* E_F)^{1/2}/\hbar$, the peaks are centered at qv_{Fn}, where $v_{Fn} = [2/m^*]^{1/2}[E_F - n\hbar\omega_c]^{1/2}$ is the velocity along $\mathbf{B}_0$ of an electron at the Fermi surface in the nth distinct Landau energy level. The nth peak in Fig. 1(b) results from the scattering of a single electron on one Landau level from inside to outside the Fermi surface. Under these conditions there are also collective modes in the frequency range $(0, qv_F)$. These modes are acoustic plasma oscillations[27] which result from motion of the carriers on the various Landau levels in such a way as to preserve charge neutrality. In the long-wavelength limit, these acoustic modes occur at frequencies between the single-particle excitation peaks [see Fig. 1(b)]. In effect, electrons in different Landau levels act as a different kind of carriers as is the case with classical ion-acoustic modes.

The other characteristic geometry $\mathbf{q} \perp \mathbf{B}_0$ also has interesting features. For this configuration (and this level of approximation) when $q_{\perp} r_c \lesssim 1$ the plasma will exhibit scattering *only* from collective modes, the hybrid (magnetoplasma wave) mode near $(\omega_c^2 + \omega_p^2)^{1/2}$, and the Bernstein modes near $N\omega_c (N \geq 2)$ [see Fig. 1(c)]. Here, r_c is an appropriate characteristic cyclotron radius. This is true for both Maxwell[5] and Fermi gases. There is no scattering intensity due to single-particle excitations. For example, for the low-temperature Fermi plasma there is no inter-Landau-level scattering. In this regime, the single-particle scattering is *completely screened* by the Coulomb interactions. This screening is apparently a general feature of the RPA treatment of Coulomb interactions and will be discussed further in the sections on III-V semiconductors. In the long-wavelength limit $q_{\perp} r_c \ll 1$, the scattering from the Bernstein modes is weak $[\propto (q_{\perp}^2 r_c^2)^N]$ compared to that from hybrid mode except for ranges of parameters where the two types of modes are strongly coupled.[5] This result does not necessarily follow for more complicated semiconductors (see Sec. IV). For $q_{\perp} r_c \to \infty$, the scattering goes over smoothly to a set of peaks at $\omega = \omega_c, 2\omega_c, 3\omega_c$, etc. The electrons now scatter as if they were a set of independent particles.[5]

For $\mathbf{q}$ oblique to $\mathbf{B}_0$ and $\omega_c > q_{\|} v_F$, Eq. (16) does predict inter-Landau-level scattering centered at $\omega = N\omega_c$. However, the cross section is weak in the long-wavelength limit, going to zero both as $q_{\perp} \to 0$ and as $q_{\|} \to 0$.

IV. COMPLEX SEMICONDUCTORS

In this section we will illustrate the usefulness of Eqs. (11)–(15) by example calculations for electrons in complex semiconductors. Spin-orbit coupling and other energy band complexities will not be discarded. Light scattering in such real semiconductors couples to a considerably greater variety of excitations than scattering for the ideal semiconductor of Sec. III. Detailed and/or quantitative computations will not

[25] N. D. Mermin and E. Canel, Ann. Phys. (N. Y.) **25**, 247 (1964); M. J. Stephen, Phys. Rev. **129**, 997 (1963).
[26] I. B. Bernstein, Phys. Rev. **109**, 10 (1958).

[27] A. L. McWhorter and W. G. May, IBM J. Res. Develop. **8**, 285 (1965); S. L. Ginzburg, O. V. Konstantinov, and V. I. Perel, Fiz. Tverd. Tela **9**, 2139 (1967) [English transl.: Soviet Phys.—Solid State **9**, 1684 (1968)]; O. V. Konstantinov and V. I. Perel, Zh. Eksperim i. Teor. Fiz. **53**, 2034 (1967) [English transl.: Soviet Phys.—JETP **26**, 1151 (1968)]; G. Benford and D. Book, Phys. Rev. Letters **21**, 898 (1968).

be made. The cross-section expressions given in Sec. II are complicated; direct computation for a given material and experimental geometry can be quite tedious. The evaluation for given materials of the parameter γ alone has been the subject of a number of authors.[2,3,9] However, we will find it possible to discern some important and rather general features. We generally have in mind in the following discussion scattering from conduction electrons in a nondegenerate spherical bands and will use the band models for the III-V semiconductor compounds of the InSb type for specific points, We will also consider a low-temperature ($\kappa T \ll E_F, \hbar\omega_c$), degenerate gas unless otherwise stated. The nature of the scattering and the excitations involved will be discussed for the two geometries, $\mathbf{q}$ perpendicular and $\mathbf{q}$ parallel to $\mathbf{B}_0$.

Before going on to the discussion of the two characteristic geometries, it is useful to find more explicit expressions for the variables appearing in the cross-section expressions given in Sec. II. We assume that the energy bands of the material being considered have at least cylindrical symmetry about the direction of the magnetic field. Then the wave functions can be written[28] as a linear combination of products of functions of the form $|\alpha\rangle = \sum_l u_l(r) f_{l\alpha}(r)$, where the u_l have the periodicity of the crystal (typically, the u_l are band edge functions) and the $f_{l\alpha}$ are Landau level wave functions of the form

$$f_{l\alpha} \simeq e^{ik_z z} e^{ik_y y} \Phi_{ml\alpha}(x - x_0).$$

Here, $\Phi_{ml\alpha}$ is the one-dimensional harmonic-oscillator wave function, k_z is the quantum number giving the electron momentum along the magnetic field $\mathbf{B}_0 = B_0\hat{z}$, and k_y is the quantum number giving the electron-orbit center $x_0 = (\hbar k_y c/eB_0)$. Under these conditions, the energies E_α are independent of k_y, as is the case for a free electron. The plane-wave nature of the f_l in the y and z directions allows one to perform directly some of the matrix element integrations and then some of the quantum-number sums of k_y in Eqs. (11)–(15). For $m_{\alpha\beta}$ we find

$$m_{\alpha\beta} = \delta(k_{\alpha z} - k_{\beta z} - q_z)\delta(k_{\alpha y} - k_{\beta y} - q_y) e^{-i\bar{q}_x k_{\beta y}/2} \times (2\pi)^2 \langle \alpha; x - \bar{q}_y | e^{iq_x x} | \beta; x \rangle, \quad (17)$$

where $|\alpha; x - \bar{q}_y\rangle$ denotes

$$\sum_l u_l(r) f_{l\alpha}'\left(x - \frac{\hbar q_y c}{eB_0}\right)$$

where f_l' is f_l with $k_y = 0 = k_z$. Then assuming that the largest terms in $\gamma_{\alpha\beta}$ result from the sum over interband matrix elements of $[\boldsymbol{\pi} - (e/c)\mathbf{A}_0]$,[29] yields

$$\gamma_{\alpha\beta} = \frac{e^2}{mc^2}\Bigg[m_{\alpha\beta}\boldsymbol{\varepsilon}_I \cdot \boldsymbol{\varepsilon}_F + (2\pi)^4 \delta(k_{\alpha z} - k_{\beta z} - q_z)\delta(k_{\alpha y} - k_{\beta y} - q_y) \times \sum_i \Bigg(\frac{\langle \alpha; x - \bar{q}_y | \boldsymbol{\varepsilon}_F \cdot \mathbf{p} e^{-i\mathbf{k}_F \cdot \mathbf{r}} | i; x - \bar{k}_{Iy} \rangle \langle i; x - \bar{k}_{Iy} | \boldsymbol{\varepsilon}_I \cdot \mathbf{p} e^{i\mathbf{k}_I \cdot \mathbf{r}} | \beta; x \rangle}{E_\beta - E_i(k_{\beta z} + k_{Iz}) + \hbar\omega_I} + \frac{\langle \alpha; x - \bar{q}_y | \boldsymbol{\varepsilon}_I \cdot \mathbf{p} e^{i\mathbf{k}_I \cdot \mathbf{r}} | i; x + \bar{k}_{Fy} \rangle \langle i; x + \bar{k}_{Fy} | \boldsymbol{\varepsilon}_F \cdot \mathbf{p} e^{-i\mathbf{k}_F \cdot \mathbf{r}} | \beta; x \rangle}{E_\alpha - E_i(k_{\beta z} - k_{Fz}) - \hbar\omega_I} \Bigg) \Bigg]. \quad (18)$$

It has been assumed that the wavelength of the light is long enough that factors of the form $e^{i\mathbf{k}\cdot\mathbf{r}}$ are slowly varying over a unit cell, and that $\mathbf{p}$ operates only on cell periodic functions.

Within the framework of usual semiconductor band-structure-calculation wave functions, Eqs. (17) and (18) are general. These two expressions can then be used with Eqs. (11)–(15) to calculate the scattering cross section. However, it is instructive to specialize the notation slightly for further calculations. We are interested in scattering from one-band excitations. Generally, even when effects such as spin-orbit coupling and nonparabolicity are strong, we can label[28] the states $|\alpha\rangle$ in a meaningful way by the set of free-electron quantum numbers (n, k_y, k_z, σ), where σ denotes spin component along the magnetic field. This labeling scheme is particularly appropriate for conduction electrons in the III-V semiconductors.[28] For low electron density and magnetic field these quantum numbers are just those of the effective-mass (and effective g-factor) states. Equations (12)–(15) become

$$L_2 = \sum_{\substack{n,n' \\ \sigma,\sigma'}} \int \frac{dk_z}{2\pi} |\gamma(n, k_z, \sigma; n', k_z + q_z, \sigma')|^2 \times \left(\frac{\rho(n, k_z, \sigma) - \rho(n', k_z + q_z, \sigma')}{\hbar\omega + E(n, k_z) - E(n', k_z + q_z, \sigma')} \right), \quad (19)$$

$$L_1 = \sum_{\substack{n,n' \\ \sigma,\sigma'}} \int \frac{dk_z}{2\pi} \gamma(n, k_z, \sigma; n', k_z + q_z, \sigma') m_{n\sigma, n'\sigma'}^* \times \left(\frac{\rho(n, k_z\sigma) - \rho(n', k_z + q_z, \sigma')}{\hbar\omega + E(n, k_z, \sigma) - E(n', k_z + q_z, \sigma')} \right), \quad (20)$$

[28] Y. Yafet, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1963), Vol. 14, p. 1.

[29] As a practical matter it is usually a good approximation to replace $[\boldsymbol{\pi} - (e/c)\mathbf{A}_0]$ by $\mathbf{p}$ in calculating interband matrix elements. See, C. Kittel, *Quantum Theory of Solids* (John Wiley & Sons, Inc., New York, 1964), p. 281.

$$L_1=\sum_{\substack{n,n'\\ \sigma,\sigma'}}\int\frac{dk_z}{2\pi}\gamma^*(n,k_z,\sigma;n',k_z+q_z,\sigma')m_{n\sigma,n'\sigma'}\times\left(\frac{\rho(n,k_z,\sigma)-\rho(n',k_z+q_z,\sigma')}{\hbar\omega+E(n,k_z,\sigma)-E(n',k_z+q_z,\sigma')}\right),\quad(21)$$

and

$$\epsilon=1-\frac{4\pi e^2}{q^2}\sum_{\substack{n,n'\\ \sigma,\sigma'}}\int\frac{dk_z}{2\pi}|m_{n\sigma,n'\sigma'}|^2\times\left(\frac{\rho(n,k_z,\sigma)-\rho(n',k_z+q_z,\sigma')}{\hbar\omega+E(n,k_z,\sigma)-E(n',k_z+q_z,\sigma')}\right),\quad(22)$$

where

$$\rho=\int\frac{dk_y}{2\pi}n_\alpha$$

is the number of electrons with quantum numbers (n,k_z,σ). These equations are now in a relatively workable form, for example, calculations. Similar equations derived by Wolff[6] for nonparabolicity-induced scattering are a special case of the more general Eqs. (19)–(22). We have immediately features which differentiate scattering from complex semiconductors from the free-electron case and special cases such as those considered by Wolff[6] and others.[3–5] The spin variables σ in Eqs. (19)–(22) cannot be summed over in the usual direct manner. Due to spin-orbit coupling, the matrix elements γ and m are not necessarily diagonal in σ. The excitation light and the Coulomb interactions can flip an electron's spin. Inelastic scattering due to single-electron spin-flip excitations is a strong and well-known process.[3,8,9] The importance of Coulomb spin-flip excitations has only been recently pointed out[30] and will be discussed below.

A. Scattering Parallel to Magnetic Field ($q\|B_0$)

When the scattering wave vector is parallel to the magnetic field, the matrix element $m_{\alpha\beta}$ conserves Landau level and spin quantum numbers, $m_{\alpha\alpha'}\propto\delta_{nn'}\delta_{\sigma\sigma'}$. However, this is not true in general for γ. In fact, $\gamma_{n\sigma,n'\sigma'}$ is nonzero in the III-V semiconductors for $\Delta n\lesssim 2$, $\Delta\sigma=0$ and $\Delta n=0$, $\Delta\sigma=1$ even in the dipole approximation ($q=0$). This means that there are inter-Landau-level ($\omega=\Delta n\omega_c$) scattering contributions to L_2 which are (to a first approximation) independent of $\mathbf{q}$. This is to be contrasted with the free-electron gas which exhibits no inter-Landau scattering for $\mathbf{q}\|\mathbf{B}_0$ and no spin-flip scattering in any geometry. The second term on the right-hand side of Eq. (11) makes no contribution to the Landau level or spin-flip scattering. Thus, the strength can be calculated using the results of one-electron calculations,[2,3] merely adding statistical weighting factors. Also, in this geometry when $\omega_p>qv_F$ one expects plasmon ($\omega\approx\omega_p$) scattering, which results from a zero of ϵ in the second term on the right-hand side of Eq. (11), $d\sigma\propto\mathrm{Im}(1/\epsilon)$. The first term ($L_2$) gives no contribution. Thus, for the cases considered above the first and the second terms give separate and independent contributions to the pertinent type of scattering: single-particle inter-Landau level and spin-flip scattering, on the one hand, and collective mode (plasmon) scattering, on the other. This is also true for scattering from the acoustic plasmons (see Sec. II) which derives from a zero of ϵ. This is not true for the quasielastic single-particle scattering; both terms in Eq. (11) contribute. The second term can screen the first, i.e., it reduces the strength of the scattering which one would calculate in the absence of Coulomb interactions. For wavelengths long compared to the screening length ($B_0=0$), the reduction in charge density fluctuation scattering is very large for a free-electron gas.[6,12] Solid state effects contained in γ can enhance this scattering to an observable level in semiconductors.[6,11,12] Note that, in general, the quasielastic scattering spectra for the real semiconductor Fermi magnetoplasma will differ considerably from the simple semiconductor plasma spectra described in Sec. III. First, it will have both charge density and spin density fluctuation contributions[12] (for $\omega_s\ll qv_F$). Also, in high magnetic fields its peaks will be near $qv_{Fn\sigma}$ where $v_{Fn\sigma}=[2/m^*]^{1/2}[E_F-(n+\frac{1}{2})\hbar\omega_c-\frac{1}{2}\sigma\hbar\omega_s]^{1/2}$ and ω_s is the electron spin-flip frequency.

B. Scattering Perpendicular to Magnetic Field ($q\perp B_0$)

We restrict our attention in this geometry to the regime $(q_1r_c)\lesssim 1$. In the free-electron case this geometry yields only scattering from the collective modes. This may no longer be true for the semiconductor plasma. There is scattering from the collective modes [$d\sigma\propto\mathrm{Im}(1/\epsilon)$] which derives from the second term on the right-hand side of Eq. (11). However, there are also single-electron scattering contributions in both terms on the right-hand side of Eq. (11). For the simple semiconductor plasma $\gamma_{\alpha\beta}\propto m_{\alpha\beta}$ and the single-particle, contributions completely cancel, yielding zero. This result does not necessarily follow for complex semiconductors.

In what follows, the usefulness of the derived expressions will be illustrated by example calculations of the strengths of the cross section for scattering from inter-Landau-level single-electron excitations and Bernstein magnetoplasma waves. In some cases, the results differ considerably from the conclusions of previous authors, illustrating the importance of Coulomb interactions and proper treatment of complex semiconductor effects. For convenience, a parabolic approximation for the electron energies will be taken:

$$E(n,k_z,\sigma)=\hbar^2k_z^2/2m^*+(n+\tfrac{1}{2})\hbar\omega_c+\tfrac{1}{2}\sigma\hbar\omega_s.$$

This is a reasonable approximation for the phenomena considered below. We are just neglecting terms of order

[30] F. A. Blum, Phys. Rev. Letters **23**, 73 (1969).

$[(\hbar\omega_c)^2/E_g]$, $[(E_F)^2/E_g]$, etc., in the energy denominators of Eqs. (19)–(22). For perpendicular scattering ($q_z=0$) both γ and m conserve k_z, i.e., they are proportional to $\delta(k_{\alpha z}-k_{\beta z})$. Thus, the energy denominators appearing in Eq. (19)–(22) are rather simple and independent of k_z, having the form $[\omega\pm(N\omega_c\pm K\omega_s)]$, where N is a positive integer and $K=0$, 1. This result reduces the complexity of these equations considerably and facilitates the calculations given below.

1. Single-Particle Excitations

Scattering peaks due to single-particle excitations occur at the frequencies, ω_s, ω_c, $\omega_c\pm\omega_s$, $2\omega_c$, $2\omega_c\pm\omega_s$, $3\omega_c$, $3\omega_c\pm\omega_s$, ... Neglecting for the moment the spin-flip contributions to $m_{\alpha\beta}$, all peaks at frequencies involving ω_s derive from L_2. These are the spin-flip transition ($\omega=\omega_s$) and combination transitions ($\omega=N\omega_c\pm\omega_s$). Their strengths are given by conventional one-electron results[3] for $|\gamma|^2$ weighted by the appropriate statistical occupation factors. The remaining peaks are the so-called Landau-Raman[2,3,8] ones occurring at $\omega=N\omega_c$ and correspond to excitation of electrons between Landau levels with the same spin quantum number.

The cross section for Landau-Raman scattering is screened by the Coulomb interactions. The importance of this screening is shown by the following example: Following Wolff[6] and isolating our attention on the behavior of $d\sigma$ at $\omega=N\omega_c$, we note that the functions L_i have the form

$$L_i=\sum_{N=-\infty}^{\infty}\left(\frac{l_i(N)}{\omega-N\omega_c}\right).$$

Then the cross section for scattering at $\omega=N\omega_c$ is given by

$$\left(\frac{d^2\sigma}{d\Omega d\omega}\right)_{N\omega_c}=\frac{\omega_F}{\omega_I}(n_\omega+1)\sum_N\left[l_2(N)-\frac{l_1(N)\tilde{l}_1(N)}{l_0(N)}\right]\times\delta(\omega-N\omega_c), \quad (23)$$

where l_0 is defined by $\epsilon\equiv[1-(4\pi e^2/q^2)L_0]$. Equation (23) has some striking features. The second term on the right-hand side which screens the one-electron contribution l_2 is *not* proportional to $(4\pi e^2/q^2)$. That is, the strength of the screening is not proportional to the self-consistent field "coupling constant" $(4\pi e^2/q^2)$. This is in direct contrast to the $B_0=0$ case where screening in the long-wavelength limit reduces the single-electron quasielastic scattering strength by a factor $\approx(q/q_s)^2$ (q_s is the appropriate screening wave vector). This feature of Eq. (23) results from the RPA and leads to *total* screening for a free-electron gas and for electrons in our model semiconductor (see Sec. III), for which we have $\gamma\propto m$. This proportionality yields $l_2=R^2l_0$ and $(l_1,\tilde{l}_1)=Rl_0$, so that the right-hand side of Eq. (23) is zero. Here, $R=\{E_g^2/[E_g^2-(\hbar\omega_I)^2]\}(e^2/m^*c^2)(\boldsymbol{\varepsilon}_I\cdot\boldsymbol{\varepsilon}_F)$ is a constant for the simple semiconductor case. For a complex semiconductor plasma, these equalities do not necessarily hold. The screening will be only partial. For illustration, consider the case where the magnetic field is so large that only the lowest Landau orbital level ($n=0$) is occupied. This situation has been considered in several one electron calculations.[3] The cross section then has the particularly simple form

$$d\sigma_{(N\omega_c)}\simeq\left[\int dk_z\rho|\gamma_{0,N}|^2-\left|\int dk_z\rho\gamma_{0,N}\right|^2\Big/\int dk_z\rho\right]. \quad (24)$$

It is appropriate to use for $\gamma_{0,N}$ expressions calculated in one-electron theories.[3] The $N=2$ Landau-Raman process in III-V semiconductors has a significant cross section even in the band-edge ($k_z=0$) approximation, which is commonly made.[3] The cross section per particle, given by the noninteracting electrons result, is (for $\omega_I\ll E_g$)

$$\sigma_{2\omega_c}=O[\sigma_T^*(\hbar\omega_c/E_g)^2], \quad (25)$$

where $\sigma_T^*=(e^2/m^*c^2)^2$ is an effective Thomson cross section and E_g is the energy band gap. However, examination of Eq. (24) shows that if $\gamma_{0,N}$ is taken independent of k_z, the right-hand side is *zero*. Again, we have the phenomena of total screening. It should be noted that nonparabolicity effects which make ω_c depend on k_z, can modify this screening effect. If the k_z dependence of γ is accounted for by inclusion of the k_z-dependent terms in the energy denominators of Eq. (18), we find for interacting electrons

$$\sigma_{2\omega_c}=O[(2/15)\sigma_T^*(E_F/E_g)^2], \quad (26)$$

where we assume k_I, $k_F\ll k$ (Fermi) and E_F is the Fermi energy measured with respect to the $k_z=0$ energy of the lowest Landau level. This Fermi energy decreases with increasing magnetic field as $(1/B_0)$, leading to a cross section which goes as B_0^{-2}. This is in direct contrast with the result for noninteracting electrons [Eq. (25)] which gives a strength increasing as B_0^2. Considering the experiments of Slusher *et al.*[8] on inter-Landau-level scattering, the example considered is an oversimplification. Their experiments were not performed with $\mathbf{q}\perp\mathbf{B}_0$ and several Landau levels were occupied. In this case, the screening is less dramatic and evaluation requires a complicated calculation. However, it is clear that the screening is there and can have important consequences concerning the magnetic field and electron density dependence of observed cross sections. Also, in a typical experiment the q's are so small $[(qr_c)\ll1]$ that, even if one has $\mathbf{q}\perp\mathbf{B}_0$, one expects scattering from a Bernstein magnetoplasma wave very *near* $\omega=2\omega_c$. Due to lifetime broadening, scattering from this mode will not generally be distinguishable from scattering from the single-particle excitation *at* $\omega=2\omega_c$. Scattering

from the magnetoplasma waves is the subject of Sec. IV D.

For scattering via a single-electron excitation from the $n=0$ to the $n=1$ Landau level, γ is proportional to k_z and is zero in the band-edge approximation.[3] The many-electron contribution to Eq. (24) merely reduces the cross section from $\sigma_{\omega_c}=O[\sigma_T^*(\hbar\omega_c/E_g)(E_F/E_g)]$ by the factor $\frac{1}{6}$. Groves and Wright[31] have found by detailed calculation, including intraband terms, that σ_{ω_c} (for 10.6-μ radiation) is actually much smaller for InSb than that predicted by the above equation.

2. Magnetoplasma Waves

The cross section for scattering from well-defined collective modes of the plasma is

$$\frac{d^2\sigma}{d\Omega d\omega}=\frac{\omega_F}{\omega_I}\frac{(n_\omega+1)}{\pi}|L_1|^2\frac{4\pi e^2}{q^2}\operatorname{Im}\left(\frac{1}{\epsilon}\right). \tag{27}$$

Again, considering the case of a nondegenerate, spherically symmetric conduction band, we can use what is basically the free-electron ($m\to m^*$, $g\to g^*$) dielectric constant,[25] being consistent with our neglect of nonparabolicity in the energy denominators of the L functions. In the long-wavelength limit ($q_\perp r_c\ll 1$) which is appropriate in typical experiments,[8] the magnetoplasma (charge density) waves are those mentioned in Sec. II: the magnetoplasmon $\omega^2=\omega_p^2+\omega_c^2+O(q_\perp^2r_c^2)$, and the Bernstein modes $\omega=N\omega_c+O[(q_\perp^2r_c^2)^N]$, $N\geq 2$. Since the scattering from the Bernstein modes near $N\omega_c$ is not likely to be distinguishable experimentally from single-electron scattering at $N\omega_c$, it is instructive to consider the relative strength of these two scattering processes. Using Mermin and Canel's[25] result for the response function residue for the free-electron gas, one can show that

$$\frac{d\sigma}{d\Omega}\simeq r_N\frac{|\gamma_N|^2}{(q_\perp^2r_c^2)^N}, \tag{28}$$

for the Bernstein mode near $N\omega_c$. In Eq. (28), $\gamma_N=O(\gamma_{n,n+N})$ and $r_N\propto(q_\perp^2r_c^2)^N$ is the residue factor defined by Mermin and Canel. Reducing Eq. (28), we find

$$\frac{d\sigma}{d\Omega}\simeq\rho_0|\gamma_N|^2, \tag{29}$$

where ρ_0 is the electron density, and we assume $\omega_p\simeq\omega_c$. The point here is that, in terms of the small parameter $(q_\perp^2r_c^2)$, the cross section is of order 1; factors involving the relative population of the various Landau levels are common to both types of scattering and have been dropped. For a free-electron gas $|\gamma_N|^2\simeq\sigma_T^*(q_\perp^2r_c^2)^N$ and the scattering cross section $(d\sigma/d\Omega)\simeq\rho\sigma_{0T}^*(q_\perp^2r_c^2)^N$ is quite small. This result applies also to the simple semiconductor for $\omega_I\ll E_g$. The important thing to note is that Eq. (29) is exactly the estimate one would make for the inter-Landau-level scattering (in the absence of significant screening). For example, consider the case $N=2$ in the III-V semiconductors. We know that $(\gamma_N)^2\simeq\sigma_T^*(\hbar\omega_c/E_g)^2$, yielding

$$\frac{d\sigma}{d\Omega}\simeq\rho_0\sigma_T^*\left(\frac{\hbar\omega_c}{E_g}\right)^2, \tag{30}$$

for the Bernstein mode scattering strength near $2\omega_c$. Equation (30) is identical with estimates given[3] for the Landau-Raman scattering at $2\omega_c$. The conclusion that inter-Landau-level single-electron excitation and Bernstein mode excitations scatter with equivalent strengths is important for interpretation of experiments. Furthermore, this result is contrary to Wolff's[6] conclusion for the $2\omega_c$ nonparabolicity induced scattering. Wolff found the screening of the single-electron excitations at $2\omega_c$ so strong in his case that the dominant scattering was from the Bernstein wave. It appears that his result is not a general one for semiconductors.

Finally, it should be mentioned that the properties of and scattering from the Bernstein waves in the geometry $\mathbf{q}\perp\mathbf{B}_0$ are somewhat special. For $\mathbf{q}$ oblique to $\mathbf{B}_0$ the Bernstein waves tend to suffer collisionless damping.[25] It is also interesting to note that for $\omega_p^2\approx(N^2-1)\omega_c^2$, the Nth Bernstein wave and the magnetoplasmon will become coupled modes. Although this coupling is rather weak, effects due to it have been observed[32] and it can significantly influence the scattering spectrum, particularly for long relaxation times.[5]

3. Spin Waves

To the level of approximation considered above, $m_{\alpha\beta}$ is diagonal in the spin quantum number. Spin-flip excitations result only from the interband terms in $\gamma_{\alpha\beta}$. The many-electron (second) term on the right-hand side of Eq. (11) makes no contribution to spin-excitation phenomena. This results because Coulomb interactions in the RPA do not couple the motion of electrons with opposite spins, if the one electron states are eigenstates of σ_z the component of the Pauli-spin operator $\boldsymbol{\sigma}$ along $\mathbf{B}_0$. However, it has been pointed out[30] recently that due to the action of spin-orbit coupling, $m_{\alpha\beta}$ effectively does not conserve σ_z. This follows since spin-orbit coupling mixes the space and spin character of the electron wave functions so that they are no longer eigenstates of σ_z. This effect has novel consequences. First, there are spin-wave[30] solutions to $\epsilon=0$ which one might observe in a light scattering experiment. Spin waves would ordinarily be expected only in magnetic semiconductors or under conditions where

[31] S. H. Groves and G. B. Wright, Solid State Research Report, Lincoln Laboratory, M.I.T., 1968:4, p. 43 (unpublished).

[32] C. K. N. Patel and R. E. Slusher, Phys. Rev. Letters **21**, 1563 (1968).

weak exchange interactions of the Fermi-liquid type[33] are important. The spin-orbit-induced spin waves exist even in the absence of explicit exchange interactions. Furthermore, screening of the single-particle spin-excitation processes results. In the long-wavelength limit for $\mathbf{q}\perp\mathbf{B}_0$ in III-V semiconductors, the fundamental spin-wave mode occurs at the frequency

$$\omega=\omega_s+\delta\omega+O(q_1^2r_c^2),$$

where $\delta\omega$ is independent of q_1 and vanishes in the absence of spin-orbit coupling. For InSb with a donor concentration of about 10^{17} cm^{-3} and a magnetic field around 100 kG, $(\delta\omega/\omega)\approx0.01$ and $\omega_s\approx190$ cm^{-1}. Although this frequency shift (from the single-particle spin-flip line) is small, effects due to these newly predicted spin waves might be observed in a high-resolution infrared-light scattering experiment. Rather narrow spin-flip linewidths (as narrow as a few cm^{-1}) have been observed[8] in scattering experiments in InSb and InAs with $\mathbf{q}$ oblique to $\mathbf{B}_0$. An analysis in exact analogy with that given for the Bernstein modes in Sec. IV B 2 shows that the total cross section $(d\sigma/d\Omega)$ for scattering from the fundamental spin-wave mode is of the same order as the cross section for single-particle spin-flip scattering.

There are higher-order spin-wave roots near combined transition (orbital plus a spin change) energies $\hbar(N\omega_c\pm\omega_s)$, $N\geq1$. For these combined spin waves, $\omega=(N\omega_c\pm\omega_s)+O[(q_1^2r_c^2)^N]$, leading to a frequency shift [from the combination single-electron excitation line at $(N\omega_c\pm\omega_s)$] which vanishes in the long-wavelength limit. The combined spin waves suffer the problems characteristic of Bernstein modes: (a) Because of their small frequency shifts their scattering peaks will tend to be indistinguishable from the single-electron excitation lines, due to relaxation broadening; (b) they will tend to suffer damping for $\mathbf{q}$ oblique to $\mathbf{B}_0$. Indeed, there is a striking similarity between the long-wavelength set of spin-wave modes formed by the fundamental $(\omega\approx\omega_s+\delta\omega)$ plus combined $(\omega\approx N\omega_c\pm\omega_s, N\geq1)$ excitations and the magnetoplasma wave set consisting of the magnetoplasmon $(\omega^2=\omega_p^2+\omega_c^2)$ plus the Bernstein waves $(\omega\approx N\omega_c, N\geq2)$.

V. DISCUSSION

Detailed quantitative calculations of scattering cross sections have not been attempted in this paper. Such calculations would indeed be tedious, if not complicated, and are probably warranted only for detailed comparison with experimental results. However, the general form of the cross section expressions given in Sec. II has proved itself quite useful in categorizing the various inelastic scattering processes and studying some of their general properties. These expressions yield a rich variety of semiconductor magnetoplasma excitations which one might hope to observe by light scattering. Light scattering is a most attractive experimental tool by virtue of the fact that the light couples to so many types of excitations: quasielastic single-electron excitations, acoustic plasma waves, optical plasma waves (including Bernstein waves), inter-Landau-level excitations, spin-flip excitations, combined excitations, and spin waves (fundamental and combined). Experiments to date have clearly only scratched the surface of rather fertile ground. Only the inter-Landau-level and spin-flip excitations, and the magnetoplasmon have been observed[3] directly in magnetoplasma light scattering. A major virtue of the theoretical treatment given in this paper is that it does in fact encompass all the above excitations. Furthermore, the treatment is sufficiently general to permit one to calculate scattering properties which are considerably more realistic than those calculated on the basis of most prior theoretical treatments. The complexities of semiconductor band structure, many-electron screening effects, and collective mode scattering can only be dealt with using results such as those derived here and/or those given by McWhorter and Argyres.[7]

The results of Sec. IV can serve as a useful guide in planning magnetoplasma scattering experiments. Because of the separation and/or isolation of various scattering processes, it is clearly desirable to perform experiments in the two characteristic geometries, scattering perpendicular and parallel to the magnetic field. Most experiments have been performed with $\mathbf{q}$ oblique to the magnetic field.[3] There is a clear advantage to studying inter-Landau level and spin-flip scattering in the III-V semiconductors in the geometry $\mathbf{q}\|\mathbf{B}_0$. For these materials, the one-electron matrix elements are nonzero in the dipole approximation. Thus, the one-electron cross sections are independent of $\mathbf{q}$. A non-interacting electron theory then gives the correct properties for this scattering when $\mathbf{q}\|\mathbf{B}_0$. However, when $\mathbf{q}$ is oblique to $\mathbf{B}_0$, many-electron screening effects come into play and enormously complicate the interpretation of experimental results. For $\mathbf{q}\perp\mathbf{B}_0$, one has the additional complication of scattering from the Bernstein waves. Of course, it would be of considerable interest to study scattering near $N\omega_c, \omega_s$, etc., in both geometries, looking for differences of the kind discussed in Sec. IV. Finally, one would hope that light scattering experiments would produce observations of excitations such as the acoustic plasma waves and spin waves which have not been detected heretofore by any means.

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[33] Spin waves observed in the alkali metals [G. Dunifer, S. Schultz, and P. H. Schmidt, J. Appl. Phys. **39**, 397 (1968)] have been attributed to the existence of significant short-range electron-electron exchange interactions [P. M. Platzman and P. A. Wolff, Phys. Rev. Letters **18**, 280 (1967)].