# <span id="page-0-1"></span>Carrier-Induced Band-Gap Variation and Point Defects in  $\text{Zn}_3\text{N}_2$  from First Principles

Yu Kumagai,<sup>1,2[,\\*](#page-0-0)</sup> Kou Harada,<sup>3</sup> Hirofumi Akamatsu,<sup>3</sup> Kosuke Matsuzaki,<sup>1</sup> and Fumiyasu Oba<sup>3,1,4</sup>

 $<sup>1</sup>$ Materials Research Center for Element Strategy, Tokyo Institute of Technology,</sup>

Yokohama 226-8503, Japan<br>PRESTO, Japan Science and Technology Agency, Tokyo 113-8656, Japan<sup>2</sup>

 $3$ Laboratory for Materials and Structures, Institute of Innovative Research,

Tokyo Institute of Technology, Yokohama 226-8503, Japan <sup>4</sup>

Center for Materials Research by Information Integration, National Institute for Materials Science,

Tsukuba 305-0047, Japan

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The zinc nitride  $Zn_3N_2$  is composed of inexpensive and earth-abundant Zn and N elements and shows high electron mobility exceeding 100 cm<sup>2</sup> V<sup>-1</sup> s<sup>-1</sup>. Although various technological applications of Zn<sub>3</sub>N<sub>2</sub> have been suggested so far, the synthesis of high-quality  $Zn_3N_2$  samples, especially single crystals, is still challenging, and therefore its basic properties are not yet well understood. Indeed, the reported band gaps of as-grown  $Z_{13}N_2$  widely scatter from 0.85 to 3.2 eV. In this study, we investigate the large gap variation of  $Zn_3N_2$  in terms of the Burstein-Moss (BM) effect and point-defect energetics using first-principles calculations. First, we discuss the relation between electron carrier concentration and optical gaps based on the electronic structure obtained using the Heyd-Scuseria-Ernzerhof hybrid functional. The calculated fundamental band gap is  $0.84 \text{ eV}$  in a direct-type band structure. Second, thermodynamic stability of  $\text{Zn}_3\text{N}_2$ is assessed using the ideal-gas model in conjunction with the rigid-rotor model for gas phases and firstprinciples phonon calculations for solid phases. Third, carrier generation and compensation by native point defects and unintentionally introduced oxygen and hydrogen impurities are discussed. The results suggest that a significant BM shift occurs mainly due to oxygen substitutions on nitrogen sites and hydrogen interstitials. However, gaps larger than 2.0 eV would not be due to the BM shift because of the Fermi-level pinning caused by acceptorlike zinc vacancies and hydrogen-on-zinc impurities. Furthermore, we discuss details of peculiar defects such as a nitrogen-on-zinc antisite with azidelike atomic and electronic structures.

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### I. INTRODUCTION

Nitrides possess various advantages for practical use as semiconductors. The constituent nitrogen is abundant and nontoxic. Moreover, since higher energy levels of N  $2p$  states than of O  $2p$  ones lead to higher valenceband maxima (VBM), nitrides can be doped into  $p$  type even without large cationic orbital contributions such as Cu  $3d$  and Sn  $5sp$  [\[1,2\]](#page-9-0). It also makes the band gaps narrower than oxides, which makes some nitrides suitable for light-absorbing layers of photovoltaic solar cells and photoelectrochemical cells [\[2\].](#page-9-1) Thus, various nitride semiconductors and their alloys have been reported and investigated aiming at a variety of applications such as optoelectronics, photovoltaics, power devices, and so on [2–[12\]](#page-9-1).

The foremost important nitrides are GaN and alloys based upon it that have been developed for commercial optoelectronic devices [7–[9\].](#page-9-2) Other nitrides have been much less investigated but are currently attracting growing research interest. In particular, thanks to recent progress in crystal growth techniques, high-quality nitride samples including metastable ones—can now be prepared. One of the attractive nitrides is  $Zn_3N_2$  [\[13\],](#page-9-3) whose biggest advantage is its high electron mobility, which exceeds 100 cm<sup>2</sup> V<sup>-1</sup> s<sup>-1</sup> [14–[17\]](#page-9-4) (the record is 395 cm<sup>2</sup> V<sup>-1</sup> s<sup>-1</sup> [\[15\]](#page-9-5)). There have been reports suggesting the use of  $Zn_3N_2$  as transparent conductors [\[18\],](#page-9-6) channel layers for optoelectronic devices [\[19\]](#page-9-7), negative electrodes in Li-ion batteries [\[20\]](#page-9-8), and precursor films for *p*-type doped ZnO [\[21\].](#page-9-9) Thus far,  $Zn_3N_2$ samples have been synthesized using various techniques, such as pulsed-laser deposition [\[22,23\]](#page-9-10), molecular beam epitaxy [\[15,16,24\],](#page-9-5) chemical vapor deposition [\[16,25\]](#page-9-11), electrochemical processes [\[26\]](#page-9-12), sputtering [\[17,27](#page-9-13)–30], and ammonolysis reactions [\[31](#page-9-14)–33]. However, synthesis of high-quality  $Zn_3N_2$  samples, especially single crystals, is

<span id="page-0-0"></span>[<sup>\\*</sup>](#page-0-1) yuuukuma@gmail.com

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still challenging, presumably due to its nearly zero formation enthalpy [\[34\],](#page-10-0) and hence its basic properties are not yet well understood.

The most controversial but technologically important physical property of  $Zn_3N_2$  is the band gap. The reported band gaps of as-grown  $Zn_3N_2$  range from 0.85 to 3.2 eV [14–16,26,28–[31,33,35,36\]](#page-9-4). Indeed, experimental gaps of some other nitride semiconductors notoriously scatter as well. The most memorable example is InN; for several decades, its gap had been believed to be around 2.0 eV, but it was revised to 0.6–0.7 eV after 2002 [37–[39\].](#page-10-1) The same case holds for ScN and  $\text{ZnSnN}_2$ ; the reported gap ranges are 0.9–1.5 and 1.4–2.9 eV, respectively [\[11,40\].](#page-9-15) Such large variations are attributed mainly to the Burstein-Moss (BM) effect [\[41,42\],](#page-10-2) where the carrier concentration is so high that the Fermi level shifts into the host valence or conduction band, leading to extended optical gaps. Some experimental studies have demonstrated that the optical gaps of nitrides change as a function of carrier electron concentration. The experimental results of InN, GaN, ScN, and  $ZnSnN<sub>2</sub>$  have also been examined by using firstprinciples calculations in terms of the BM effect [\[11,39,40\]](#page-9-15).

Likewise, the BM effect has been considered as the most likely cause for the wide variety of reported gaps for  $\text{Zn}_3\text{N}_2$ , although the variation range is much larger than those of other nitrides. Another suggested possibility includes oxygen impurities leading to a larger ionicity, which may cause wider band gaps [\[28,36\].](#page-9-16) Besides, most  $Zn_3N_2$  samples have been found to be unintentionally  $n$  type with a carrier concentration of  $10^{18}$ – $10^{20}$  cm<sup>-3</sup>. It is widely accepted that nitrogen vacancies and/or oxygen substitutions at nitrogen sites are responsible for the generation of such high carrier electrons [\[16,18,19,27,30,36\].](#page-9-11) However, there is still no clear theoretical support; researchers have investigated native defects [\[43\],](#page-10-3) oxygen impurities [\[44\]](#page-10-4), and copper, silver, and gold dopants  $[45]$  in  $Zn_3N_2$  in the neutral charge state using first-principles calculations. However, they adopted the local-density approximation (LDA) and/or the generalized-gradient approximation (GGA), both of which predict  $Zn_3N_2$  to be (nearly) metallic and so are not suited for predicting defect properties even qualitatively. Furthermore, calculations of charged defects are essential for identifying the source of the carrier electrons [\[46\]](#page-10-6). Such calculations can also provide insight into the possibility of controlling a carrier electron concentration in  $\text{Zn}_3\text{N}_2$ , which is indispensable for device applications such as transistors.

In this paper, our main purpose is twofold. First, we discuss the relation between the electron carrier concentration  $N_e$  and the optical gap determined from absorption processes  $E_g^{\text{opt}}$  based on the calculated band structure and density of states (DOS) (Sec. [III B\)](#page-3-0). We use the Heyd-Scuseria-Ernzerhof (HSE06) hybrid functional, which has been reported to accurately predict the band gaps of semiconductors, especially nitrides [\[11,12,40\].](#page-9-15) We also

compare our results with measured  $E_g^{\text{opt}}$  as a function of  $N_e$ . Second, we show the calculation results of native point defects and unintentionally introduced oxygen and hydrogen into  $Zn_3N_2$  (Sec. [III D](#page-4-0)). Image-charge interactions under periodic boundary conditions are corrected by using our extended Freysoldt–Neugebauer–Van de Walle (FNV) scheme, which is verified through a series of different supercell calculations with GGA with the Hubbard U correction (see the Appendix) [\[47\].](#page-10-7) Such defect calculations allow us to reveal the dominant point defects generating the carrier electrons and, therefore, causing the BM shift. In addition, we also discuss the variable range of the Fermi level in view of the Fermi-level pinning. Furthermore, it is well known that nitrides tend to be thermodynamically less stable than oxides. Thus, in Sec. [III C](#page-4-1), we examine the thermodynamic stability of  $Zn_3N_2$  using the ideal-gas model in conjunction with the rigid-rotor model for gas phases and first-principles phonon calculations for solid phases. This analysis also provides the chemical potentials used in the discussion of defect energetics.

#### II. METHODS

### A. Computational details

The calculations are performed using the projector augmented-wave (PAW) method [\[48\]](#page-10-8) as implemented in the Vienna ab initio simulation package [\[49,50\]](#page-10-9). PAW data sets with radial cutoffs of 1.22, 0.79, 0.80, and 0.58 Å for Zn, N, O, and H, respectively, are employed. Zn 3d and 4s, N 2s and  $2p$ , O 2s and  $2p$ , and H 1s orbitals are considered as valence electrons. For the perfect crystal, the lattice constant and the internal atomic positions are fully optimized until the residual stresses and forces converge to less than 0.01 GPa and 0.01 eV/Å, respectively. In this study, we adopt the HSE06 hybrid functional [51–[53\],](#page-10-10) which describes the electronic structures of semiconductors more accurately than LDA or GGA [53–[56\]](#page-10-11) and has been applied to their point defects [\[12,57](#page-9-17)–60]. The screening parameter and mixing of Hartree-Fock exchange are set at the HSE06 values of 0.208 Å<sup> $-1$ </sup> and 0.25 [\[53\].](#page-10-11) Defect calculations are performed at the theoretical lattice constant of perfect  $Zn_3N_2$ . The internal atomic positions are relaxed until the residual forces reduce to less than  $0.04 \text{ eV/A}$ . Wave functions are expanded with a planewave basis set. Cutoff energies are set to 550 and 400 eV for the calculations of perfect  $Zn_3N_2$  and defective systems, respectively. Spin polarization is considered for all of the defects.

### B. Thermodynamic stability

<span id="page-1-0"></span>In this study, we discuss the thermodynamic stability of  $\text{Zn}_3\text{N}_2$  at finite temperatures. The entropic contributions of  $N_2$  and  $NH_3$  are obtained using the ideal-gas model in conjunction with the rigid-rotor model [\[61\]](#page-10-12).

The characteristic rotational temperatures and vibrational temperatures measured by experiments are taken from Ref. [\[61\].](#page-10-12) For solid phases of antibixbyite  $\text{Zn}_3\text{N}_2$ , wurtzite ZnO, and hexagonal close-packed Zn, we calculate the Gibbs free energies within the quasiharmonic approximation [\[62\]](#page-10-13). The PBEsol functional (the Perdew-Burke-Ernzerhof functional parametrized for solids) [\[63\]](#page-10-14) is employed for the calculations of cell-volume-dependent phonon frequencies. For insulating  $Zn_3N_2$  and  $ZnO$ , on-site Coulomb interactions on the Zn 3d states are taken into account by the  $+U$  correction according to Dudarev's formalism [\[64\].](#page-10-15) The effective U value  $U_{\text{eff}} = U - J$  is set at 5 eV, with which the Zn 3d band position with respect to the VBM is very close to that calculated by the HSE06 functional [\[55\].](#page-10-16) We use  $2 \times 2 \times 2$  supercells for  $\text{Zn}_3\text{N}_2$  and  $3 \times 3 \times 2$  supercells for ZnO and Zn. The cell volume is varied in the range of about 85% to 115% of the equilibrium volume, depending on the compounds. At the various cell volumes, the atomic coordinates are optimized until the forces converge to less than  $0.001 \text{ eV}/\text{\AA}$ , and finite displacements are then introduced. Using the PHONOPY code [\[65\]](#page-10-17), the phonon frequencies are computed from the calculated force constants. The Gibbs free energies at zero pressure and given temperatures are derived via minimization of Helmholtz free energies with respect to volume.

#### C. Point defects

The formation energy of a point defect is calculated as [\[47\]](#page-10-7)

$$
E_f[D^q] = (E[D^q] + E_{\text{corr}}[D^q])
$$

$$
-E_P - \sum n_i \mu_i + q(\epsilon_{\text{VBM}} + \Delta \epsilon_F), \quad (1)
$$

where  $E[D^q]$  and  $E_P$  are the total energies of the supercell with defect  $D$  in charge state  $q$  and the perfect-crystal supercell without any defect, respectively.  $n_i$  is the number of removed  $(n_i < 0)$  or added  $(n_i > 0)$  *i*-type atoms and  $\mu_i$ refers to the chemical potential. The chemical potentials depend on the growth condition, which is discussed in detail in Sec. [III C](#page-4-1).  $\epsilon_{VBM}$  is the energy level of the VBM in perfect  $Zn_3N_2$ .  $\Delta \epsilon_F$  represents the Fermi level with respect to the VBM. Note that, when discussing the BM effect,  $\Delta \epsilon_F$  not only varies inside the band gap but also moves into the conduction band. Therefore, we need to take special care for the defect calculations, which are dealt with in Sec. [III D 1](#page-4-2). Except for the exchange-correlation functional approximated, an error in the defect-formation energy mainly comes from the spurious electrostatic interactions under periodic boundary conditions.  $E_{\text{corr}}[D^q]$  corresponds to its correction energy. We emphasize that the potential alignment is not considered because it is unnecessary when the imagecharge correction is properly applied [\[47\]](#page-10-7). We estimate  $E_{\text{corr}}[D^q]$  using our extended FNV scheme, which can correct energies of charged defects accurately [\[47,66,67\]](#page-10-7). Based on the careful cell-size test, we adopt the primitive cell with 40 atoms for defect calculations (see the Appendix for details). The defect eigenvalues are also corrected within the point-charge approximation as discussed in Refs. [\[58,68\]](#page-10-18).

# III. RESULTS AND DISCUSSION

### A. Fundamental properties of the perfect crystal

Figure [1](#page-2-0) presents the crystal structure of the cubic antibixbyite  $Zn_3N_2$  with the space group of  $Ia\overline{3}$ , which can also be regarded as a defective antifluoride structure. As seen, each Zn atom is surrounded by four N atoms, and each N atom by six Zn atoms. The calculated theoretical lattice constant is 9.76 Å, agreeing very well with the experimental values of 9.77–9.78 Å [31–[33\].](#page-9-14)

<span id="page-2-0"></span>

FIG. 1. Antibixbyite crystal structure of  $\text{Zn}_3\text{N}_2$ . The conventional unit cell shown includes 80 atoms, whereas the primitive cell 40 atoms. The small gray balls represent Zn atoms, and the large blue ones N atoms. Two inequivalent nitrogen sites are distinguished as N1 (8b) and N2 (24d). The bond distances from neighboring Zn are also shown in Å and equivalent bonds are grouped using the same colors. For the defect calculations, interstitial sites coordinated by cation zinc atoms (two inequivalent sites) and anion nitrogen atoms are considered; they are depicted as  $i(c1)$ ,  $i(c2)$ , and  $i(a)$ .

<span id="page-2-1"></span>

FIG. 2. Calculated band structure and DOS of  $Zn_3N_2$  using the HSE06 functional at the theoretical lattice constant. The zero of energy is set at the VBM. The horizontal dashed lines indicate the VBM and conduction-band minimum (CBM), located at the Γ point. The band path is taken from Ref. [\[69\].](#page-11-0) The partial DOSs of Zn and N are summed from all of the Zn sites and N sites, respectively. The fitting range used for estimating the effective masses in this study is also shown.

<span id="page-3-1"></span>

FIG. 3. Optical gap determined from absorption processes as a function of carrier electron concentration estimated from the band structure and the DOS computed with the HSE06 functional (see the text for details). The same information calculated within the parabolic band approximation is shown by a dashed line (see the text for details). A horizontal dotted line indicates the calculated fundamental gap. The compiled experimental data measured from samples grown by metal-organic chemical vapor deposition (MOCVD), molecular beam epitaxy (MBE), and radio-frequency magnetron sputtering (rf mag. sputt.) are also shown by points [\[14,16,18,70\]](#page-9-4). A horizontal arrow shows a range of experimentally reported electron carrier concentration, and a vertical one the range of the corresponding optical gaps. Note that the variation of the optical gap is mainly attributed to the shift of the Fermi level, as the change in the valence band is very small along the Γ-N line (see Fig. [2\)](#page-2-1). For instance, the difference between the changes of the optical gap and the Fermi level is 0.05 eV at an electron carrier concentration of  $3.5 \times 10^{20}$  cm<sup>-3</sup>.

Figure [2](#page-2-1) shows the calculated band structure and DOS of  $Zn_3N_2$ . The band structure shows a direct gap at the Γ point. The fundamental gap is estimated at 0.84 eV, consistent with the reported HSE06 gap of 0.86 eV [\[55\]](#page-10-16). We previously found that the HSE06 functional performs better than the  $GW_0$  approximation for predicting the band gaps of several nitrides [\[12\]](#page-9-17). Deng *et al.* [\[40\]](#page-10-19) and Lahourcade *et al.* [\[11\]](#page-9-15) have also reported that the HSE06 functional can more accurately predict the experimental gaps of ScN and  $ZnSnN<sub>2</sub>$ , respectively, than the GW approximations. Thus, the fundamental gap of pristine  $Zn_3N_2$  is expected to be approximately 0.84 eV. The site-projected DOS shows strong hybridization between the closed-shell Zn  $3d$  and N  $2p$  states in the valence band, which is expected to reduce the hole effective mass compared to other nitrides [\[12\].](#page-9-17) Actually, in addition to a very low electron effective mass  $m_e$  of 0.08 $m_0$  originating from unoccupied Zn 4s states, where  $m_0$  means the freeelectron rest mass, the effective mass of the heavy hole  $m_h$ of 0.99 $m_0$  is nearly half of GaN [\[12\]](#page-9-17).

#### B. Carrier-induced optical-gap variation

<span id="page-3-0"></span>In Fig. [3,](#page-3-1) we show an optical gap determined from absorption processes  $E_g^{\text{opt}}$  as a function of electron carrier concentration  $N_e$  calculated from the band structure and

DOS in Fig. [2](#page-2-1). For estimating  $E_g^{\text{opt}}$ , we should keep the following in mind: (1) The electronic transition from the highest valence band to the lowest conduction band is dipole allowed. (2) Since only a vertical transition is optically allowed when the phonon-assisted transition is ignored, the valence-band curvature needs to be taken into account (see the inset of Fig. [3](#page-3-1)). Thus,  $E_g^{\text{opt}}$  is computed by the following two steps: First, a relation between the Fermi-level position in the conduction band and  $N_e$  is calculated from the DOS. Then, we calculate a relation between  $E_g^{\text{opt}}$  and the Fermi-level position from the band structure. Note that the temperature effect is ignored in this subsection because we have found that it is small compared to the energy scale discussed here.

We also calculate  $E_g^{\text{opt}}$  against  $N_e$  within the parabolic approximation frequently used for the analysis of experimental data by adding

$$
\Delta E = \frac{\hbar^2}{2} \left( \frac{1}{m_e} + \frac{1}{m_h} \right) (3\pi^2 N_e)^{2/3}
$$
 (2)

to the fundamental gap of 0.84 eV. Note that this equation is derived from the free-electron model and leads to the well-known relationship  $\Delta E \sim N_e^{2/3}$ . The carrier effective masses depend on the region for fitting the band structure. Here, we adopt the band along the Γ-N direction giving the heaviest effective hole mass, with a relatively longer distance of 1.16 Å<sup>-1</sup> (Fig. [2\)](#page-2-1), which gives  $m_e = 0.20m_0$ and  $m_h = 8.21m_0$ . The discrepancy from the effective masses in Ref. [\[12\]](#page-9-17) is attributed to the difference of the fitting range and direction. Indeed, when the masses are evaluated in the vicinity of the band edges, we obtain  $m_e = 0.08m_0$  and  $m_h = 0.99m_0$ , which are the same as reported values in the literature [\[12\]](#page-9-17).

One can see that our calculated  $E_g^{\text{opt}}$  against  $N_e$  accurately reproduces the experimental reports by Suda and Kakishita [\[16\]](#page-9-11) and Futsuhara et al. [\[14\].](#page-9-4) Conversely, the parabolic approximation relatively fails to match up with the experiments. The discrepancy between directly calculated  $E_g^{\text{opt}}$  and that of the parabolic approximation is not so small. This is because the conduction band varies not quadratically but nearly linearly from the  $\Gamma$  point to nearby special points in the relevant energy range, as seen in Fig. [2](#page-2-1). It has also been reported that the parabolic approximation does not work well for InN either [\[38\]](#page-10-20).

The reported carrier concentrations lie mostly in the range  $10^{18}$ –3.5 ×  $10^{20}$  cm<sup>-3</sup> [14–[17,28,35,36\],](#page-9-4) which correspond to band gaps of 1.0–1.9 eV, as shown in Fig. [3](#page-3-1). Obviously, gaps larger than 2.0 eV depart from our calculated  $E_g^{\text{opt}}$ , and other experimental values by about 1 eV. Furthermore, because of the Fermi-level pinning caused by acceptor-type defects, as is shown in Sec. [III D](#page-4-0), it should be difficult to lift the Fermi level to higher than

<span id="page-4-3"></span>

FIG. 4. (a) Temperature dependence of sums of the zero-point vibrational energies (ZPVE) and entropy contributions to the Gibbs free energies. The values of solids (the dashed lines) are calculated using the  $PBEsol(+U)$  functional (see the text for details), whereas those for gas phases (the solid lines) are quantified based on the characteristic rotational temperatures and vibrational temperatures. The pressures of the gas phases are set to  $10^4$ ,  $10^5$ , and  $10^6$  Pa. (b) Gibbs free energy of formation of  $Zn_3N_2$ , i.e., the free energy of reaction,  $3Zn + N_2 \rightarrow Zn_3N_2$ . Note that the total energy contributions are calculated using the HSE06 functional. (c) Calculated Gibbs free energies of  $\text{Zn}_3\text{N}_2$  with respect to Zn and N<sub>2</sub> at 10, 300, and 700 K estimated from (b). The intersections between the dotted line A and the y axes correspond to Zn and N chemical potentials at the condition where  $Zn_3N_2$  is grown at the Zn-rich condition at 300 K.

1.0 eV from the CBM, corresponding to  $E_g^{\text{opt}} = 1.9 \text{ eV}$ . We return to a discussion of such large optical gaps in Sec. [III D](#page-4-0).

It is also known that the band-gap renormalization may be significant when such a high concentration of carrier electrons is introduced. Therefore, we calculate its effect by adding electrons into pristine  $Zn_3N_2$  up to  $3.3 \times 10^{20}$  cm<sup>-3</sup>, which is nearly the maximum value of the electron carrier concentration reported. As a result, we find that the variation of the fundamental band gap is only 0.08 eV, whereas the variation caused by the BM shift is more than 1 eV. Thus, the renormalization effect would not be significant in  $\text{Zn}_3\text{N}_2$ .

### <span id="page-4-1"></span>C. Thermodynamic stability of  $\text{Zn}_3\text{N}_2$  and chemical potentials of the related compounds

In this subsection, we discuss thermodynamic stability of  $Zn_3N_2$ . It is well known that nitrides are difficult to grow compared to oxides. This is mainly due to the strong stability of the  $N_2$  molecule with a triple bond. Discussions of the stability of nitrides at finite temperatures based on first-principles calculations have not been common in the past. Here, we report it by taking  $Zn_3N_2$  as an example.

Figure [4\(a\)](#page-4-3) shows sums of zero-point vibrational energies and entropy contributions to the Gibbs free energies for  $Zn_3N_2$ ,  $Zn$ ,  $N_2$ ,  $ZnO$ , and  $NH_3$  as a function of temperature; the latter two are later used for determining the chemical potentials of O and H. By adding these contributions to the total energies, the Gibbs free energies are obtained. One can see that the phonon contributions in solids are small at around room temperature  $(< 0.1$  eV) as a result of cancellation of the zero-point vibrational energies and phonon entropy contributions. The variation of the free energy is highest for the  $N_2$  gas phase because the entropic contribution by the translation degree of freedom per atom is high.

The Gibbs free energy of formation for  $Zn_3N_2$  as a function of temperature at  $p_{N_2} = 10^5$  Pa is shown in Fig. [4\(b\)](#page-4-3). Note that the total energies are calculated using the HSE06 functional, and the phonon frequencies of the solids from PBEsol $(+U)$  as described in Sec. [II B](#page-1-0). As seen,  $Zn_3N_2$  is thermodynamically unstable in a whole range of temperatures and becomes less stable as increasing temperature. Indeed, the growth of nitrides is usually cumbersome and typically requires the experimental techniques that can achieve higher nitrogen chemical potential than that of the  $N_2$  gas phase at the standard state, such as molecular beam epitaxy, ammonolysis reaction, and high-pressure synthesis [\[2\].](#page-9-1) However, once the nitrides are synthesized, they can be kinetically stable thanks to the strong chemical bonding between cations and nitrogen ions [\[2\]](#page-9-1).

#### D. Point defects

<span id="page-4-0"></span>Here, we discuss formation energies and thermodynamic transition levels (TTLs) of native point defects and unintentionally doped oxygen and hydrogen in  $Zn_3N_2$ . We consider the vacancies, antisites, and interstitials shown in Fig. [1](#page-2-0) for native defects, and substitutional and interstitial sites for oxygen and hydrogen impurities. A wide range of charge states is considered for each defect, and symmetry constraints are removed.

## <span id="page-4-2"></span>1. Procedure of defect calculations at the Fermi level located above the CBM

In this study, we want to identify dominant defects introducing carrier electrons and causing the BM shift. To this end, we need to calculate the defect-formation energies and TTLs at the Fermi level, which are a few eV above the CBM. This conclusion is, however, not possible when the k-point sampling mesh covers the band edges. More specifically, let us think about  $\text{Zn}_3\text{N}_2$ , in which the VBM and CBM locate at the  $\Gamma$  point. When the  $\Gamma$  point is included in the k-point sampling, a filled electronic state existing above the CBM releases electrons to the CBM

<span id="page-5-0"></span>

FIG. 5. Schematic illustrations of (a) the band structure of  $\text{Zn}_3\text{N}_2$  and (b) the formation energies of donorlike defect D. Note that when a  $2 \times 2 \times 2$  MP mesh is used, the computed band gap is spuriously increased, and the TTLs are calculated appropriately within the spurious gap (see the text for details).

and forms a hydrogenlike state, as illustrated in Fig. [5\(a\)](#page-5-0). In such a case, the TTL is located adjacent to the CBM [Fig. [5\(b\)](#page-5-0)], which is not appropriate when the Fermi level moves above the CBM. Thus, we instead use a  $2 \times 2 \times 2$ Monkhorst-Pack (MP) k-point sampling mesh [\[71\]](#page-11-1) for point-defect calculations, which excludes the Γ point. The band gap is then spuriously increased as depicted in Fig. [5\(a\)](#page-5-0), and, consequently, the TTLs of the donor states are properly calculated at the Fermi level, being within the spurious gap. The same procedure was also employed by Janotti et al. for the study of point defects in InN [\[72\]](#page-11-2). Since the spurious CBM locates at 2.49 eVabove the fundamental VBM when using the HSE06 functional, we show the defect-formation energies up to  $\Delta \epsilon_F = 2.0 \text{ eV}$  in this study.

### 2. Native point defects

Figure [6](#page-5-1) shows a plot of calculated formation energies for native point defects against the Fermi level in  $\text{Zn}_3\text{N}_2$ . Since  $\text{Zn}_3\text{N}_2$  is unstable compared to  $\text{Zn}$  metal and  $\text{N}_2$ gas at the entire temperature range, as discussed in Sec. [III C,](#page-4-1) we set the chemical potentials at the condition where  $Zn$  and  $Zn_3N_2$  coexist at 300 K [denoted as  $A$  in Fig. [4\(c\)\]](#page-4-3). This situation is proximate to the Zn-rich equilibrium condition, which is usually considered for defect calculations.

It is noteworthy that all of the defects except for zinc vacancies  $(V_{Zn})$  behave as donors when the Fermi level is within the fundamental gap. Among the donors, nitrogen vacancies  $(V_N)$  at the N1 site have the lowest formation energies. However, the other defects may also be present in large numbers if we can increase  $\mu_N$  as discussed in Ref. [\[2\]](#page-9-1). Although  $V_{\text{N1}}$ <sup>+</sup> and  $V_{\text{N2}}$ <sup>+</sup> show nearly the same formation energies, their  $3+$  charge states, being dominant for Fermi-level positions in the fundamental gap, are very different. Indeed, Paniconi et al. reported that neutrondiffraction data indicate that the N1 site shows lesser site occupancy than the N2 site in their samples, with relatively

<span id="page-5-1"></span>

FIG. 6. Formation energies of native point defects in  $\text{Zn}_3\text{N}_2$  as a function of the Fermi level, ranging from the VBM, which is set to zero, to 2 eV so that the BM shift is discussed. V's in defect species denote vacancies, and the subscripts defect sites, where  $i$ indicates interstitial sites. The charge states of defects correspond to the gradients. Only the most energetically favorable charge state at a given Fermi level is shown for each defect. Positive and negative charge states indicate donor and acceptor behavior, respectively. The Fermi level at which the favorable charge state changes corresponds to the positions of the donor or acceptor levels, which are designated by filled circles.

low experimental band gaps of approximately 0.9 eV [\[33\]](#page-9-18). Neither  $V_{\text{N1}}^{2+}$  nor  $V_{\text{N2}}^{2+}$  appear in Fig. [6,](#page-5-1) meaning their negative-U characteristics that is common for anion vacan-cies [\[73\].](#page-11-3) On the other hand,  $V_{Zn}$  becomes a shallow acceptor center, in contrast to  $V_{Zn}$  in ZnO with deep acceptor levels [\[57,74\],](#page-10-21) and constructs a Fermi-level pinning at  $\Delta \epsilon_F = 1.9$  eV, from which the Fermi level cannot be lifted due to the spontaneous generation of acceptor  $V_{Zn}$ .

Among the interstitials, nitrogen interstitials  $(N_i)$  show unusual behaviors in view of the ionic picture. First,  $N_{i(c1)}$ and  $N_{i(c2)}$  are very high in energy, although the  $i(c1)$  and  $i(c2)$  sites are surrounded by Zn cations. By contrast, N<sub>i(a)</sub> shows much lower energy and acts as a donor center at  $E_F$  < 1.5 eV. This is counterintuitive because N atoms usually attract electrons as anions. To take a closer look, we investigate the atomic and electronic structures of  $N_{i(a)}$  as shown in Figs. [7](#page-6-0) and [8.](#page-6-1) One can see that  $N_{i(a)}$  locates near a host N atom with a strong bond in all charge states; the N-N distances are close to that of a  $N_2$  molecule, indicating that  $N_{i(a)}$  comprises a diatomic molecule. N 2p orbitals of the N<sub>2</sub> molecule form a triple bond, i.e.,  $\sigma$  and  $\pi$  bonds, and  $\sigma^*$ and  $\pi^*$  antibonds, as shown in Fig. [8](#page-6-1). The  $\sigma$  and  $\pi$  bonds of the  $N<sub>2</sub>$  dimerlike defect locate about 10 and 12 eV below the VBM of  $\text{Zn}_3\text{N}_2$  and the  $\sigma^*$  bond about 5 eV above the CBM. On the other hand, the  $\pi^*$  bonds exist around the band edges when occupied, and three electrons originating from the host  $N^{3-}$  are located at  $\pi^*$ . Thus, one more electron can be added to  $\pi^*$  and three electrons can be removed. Consequently,  $N_{i(a)}$  can be mainly in the −, +,

<span id="page-6-0"></span>

FIG. 7. Local atomic structures of (a) perfect  $Zn_3N_2$ , (b),(c)  $N_{i(a)}$  in the − and 3+ charge states, (d)  $N_{Zn}$  before geometry optimization, and (e),(f)  $N_{Zn}$  in the  $+$  and 3+ charge states. The N─N bond distances are also shown in Å. Note that the calculated N–N bond distances in  $N_2$  and  $N_3$ <sup>–</sup> molecules by the HSE06 functional are 1.11 and 1.18 Å.

 $2+$ , or  $3+$  charge states, as shown in Fig. [6.](#page-5-1) Furthermore, as more electrons are donated to  $\pi^*$ , the N—N bonds are weakened more and become longer. Regarding zinc interstitials (Zn<sub>i</sub>), Zn<sub>i(c2)</sub> finally transitions to the  $i(a)$  site via geometry optimization, and it is not shown in Fig. [6](#page-5-1). Both  $\text{Zn}_{i(c1)}$  and  $\text{Zn}_{i(a)}$  take the 2+ charge state in a wide Fermilevel range, and  $\text{Zn}_{i(a)}$  is lower in energy, probably thanks to the attractive electrostatic interaction from nearby N<sup>3−</sup> ions, and can be present in significant concentration at a lower Fermi level.

Zinc-on-nitrogen antisites  $(Zn_N)$  show higher energies and are not important in  $Zn_3N_2$ . On the contrary,

<span id="page-6-1"></span>

FIG. 8. (a) Calculated orbital levels of a N atom and a  $N_2$ molecule using the HSE06 functional. The energies are shown with respect to the vacuum level. Although  $N$  2s orbitals hybridize with  $\sigma$  and  $\sigma^*$ , this is not explicitly stated here for the sake of simplicity. (b) Spin-resolved-defect-induced singleparticle levels of  $N_{i(a)}$  and  $N_{Zn}$ . Note that  $N_{i(a)}^{3+}$  and  $N_{Zn}^{3+}$  do not have in-gap states, so they are not shown here. The dashed lines indicate the deep levels located outside the spurious band gap before applying corrections for the defect eigenvalues (see the text for details).

nitrogen-on-zinc antisites  $(N_{Zn})$  have lower formation energies and are donors at a lower Fermi level, which is also not compatible with the ionic view. Figures [7](#page-6-0) and [8](#page-6-1) show the atomic and electronic structures of  $N_{Zn}$ . Intriguingly, it is seen that  $N_{Zn}^{3+}$  constructs a linear bond with two N atoms, namely, an azidelike structure. The bond distances between the N atoms shown in Fig. [7](#page-6-0) are also very close to that of the azide  $N_3$ <sup>-</sup> molecule. When constructing the azidelike structure, six electrons spill out from two host N<sup>3−</sup> ions. Among these electrons, one and two electrons move to  $N_3$ <sup>-</sup> and  $V_{Zn}$ <sup>2-</sup>, respectively, and three electrons remain. Therefore, the azidelike  $N_{Zn}$  defect takes the 3+ charge state. Conversely,  $N_{Zn}$  in the 2+ or lower charge state forms a N─N bond with a nearby N atom, as in the case of  $N_{i(a)}$ .

#### 3. Oxygen and hydrogen impurities

Oxygen has been considered to be a prime source of the BM effect in  $\text{Zn}_3\text{N}_2$ . Hydrogen is also commonly unintentionally incorporated into nitrides and becomes donor centers, although it is not yet regarded as a source of the BM effect in  $Zn_3N_2$ . Therefore, we investigate their behaviors as impurities in  $\text{Zn}_3\text{N}_2$ .

Figure [9](#page-6-2) shows calculated formation energies for oxygen impurities in  $Zn_3N_2$ . The chemical potential of O is determined at the condition where Zn and ZnO coexist at 300 K [see Fig.  $4(a)$ ]. The results show that O atoms preferentially locate at the nitrogen sites and show single donor states. The oxygen-on-nitrogen defects  $(O<sub>N</sub>)$  have significantly lower formation energies than  $V<sub>N</sub>$  at a higher Fermi level. Near  $O_N^+$ , the Zn–O bonds are about 0.1 Å longer than the original Zn─N bonds, although the ionic radius of O is slightly smaller than that of N. (The ionic radius of four-coordinated  $O^{2-}$  is 1.24 Å, whereas that of  $N^{3-}$  is 1.32 Å [\[75\]](#page-11-4).) This is because the ionic attraction by

<span id="page-6-2"></span>

FIG. 9. Same as in Fig. [6](#page-5-1), but for oxygen impurities. The formation energies of native defects with lower energies, i.e., vacancies, are also shown for comparison.

<span id="page-7-0"></span>

FIG. 10. Same as in Fig. [9,](#page-6-2) but for hydrogen impurities.

 $O^{2-}$  is weaker than that by N<sup>3−</sup>. On the other hand, O atoms at the interstitial sites  $(O_i)$  are acceptors but are very high in energy.

Figure [10](#page-7-0) presents a plot of formation energies for hydrogen impurities. According to the Open Quantum Materials Database [\[76\],](#page-11-5)  $\text{Zn}_3\text{N}_2$  competes with  $\text{Zn}_3$ , NH<sub>3</sub>, and  $N_2$  molecules in the Zn-N-H ternary phase diagram at 0 K, when  $Zn_3N_2$  is assumed to be slightly stable with respect to Zn and  $N_2$ . Thus, the chemical potential of H is determined at the condition where  $\text{Zn}$ ,  $\text{Zn}_3\text{N}_2$ , and  $\text{NH}_3$ coexist at 300 K and  $p_{NH_3} = 10^5$  Pa. The results show that hydrogen atoms preferentially locate at the  $i(a)$  sites in the  $+$  charge state, and its energy is very low and even comparable to  $O_N$ . Hydrogen-on-zinc defects  $(H_{Zn})$  show acceptor behaviors and a slightly higher pinning level compared to  $V_{Zn}$ . H atoms in the H<sub>Zn</sub>, H<sub>i(a)</sub>, H<sub>i(c1)</sub>, and  $H_{i(c2)}$  defects show protonic characteristics and strongly bind to a neighboring N atom with N─H bond distances of 1.02–1.04 Å, which are very close to that of the  $NH_3$ molecule (1.01 Å by the HSE06 functional). Hydrogen-onnitrogen defects  $(H_N)$  are mostly double donors. We analyzed their atomic and electronic structures and have found that  $H_{N1}$  stays at the symmetric substitutional site with constructing the so-called hydrogen multicenter bond-ing as in InN [\[77\].](#page-11-6) On the other hand,  $H_{N2}$  shows a similar tendency but locates near two Zn atoms with a localized state. As a result,  $H_{N2}$  shows the  $2 + /0$  transition level at 1.9 eV. These defects, however, are higher in energy than  $H_i$  in the *n*-type regime.

The point-defect calculations shown ensure that the BM shift in  $Zn_3N_2$  is mainly attributed to  $O_N^+$  and/or  $H_{i(a)}^+$ , rather than  $V_N$ . This conclusion is the same as that of InN [\[77\].](#page-11-6) If, however, the nitrogen chemical potential is increased as discussed in Ref. [\[2\],](#page-9-1) the energy of  $V<sub>N</sub>$ increases, whereas the energy of  $V_{Zn}$  decreases. As a consequence, the Fermi level can be dropped near the CBM, which might happen when  $\text{Zn}_3\text{N}_2$  with a relatively low optical gap is grown.

### E. Origin of larger experimental optical gaps

Based on our theoretical calculations, it seems difficult to lift the Fermi level over 2.0 eV from the VBM in view of the Fermi-level pinning caused by  $V_{Zn}$  and  $H_{Zn}$ . In addition, we need to introduce electrons in a high concentration such as  $5 \times 10^{20}$  cm<sup>-3</sup> for band gaps larger than 2 eV (see Fig. [3\)](#page-3-1). Thus, such high band gaps are not attributed to the BM effect. Here, we discuss alternative possibilities to explain the larger optical gaps observed in experiments.

First, we consider the possibility that charged defects increase the fundamental gaps. This is called band-gap renormalization (BGR), although it usually decreases the band gaps. To check this effect, we calculate the fundamental gaps of the models with the dominant defects  $O_{N1}^+$ ,  $O_{N2}$ <sup>+</sup>, and  $H_{i(a)}$ <sup>+</sup>, but we find that these reduce the fundamental gaps by less than 0.1 eV. Therefore, BGR is not considered to be a source of the large gaps.

Another possibility is that samples are not pristine  $Zn_3N_2$ but partly include zinc oxide and/or oxinitride phases. To check this possibility, we calculate a model in which one Zn atom in the primitive cell is removed and two neighboring N atoms are replaced by O atoms. Note that charge neutrality is preserved in this model as  $V_{\text{Zn}}^{2-} + 2O_N^{+}$  is introduced. The fraction of oxygen at anion sites is then  $O/(O+N) = 2/16 = 12.5\%$ , which is slightly higher than those of intentionally oxygen-doped samples made by Cao et al. [\[18\]](#page-9-6). The calculated fundamental gap of this model is 0.91 eV, which is only 0.07 eV higher than that of pristine  $Zn_3N_2$ . Thus, small fraction of alloying with  $ZnO$  is not a cause of the large gaps. However, if a majority of the sample is oxidized to ZnO,  $\text{Zn}(\text{OH})_2$ , or their mixed phase, the band gaps are drastically changed, as discussed by Trapalis et al. [\[36\].](#page-10-22)

The third possible origin is the uncertainty of the fitting of the absorption spectra. Most groups have estimated band gaps by extrapolating the optical absorption spectra with a Tauc plot with an expression of  $(\alpha h\nu)^2$ , where  $\alpha$  is the absorption coefficient and  $h\nu$  the photon energy. However, the choice of fitting range of the spectra is widespread; in general, the wider the fitting range is, the larger the derived gaps are. Trapalis et al. developed a similar argument and found that the extrapolated band gap can also be varied when adopting the Cody plot with an expression of  $(\alpha/h\nu)^2$  [\[36\].](#page-10-22)

Although we still cannot decisively conclude the origins of larger gaps observed in  $\text{Zn}_3\text{N}_2$ , we hope our calculations and discussion help further experimental analyses.

### IV. CONCLUSIONS

In this paper, we investigate the zinc nitride  $Zn_3N_2$  using the HSE06 hybrid functional, which has been reported to accurately predict the band gaps of semiconductors, especially nitrides. First, we show the calculated density of states and band structure and find that the band gap is 0.84 eV in a direct-type band structure. We then discuss the

relation between the electron carrier concentration and the optical gap. The result indicates that the reported optical gaps between 1 and 2 eV can be mainly attributed to the BM effect. Furthermore, the parabolic approximation used for analyzing the BM shift is found to not work well for  $Zn_3N_2$ . Second, we assess thermodynamic stability of  $Zn_3N_2$  using the ideal-gas model in conjunction with the rigid-rotor model for gas phases and first-principles phonon calculations for solid phases. It is found that  $Zn_3N_2$  is slightly unstable with respect to the Zn metal and  $N_2$  gas phases at  $p_{\text{N}_2} = 10^5$  Pa at low temperature and becomes less stable at increased temperature. Third, carrier generation and compensation by native point defects and unintentionally introduced oxygen and hydrogen impurities are discussed. The results suggest that the BM shift occurs even without intentional donor doping due to oxygen substitutions on nitrogen sites and/or hydrogen interstitials. However, reported gaps larger than 2.0 eV are not due to the BM shift because of the Fermi-level pinning caused by zinc vacancies and hydrogen-on-zinc impurities. We also discuss details of peculiar defects such as nitrogen interstitials and nitrogen-on-zinc antisites with  $N_2$  molecularlike or azidelike atomic and electronic structures. Finally, we develop an argument for the possible origins of reported optical gaps larger than 2.0 eV; the likely cause is the choice of the range of absorption spectra used for the Tauc plots.

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## APPENDIX: ENERGY CORRECTIONS FOR POINT DEFECTS

Here, we address errors in defect-formation energies caused by finite sizes of supercells, and also how accurately they are corrected using a posteriori electrostatics-based corrections. More details are discussed in Ref. [\[47\]](#page-10-7).

Our extension to the FNV correction scheme uses a static dielectric tensor, namely, the sum of the ionclamped dielectric tensor  $(e^{ele})$  and the ionic contribution  $(\epsilon^{\text{ion}})$ , to account for electrostatic-potential screening [\[47\].](#page-10-7) For the test calculations using  $PBE + U$  [\[64,78\]](#page-10-15), we obtain them using density-functional perturbation theory [\[79,80\].](#page-11-7) The electronic part includes local-field effects. In cubic systems, only one diagonal component is independent. The calculated  $\epsilon^{\text{ele}}$  and  $\epsilon^{\text{ion}}$  using PBE + U  $(U_{\text{eff}} = 5 \text{ eV})$  are 8.30 and 14.60, respectively. In the

<span id="page-8-0"></span>

FIG. 11. Uncorrected, point-charge-corrected, and extended FNV-corrected relative defect-formation energies of  $V<sub>N</sub><sup>3+</sup>$  in  $Zn_3N_2$  as a function of the inverse of the cube root of the number of atoms ( $N_{\text{atoms}}$ ). Calculations are performed using PBE + U, and  $2 \times 2 \times 2$  MP k-point sampling is adopted. The zero of the relative energies is set to the extrapolated value of the infinite interdefect distance limit by fitting the defect-formation energies with a function of  $aN_{\text{atoms}}^{-1} + bN_{\text{atoms}}^{-1/3} + c$ . The upper values are the numbers of atoms in the supercells before vacancy introduction.

HSE06 defect calculations, we use the summation of the electronic part calculated using the HSE06 functional and the ionic part calculated using  $PBE + U$  at the HSE06 theoretical structure. This approximation is justified because the ionic contributions are determined from the zone-center phonon frequencies and the Born effective charges [\[81\],](#page-11-8) both of which are expected to be nearly the same between the HSE06 and  $PBE + U$ calculations at the same lattice constant.  $\epsilon^{ele}$  using the HSE06 functional with a finite-electric-field approach is 7.29, which is slightly smaller than the PBE  $+ U$  value due to the increase of the band gap, whereas  $\epsilon^{ion}$  from PBE + U is  $\epsilon^{ion} = 17.14$ .

Figure [11](#page-8-0) presents the relative formation energies of  $V_{\text{N}}^{3+}$  in  $\text{Zn}_3\text{N}_2$  with no corrections, point-charge corrections, and extended FNV corrections as a function of supercell sizes.  $V_N^{3+}$  has the largest absolute charge state, and it is expected to have the largest error in this study. One can see that, while the point-charge corrections largely overestimate the defect formation energies, especially at smaller cells, the FNV corrections drastically improve the energies; the remaining error is less than 0.05 eV—even with a 40-atom primitive cell. Therefore, we adopt the primitive cell for defect calculations. We should emphasize that Zn 3d orbitals are treated as valence electrons; therefore, the number of valence electrons in the primitive cell amounts to 368, which is computationally demanding when adopting the HSE06 functional.

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