

# Subwavelength-Sized Narrow-Band Anechoic Waveguide Terminations

Arturo Santillán,\* Emil Aerenlund, and Sergey I. Bozhevolnyi

Center for Nano Optics, University of Southern Denmark,  
Campusvej 55, DK-5230 Odense M, Denmark

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We propose and demonstrate the use of a pair of detuned acoustic resonators to efficiently absorb narrow-band sound waves in a terminated waveguide. The suggested configuration is relatively simple and advantageous for usage at low frequencies, since the dimensions of the resonators are very small compared to the wavelength. We present a theoretical description based on lumped parameters to calculate the absorption coefficient, which agrees very well with experimental data. The experimental results verify that the anechoic (reflection approximately  $-38$  dB) narrow-band ( $\Delta f/f \sim 0.1$ ) termination with deeply subwavelength ( $< \lambda/10$ ) sizes can be realized at a target frequency, suggesting thereby applications for noise control and sensing. As an illustration of possible applications for sound absorption in a room, we demonstrate by use of numerical simulations that a given axial resonant excitation in a room can be practically eliminated. Thus, a reduction of approximately 24 dB in the average acoustic energy is achieved in the room when using only four Helmholtz resonators. We also discuss various scenarios of noise control in rooms.

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## I. INTRODUCTION

The absorption of low-frequency sound is a challenging problem because it normally implies the use of bulky elements with dimensions of the order of the corresponding wavelength. The approach chosen for sound absorption can vary between different application domains. In rooms, for example, acoustic resonators are often used for attenuation of acoustic modes [1–4], although the achieved absorption is normally quite limited. To reduce the propagation of noise along waveguides, for example, in ventilation ducts, the use of active control is considered an appropriate option for low frequencies [5–8]. In recent years, acoustic metamaterials have been proposed as a promising alternative to deal with this problem [9–12], and subwavelength systems for efficient narrow-band sound absorption at low frequencies in waveguides have attracted growing attention [13–17].

In this paper, we propose and experimentally demonstrate an approach to efficient noise control at low frequencies in waveguides, with the anechoic termination in a narrow frequency band being realized with subwavelength elements. We begin with a theoretical description assuming an acoustic subsystem to be attached as a side branch to a waveguide that ends in a rigid wall at a certain distance from the subsystem position. This allows us to obtain optimal values of the subsystem (internal) absorption, inherent transmission, and reflection, which ensure the anechoic termination. We then suggest that a single Helmholtz resonator or a pair of detuned Helmholtz resonators can be used as the *optimal* acoustic subsystem,

noting that the latter represents a much simpler solution. Because of a rather fast variation of the response phase of Helmholtz resonators, the anechoic termination can be realized in a very narrow frequency band. The effect of the distance between the position of the *optimal* acoustic element and the closed end is also considered and found that practically total absorption can also be achieved for small values of this distance. We present experimental results with a pair of detuned Helmholtz resonators demonstrating the anechoic (reflection approximately  $-38$  dB) narrow-band ( $\Delta f \sim 90$  Hz) termination with deeply subwavelength ( $< \lambda/10$ ) sizes realized at the target frequency of 925 Hz. Finally, we show numerical results to illustrate the application of the method in a small rectangular room, in which an axial room mode with eigenfrequency equal to 57.2 Hz is completely suppressed by using four resonators. In this way, the absorption of sound on one wall of the room is maximized.

A system formed by two detuned Helmholtz resonators at the same axial position as the one considered in this paper has very recently been studied [18], achieving a maximum absorption coefficient of 0.5 for an incident (from one side) wave in an infinite waveguide and predicting total absorption for two mutually symmetric incident waves, one from each of the two sides of the infinite waveguide. Our approach is more general; we show below that total absorption can be achieved also when the two waves are not mutually symmetric or with only one Helmholtz resonator.

## II. THEORY

Let us assume an infinite straight waveguide with a constant cross-sectional area  $S$ , which has an acoustic

\*Corresponding author.  
aos@iti.sdu.dk

subsystem attached as side branch. Take the  $x$  direction along the axis of the waveguide and the origin at the position of the center of the branch to the acoustic subsystem.

Assume an incident harmonic sound wave,  $p_i = A_i \exp[j(\omega t - kx)]$  in  $x < 0$ , with angular frequency  $\omega$  and complex amplitude  $A_i$ , and consider that the wavelength is much larger than the transversal dimensions of the waveguide that only plane waves propagate. Thus, at  $x = 0$ , part of the acoustic energy is reflected as a wave represented by  $A_r \exp[j(\omega t + kx)]$ , and the other part of the acoustic energy continues along the pipe giving the transmitted wave  $A_t \exp[j(\omega t - kx)]$ . The pressure reflection coefficient can be expressed as [19]

$$R = \frac{A_r}{A_i} = -\frac{Z}{Z + 2Z_b}, \quad (1)$$

where  $Z_b$  is the input acoustic impedance of the acoustic subsystem,  $Z = \rho c/S$ , with  $\rho$  being the density of the medium, and  $c$  the sound speed. In addition, the pressure transmission coefficient is given by [19]

$$T = \frac{A_t}{A_i} = \frac{2Z_b}{Z + 2Z_b}. \quad (2)$$

Now let us assume a semi-infinite waveguide, which has a hard termination at  $x = L$  and the rest of the conditions are the same as before. Here, the acoustic field inside the pipe in steady state can be determined by following the propagation of the original incident wave and taking into account all the multiple reflections and transmissions (“subwaves”) that occur at the position of the acoustic subsystem ( $x = 0$ ) and all the multiple reflections at the hard end. Consider that there is no absorption on the walls of the pipe.

We can determine the resulting acoustic wave reflected at  $x = 0$  by adding the infinite number of subwaves propagating in the semispace  $x \leq 0$  as follows:

$$\begin{aligned} p_r &= A_i e^{j(\omega t + kx)} [R + T^2 e^{-j2kL} (1 + R e^{-j2kL} \\ &\quad + R^2 e^{-j4kL} + \dots)] \\ &= A_i e^{j(\omega t + kx)} \left[ R + T^2 e^{-j2kL} \frac{1}{1 - R e^{-j2kL}} \right]. \end{aligned} \quad (3)$$

By adding all the subwaves moving in the positive  $x$  direction in the space  $0 \leq x \leq L$ , we get

$$\begin{aligned} p_+ &= A_i e^{j\omega t} [T e^{-jkx} + T R e^{-jk(x+2L)} + T R^2 e^{-jk(x+4L)} \\ &\quad + T R^3 e^{-jk(x+6L)} + \dots] \\ &= A_i T e^{j(\omega t - kx)} \frac{1}{1 - R e^{-j2kL}}. \end{aligned} \quad (4)$$

The resulting wave propagating in the negative  $x$  direction in the interval  $0 \leq x \leq L$  is given by

$$\begin{aligned} p_- &= A_i e^{j\omega t} [T e^{jk(x-2L)} + T R e^{jk(x-4L)} + T R^2 e^{-jk(x-6L)} \\ &\quad + T R^3 e^{-jk(x-8L)} + \dots] \\ &= A_i T e^{j(\omega t + kx)} e^{-j2kL} \frac{1}{1 - R e^{-j2kL}}. \end{aligned} \quad (5)$$

The superposition of the two waves given by Eqs. (4) and (5) results in a standing wave described by

$$p_+ + p_- = 2A_i T \cos[k(L - x)] \frac{e^{-jkL}}{1 - R e^{-j2kL}}. \quad (6)$$

For an anechoic termination in the waveguide, the reflected wave in Eq. (3) has to be zero, which means that

$$T^2 = -R e^{j2kL} (1 - R e^{-j2kL}). \quad (7)$$

In addition, since the sound pressure has to be a continuous function in space, the complex pressure amplitude of the standing wave described by Eq. (6) has to be equal to  $A_i$  at  $x = 0$ . This condition implies that

$$T \frac{e^{-jkL}}{1 - R e^{-j2kL}} 2 \cos(kL) = 1. \quad (8)$$

By combining Eqs. (7) and (8), we obtain the optimal values of the inherent pressure reflection and transmission coefficients of the acoustic side-branch acoustic subsystem with which we achieve an anechoic termination in the waveguide, respectively, given by

$$R_{\text{op}} = \frac{1}{e^{-j2kL} - 4\cos^2(kL)} \quad (9)$$

and

$$T_{\text{op}} = -\frac{2e^{jkL} \cos(kL)}{e^{-j2kL} - 4\cos^2(kL)}. \quad (10)$$

Consider the simple cases  $kL = 0$  and  $kL = \pi$ , which are mutually equivalent. Here,  $R_{\text{op}} = -1/3$  and  $T_{\text{op}} = 2/3$ . These optimal values can be realized, for instance, with a Helmholtz resonator as the side-branch acoustic subsystem. For this case, the acoustic impedance of the resonator is given by  $Z_b = R_a + j(\omega M - K/\omega)$ , where  $R_a$ ,  $M$ ,  $K$  are, respectively, the acoustic resistance, inertance, and stiffness. In addition,  $M = \rho L_{\text{ef}}/S_{\text{res}}$  and  $K = \rho c^2/V$ , where  $V$  is the volume of the resonator,  $S_{\text{res}}$  is the cross-sectional area of the neck, and  $L_{\text{ef}}$  its effective length. The acoustic resistance of the Helmholtz resonator arises from the thermoviscous losses on its walls and the radiated acoustic energy. The former is normally the main contribution and can be reduced by increasing the cross-sectional area of the resonator.

At the resonance frequency of the Helmholtz resonator, its input acoustic impedance is equal to its acoustic resistance ( $Z_b = R_a$ ). If we make a resonator for which  $R_a = Z$  at that frequency, substitution in Eqs. (1) and (2) gives  $R = -1/3$  and  $T = 2/3$ , which are the desired optimal values.

Therefore, to obtain an anechoic termination, we have to construct a Helmholtz resonator with  $R_a = Z$  and the target resonance frequency. However, both the resonance frequency and the acoustic resistance of the resonator depend on its geometric parameters. Another simpler option to create an acoustic subsystem with the optimal inherent values is to use a pair of detuned Helmholtz resonators. These two acoustic resonators can be combined in a similar way as two electrical loads connected in parallel, and they can be represented as an equivalent single acoustic subsystem attached as a side branch to the waveguide. It has been demonstrated that two detuned Helmholtz resonators side attached to an acoustic pipe produce an induced transparency effect in a narrow frequency band (see, for instance, Refs. [20–24]). The energy transmitted in this band of induced transparency depends on the difference between the two resonance frequencies. As a result, the values of the transmitted and reflected pressure acoustic coefficients  $T$  and  $R$  can easily be changed by the value of the detuning, which can just be modified by adjusting only the volumes of the resonators.

### III. RESULTS

#### A. Waveguide with anechoic termination

We start with two identical resonators for which the neck has a diameter of 4.4 mm and a length of approximately 1 mm (Fig. 1). As a waveguide, we use a 4-m-long pipe with an inner diameter of 2 cm and a wall 5 mm thick. A driver from a horn loudspeaker is attached to one end of the pipe

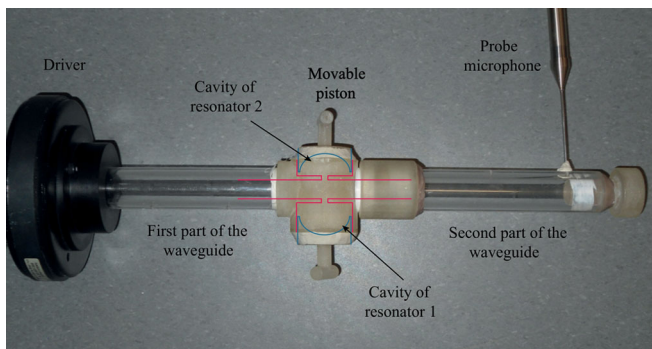


FIG. 1. Illustration of the experimental setup. The red and blue lines indicate the interior of the two Helmholtz resonators. To determine the transmission spectra shown in Fig. 2, the two short parts of the waveguide seen in the picture are substituted by two long pipes of 2 m each, and a half-inch microphone is attached at the right end of the waveguide instead of the probe microphone.

to generate a sound wave, and a half-inch-measurement microphone is placed at the other end. The two resonators are mounted as side branches to the pipe, one in front of the other, and approximately in the middle between the two ends of the waveguide. We determine experimentally the spectrum of transmission for this pair of detuned resonators following the same procedure that we describe in a previous paper [20]. By trial and error, we adjust the volume of each resonator by means of a movable piston until we produce a maximum energy transmission of approximately  $4/9$  at about 1 kHz in the transparency window [see Figs. 2(a) and 2(b)].

It is interesting to observe that the individual behavior of each of the two resonators used in our experiment is very different from the effect of induced transparency [see Figs. 2(c) and 2(d)]. The resonance frequency of the first resonator is 891 Hz, at which the transmission is equal to 0.014; for the second resonator, the resonance frequency is 1124 Hz, and the minimum transmission is 0.020. The average ambient temperature in the laboratory is 28 °C, from which we estimate the sound speed to be 347.8 m/s. From the measured transmission spectra, we also determine the following parameters for the first resonator:  $L_{\text{ef}} = 4.69$  mm,  $V = 12.51$  cm<sup>3</sup>, and  $R_a = 211.2$  Pa m/s. The calculated values for the second resonator are  $L_{\text{ef}} = 4.47$  mm,  $V = 8.26$  cm<sup>3</sup>, and  $R_a = 263.1$  Pa m/s. We use these values in our mathematical model based on lumped parameters [20] to calculate the theoretical curves in Fig. 2. It can be observed that the theoretical curves match very well the corresponding experimental curves of the transmission spectrum and the phase of the transmitted wave for each of the individual resonators. The agreement between the theoretical and experimental curves in Figs. 2(a) and 2(b) is also quite good, with slight expected deviations due to the interaction between the two resonators, which is not included in our mathematical model. The behavior of one resonator is slightly affected by the presence of the other, but in our model, we neglect this effect.

With the parameters of the resonators mentioned in the previous paragraph and  $kL = \pi$ , our model predicts near total sound absorption at 1012 Hz. If  $L = 3\lambda_{\text{op}}/8$ , where  $\lambda_{\text{op}}$  is the wavelength at 1012 Hz, the efficient absorption occurs at 1034 Hz (see Fig. 3); for  $L = 5\lambda_{\text{op}}/8$ , the reflection is practically zero at 992 Hz. Thus, small variations from the desired value of  $L = \lambda_{\text{op}}/2$  result in slight frequency shifts for the target value. Nevertheless, one can also observe that all the energy is reflected at the target frequency of 1012 Hz if  $L = \lambda_{\text{op}}/4$ . As predicted,  $L = 0$  gives practically the same result as  $L = \lambda_{\text{op}}/2$  about the frequency of 1012 Hz. In the particular cases  $kL = 0$  and  $kL = \pi$ , we have the equivalent to two symmetric incident waves arriving at the position of the detuned resonators from the sides of an infinite waveguide as studied in Ref. [18], and the results are the same.

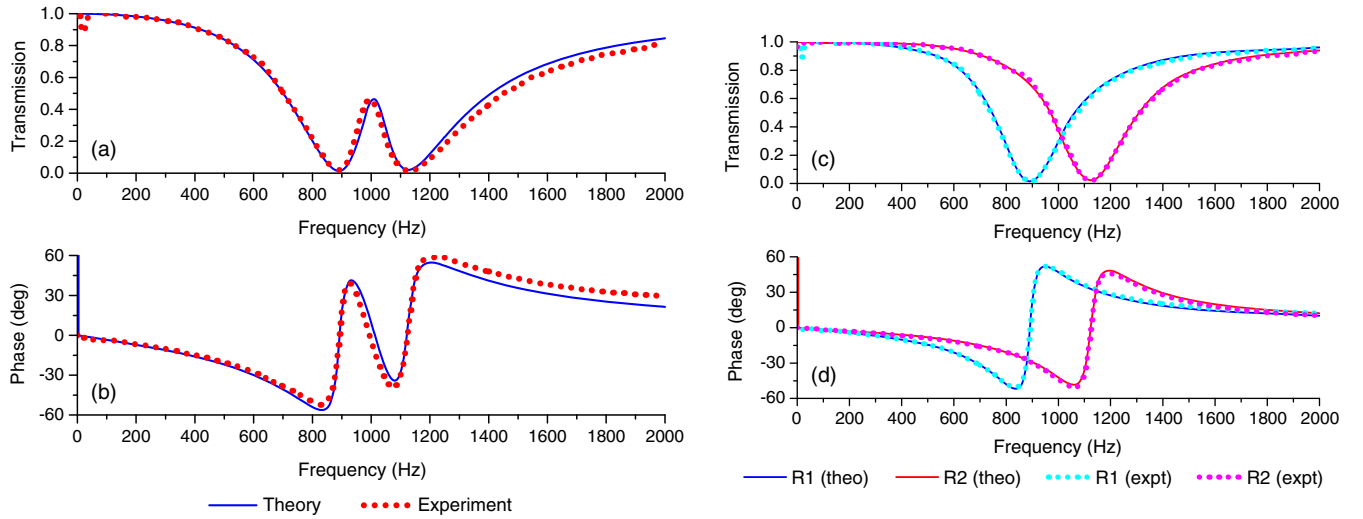


FIG. 2. (a) Transmission spectrum for a pair of detuned resonators attached to a long waveguide as side branches at the same axial position and (b) the corresponding phase of the transmitted wave. (c) Individual transmission spectrum measured for each of the two resonators and (d) the phases of the corresponding transmitted waves.

To verify experimentally the result for  $L = \lambda_{op}/2$  shown in Fig. 3, we use a short waveguide (the one seen in Fig. 1). The distance between the mouth of the driver and the position of the openings of the resonators is 20 cm, and the section of the pipe from the position of the resonators to the hard end is equal 17.48 cm, which corresponds to half of the wavelength of the target frequency of 995 Hz. At this frequency, the maximum transmission in the transparency window of the experimental curve in Fig. 2(a) is obtained.

The driver is excited with white noise to determine the incident and reflected waves as a function of frequency by using the two-microphone method [25]. We measure the

sound pressure inside the pipe at the positions  $x = -71.5$  mm and  $x = -126.5$  mm using a probe microphone with an outer diameter of 2 mm. The frequency response function between the input white noise fed to the driver and the output signal from the microphone is determined for each of the two positions. Each of these frequency response functions is obtained by the average of 960 sampled spectra taken in a measurement time of 1 min.

The results show that the pair of detuned resonators actually works as an efficient sound absorber in a narrow frequency band as predicted (Fig. 4). The experimental curve of the energy reflection coefficient has a very small

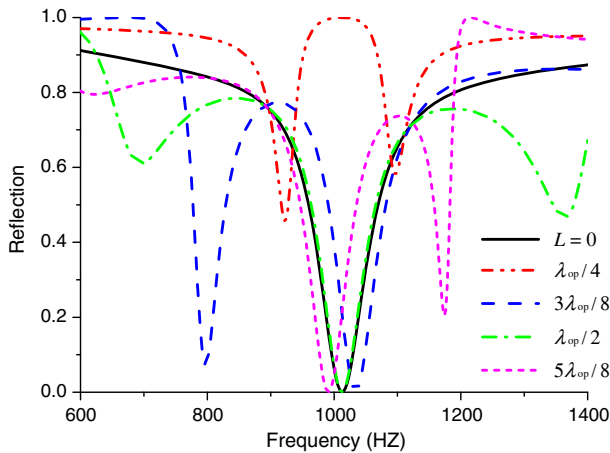


FIG. 3. Theoretical reflection spectra for a waveguide with a pair of detuned resonators and a hard termination. The target frequency for a minimum reflection (maximum absorption in the resonators) is 1012 Hz, for which the wavelength is represented as  $\lambda_{op}$ . The curves correspond to different values of the distance  $L$  between the position of the resonators and the hard end of the waveguide indicated as fractions of  $\lambda_{op}$ .

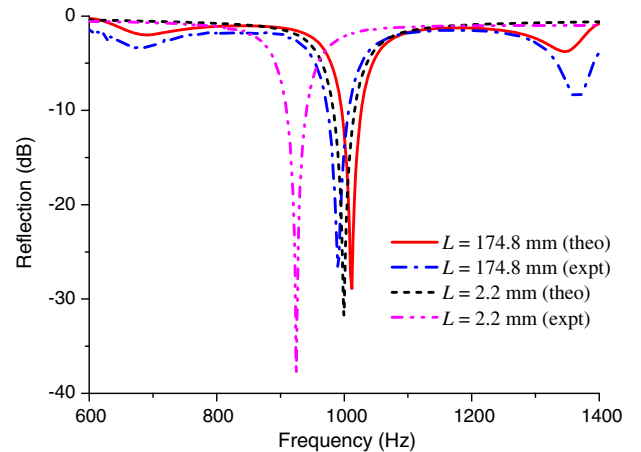


FIG. 4. Spectrum of the energy reflection coefficient for a pair of detuned resonators side branch attached to a pipe with a hard end. The distance  $L$  between the position of the resonators and the closed end is equal to half of the wavelength (174.8 mm) at the target frequency in a first case, and the same as the radius of the neck of the resonators (2.2 mm) in a second case.

value equal to 0.002 at the frequency of 990 Hz. In addition, there is quite good agreement between the experimental results and the theoretical predictions. However, the minimum reflection appears at a slightly higher frequency in the theoretical curve. This difference is explained by the interaction between the resonators, which is not included in our model.

We also carry out an experiment with the hard termination of the waveguide placed just after the edge of the mouths of the two resonators. In our theoretical model, we set the value  $L$  equal to the radius of the neck of the resonators, 2.2 mm; the result gives a very high absorption of the incident wave at almost the same frequency as in the case  $kL = \pi$ . It should be taken into account that the presence of the wall slightly increases the effective length of the neck of each resonator. Therefore, the resonance frequencies of the resonators are shifted to lower values. It is interesting to observe that the experimental curve of reflection obtained with the hard termination just after the edge of the mouths of the resonators has a pronounced trough of approximately  $-38$  dB at 925 Hz (Fig. 4). This result confirms that two detuned Helmholtz resonators placed on a hard end of a waveguide can behave almost as a perfect sound absorber, which can provide a more convenient array for practical applications.

Helmholtz resonators are used at low frequencies to damp individual and distinct modes of a large enclosure at low frequencies; however, the total sound absorption is relatively small due to the strong effect of the enclosure resonance on the acoustic resistance of the mouth of the resonators. The interaction of a Helmholtz resonator and one mode of a room, with the resonator tuned to the same frequency of the mode, normally creates two new coupled modes with natural frequencies different from the original natural frequency of the mode. As a result, the absorbed acoustic power is limited to about 6 dB [1]. Therefore, it is interesting to study if higher sound absorption can be obtained with Helmholtz resonators placed on a wall of a slightly damped room by an appropriate adjustment of their acoustic resistance.

## B. Sound absorption in a room

The method described in this paper can be used in more complex situations, for instance, to efficiently absorb the acoustic energy of standing waves between two parallel surfaces in a room. In this section, we present an illustrative example of the use of the method, in which one axial mode of a small rectangular room is practically eliminated.

### 1. The model

Consider a slightly damped rectangular room with dimension  $L_x = 6$  m,  $L_y = 4.45$  m, and  $L_z = 2.8$  m. The sound speed is assumed to be 343 m/s, and the mean density equal to  $1.2$  kg/m<sup>3</sup>. The purpose in this example is to suppress the second axial mode in the  $x$  direction, which

corresponds to a frequency of 57.2 Hz, when the room is excited by means of a sound source located in one of the corners of the room (Fig. 5). The results are obtained by means of numerical simulations using the program Comsol Multiphysics. In the model, the sound source is implemented as a point monopole with a constant reference free-space acoustic power equal to 10 mW. For simplicity, we assume that all the hard boundaries of the room have characteristic acoustic impedance 500 times larger than the characteristic impedance of air and that this impedance is independent of frequency, which is a good approximation in the low-frequency interval of interest.

We want the energy of the axial mode at 57.2 Hz to be efficiently absorbed by means of a set of Helmholtz resonators. We analyze the simple case of four resonators, each of them placed near one of the corners of the wall of the room perpendicular to the  $x$  axis. To determine the optimal values of the parameters of the resonators, first we consider an infinite waveguide with a cross section of  $4.45 \times 2.8$  m<sup>2</sup> and hard surfaces. A part of the waveguide with a length of 12 m is modeled. We start with four identical Helmholtz resonators attached to the waveguide as side branches at the same axial position  $x = 0$ , as shown in Fig. 6. To simplify the model, the losses of acoustic energy in the resonators are introduced by assuming that their necks are filled with poroacoustic material, for which the *Delany-Bazley-Miki* model implemented in Comsol is used. In this way, the energy losses in the resonators are set by means of the flow resistivity of the poroacoustic material model.

An incident plane wave with a frequency of 57.2 Hz and amplitude of 1 Pa is modeled in the waveguide approaching the resonators from the left. The parameters of the resonators are adjusted to obtain a resonance frequency of approximately 57.2 Hz and a transmitted wave with an

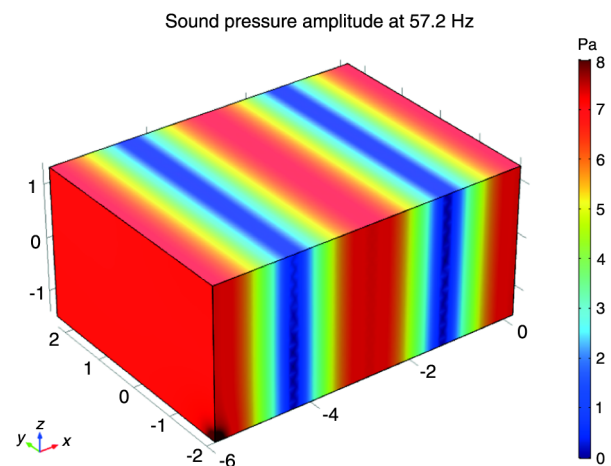


FIG. 5. Spatial distribution of the sound pressure amplitude in a rectangular room corresponding to its second  $x$ -axial mode. The sound field is generated by a point monopole source located in the corner at  $(-6, -2.2, -1.4)$  m.

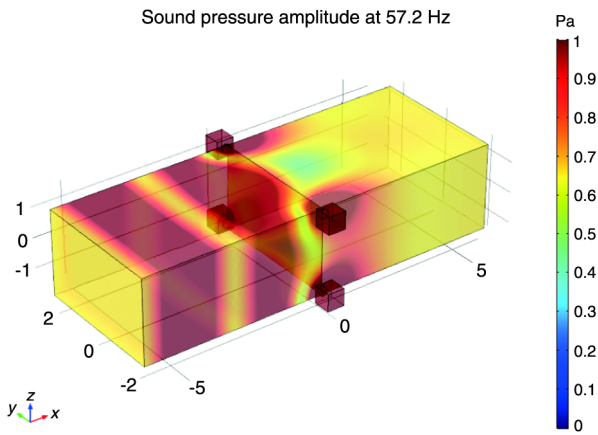


FIG. 6. Propagation of a plane wave along an infinite waveguide with a rectangular cross-sectional area of  $4.45 \times 2.8 \text{ m}^2$  and four identical Helmholtz resonators attached to the waveguide as side branches at  $x = 0$ . Only a section of the waveguide is used in the model. The incident plane wave moves in the positive  $x$  direction.

amplitude of  $2/3 \text{ Pa}$  at the same frequency, which gives the desired optimal value of the transmission coefficient (see Fig. 6). The optimal values of the parameters of the resonators are radius of the neck =  $15 \text{ cm}$ , length of the neck =  $9 \text{ cm}$ , a cubic volume with a side length of  $57.1 \text{ cm}$ , and a flow resistance in the neck of  $46 \text{ Pa s/m}^2$ .

## 2. Effect of the optimal Helmholtz resonators

With the introduction of the optimal resonators in the room model, the characteristic standing wave pattern of the axial model at  $57.2 \text{ Hz}$  disappears (Fig. 7). In addition, the sound pressure amplitude is distributed in a more homogeneous way compared to the case without the resonators, and the amplitude of the sound pressure is

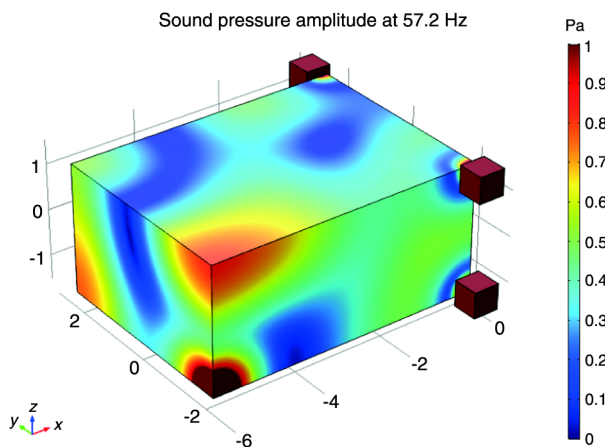


FIG. 7. Distribution of the sound pressure amplitude on the surfaces of a rectangular room with four Helmholtz resonators used to efficiently absorb the acoustic energy impinging on the wall at  $x = 0$  for a sound wave of  $57.2 \text{ Hz}$ .

significantly lower. It is interesting to observe that the highest values of the sound pressure amplitude are located around the position of the sound source and also inside the resonators. Therefore, sound is efficiently absorbed by the four resonators.

The cross-sectional area of the necks of the resonators has a significant effect on the coupling between the room and the resonators. As an illustration, we carry out simulations by using a second set of larger resonators, which have a neck radius of  $31.1 \text{ cm}$ . We again use four identical resonators and follow the same procedure mentioned above to determine the optimal parameters that produce a transmitted plane wave with amplitude equal to  $2/3 \text{ Pa}$  with the resonators attached as side branches to an infinite waveguide. As the optimal parameters of the resonators, we obtain the length of the neck equal to  $10 \text{ cm}$ , a cubic volume with a side length of  $76.9 \text{ cm}$ , and a flow resistance in the neck of  $302 \text{ Pa s/m}^2$ . To evaluate the global response of the room, we determine the average value of the square of the sound pressure amplitude over the volume of the room as a function of frequency. As can be observed in Fig. 8, the high peak of the axial mode of  $57.2 \text{ Hz}$  is removed by means of the set of four small resonators discussed above, and improved results are achieved with the set of four large resonators. The average square sound pressure at  $57.2 \text{ Hz}$  is reduced by approximately  $24 \text{ dB}$  by means of the small resonators. Since the potential acoustic energy is proportional to the square

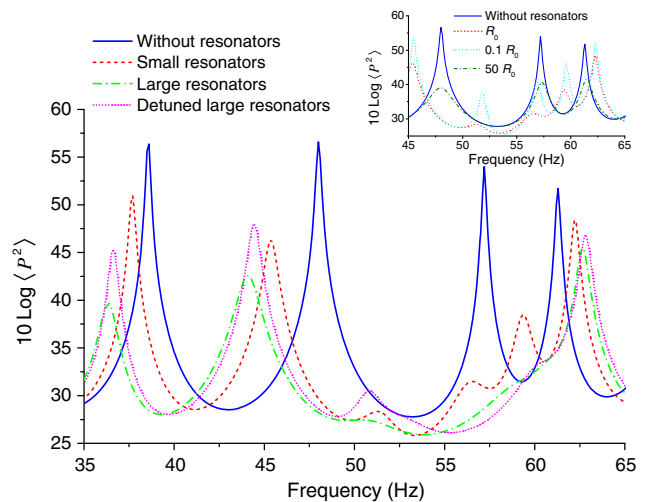


FIG. 8. Square of the sound pressure amplitude averaged over the total volume of the modeled room as a function of frequency for three different sets of four resonators. In each set, all the resonators have the same dimensions in their necks. The radius of the necks is  $15 \text{ cm}$  for the small resonators and  $31.1 \text{ cm}$  for the large and for the detuned ones. The value of the average value of the square sound pressure without resonators is also included. The inset shows the effect of different values of acoustic losses in the set of small resonators, where  $R_0$  corresponds to the optimal value.

sound pressure, the result implies that the average potential acoustic energy stored in the room is decreased by a factor of about 251 due to the absorption of energy in the resonators. A new small peak appears in the curve corresponding to the small resonators near the frequency of 59 Hz due to the coupling of the resonator and the room; however, this peak is only about 6 dB higher than the value without the resonators. Two more new peaks can also be observed at approximately 51 and 56 Hz, but the amplitudes of these peaks are lower than the respective values in the room without resonators. Nevertheless, there are no new peaks in the curve obtained with the set of large resonators. One can also observe in Fig. 8 that the resonance frequencies of other modes in the room are shifted when the resonators are used, and the stored energy of those modes is also reduced.

As mentioned before, the process to obtain the optimal acoustic resistance in the resonators can be simplified in a practical situation by using detuned resonators; in this case, one needs to adjust only the individual resonance frequencies of the resonators until the optimal combined value of the acoustic losses at the target frequency is obtained. With detuned resonators, the dimensions of the necks and the acoustic absorbent material can be kept fixed, and only the volumes of the resonators can be adjusted. As an example, we consider two pairs of detuned resonators for the absorption of the second  $x$ -axial mode presented in this example. The necks of the four resonators have the same dimensions: a radius of 31.11 cm and a length of 10 cm. Their resonance frequencies are adjusted by changing the volumes of the resonators.

To determine the optimal parameters in the resonators, we again model the propagation of a plane wave in an infinite waveguide similar to the one in Fig. 6. We identify a transparency window in the transmission vs frequency curve and change the volumes of the resonators until the maximum transmission in the transparency window is equal to  $4/9$ . To simplify the identification of the maximum value of transmission in the transparency window, a flow resistance equal to  $43.14 \text{ Pa s/m}^2$  is assumed in the necks of the two resonators with the low resonance frequency and a flow resistance of  $274.54 \text{ Pa s/m}^2$  for the two resonators with high resonance frequency. The resonators are placed in a similar distribution as in the previous cases, near the four corners of the wall at  $x = 0$ . The two resonators located near the corners at  $(0, 2.2, -1.4)$  and  $(0, -2.2, 1.4)$  are the ones with the small volumes (high frequencies). The final volumes of the resonators are two parallelepipeds with cross-sectional area of  $76.9 \times 76.9 \text{ cm}^2$  and a length of 77.67 cm for the resonators with low frequency and 67.13 cm for the resonators with high frequency. As can be seen in Fig. 8, this set of two pairs of detuned resonators also gives a significant reduction of the acoustic energy at 57.2 Hz. It should be mentioned that it is not possible to use detuned resonators with smaller neck diameters under the

studied configuration since a clear transparency window is not observed.

The optimal value of the flow resistance in the neck of the resonators provides a good approximation to the configuration that gives the maximum sound absorption at the target frequency in the room. If the flow resistance in the necks of the resonators is changed, the sound absorption at the target frequency is reduced (see inset in Fig. 8). With lower flow resistance, the mechanical coupling between the resonators and the room increases, which is reflected as the generation of new peaks in the curve of the acoustic energy stored in the room. High flow resistance reduces the mechanical coupling, but the sound absorption is also decreased. In this case, the original resonance frequencies of the room are only slightly shifted.

#### IV. DISCUSSION

In the work described in this paper, a waveguide with a closed end is used to have zero transmission for practical purposes. However, a pair of detuned resonators can be used for narrow-band acoustic noise absorption in a waveguide with an open termination. A Helmholtz resonator with low internal acoustic resistance placed as a side branch on a waveguide behaves almost as a perfect reflector at its resonance frequency. Therefore, one can use a Helmholtz resonator to simulate a rigid surface in a waveguide and place a pair of detuned resonators to efficiently absorb the acoustic energy of the incident wave. In this way, the flow of air will not be blocked, and the proposed configuration can be used in air ventilation and heat-exchange systems to reduce noise.

It should be noted that if the transversal dimensions of the waveguide are very small compared with the wavelength, most of the acoustic energy is reflected at an open termination. By using the considered pair of detuned resonators and adjusting the distance  $L$  to compensate for the phase shift of  $180^\circ$ , one can also obtain a very efficient sound absorption.

It should be mentioned that the example of the suppression of a room mode is presented as an illustration of the idea of sound absorption. In a practical application, a more appropriate solution can be explored, for instance, using more than four resonators with much smaller size, distribution of resonators in a more uniform way on a wall, or use of several pairs of slightly separated detuned resonators. In addition, the absorption of sound in a room is a more complex problem than the case of a waveguide, and it deserves further investigation, particularly, the use of pairs of detuned resonators and the absorption of the energy of other modes different from the axial ones.

#### V. CONCLUSIONS

In summary, we demonstrate that a pair of detuned Helmholtz resonators can be used as an efficient acoustic

absorbent device with a narrow frequency band in a waveguide. The resonators have to be tuned to obtain the appropriate value of each resonance frequency, but this can be done in an easy way by adjusting their volumes. A theoretical description is provided, which agrees very well with experimental results. The proposed method of sound absorption can be used to attenuate noise generated by motors since very commonly they produce noise with a narrow frequency band. Another application can be for sensing purposes, where a change in the frequency of the sound of a device, for instance, generated by a malfunction, can produce a large transmission. We further emphasize that the narrow-band anechoic termination can be realized with very small distances between the rigid-wall termination and the (opposite side-branched) detuned resonators. This remarkable feature suggests interesting possibilities for sound attenuation of distinct modes in rooms. It should also be mentioned that the studied pair of detuned resonators can be used as a unit for the elaboration of acoustic metamaterials [20,21]. We conduct further research in this area.

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- [1] F. J. Fahy and C. Schofield, A note on the interaction between a Helmholtz resonator and an acoustic mode of an enclosure, *J. Sound Vib.* **72**, 365 (1980).
- [2] D. Li, L. Cheng, G. H. Yu, and J. S. Viperman, Noise control in enclosures: Modeling and experiments with T-shaped acoustic resonators, *J. Acoust. Soc. Am.* **122**, 2615 (2007).
- [3] O. Inácio, L. Henrique, and J. Antunes, Design of duct cross sectional areas in bass-trapping resonators for control rooms, *Noise Control Engineering Journal* **55**, 172 (2007).
- [4] A. Doria, Control of acoustic vibrations of an enclosure by means of multiple resonators, *J. Sound Vib.* **181**, 673 (1995).
- [5] N. Han and C. Wang, Time-domain simulation for active noise control in a two-dimensional duct, *Noise Control Engineering Journal* **63**, 59 (2015).
- [6] P. Gardonio and J. Rohlffing, Modular feed-forward active noise control units for ventilation ducts, *J. Acoust. Soc. Am.* **136**, 3051 (2014).
- [7] Y.-J. Chan, L. Huang, and J. Lam, Effects of secondary loudspeaker properties on broadband feedforward active duct noise control, *J. Acoust. Soc. Am.* **134**, 257 (2013).
- [8] M. Kato, Active noise control in a duct by an analog neural network circuit, *Appl. Acoust.* **72**, 732 (2011).
- [9] J. Mei, G. Ma, M. Yang, Z. Yang, W. Wen, and P. Sheng, Dark acoustic metamaterials as super absorbers for low-frequency sound, *Nat. Commun.* **3**, 756 (2012).
- [10] J.-P. Groby, W. Huang, A. Lardeau, and Y. Aurégan, The use of slow waves to design simple sound absorbing materials, *J. Appl. Phys.* **117**, 124903 (2015).
- [11] F. Ma, J. H. Wu, and M. Huang, One-dimensional rigid film acoustic metamaterials, *J. Phys. D* **48**, 465305 (2015).
- [12] G. Ma, M. Yang, Z. Yang, and P. Sheng, Low-frequency narrow-band acoustic filter with large orifice, *Appl. Phys. Lett.* **103**, 011903 (2013).
- [13] G. Ma, M. Yang, S. Xiao, Z. Yang, and P. Sheng, Acoustic metasurface with hybrid resonances, *Nat. Mater.* **13**, 873 (2014).
- [14] M. Yang, C. Meng, C. Fu, Y. Li, Z. Yang, and P. Sheng, Subwavelength total acoustic absorption with degenerate resonators, *Appl. Phys. Lett.* **107**, 104104 (2015).
- [15] P. Wei, C. Croëenne, S. T. Chu, and J. Li, Symmetrical and anti-symmetrical coherent perfect absorption for acoustic waves, *Appl. Phys. Lett.* **104**, 121902 (2014).
- [16] V. Romero-García, G. Theocharis, O. Richoux, A. Merkel, V. Tournat, and V. Pagneux, Perfect and broadband acoustic absorption by critical coupling, [arXiv:1501.0019](https://arxiv.org/abs/1501.0019).
- [17] J. R. Piper, V. Liu, and S. Fan, Total absorption by degenerate critical coupling, *Appl. Phys. Lett.* **104**, 251110 (2014).
- [18] A. Merkel, G. Theocharis, O. Richoux, V. Romero-García, and V. Pagneux, Control of acoustic absorption in one-dimensional scattering by resonant scatterers, *Appl. Phys. Lett.* **107**, 244102 (2015).
- [19] L. E. Kinsler, A. R. Frey, A. B. Coppens, and J. V. Sanders, *Fundamentals of Acoustics* (John Wiley & Sons, New York, 2000), p. 291.
- [20] A. Santillan and S. I. Bozhevolnyi, Acoustic transparency and slow sound using detuned acoustic resonators, *Phys. Rev. B* **84**, 064304 (2011).
- [21] A. Santillan and S. I. Bozhevolnyi, Demonstration of slow sound propagation and acoustic transparency with a series of detuned resonators, *Phys. Rev. B* **89**, 184301 (2014).
- [22] E. H. El Boudouti, T. Mrabti, H. Al-Wahsh, B. Djafari-Rouhani, A. Akjouj, and L. Dobrzynski, Transmission gaps and Fano resonances in an acoustic waveguide: Analytical model, *J. Phys. Condens. Matter* **20**, 255212 (2008).
- [23] W. Tan, C. Z. Yang, H. S. Liu, Z. G. Wang, H. Q. Lin, and H. Chen, Manipulating classical waves with an analogue of quantum interference in a V-type atom, *Europhys. Lett.* **97**, 24003 (2012).
- [24] A. Mouadili, E. H. El Boudouti, A. Soltani, A. Talbi, B. Djafari-Rouhani, A. Akjouj, and K. Haddadi, Electromagnetically induced absorption in detuned stub waveguides: A simple analytical and experimental model, *J. Phys. Condens. Matter* **26**, 505901 (2014).
- [25] J. Y. Chung and D. A. Blaser, Transfer function method of measuring in-duct acoustic properties. I. Theory, *J. Acoust. Soc. Am.* **68**, 907 (1980).