



Nontensorial Transformation Optics

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We present an alternative version of transformation optics that allows us to mold the flow of light without rotating or scaling the electromagnetic fields. The resulting media experience unusual force densities, are nonreciprocal, and exhibit loss or gain. Because of these singular features, a variety of effects and devices unreachable by standard transformation optics can be achieved, including reflectionless light compression, optical modes with arbitrary in-plane polarization, and special isolators.

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I. INTRODUCTION

Following the general principle of relativity, physical laws must have the same form in all admissible reference frames. Electrodynamics provides a paradigmatic example of this feature (if only one understands the metric as an abstract object). In fact, Maxwell's equations admit a tensorial (T) representation that preserves its form under any coordinate transformation [1]. A recently developed branch of physics known as transformation optics (TO) exploits this form independence to find connections between the electromagnetic fields supported by different media. As a particular case, in an isotropic medium with permittivity ϵ_0 and permeability μ_0 , the time-harmonic Faraday's and Ampère's laws can be written in arbitrary spatial coordinates [e.g., arising from a transformation $\bar{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ of the Cartesian coordinates \mathbf{x}] as

$$\epsilon^{\bar{i}\bar{j}\bar{k}} \partial E_{\bar{k}} / \partial x^{\bar{j}} = -i\omega\mu_0 \sqrt{\gamma} \gamma^{\bar{i}\bar{j}} H_{\bar{j}}, \quad (1)$$

$$\epsilon^{\bar{i}\bar{j}\bar{k}} \partial H_{\bar{k}} / \partial x^{\bar{j}} = i\omega\epsilon_0 \sqrt{\gamma} \gamma^{\bar{i}\bar{j}} E_{\bar{j}}. \quad (2)$$

Here ω is the angular frequency, ϵ^{ijk} the Levi-Civita symbol, $E_{\bar{k}}$ and $H_{\bar{k}}$ the components of the electric and magnetic fields in the chosen coordinate system, $\gamma_{\bar{i}\bar{j}}$ the spatial metric associated with such a system, and γ its determinant [2]. Note that the other two Maxwell's equations (Gauss's laws) are automatically satisfied for $\omega \neq 0$, our case throughout this work. TO amounts to interpreting the coordinate-transformed version of Maxwell's equations, now on just Eqs. (1) and (2), as representing a different medium with different fields living in, which written in the original coordinate system (Cartesian in our case) are taken to be

$$E_{\bar{k}}(\mathbf{x}) = \Lambda_{\bar{k}}^k E_{0k}[\mathbf{f}^{-1}(\mathbf{x})], \quad (3)$$

$$H_{\bar{k}}(\mathbf{x}) = \Lambda_{\bar{k}}^k H_{0k}[\mathbf{f}^{-1}(\mathbf{x})], \quad (4)$$

$$\epsilon^{ij}/\epsilon_0 = \mu^{ij}/\mu_0 = \sqrt{\gamma} \gamma^{ij}, \quad (5)$$

where $\Lambda_{\bar{i}}^i = \partial x^i / \partial \bar{x}^{\bar{i}}$ and \mathbf{E}_0 and \mathbf{H}_0 are the electric and magnetic fields in the original untransformed equations.

The fact that the fields \mathbf{E}_0 and \mathbf{H}_0 supported by the original medium ϵ_0, μ_0 (so-called virtual medium) and the fields \mathbf{E} and \mathbf{H} supported by the transformed medium ϵ^{ij}, μ^{ij} (so-called physical medium) are connected by a coordinate transformation, has allowed the design of unprecedented devices, such as invisibility cloaks (see Fig. 1 for an example) [3,4], optical wormholes [5], and many others [2,6,7].

According to Eqs. (3)–(5), each element in the new set of reinterpreted equations is obtained from its original counterpart through the proper T transformation, which is linked to its assumed physical nature; the permittivity and permeability are transformed as tensor densities and the fields as covectors. Although several extensions of the original TO technique have been proposed, for example, to work with complex coordinates [8,9] or in Fourier space [10], the fact that fields and medium must transform following Eqs. (3)–(5) under coordinate changes, has been invariably accepted. This implies that the fields are rotated and scaled inhomogeneously, depending on the applied mapping, as it would happen after a change of reference frame [Fig. 1(b)]. Although this effect is usually desired or irrelevant, a different, nontensorial (NT), transformation rule might be of interest in some cases. Actually, the process of transforming the equations to find connections between different field distributions is not subjected to relativity or any physical law. The only requirement is that the transformed fields are the solution of Maxwell's equations for a given medium. Therefore, it is reasonable to explore the use of unphysical (NT) transformation laws within the TO context, as it may lead to novel physical effects. This is the purpose of this work. As discussed

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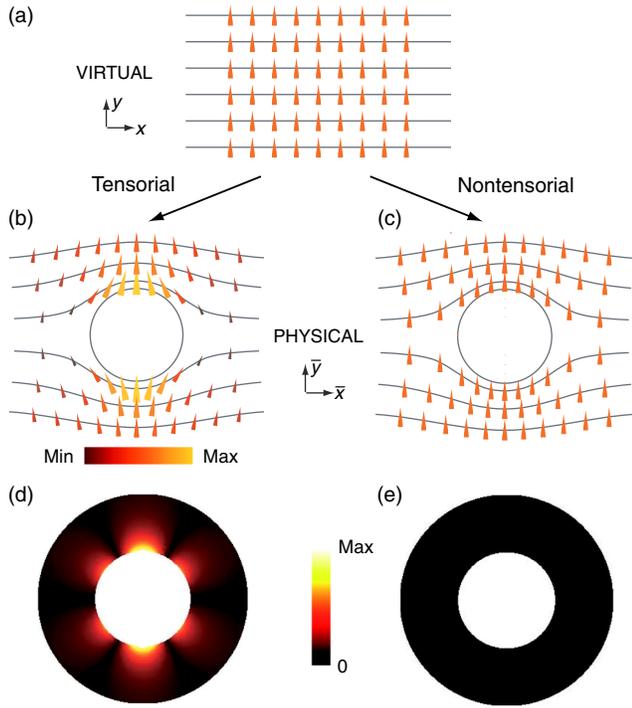


FIG. 1. A TO medium resulting from the Joukowski mapping $z = 1 + 1/\bar{z}$ ($z = x + iy$, $\bar{z} = \bar{x} + i\bar{y}$) acts as an invisibility cloak for x -propagating light waves or rays, which are made to surround a circular region without producing scattering or casting shadows. (a) Original field and rays in virtual space. (b),(c) Transformed field and rays associated with the medium prescribed by (b) standard TO and (c) the proposed NT TO technique. (d),(e) Magnitude of the calculated force density in the standard and proposed medium, respectively.

below, the formalism we will develop here around this idea has potential applications in high-energy electrodynamics, lensing, microscopy, particle acceleration, nanophotonics, and optical communications.

II. THEORY

Specifically, we focus on the possibility of achieving transformation media supporting the following transformation for the fields:

$$E_k(\mathbf{x}) = E_{0k}[\mathbf{f}^{-1}(\mathbf{x})], \quad (6)$$

$$H_k(\mathbf{x}) = H_{0k}[\mathbf{f}^{-1}(\mathbf{x})]. \quad (7)$$

That is, in this case, each field component would be transformed independently as if it were a scalar. This implies that the fields are simply transported, without being scaled or rotated as in standard TO [see Fig. 1(c)], and in contrast with previous *field* transformations based either on tensorial complex coordinate transformations [8] or on direct reciprocal linear transformations of the fields [11].

To this end, we start by expressing the new fields in terms of the old ones in the transformed Faraday's law

$$\epsilon^{\bar{i}\bar{j}\bar{k}} \partial(\Lambda_{\bar{k}}^k E_{0k}) / \partial \bar{x}^{\bar{j}} = -i\omega\mu_0 \sqrt{gg^{\bar{j}\bar{j}}} \Lambda_{\bar{j}}^k H_{0k}. \quad (8)$$

The question now is whether this equation can be made to correspond to Faraday's law for E_{0k} and H_{0k} when \bar{x}^i are interpreted as the original coordinates. Remarkably, we have identified a quite general situation in which this is the case: a conformal transformation of the xy plane, in which the electric or magnetic field lies, of an initially z -invariant problem. This kind of mapping, together with quasiconformal mappings that effectively behave as conformal ones, constitutes a wide class of transformations that has been extensively used in TO, proving to have many applications [3,7,12–22]. The Jacobian of this transformation is of the form

$$\Lambda_{\bar{i}}^i = \begin{pmatrix} a & b & 0 \\ -b & a & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (9)$$

This implies that $\partial E_{0k} / \partial \bar{x}^3 = 0$, since $\Lambda_3^1 = \Lambda_3^2 = 0$, and $\partial E_{0k} / \partial x^3 = 0$, due to the z invariance of the initial problem. Given that $\epsilon^{\bar{i}\bar{j}\bar{k}} \partial \Lambda_{\bar{k}}^k / \partial \bar{x}^{\bar{j}} E_{0k} = 0$, the left-hand member of Eq. (8) is just $\epsilon^{\bar{i}\bar{j}\bar{k}} \Lambda_{\bar{k}}^k \partial E_{0k} / \partial \bar{x}^{\bar{j}}$. Using Eq. (9),

$$\begin{aligned} \epsilon^{\bar{i}\bar{j}\bar{k}} \Lambda_{\bar{k}}^k \frac{\partial E_{0k}}{\partial \bar{x}^{\bar{j}}} &= \delta_1^{\bar{i}} \Lambda_3^1 \frac{\partial E_{03}}{\partial \bar{x}^2} - \delta_2^{\bar{i}} \Lambda_3^2 \frac{\partial E_{03}}{\partial \bar{x}^1} \\ &+ \delta_3^{\bar{i}} \Lambda_1^1 \left(\frac{\partial E_{02}}{\partial \bar{x}^1} - \frac{\partial E_{01}}{\partial \bar{x}^2} \right) \\ &+ \delta_3^{\bar{i}} \Lambda_2^1 \left(\frac{\partial E_{01}}{\partial \bar{x}^1} + \frac{\partial E_{02}}{\partial \bar{x}^2} \right). \end{aligned} \quad (10)$$

The last term in brackets simplifies to $-i\omega\mu_0 \Lambda_2^1 H_{03}$, since $\partial E_{01} / \partial x^1 + \partial E_{02} / \partial x^2 = \nabla \cdot \mathbf{E}_0 = 0$ and $\partial E_{02} / \partial x^1 - \partial E_{01} / \partial x^2 = (\nabla \times \mathbf{E}_0)_3$. Thus, defining $M_i^i = \text{diag}(1, 1, \Lambda_1^1)$, Eq. (10) can be expressed as

$$\epsilon^{\bar{i}\bar{j}\bar{k}} \Lambda_{\bar{k}}^k \frac{\partial E_{0k}}{\partial \bar{x}^{\bar{j}}} = M_i^i \epsilon^{\bar{i}\bar{j}\bar{k}} \frac{\partial E_{0\bar{k}}}{\partial \bar{x}^{\bar{j}}} - \delta_3^{\bar{i}} i\omega\mu_0 (\Lambda_2^1)^2 H_{03}. \quad (11)$$

Finally, from Eqs. (8) and (11), we obtain

$$\epsilon^{\bar{i}\bar{j}\bar{k}} \frac{\partial E_{0\bar{k}}}{\partial \bar{x}^{\bar{j}}} = -i\omega \bar{\mu}^{\bar{i}\bar{k}} H_{0\bar{k}}, \quad (12)$$

$$\bar{\mu}^{\bar{i}\bar{k}} = \mu_0 (M^{-1})_i^{\bar{i}} [\sqrt{gg^{\bar{j}\bar{j}}} \Lambda_{\bar{j}}^{\bar{k}} - \delta_3^{\bar{k}} \delta_3^{\bar{i}} (\Lambda_2^1)^2]. \quad (13)$$

Proceeding in a similar manner with Ampère's law yields analogous results, with $\epsilon^{\bar{i}\bar{k}} / \epsilon_0 = \bar{\mu}^{\bar{i}\bar{k}} / \mu_0$. Therefore, Eq. (13) represents the medium that implements the transformation

given by Eqs. (6) and (7), which after some manipulations can be expressed as

$$\frac{\epsilon^{ij}}{\epsilon_0} = \frac{\mu^{ij}}{\mu_0} = \begin{pmatrix} \Lambda_1^1 & -\Lambda_2^1 & 0 \\ \Lambda_2^1 & \Lambda_1^1 & 0 \\ 0 & 0 & \Lambda_1^1 \end{pmatrix}. \quad (14)$$

This result uncovers a whole class of media unreachable by the standard formalism. In particular, it shows that for any conformal mapping there is an alternative medium (different from the one prescribed by the standard theory) that also implements such a mapping, but without rotating or scaling the electromagnetic fields. This feature opens the door to a variety of effects and functionalities unattainable with the standard theory. To see this, compare the medium given by Eq. (14) with that prescribed by the standard formula, Eq. (5), which for the considered conformal mapping reduces to

$$\epsilon^{ij}/\epsilon_0 = \mu^{ij}/\mu_0 = \text{diag}[1, 1, (\Lambda_1^1)^2 + (\Lambda_2^1)^2]. \quad (15)$$

A first difference is related to the time-average force density \mathbf{F} experienced by each medium under plane-wave illumination. Although the correct expression for \mathbf{F} in media is still a subject of debate [23], one of the main candidates, which has been used to estimate the force density undergone by lossy particles [24], as well as transformation media [25], is the Chu force [23]

$$\mathbf{F} = \frac{1}{2} \text{Re} \{ \epsilon_0 (\nabla \cdot \mathbf{E}) \mathbf{E}^* + \mu_0 (\nabla \cdot \mathbf{H}) \mathbf{H}^* - i\omega(\epsilon - \epsilon_0) \mathbf{E} \times \mu_0 \mathbf{H}^* + i\omega(\mu - \mu_0) \mathbf{H} \times \epsilon_0 \mathbf{E}^* \}. \quad (16)$$

For real-valued permittivity and permeability distributions satisfying the relation $\epsilon^{ij}\mu_0 = \mu^{ij}\epsilon_0$, as those given by Eqs. (14) and (15), the force due to the last two terms is zero. We calculate the contribution of the first two terms for TM waves (z -polarized magnetic field). Analogous results are obtained for the TE case. Because of the z invariance of the problem, $\nabla \cdot \mathbf{H} = 0$. Without loss of generality, the electric field in virtual space can be expressed as $\mathbf{E}_0 = e^{ik_0x} \hat{\mathbf{y}}$. Hence, for the standard T medium we have $\mathbf{E}_T = (\Lambda_2^1 \hat{\mathbf{x}} + \Lambda_1^1 \hat{\mathbf{y}}) e^{ik_0 f_x^{-1}(x,y)}$, which upon substitution in Eq. (16) yields $\mathbf{F} = \partial \Lambda_2^1 / \partial x (\Lambda_2^1 \hat{\mathbf{x}} + \Lambda_1^1 \hat{\mathbf{y}})$. In the NT case, $\mathbf{E}_{NT} = e^{ik_0 f_x^{-1}(x,y)} \hat{\mathbf{y}}$, and we find that $\mathbf{F} = \mathbf{0}$, i.e., unlike standard transformation media, NT transformation media are not subjected to any stress under the assumption of the Chu force (see Fig. 1). In general, T and NT transformations will give rise to different results for any version of \mathbf{F} . This could be important for high-energy applications, in which the electromagnetic force might be large enough to damage the device.

Another difference is that the T medium satisfies the reciprocity conditions $\epsilon^{ij} = \epsilon^{ji}$, $\mu^{ij} = \mu^{ji}$, and the lossless conditions $\epsilon^{ij} = (\epsilon^{ji})^*$, $\mu^{ij} = (\mu^{ji})^*$ [26], while the NT medium does not. This is to be expected, as we can, for example, squeeze a certain field distribution without changing its amplitude. Achieving this effect for all propagation directions requires asymmetric absorption-amplification. For example, if a wave incident from the left is squeezed without increasing its intensity, and hence absorbed, the device must equally amplify a field coming from the right so that it is expanded without decreasing its amplitude. The presence of such an asymmetric loss or gain could certainly complicate implementation as compared with standard transformation media. In this respect, metamaterials may provide a feasible solution [9]. A promising example was introduced in Ref. [27], in which a metamaterial with an antisymmetric permittivity tensor having off-diagonal components with both imaginary and real parts, while requiring no bias magnetic field, was developed.

Let us now show how these special features can give rise to interesting effects and applications not attainable with standard TO.

III. APPLICATIONS

A first application is related to the compression-expansion of electromagnetic fields, for which TO provides an obvious solution [16,28,29]. Unfortunately, to transfer any light alteration from the transformed medium to the background, the transformation must be discontinuous at their boundary, and it is well known that such discontinuities introduce reflections [16,28].

Consider, for instance, an optical squeezer based on the quasiconformal mapping shown in Fig. 2(a). The device

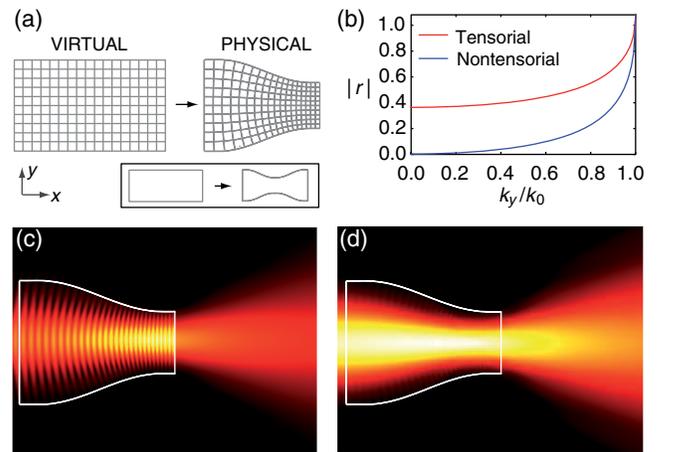


FIG. 2. Optical squeezers based on a quasiconformal mapping. (a) Employed mapping ($F = 2$). (b) Reflection coefficient. (c), (d) Norm of the electric field for the T and NT implementations (calculated with COMSOL Multiphysics). The source is a right-propagating Gaussian beam.

gradually compresses a beam entering from the left, delivering a squeezed version at the right exit. Intuitively, in a standard TO implementation, energy density is increased at the squeezer's right end (the amplitude of the fields increases) and, since energy density is lower in the background medium, a reflected wave must appear to conserve power flux density. This can be clearly seen in Fig. 2(c), where reflections at the output boundary generate a standing wave in a squeezer based on Eq. (15). In order to minimize these reflections, the so-called impedance-tunable technique is proposed [30]. However, the achieved reflection reduction is not complete and is frequency and polarization dependent. Remarkably, in our NT version of TO, the amplitude of the fields, and consequently, energy density, do not change, which suggests that it could be used to build reflectionless squeezers. In fact, no reflections appear when the medium given by Eq. (14) is employed to build the squeezer [Fig. 2(d)]. From [16], for a conformal transformation and a homogeneous isotropic exterior medium, the squeezer's reflection coefficient r fulfills

$$|r| = \frac{\sqrt{\xi^{22}(k_0^2 \xi^{33} \xi^{11} - k_y^2)/\xi^{11}} - \xi^{22} \sqrt{k_0^2 - k_y^2}}{\sqrt{\xi^{22}(k_0^2 \xi^{33} \xi^{11} - k_y^2)/\xi^{11}} + \xi^{22} \sqrt{k_0^2 - k_y^2}} \quad (17)$$

for both polarizations, where ξ^{ij} is equal to the ϵ^{ij}/ϵ_0 associated with a transformation $\bar{x} = x/F$, $\bar{y} = y/F$, F being the compression factor at the output boundary. Using Eqs. (14) and (15), we can obtain the values of r as a function of k_y (oblique incidence) for the T and NT squeezers [Fig. 2(b)]. It can be seen that the former always introduces reflections, while the latter is close to zero in a wide range of incidence angles (reflections appear at high angles to satisfy phase matching). In particular, at normal incidence $|r| = |1 - F|/(1 + F)$ for the T squeezer, while $r = 0$ for the NT one, regardless of the frequency or polarization.

This kind of reflectionless squeezer might provide an enhanced performance in applications such as hyperlensing [16], fiber-to-chip coupling [31], or even surface plasmon excitation [32]. Note that, for continuous transformations along the propagation path, NT media exhibit balanced loss-gain, since they do not alter the outer fields and thus do not supply or absorb energy [e.g., if we compress and expand space as in the inset of Fig. 2(a)].

A second unusual effect is related to uniform rotations of space $\bar{x} = x \cos \phi - y \sin \phi$, $\bar{y} = x \sin \phi + y \cos \phi$. In standard TO, such rotations do not have any physical consequence, that is, we obtain $\epsilon^{ij}/\epsilon_0 = \mu^{ij}/\mu_0 = \delta^{ij}$ from Eq. (15). This can be easily understood by looking at Figs. 3(a) and 3(b). Since every vector in virtual space is rotated, including the fields and wave vector \mathbf{k} associated with any fundamental mode, the transformation can be seen just as a change of perspective. However, in the NT case,

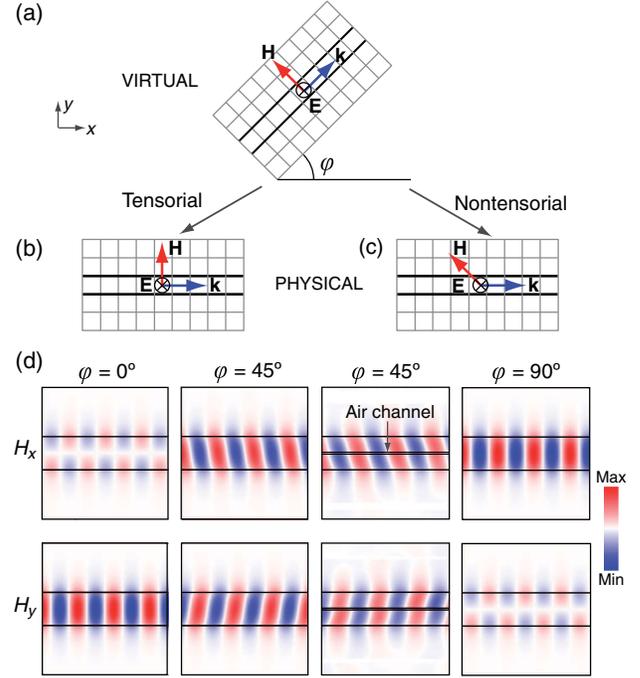


FIG. 3. Uniform space rotation. (a) Cartesian grid and fields in virtual space. Thicker lines may represent a waveguide. (b),(c) Transformed grid and fields in (b) a standard TO medium and (c) an NT medium. (d) In-plane field of the transformed quasi-TM mode supported by a rotated dielectric waveguide (core width: $1 \mu\text{m}$, free-space wavelength: $1.55 \mu\text{m}$) implemented with an NT medium. COMSOL simulations for $\phi = 0^\circ$ (original untransformed mode), $\phi = 45^\circ$ (with and without a 50-nm air gap), and $\phi = 90^\circ$ are shown. The refractive indexes of the original core and cladding are 1.5 and 1.

the fields keep their orientation while \mathbf{k} rotates, modifying the in-plane polarization of the original modes [Fig. 3(c)]. In fact, it can be easily checked that the NT medium obtained with Eq. (14) is different from the original one. This effect can be used, for example, to engineer the modes supported by dielectric waveguides, as shown in Fig. 3(d). In this way, the original quasitransverse in-plane polarization can be rotated, even achieving quasilongitudinal electric or magnetic modes. Moreover, simulations show that a narrow air channel can be created at the center of the waveguide without significantly altering the fields. This special field directionality could have applications in particle acceleration and optical tweezing, among others [33].

As a final example, consider the nonuniform rotation of space induced by the complex exponential function $\bar{z} = \exp(z)$ (Fig. 4). This transformation is approximately continuous (reflectionless) at the bottom boundary. With a T implementation, the device basically rotates the fields as they go through it [Figs. 4(a) and 4(b)]. Therefore, electromagnetic modes are also very similar at both sides of the left boundary, so the device acts just as a reciprocal bend [7]. However, with an NT implementation, the fields

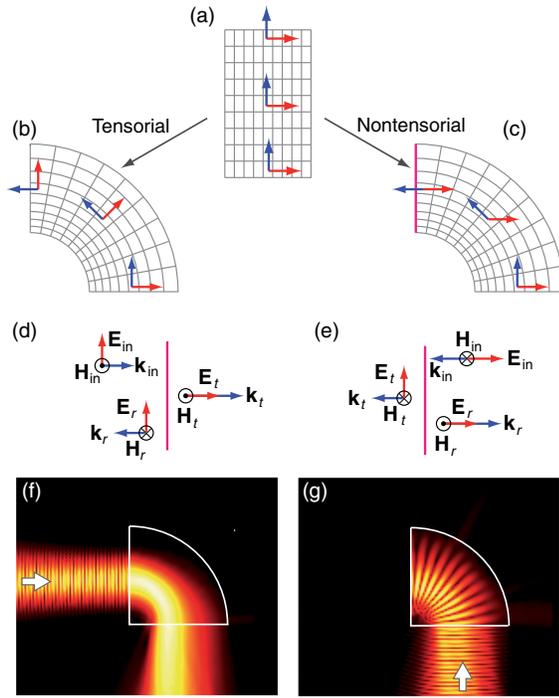


FIG. 4. Optical isolator based on the $\bar{z} = \exp(z)$ map. (a) Grid, electric field (red), and wave vector (blue) in virtual space (TM polarization). (b),(c) Transformed grid, electric field, and wave vector using (b) standard TO and (c) NT TO. (d),(e) Incident, reflected, and transmitted waves at the device left boundary for left and right incidence. (f),(g) Norm of the magnetic field in each case (calculated with COMSOL).

do not rotate [Fig. 4(c)] and, consequently, electromagnetic modes are completely different at each side of the left boundary. A nonreciprocal behavior arises as follows (we analyze the TM case, the result being the same for TE waves). Take the reflection and transmission coefficients to be r and t , respectively. According to Fig. 4(d), continuity of the tangential components implies that $r = -1$ ($\mathbf{E}_{\text{in}} + \mathbf{E}_r = \mathbf{0}$) and $T = 2$ ($\mathbf{H}_{\text{in}} - \mathbf{H}_r = \mathbf{H}_t$) for a wave impinging from the left. That is, the incident wave is completely reflected while, at the same time, it generates a transmitted wave with double amplitude [Fig. 4(f)]. Proceeding analogously for the reverse direction, we obtain $r = 1$ and $t = 0$ [Fig. 4(e)], i.e., a wave coming from the bottom is just reflected [Fig. 4(g)]. Hence, the device acts as an isolator that looks like a mirror to a left observer (no wave exits the device and right-propagating waves are completely reflected).

It is worth mentioning that this special kind of isolator is not gyrotropic (i.e., the off-diagonal components of the constitutive parameters have no imaginary part), as Eq. (14) shows. Instead, its nonreciprocal behavior arises from the device loss-gain, and thus represents an alternative to typical Faraday isolators. In addition, note that nonreciprocal devices can also be achieved with \mathcal{PT} -symmetric transformation media [9]. However, such devices present

balanced loss and gain, and cannot provide the particular behavior of the proposed NT bend. Likewise, a certain kind of unidirectional propagation based on mode conversion can be achieved using reciprocal media resulting from gauge field transformations [34]. Nonetheless, such devices are not true isolators as the proposed NT bend, in which the input and output modes are the same [35].

IV. CONCLUDING REMARKS

Our results show that a completely different class of transformation media displaying unusual effects can be obtained by assigning nonstandard transformation rules to the elements of the electromagnetic equations, opening different pathways and applications in the field of transformation optics. Besides the aforementioned potential utilities of the designed squeezer and waveguides, these optical elements could provide improvements in applications such as solid immersion microscopy, nanolithography, and optical data storage [16,33,36]. In addition, the alternative route for the creation of isolators that we have presented might be of particular interest, as this kind of device plays an essential role in microwave and nanophotonic technology, including laser protection, as well as the suppression of undesired crosstalk, signal routing, or interferences in a communication system [27,35].

Moreover, the proposed idea provides additional perspectives to the general transformation concept, as it could be applied to other branches of physics in which tensorial or vectorial quantities, such as flux densities, are involved.

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- [1] E. G. Post, *Formal Structure of Electromagnetics: General Covariance and Electromagnetics* (Interscience Publishers, New York, 1962).
 - [2] U. Leonhardt and T. Philbin, *Geometry and Light. The Science of Invisibility* (Dover Publications, New York, 2010).
 - [3] U. Leonhardt, Optical conformal mapping, *Science* **312**, 1777 (2006).
 - [4] J.B. Pendry, D. Schurig, and D.R. Smith, Controlling electromagnetic fields, *Science* **312**, 1780 (2006).
 - [5] A. Greenleaf, Y. Kurylev, M. Lassas, and G. Uhlmann, Electromagnetic Wormholes and Virtual Magnetic Monopoles from Metamaterials, *Phys. Rev. Lett.* **99**, 183901 (2007).

- [6] H. Chen, C. T. Chan, and P. Sheng, Transformation optics and metamaterials, *Nat. Mater.* **9**, 387 (2010).
- [7] L. Xu and H. Chen, Conformal transformation optics, *Nat. Photonics* **9**, 15 (2015).
- [8] B.-I. Popa and S. A. Cummer, Complex coordinates in transformation optics, *Phys. Rev. A* **84**, 063837 (2011).
- [9] G. Castaldi, S. Savoia, V. Galdi, A. Alù, and N. Engheta, \mathcal{PT} Metamaterials via Complex-Coordinate Transformation Optics, *Phys. Rev. Lett.* **110**, 173901 (2013).
- [10] G. Castaldi, V. Galdi, A. Alù, and N. Engheta, Nonlocal Transformation Optics, *Phys. Rev. Lett.* **108**, 063902 (2012).
- [11] F. Liu, Z. Liang, and J. Li, Manipulating Polarization and Impedance Signature: A Reciprocal Field Transformation Approach, *Phys. Rev. Lett.* **111**, 033901 (2013).
- [12] J. Li and J. B. Pendry, Hiding under the Carpet: A New Strategy for Cloaking, *Phys. Rev. Lett.* **101**, 203901 (2008).
- [13] N. I. Landy and W. J. Padilla, Guiding light with conformal transformations, *Opt. Express* **17**, 14872 (2009).
- [14] M. Schmiele, V. S. Varma, C. Rockstuhl, and F. Lederer, Designing optical elements from isotropic materials by using transformation optics, *Phys. Rev. A* **81**, 033837 (2010).
- [15] N. I. Landy, N. Kundtz, and D. R. Smith, Designing Optical Elements from Isotropic Materials by Using Transformation Optics, *Phys. Rev. Lett.* **105**, 193902 (2010).
- [16] C. García-Meca, M. M. Tung, J. V. Galán, R. Ortuño, F. J. Rodríguez-Fortuño, J. Martí, and A. Martínez, Squeezing and expanding light without reflections via transformation optics, *Opt. Express* **19**, 3562 (2011).
- [17] C. García-Meca, A. Martínez, and U. Leonhardt, Engineering antenna radiation patterns via quasi-conformal mappings, *Opt. Express* **19**, 23743 (2011).
- [18] Q. Wu, J. P. Turpin, and D. H. Werner, Integrated photonic systems based on transformation optics enabled gradient index devices, *Light Sci. Appl.* **1**, e38 (2012).
- [19] D. Liu, L. H. Gabrielli, M. Lipson, and S. G. Johnson, Transformation inverse design, *Opt. Express* **21**, 14223 (2013).
- [20] H. Xu, H. Sun, and B. Zhang, Waveguide design and application with transformation optics, *Sci. China Inform. Sci.* **56**, 1 (2013).
- [21] C. García-Meca, R. Ortuño, J. Martí, and A. Martínez, Full three-dimensional isotropic transformation media, *New J. Phys.* **16**, 023030 (2014).
- [22] D. Shin, Y. Urzhumov, D. Lim, K. Kim, and D. R. Smith, A versatile smart transformation optics device with auxetic elasto-electromagnetic metamaterials, *Sci. Rep.* **4**, 4084 (2014).
- [23] B. A. Kemp, Resolution of the Abraham-Minkowski debate: Implications for the electromagnetic wave theory of light in matter, *J. Appl. Phys.* **109**, 111101 (2011).
- [24] B. A. Kemp, T. M. Grzegorzczuk, and J. A. Kong, Optical Momentum Transfer to Absorbing Mie Particles, *Phys. Rev. Lett.* **97**, 133902 (2006).
- [25] H. Chen, B. Zhang, Y. Luo, B. A. Kemp, J. Zhang, L. Ran, and B.-I. Wu, Lorentz force and radiation pressure on a spherical cloak, *Phys. Rev. A* **80**, 011808(R) (2009).
- [26] F. Capolino, *Theory and Phenomena of Metamaterials* (CRC Press, Boca Raton, FL, 2009).
- [27] Z. Wang, Z. Wang, J. Wang, B. Zhang, J. Huangfu, J. D. Joannopoulos, M. Soljačić, and L. Ran, Gyrotropic response in the absence of a bias field, *Proc. Natl. Acad. Sci. U.S.A.* **109**, 13194 (2012).
- [28] M. Rahm, D. A. Roberts, J. B. Pendry, and D. R. Smith, Transformation-optical design of adaptive beam bends and beam expanders, *Opt. Express* **16**, 11555 (2008).
- [29] P.-H. Tichit, S. N. Burokur, and A. de Lustrac, Waveguide taper engineering using coordinate transformation technology, *Opt. Express* **18**, 767 (2010).
- [30] J. Cao, L. Zhang, S. Yan, and X. Sun, Reflectionless design of optical elements using impedance-tunable transformation optics, *Appl. Phys. Lett.* **104**, 191102 (2014).
- [31] P. Markov, J. G. Valentine, and S. M. Weiss, Fiber-to-chip coupler designed using an optical transformation, *Opt. Express* **20**, 14705 (2012).
- [32] C. García-Meca, R. Ortuño, J. Martí, and A. Martínez, Exciting surface plasmons with transformation media, *Plasmonics* **7**, 701 (2012).
- [33] J. B. Driscoll, X. Liu, S. Yasseri, I. Hsieh, J. I. Dadap, and R. M. Osgood, Large longitudinal electric fields (E_z) in silicon nanowire waveguides, *Opt. Express* **17**, 2797 (2009).
- [34] F. Liu and J. Li, Gauge Field Optics with Anisotropic Media, *Phys. Rev. Lett.* **114**, 103902 (2015).
- [35] D. Jalas *et al.*, What is—and what is not—an optical isolator, *Nat. Photonics* **7**, 579 (2013).
- [36] C. García-Meca and M. M. Tung, The variational principle in transformation optics engineering and some applications, *Math. Comput. Modell.* **57**, 1773 (2013).