Current Filamentation in Large $Bi_2Sr_2CaCu_2O_{8+\delta}$ Mesa Devices Observed via Luminescent and Scanning Laser Thermal Microscopy

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We study the self-heating of a large stack of $Bi_2Sr_2CaCu_2O_{8+\delta}$ intrinsic Josephson junctions, of a configuration designed for terahertz generation. We find good qualitative agreement between direct thermoluminescent measurements of the device surface temperature and low-temperature scanning laser microscopy images. In particular, the two techniques both reveal a mode of thermal instability through the asymmetric nucleation of a small hot spot near a corner or edge of the sample. This behavior conforms with a theoretical stability analysis, and the radius of the hot spot is in excellent agreement with theoretical predictions, as is its growth with increasing bias current and bath temperature. Narrow hot spots may offer a possible means of enhancing the terahertz emission power from this type of device.

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I. INTRODUCTION

Stacks of intrinsic Josephson junctions (IJJs) in the high-temperature superconductor $Bi_2Sr_2CaCu_2O_{8+\delta}$ [1] form a promising compact source of coherent terahertz (THz) radiation [2,3]. For a number of reasons, it is essential to understand self-heating behavior in this type of device. Because of the low thermal conductivity of $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Bi-2212), self-heating in large stacks of IJJs is substantial when they are biased to current-voltage conditions typical for THz generation [4–7]. Excessive self-heating may obviously be detrimental to THz emission, if the device entirely self-heats above its superconducting critical temperature. Theoretical predictions [6,7] as well as experimental evidence [8,9] suggest that a certain amount of self-heating may be beneficial for emission from these devices, especially if the resulting mesa temperature is very inhomogeneous. A localized thermal hot spot may act as an internal resistive shunt, promoting phase locking of the stacked junctions to the same voltage and THz emission frequency. Also, if the pattern of self-heating is highly asymmetric with respect to the cavity mode being excited, then coupling of the Josephson oscillations to free space–and thus the level of emitted THz power–is predicted to be strongly enhanced [10]. Localized hot spots are observed in low-temperature scanning laser thermal microscopy (LTSLM) studies [11–13] as well as in thermoluminescence imaging [14–16]. Indeed, the highest levels of THz power observed to date from Bi-2212 mesa devices [17,18] occur in the presence of strong self-heating typical for hot-spot formation. However, recent thermal-imaging experiments do not reveal a clear correlation between hot-spot formation and THz emission [15,16], whereas emission experiments under uniform and nonuniform biasing conditions indicate that an inhomogeneous temperature may benefit emission [19]. It is therefore crucial to understand the complex dynamic states that arise from the coupling from highly nonuniform temperature distributions and the nonlinear Josephson dynamics, as has been recently investigated numerically in a model of 1D coupled sine-Gordon equations [20]. Here we present a thermal-imaging study combining thermoluminescent micro-imaging and LTSLM on the same device. Both techniques yield the same results if the proper sensitivity function of LTSLM is taken into account. Furthermore, by careful heat sinking of the sample we extend the range of thermal imaging to lower temperatures and observe a mode of thermal hot-spot dynamics not reported in previous studies. At low temperatures, hot spots with sizes that are significantly smaller than the sample size nucleate in a corner (at the edge) of the sample and undergo on increasing bias current a sequence of multistability and hysteretic jumps. Such findings are in agreement with model calculations of the stability of current filaments [21]. At higher temperatures, the hot-spot size increases strongly, resulting in a location

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near the center of the sample; and on further increasing the temperature the hot-spot size exceeds the sample dimension and a thermal instability no longer occurs.

II. EXPERIMENT

The sample studied in this work is a Bi-2212 mesa of dimensions $300 \times 60 \times 0.8\ \mu m^3$ containing approximately 530 junctions and patterned on the surface of an optimally doped Bi-2212 crystal of approximately 4-μm thickness that has been soldered to a 0.5-mm-thick Cu substrate. In this mounting scheme, epoxies are avoided, enabling optimal heat removal from the mesa. At bath temperatures of around 60 K and under correct biasing conditions, it can generate up to 60 μW of coherent far-field radiation at approximately 0.6 THz [14]. In this work we investigate the unusual and complex current filamentation behavior, which this device exhibits at lower bath temperatures.

Bi-2212 is characterized by a rather poor c-axis thermal conductivity and a c-axis electrical conductivity, which strongly rises with increasing temperature. Therefore, under certain combinations of electrical biasing and heat transport, Bi-2212 mesas may undergo a thermal instability resulting in the formation of regions of strongly localized thermal runaway, associated with the appearance of so-called current filaments, which carry a much higher current density and which self-heat to a much higher temperature than the surrounding material. This phenomenon is not specific to superconductivity. It has been known in semiconducting materials since the 1930s [22], and the properties of current filaments have been extensively studied since then [21,23–26]. If such filaments have a characteristic length scale which is significantly smaller than the dimensions of the device, then the device may exhibit the formation, migration, and switching behavior of hot spots as shown in Figs. 1 and 2 below. For the purposes of analytically understanding hot-spot behavior, a Bi-2212 mesa is equivalent to an electrically biased sheet of semiconducting material, as has been considered in detail by Volkov and Kogan [24]. Both materials are characterized by positive $d\sigma/dT$, an in-plane thermal conductivity κ_{ab}, and a heat-removal coefficient P_0. For a Bi-2212 mesa device, heat removal occurs through the Bi-2212 base crystal [5] at a rate given by $P = P_0(T - T_0)$, where T is the (average) mesa temperature and T_0 the bath temperature.

A requirement for electrical multistability and current filamentation to arise is the appearance of a branch in the current-voltage (I-V) characteristics with negative differential conductivity, such as in S- or N- (or Z-) shaped characteristics [25,26]. Overheating in materials with $d\sigma/dT > 0$ typically results in S-shaped characteristics (see Fig. 1). Under voltage bias, this branch is not stable, and bistability arises in which the current jumps to the upper (lower) branch at the turning points of the I-V characteristics. For sufficiently high load resistance R in the external circuit such that $R > R_c$, where R_c is the c-axis resistance of the mesa, the branch with negative differential conductivity is stable against uniform fluctuations and can in principle be traced out. However, this state may be unstable towards inhomogeneous fluctuations in which the current density and temperature become dependent on coordinates x and y, that is, towards the formation of hot spots and current filaments. Stability analysis [21,24] reveals that the minimum lateral dimension of the sample for an inhomogeneous state to occur is given as $l_c \approx \pi\sqrt{\kappa_{ab}/(\sigma'_c E_c^2 - P_0/h_c)}$, where $E_c = V_{\mathrm{mesa}}/h_c$ is the c-axis electric field, h_c is the height of the mesa, σ_c is the c-axis conductivity, and $\sigma'_c = \partial\sigma_c/\partial T$ is its temperature derivative. Here, $\sigma'_c E_c^2 > P_0/h_c$, which is in fact the condition for a branch in the I-V curve with negative differential conductivity to occur. The quantities in these expressions refer to the homogeneous state, which may be difficult to access experimentally, since a sufficiently large sample is unstable against the formation of hot spots. Alternatively, near the turning point of the I-V characteristics, one can obtain the temperature distribution through an expansion up to second order in temperature around a stable point on the lower branch of the I-V curve. In this way we obtain that a hot spot (HS) in a strip-shaped sample has a characteristic half-width of $x_{\mathrm{HS}} = \sqrt{\kappa_{ab}/(P_0/h_c - \sigma'_c E_c^2)}$, where the quantities are evaluated at the stable reference point. The mesa temperature around the hot spot is approximately given by $T = T_1 + (T_2 - T_1)/\cosh^2(x/2x_{\mathrm{HS}})$, where the hot spot is centered at $x = 0$ with a central temperature T_2, while T_1 is the mesa temperature far from the hot spot. If the sample is larger than l_c in both in-plane directions, a two-dimensional hot spot arises, and the heat diffusion equation must be solved numerically. A fit to the numerical solutions yields a hot-spot radius still approximately given by $r_{\mathrm{HS}} \approx \sqrt{\kappa_{ab}/(P_0/h_c - \sigma'_c E_c^2)}$ and an excess current drawn by the hot spot of $I_{\mathrm{HS}} = \iint_{\text{hot-spot}} \{J_z[T(x,y)] - J_z[T_1]\} dx dy \approx 62\sigma'_c\kappa_{ab}/\sigma''_c E_c$. In addition to material parameters (κ_{ab} and σ'_c), the hot-spot size and the critical sample size depend sensitively on details of the experimental setup through the heat-removal coefficient P_0.

The two imaging techniques that we compare in this manuscript have been demonstrated previously, although not on the same device. Thermoluminescent micro-imaging [see Fig. 1(a)] has been employed for a number of applications in the past [27], while more recently we have used it for imaging self-heating in Bi-2212 mesas [14]. As described in Ref. [14], we take advantage of the temperature dependence of the 612-nm fluorescence line of the europium chelate compound europium thenoyltrifluoroacetonate (EuTFC). A recent variation of this technique employs a coating of SiC granules in place of the EuTFC film [15,16]. The key advantage of thermoluminescent

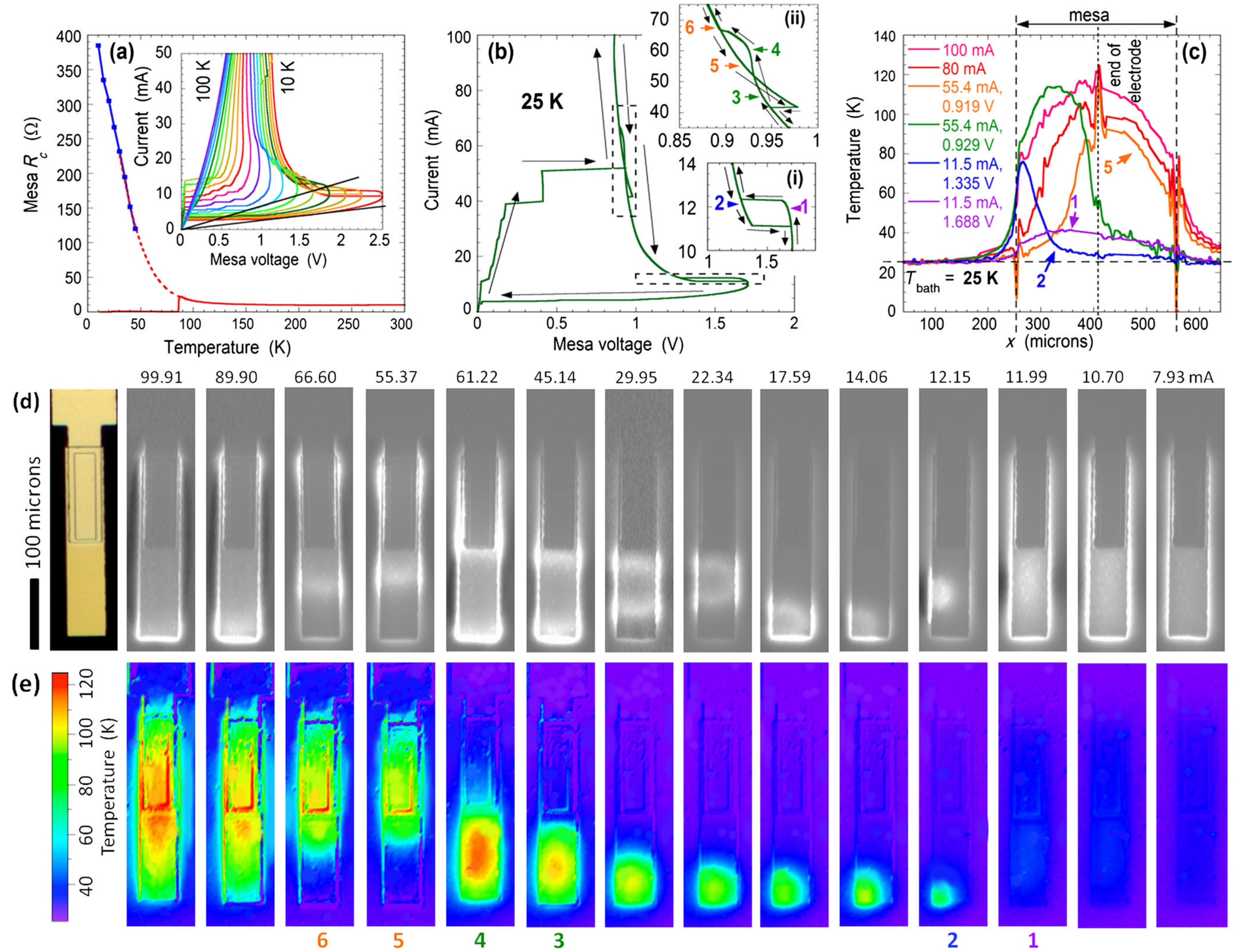

FIG. 1. (a) Main image: $R(T)$ for Bi-2212 mesa (red solid curve) measured in the absence of self-heating, using 10 μA bias current. Values for $T < T_c$ (blue squares) are determined by extrapolating from the slope of the low-bias I-V characteristic (black lines in inset). The dotted line is a fit of the form $R_c(T) = T^{-\alpha}P(T)$ used to interpolate between these two types of data. $P(T)$ is a seventh-order polynomial. Inset: I-V curves on downward current sweep at 5 K bath temperature intervals between 10 and 100 K. (b) I-V curve for the device at a bath temperature of 25 K. Black arrows indicate the current sweep direction. These I-V curves present the lower half of typical S-shaped I-V curves; the upper portion cannot be reached due to limits on the current the device can carry. Insets (i) and (ii) show regions of I-V characteristic where it is hysteretic due to nucleation and relocation of the hot spot, respectively. (c) Longitudinal temperature profiles of the mesa, taken through the center line of the hot spot in selected images from (e), which are temperature maps of the mesa surface obtained by thermoluminescent imaging. Note the distinct hot-spot (2) and no-hot-spot states (1) which exist for certain currents. Reflection of excitation light off vertical surfaces results in artifacts in the images at the edges of the mesa and the electrode. (d) The leftmost image is a conventional optical micrograph of the mesa, showing the Au thin-film electrode (500 nm thick) extending from the top end of the device. Black-and-white images are LTSLM raster scans of the mesa at the same set of bias currents. No LTSLM signal is seen for the top area of the mesa. Here the Au electrode almost completely reflects the incident laser beam and prevents it from thermally coupling to the device.

imaging is that it provides a direct quantitative map of the surface temperature of the sample.

As described in Refs. [11–13], LTSLM uses a modulated laser beam to induce a modulation of the voltage across a current-biased sample. LTSLM offers excellent sensitivity and spatial resolution; however, it is difficult to quantitatively convert these results into a sample temperature, since the observed signal depends not only upon the electrical conductivity derivative $\sigma_c'(x,y)$ and c-axis local current density $J_z(x,y)$ but also on the thermal conductivities $\kappa_{ab}[T(x,y)]$ and $\kappa_c[T(x,y)]$ as well as weakly on the heat capacity $C_p[T(x,y)]$, which have their own temperature dependencies that could, for example, induce nonmonotonic signals.

Figure 1 shows images collected on the same Bi-2212 mesa THz device via both techniques, for a range of currents, and at a bath temperature of 25 K. Figures 1(d) and 1(e) show excellent qualitative agreement between the results of the two techniques. For bath temperatures of 45 K

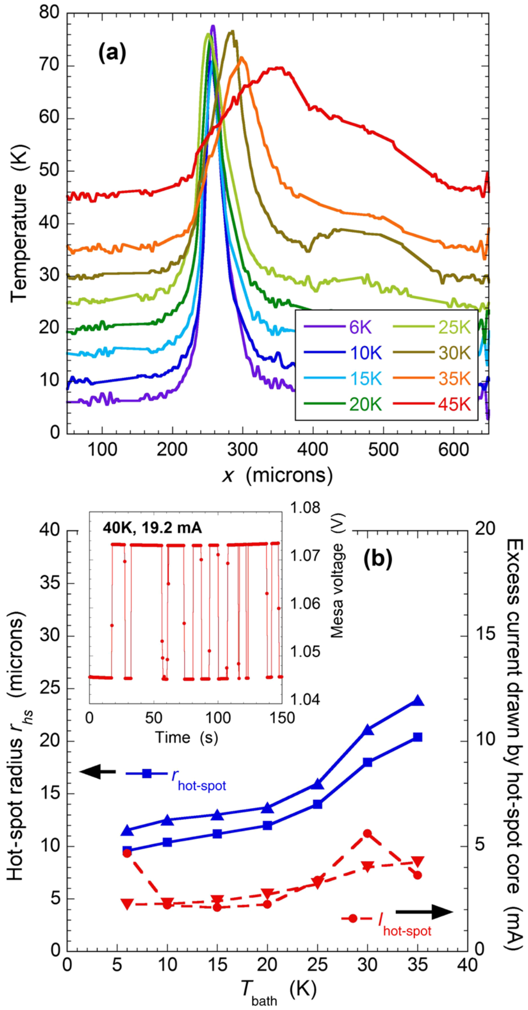

FIG. 2. (a) Longitudinal thermal cross section of the mesa, taken through the center line of the hot spot at its minimum stable size at each bath temperature. (At 45 K there is no longer a distinct hot-spot state and no corresponding *I*-*V* jumps.) (b) Main image: Hot-spot radius r_{HS} (squares, solid lines) and excess current I_{HS} (circles, dashed lines) plotted against T_{bath}. Experimental values are plotted as squares and circles, while triangles are predictions calculated by using the relations given in the main text. Inset: Random switching of the mesa voltage, at $T_{\mathrm{bath}} = 40$ K, corresponding to continual switching of the mesa in and out of a hot-spot state.

and above, thermal imaging reveals self-heating over a broad area of comparable size to the mesa itself, with self-heating strongest in the center of the mesa. This is similar to the results reported in Ref. [14] and the results of numerical simulations of Bi-2212 mesa self-heating [6,7]. However, at bath temperatures of 35 K and below, a new type of behavior appears. For sufficiently large bias currents on the return leg of the *I*-*V* characteristic (typically around 10 mA), a small hot spot hysteretically nucleates as shown in Figs. 1(a) and 3(a). At the lowest temperatures, the hot spot is clearly smaller than the sample width, making it possible to explore its 2D dynamics. The presence of the

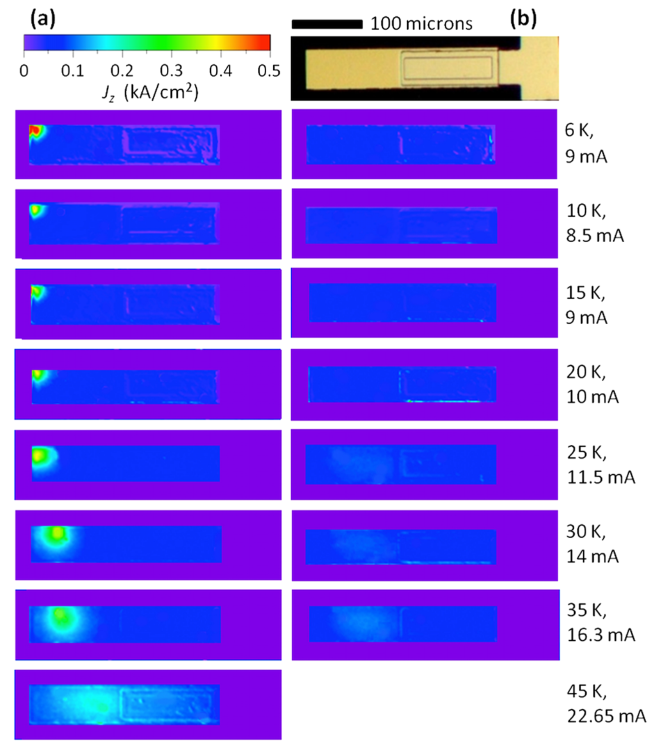

FIG. 3. (a) Current density maps calculated from thermoluminescent images of the smallest stable hot spot. (b) Corresponding maps in a no-hot-spot state at the same I_{mesa} and T_{bath}.

hot spot results in a lower mesa resistance, which is the reason for the jumps seen in the *I*-*V* curves in Figs. 1(a) (inset) and 1(b). As revealed by frames 1 and 2, the hot spot nucleates upon a current increment of less than 0.2 mA, i.e., approximately 2%, indicative of a true thermal instability. Also note that the temperature in the hot spot jumps to close to T_c, whereas over the majority of the mesa surface the temperature actually decreases [see traces 1 and 2 in Fig. 1(c)]. This is consistent with the observation that roughly half the current is confined into the hot spot [see Fig. 2(b)]. The temperature variation in trace 1 of Fig. 1(c) is caused by lateral heat removal along the surface of the base crystal; for heat removal strictly along the *c* axis, a spatially uniform temperature is expected. For certain currents and temperatures, the mesa may randomly jump between the hot-spot and no-hot-spot states, typically on a time scale of a few seconds [Fig. 2(b), inset].

In the ANL thermoluminescence cryostat, the hot spot is seen to nucleate at the corner of the mesa, while LTSLM images made in Tübingen show it nucleating along the edge of the mesa (cf. frame 2). This is likely to be due to subtle differences in the heat sinking of the sample between the two cryostats. In fact, upon increasing the temperature, the nucleation site of the hot spot shifts along the edge and finally towards the center of the mesa [see Fig. 3(a)].

Between 40 and 65 mA, there exist two hysteretic current-voltage paths, corresponding to the hot spot located near the center of the mesa, or near the left end of the mesa as represented by frames 4 and 6 in Fig. 1(d). (We note that this asymmetrical behavior is commonly seen in mesas with sufficiently strong heat removal. Further examples of the resulting thermal images are given in the Supplemental Material [28].) At currents above approximately 65 mA, the thermal state of the mesa is reversible in current and characterized by very strong overheating centered on the middle of the mesa. On decreasing the current, the hot spot remains at the center of the mesa for currents larger than approximately 40 mA, at which point it jumps to the end of the mesa. This jump is accompanied by a jump in the I-V curve; see inset (ii) in Fig. 1(b). On subsequent increasing current, the hot spot remains at the end of the mesa for currents up to approximately 60 mA.

A stability analysis of current filaments yields the surprising result that a current filament is not stable in the interior of the sample (which is assumed to have convex boundaries) but is attracted to a corner or edge [21]. On a qualitative level, the nucleation of a hot spot at a corner can be explained by realizing that this hot spot requires about $1/4$ of the current a similar hot spot located in the center of the sample would draw. Thus, upon increasing the bias along an I-V-curve such as shown in Fig. 1(b)(i), the first (i.e., lowest bias current) instability that could arise is the nucleation of a corner hot spot. With increasing current, hysteretic jumps between corner and edge filaments are expected to occur. Furthermore, the simulations show [21] that a current filament can be pinned near the center of the sample if an artificial seed of enhanced temperature is introduced, where the required temperature enhancement depends strongly on the sample size. Our thermal images at low temperature and low bias clearly reveal the nucleation of a corner filament, which with increasing bias current undergoes hysteretic jumps towards a center filament. Such a scenario can arise in a model in which the equilibrium position of the filament is determined by the competition between two energies, the attraction to the corner and the pinning at the location of enhanced background temperature near the center due to lateral cooling [see trace 1 in Fig. 1(c)]. At low temperatures and low bias the attraction to the corner wins out, whereas at high bias and elevated temperature pinning near the mesa center dominates. For mesas exhibiting stronger self-heating (e.g., those sitting on comparatively thick base crystals), the temperature pinning may always be sufficient to stabilize the hot spot near the center of the mesa, even at the lowest power dissipation at which the hot spot stably exists.

For optimally doped Bi-2212, $\sigma_c'(T)$ peaks at around 85 K. Consequently, LTSLM on Bi-2212 mesas typically gives its largest signal response near the edge of the hot spot, since, with increasing local device temperature $T(x,y)$, $\sigma_c'(x,y)$ may drop to near zero and even become negative. Thus the dark regions in the center of the mesa in Fig. 1(e) are in fact hotter than the surrounding lighter-shaded regions, as confirmed by our thermoluminescence results, and the wavelike features (for instance, at bias currents of 29.95 and 22.33 mA) are in fact a consequence of the temperature-dependent sensitivity of LTSLM.

At comparatively small bias voltages, the mesa current is an accelerating function of bias voltage [i.e., $d^2I/dV^2 \geq 0$ at bias currents below the backbending point seen in the inset in Fig. 1(a)]. We may thus estimate the mesa conductance with negligible self-heating as being the tangent to the I-V characteristic in this region (black lines on plot) as described in Ref. [14], allowing us to determine $\sigma_c(T)$ below T_c, and convert our temperature maps into maps of the c-axis quasiparticle current density.

The hot spot is imaged at the minimum current at which it is stable, at bath temperatures between 6 and 35 K, as well as at 45 K [see Fig. 3(a)], at which temperature the hot spot becomes comparable to the width of the mesa itself, and there still exists a cusp–but no hysteretic jump–in the I-V characteristic. We suggest that at a temperature near 45 K the character of the hot spot in the present device changes from two-dimensional to one-dimensional. At higher temperatures, there are no features in the I-V curves discernable [see Fig. 1(a)] and the thermal images reveal a broad temperature distribution consistent with simulations [6]. Figure 2(a) shows temperature cross sections of the hot spot in these states, while Fig. 2(b) compares predicted r_{HS} and I_{HS} (as respectively calculated from the relations given above with values measured experimentally). r_{HS} is measured from Fig. 2(a), while I_{HS} is obtained from the maps of $J_z(x,y,T_{\mathrm{bath}})$ plotted in Fig. 3. In this calculation, $\kappa_{ab}(\bar{T}_{\mathrm{HS}})$ and $\sigma_c'(\bar{T}_{\mathrm{HS}})$ are taken by averaging $T(x,y)$ over $r < r_{\mathrm{HS}}$, while $\kappa_{ab}(T)$ for optimally doped Bi-2212 is taken from Ref. [29]. From the resistance vs temperature curve and I-V characteristics plotted in Fig. 1(a), the average power density and average temperature rise in the mesa can be obtained, allowing the heat-removal coefficient P_0 to be estimated for small bias currents at which $T(x,y)$ is still comparatively uniform. P_0 varies with temperature but is typically around $6 \times 10^4\ \mathrm{W m^{-2} K^{-1}}$, consistent with the mesa being heat sunk to the metal substrate through a base crystal around 4 μm thick, assuming that $\kappa_{ab}/\kappa_c \approx 10$ [6]. For temperatures at which the hot spot is close to the corner of the mesa, I_{HS} is calculated from the quadrant located entirely inside the mesa and normalized accordingly to allow comparison with theory. Given the approximations necessarily made in these calculations, the close and systematic agreement between the theoretical and experimental values for hot-spot size and current and their growth with increasing T_{bath}–as indicated in Fig. 2(b)–is remarkable. In the center of the hot spot, local thermal runaway results in a current density up to 7 times higher than J_z far from the hot spot. Despite this, the comparatively small area of the hot spot means that it draws only 20%–50% of the entire current driven through the mesa. The data in Fig. 2(b) suggest that the maximum temperature at which a hot spot can still be observed is reached when the minimum hot-spot radius becomes larger than the width of the mesa. At higher temperatures, the sample cannot accommodate the hot spot anymore and the thermal instability does not occur.

III. DISCUSSION AND CONCLUSIONS

The behavior shown by this type of current filament may have significant implications for the design of IJJ THz sources in the future. A hot spot which is intrinsically narrow may act as an effective shunt resistance for the stack of IJJs as described previously, without substantially disrupting the resonant properties of the remainder of the cavity. In fact, a certain degree of dissipation induced by the shunt may be beneficial for synchronizing the junctions and achieving high emission power from stacks of intrinsic junctions [6] as well as from arrays of conventional junctions [30]. Furthermore, a hot spot which stably locates itself at one edge of the mesa will inherently make the Josephson critical current density asymmetric, which is predicted to strongly enhance the emission power [10]. If a mesa were to be designed such that its cavity resonance occurs at the same bath temperature and I-V conditions at which a sufficiently small hot spot nucleates, then the hot-spot behavior observed here could be exploited for the purposes of enhanced THz emission.

We measure the self-heating of a large Bi-2212 mesa device via two different techniques. We find good qualitative agreement between our direct thermoluminescent measurements of device surface temperature and our LTSLM images, confirming that the latter are primarily probing temperature-dependent changes in $\partial\sigma_c/\partial T$. In particular, the two techniques both reveal the asymmetric nucleation of a small asymmetric hot spot at low mesa temperatures where the hot-spot diameter is significantly smaller than the mesa width. The nucleation or annihilation of the hot spot causes dramatic hysteretic jumps in the mesa resistance. We find that the radius of this hot spot is in excellent agreement with theoretical predictions, as is its growth with increasing bias current and bath temperature.

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