

Extension of Babinet's relations to reflective metasurfaces: Application to the simultaneous control of wavefront and polarization

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We extend Babinet's relations, which were originally derived for transmissive metasurfaces, to reflective metasurfaces embedding planar metallic structures in a substrate containing a mirror. To verify the extended Babinet's relations, we investigate two types of reflective metasurface embedding self-complementary structures, which produce a π phase difference between two orthogonal linear polarizations under specific conditions required for the extended Babinet's relations. Both theoretical and experimental studies demonstrate a metasurface-based half-wave plate with a reflective metasurface including single self-complementary structures in the terahertz regions. In addition, we study a reflective metasurface with embedded self-complementary structures with a phase gradient in reflection, and demonstrate simultaneous implementation of anomalous reflection and polarization conversion with high efficiency.

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I. INTRODUCTION

Metamaterials and metasurfaces are artificial media composed of subwavelength structures known as meta-atoms and can be used to control electromagnetic wave propagation and polarization [1–3]. The characteristics of metamaterials depend crucially on the structural symmetry of the meta-atoms [4]. For example, metamaterials with broken-mirror-symmetry structures exhibit chiral responses [5–9], while those with anisotropic structures exhibit birefringence. Various types of anisotropic metamaterial have been proposed to realize the functions of quarter-wave and half-wave plates, which induce a phase difference between the two orthogonal linear polarizations [10–17]. The wavefront of the incident waves can be controlled in addition to the polarization by the introduction of a phase gradient into the metasurface [18,19]. Simultaneous control of the wavefront and polarization using spatial distribution of various anisotropic meta-atoms with different orientations has been reported [20–25].

Single-layer transmissive metasurfaces composed of planar metallic structures have been extensively investigated to control transmission waves in various frequency regions, and Babinet's principle has been widely used in the design of planar structures [26,27]. The transmission coefficients of the original metallic structures for linear polarization and the complementary structures for orthogonal polarization are linked by simple equations, referred to as "Babinet's relations." Babinet's relations were applied to the design of reconfigurable polarization devices using

metasurfaces incorporated with phase-changing materials, which are introduced to switch between metallic structures and their complementary structures [28–31]. In addition, unique properties of self-complementary structures can be applied to frequency-independent responses [32] and quarter-wave plates [33]. Single-layer transmissive metasurfaces, however, suffer substantial reflection loss, as discussed in Sec. II. On the other hand, reflective metasurfaces composed of planar metallic structures and a mirror provide a solution to significantly increase the efficiency. In this paper, we compare a transmissive metasurface composed of planar metallic structures with a reflective metasurface composed of the same metallic structures embedded in a substrate and a mirror, and derive conditions in which relations similar to conventional Babinet's relations are satisfied for reflective metasurfaces in the latter part of Sec. II.

According to the extended Babinet's relations, the difference between the reflection phases for two orthogonal linear polarizations is always π for reflective metasurfaces with self-complementary structures under the derived conditions. In other words, the reflective metasurface functions as a half-wave plate. To test this hypothesis, we design a reflective metasurface composed of self-complementary structures and a mirror, which operates in the terahertz regime, and demonstrate a metasurface-based half-wave plate both experimentally and using electromagnetic simulations in Sec. III. Furthermore, we apply the extended Babinet's relations to phase-gradient metasurfaces with spatially varying reflection phases. We design a reflective metasurface that includes eight types of self-complementary structure, thus realizing a linear phase

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gradient in a specific direction, and demonstrate an anomalous reflection with polarization conversion caused by phase retardation between the two orthogonal polarizations in Sec. IV. This simultaneous control of wavefront and polarization is a unique property derived from the extended Babinet's relations. The extended Babinet's relations may pave the way for efficient multifunctional metasurfaces such as reconfigurable intelligent surfaces [34–37] and polarization-encoded metasurfaces [38,39].

II. EXTENSION OF BABINET'S RELATIONS TO A REFLECTIVE METASURFACE

First, the conventional Babinet's relations for transmissive metasurfaces are reviewed. Suppose a metasurface with thin metallic structures located on the $z = 0$ plane is illuminated by TE waves with y polarization, as shown in Fig. 1(a). Additionally, we assume that the anisotropic axes are aligned in the x and y directions and that the periodicity of the structures is sufficiently small such that diffraction does not occur. Under these conditions, the incident TE waves are separated into two paths as transmitted TE waves and reflected TE waves without diffraction. Next, we consider the interaction between TM waves

and a complementary metasurface obtained with use of the impedance inversion given by $Z_s^{(c)} = Z^2/[4Z_s(x,y)]$, where Z is the wave impedance of the surrounding medium and $Z_s(x,y)$ denotes the local impedance of the original metasurface shown in Fig. 1(a). If the original metasurface comprises a perfect electric conductor $Z_s(x,y) = 0$ and insulation holes $Z_s(x,y) \rightarrow \infty$, the complementary metasurface can be obtained by swapping the metallic and insulating parts, as shown in Fig. 1(b). Babinet's principle links the transmission coefficient $\tilde{\tau}_{TE}$ for the original metasurface with the transmission coefficient $\tilde{\tau}_{TM}^{(c)}$ for the complementary metasurface as follows [27]:

$$\tilde{\tau}_{TE} + \tilde{\tau}_{TM}^{(c)} = 1. \quad (1)$$

A similar relationship is derived by exchanging the TE and TM waves as

$$\tilde{\tau}_{TM} + \tilde{\tau}_{TE}^{(c)} = 1. \quad (2)$$

These relations are referred to as ‘‘Babinet's relations for transmissive metasurfaces.’’

In this paper, we propose the reflective metasurface shown in Fig. 1(c). The metasurface embeds metallic planar structures identical to those in Fig. 1(a) in a substrate with refractive index n whose back side is covered with a metallic mirror. The reflection coefficient $\tilde{r}_i$ ($i = TE, TM$) in Fig. 1(c) is connected to the transmission coefficient $\tilde{\tau}_i$ and reflection coefficient $\tilde{\rho}_i$ of the metallic planar structure as shown in Fig. 1(a). The same relationship holds for complementary structures, as shown in Fig. 1(b), and a reflective metasurface embedding the structure, as shown in Fig. 1(d). In this section, first, the normal incidence of waves is considered for simplicity. From analyses using transfer matrices, which are presented in Appendix A, we obtain the reflection coefficient $\tilde{r}_i$ under the condition $kd = \pi/2$, where k is the wave number in the substrate, as follows:

$$\tilde{r}_i = -\frac{(n+2)\tilde{\tau}_i - 2}{(n-2)\tilde{\tau}_i + 2}, \quad (3)$$

which shows that the reflection coefficient for Fig. 1(c) is expressed as a Möbius transformation of the transmission coefficient $\tilde{\tau}_i$ for the embedded metallic structures. Under the condition $n = 2$, the above relationship is reduced to

$$\tilde{r}_i = -2\left(\tilde{\tau}_i - \frac{1}{2}\right). \quad (4)$$

The same relationship is expected for a reflective metasurface as shown in Fig. 1(d), embedding a complementary structure, as shown in Fig. 1(b), and the reflection coefficient is given by $\tilde{r}_i^{(c)} = -2(\tilde{\tau}_i^{(c)} - 1/2)$. Hence, Eqs. (1)

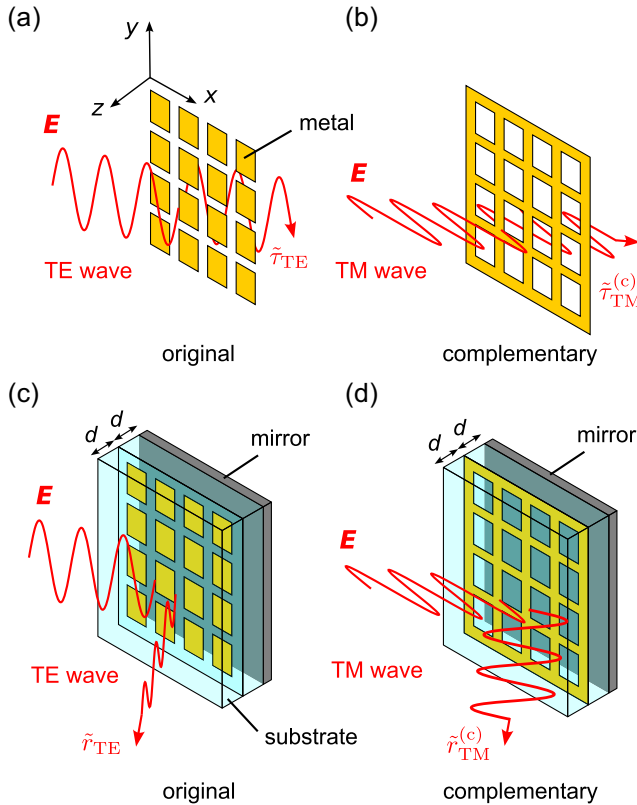


FIG. 1. Electromagnetic response of (a) planar metallic structures and (b) complementary structures. Electromagnetic response of a reflective metasurface embedding (c) planar metallic structures and (d) complementary structures.

and (2) yield the following relations:

$$\tilde{r}_{\text{TE}} + \tilde{r}_{\text{TM}}^{(c)} = 0, \quad (5)$$

$$\tilde{r}_{\text{TM}} + \tilde{r}_{\text{TE}}^{(c)} = 0. \quad (6)$$

These relationships can be regarded as Babinet's relations extended to reflective metasurfaces. Note that the relations hold under specific conditions for refractive index $n = 2$ and thickness $2d = \pi/k$ for the substrate. For oblique incidence, the condition $2d$ is modified to $2d = \pi/(k \cos \theta)$, where θ is the refraction angle in the substrate, as shown in Appendix B.

We reconsider Babinet's relations for transmissive and reflective metasurfaces by representing the transmission and reflection coefficients on a complex plane. For an arbitrary transmissive metasurface composed of planar lossless metals with transmission coefficient $\tilde{\tau}$ and reflection coefficient $\tilde{\rho}$, continuity of electric fields on the metasurface requires that $1 + \tilde{\rho} = \tilde{\tau}$ and energy conservation requires that $|\tilde{\tau}|^2 + |\tilde{\rho}|^2 = 1$. These two conditions yield

$$|\tilde{\tau}|^2 + |1 - \tilde{\tau}|^2 = 1, \quad (7)$$

which implies that the transmission coefficient $\tilde{\tau}$ is located on a circle with a radius of $1/2$ and center of $1/2$ on a complex plane. Hence, the transmission coefficients $\tilde{\tau}_{\text{TE}}$ and $\tilde{\tau}_{\text{TM}}^{(c)}$ for the transmissive metasurfaces as shown in Figs. 1(a) and 1(b), which require the conventional Babinet's relations given by Eqs. (1) and (2), are located on the opposite sides of the circle, as shown in Fig. 2(a). As clearly shown in Fig. 2(a), the transmissive metasurfaces suffer from reduced transmission for large transmission phases $\arg(\tilde{\tau})$. This substantially restricts the efficiency of the wave-plate operation, which requires large phase retardation between two polarizations. The reduction in transmission is attributed to the reflection loss. By contrast, the reflection coefficient $\tilde{r}$ of an arbitrary reflective metasurface composed of planar lossless metals and a mirror satisfies $|\tilde{r}| = 1$ because all incident energy is reflected

by the metasurfaces following the interaction between the metallic structures and the mirror. From Eq. (4), the transmission coefficients $\tilde{\tau}_{\text{TE}}$ and $\tilde{\tau}_{\text{TM}}^{(c)}$ of the transmissive metasurfaces are mapped onto the corresponding reflection coefficients $\tilde{r}_{\text{TE}}$ and $\tilde{r}_{\text{TM}}^{(c)}$, as shown in Fig. 2(b). It is confirmed that Babinet's relation extended to the reflective metasurfaces, which is given by Eq. (5), is satisfied. In contrast to transmissive metasurfaces, the reflectances of the reflective metasurfaces are always unity regardless of the phase of reflection, and polarization manipulation with high efficiency can be expected.

III. HALF-WAVE PLATE USING A REFLECTIVE METASURFACE EMBEDDING SELF-COMPLEMENTARY STRUCTURES

A. Babinet's relations for reflective metasurfaces embedding self-complementary structures

If a structure is congruent with its complementary structure, it is referred to as a "self-complementary structure." A typical example of the self-complementary structure used in our experiment is shown in Fig. 3(a). If the self-complementary structures are embedded in a reflective metasurface as shown in Fig. 3(b), the extended Babinet's relations given by Eqs. (5) and (6) are reduced to

$$\tilde{r}_{\text{TE}} + \tilde{r}_{\text{TM}} = 0, \quad (8)$$

owing to the self-complementarity $\tilde{r}_{\text{TE}} = \tilde{r}_{\text{TM}}^{(c)}$ and $\tilde{r}_{\text{TM}} = \tilde{r}_{\text{TE}}^{(c)}$. This relationship indicates that reflective metasurfaces with self-complementary structures function as half-wave plates, which exhibit a π phase difference between two orthogonal linear polarizations in a reflection mode. If the diffraction and dissipation losses are negligible, the reflectance is expected to be high, $|\tilde{r}_{\text{TE}}| = |\tilde{r}_{\text{TM}}| \sim 1$, which leads to the implementation of efficient half-wave plates.

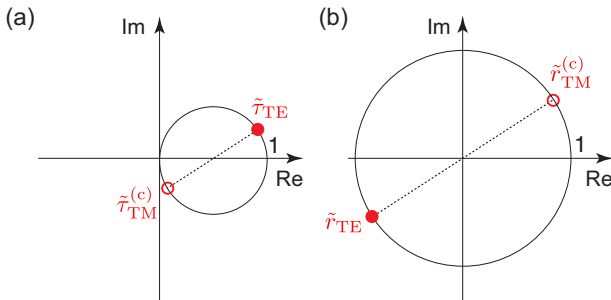


FIG. 2. (a) Transmission coefficients of the transmissive metasurface and its complementary metasurface on a complex plane. (b) Reflection coefficients of the reflective metasurface and its complementary metasurface on a complex plane.

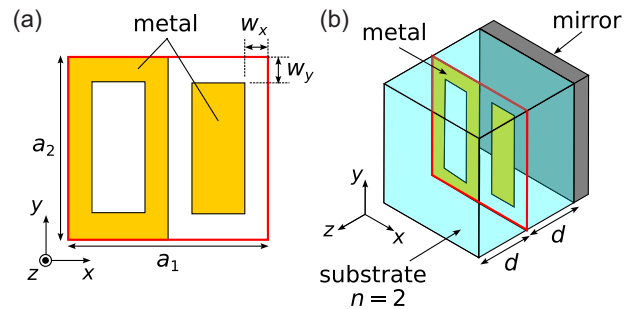


FIG. 3. (a) Unit cell of self-complementary structures composed of a planar metallic sheet. (b) Unit cell of a reflective metasurface embedding self-complementary structures.

B. Simulations

To confirm the extended Babinet's relation given by Eq. (8), numerical simulations were first conducted with the use of an electromagnetic simulator (CST MICROWAVE STUDIO) for the metasurface, as shown in Fig. 3(b). The periodicity $a_1 = a_2$ is assumed to be $200\ \mu\text{m}$, and the other dimensions are set to $w_x = 30\ \mu\text{m}$, $w_y = 50\ \mu\text{m}$, and $d = 100\ \mu\text{m}$. We assume that the incident plane and incident angle θ_i are the x - z plane and 45° , respectively. Under these conditions, the lowest diffraction frequency $c_0/a/(1 + \sin\theta_i)$ is $0.88\ \text{THz}$. The incident waves are refracted at angle $\theta = \sin^{-1}(\sin\theta_i/n) = 20.7^\circ$ in the substrate with refractive index $n = 2$, and the estimated operating frequency $f_0 = 0.40\ \text{THz}$ is derived from $2d = \pi/(k \cos\theta)$. The metal constituting the self-complementary structures was assumed to be aluminum with a thickness of $300\ \text{nm}$ and conductivity of $3.56 \times 10^7\ \text{S/m}$, and the mirror was assumed to be gold with a thickness of $200\ \text{nm}$ and conductivity of $4.56 \times 10^7\ \text{S/m}$. Floquet boundary conditions were applied along the x and y directions to simulate the interaction between the metasurface and the plane waves incident at $\theta_i = 45^\circ$. The input port was located $1000\ \mu\text{m}$ from the top surface of the substrate.

The calculated reflection amplitudes $|\tilde{r}_{\text{TE}}|$ and $|\tilde{r}_{\text{TM}}|$ for the TE and TM waves are indicated by the dashed lines in Figs. 4(a) and 4(b), respectively. This frequency region, which is lower than the diffraction frequency of $0.88\ \text{THz}$, contains no diffraction; consequently, the reflection amplitudes are close to unity. The small depressions observed above $0.50\ \text{THz}$ are caused by resonances involving Joule losses in the metallic structures and mirror with finite conductivity. The dashed line in Fig. 4(c) shows the calculated phase difference defined by $\Delta\theta = \arg(\tilde{r}_{\text{TE}}/\tilde{r}_{\text{TM}})$. As expected from the extended Babinet's relation expressed by Eq. (8), the phase difference between the TE and TM waves approaches π at an operating frequency of $0.40\ \text{THz}$, denoted by the gray line. The slight deviation from π is attributed to losses that are not considered in the derivation of Babinet's relation.

C. Experimental demonstration

For the experimental demonstration, synthetic quartz was used as the substrate with refractive index $n = 2.0$ [40]. The metasurface was fabricated by the following procedure. First, titanium and gold films of thickness 10 and $200\ \text{nm}$, respectively, were deposited as a mirror on a 1-mm -thick quartz substrate via electron-beam evaporation. Second, the substrate containing the mirror and a thin quartz substrate were bonded with an optical adhesive (Norland, NOA89H). The thickness of the thin quartz substrate was $93\ \mu\text{m}$, and the total thickness, including the adhesive layer, was $108\ \mu\text{m}$. Third, an aluminum film with a thickness of $300\ \text{nm}$ was deposited via electron-beam evaporation. Self-complementary patterns were fabricated

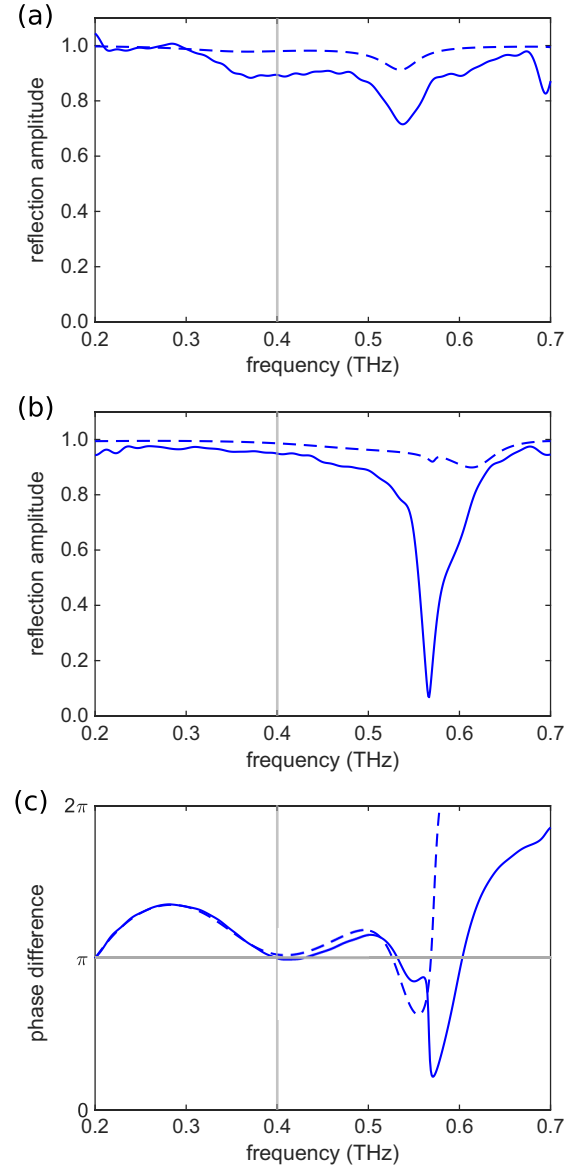


FIG. 4. Amplitude and phase responses of a reflective metasurface with $w_x = 30\ \mu\text{m}$ and $w_y = 50\ \mu\text{m}$. Solid lines and dashed lines represent experimental and simulation results, respectively. (a) Reflection-amplitude spectra $|\tilde{r}_{\text{TE}}|$ for TE waves. (b) Reflection-amplitude spectra $|\tilde{r}_{\text{TM}}|$ for TM waves. (c) Phase difference $\arg(\tilde{r}_{\text{TE}}/\tilde{r}_{\text{TM}})$ between reflected TE waves and reflected TM waves.

via wet etching with a mixture comprising phosphoric acid, nitric acid, acetic acid, and water, in a 16:1:2:1 ratio, which does not erode the adhesive layer. Next, the patterned substrate was bonded to a thin quartz substrate, whose thickness was $92\ \mu\text{m}$. The distance between the embedded aluminum structures and the top surface was $104\ \mu\text{m}$. Finally, a 300-nm -thick aluminum film was deposited on half of the sample to form a reference mirror, which was used to estimate the reflection coefficients. A photograph

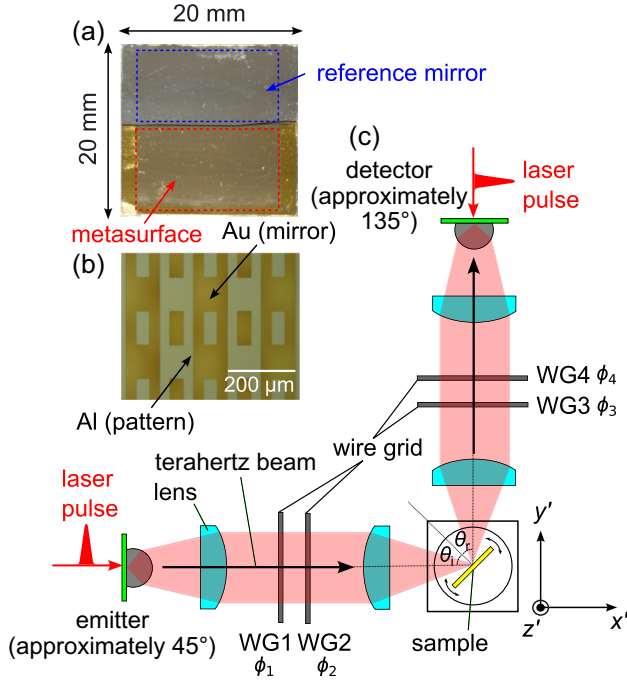


FIG. 5. (a) Photograph of the fabricated sample. (b) Microphotograph of the metasurface parts. (c) Experimental setup.

of the fabricated sample is presented in Fig. 5(a). The reference mirror was placed in the upper half of the sample. The metasurface embedding the self-complementary structures was placed in the lower half of that area. A microphotograph of the metasurface is shown in Fig. 5(b). The white area corresponds to the aluminum patterns that form self-complementary structures, and the gold area corresponds to the mirror made of the gold film, which can be seen through the aluminum structures.

We used terahertz time-domain spectroscopy (TDS) in the reflective setup as shown in Fig. 5(c) to acquire the reflection spectra of the fabricated metasurface for TE and TM waves. The incident terahertz waves propagate along the x' direction, and the reflected waves from the sample propagate along the y' direction. In this experiment, the incident angle θ_i and reflection angle θ_r were fixed at 45° . We introduce polarization angles ϕ_i and ϕ_r for the incident waves and reflected waves, respectively. The polarization directions are defined as $\cos \phi_i \hat{e}_{y'} + \sin \phi_i \hat{e}_{z'}$ for the incident waves and $\cos \phi_r \hat{e}_{x'} + \sin \phi_r \hat{e}_{z'}$ for the reflected waves, where $\hat{e}_{x'}$, $\hat{e}_{y'}$, and $\hat{e}_{z'}$ are the unit vectors along the x' , y' , and z' directions, respectively. According to this definition, when the incident waves with $\phi_i = 45^\circ$ are reflected by a uniform mirror, the reflected waves have $\phi_r = 45^\circ$.

The gaps between electrodes of photoconductive antennas used as the emitter and detector were approximately aligned at 45° and 135° , respectively. The transmission axes of the four wire grids labeled “WG1,” “WG2,”

“WG3,” and “WG4” are represented as ϕ_1 , ϕ_2 , ϕ_3 , and ϕ_4 , respectively. We fixed $\phi_1 = 45^\circ$ and $\phi_4 = 135^\circ$ and changed the orientations of WG2 and WG3 depending on the polarizations to be measured. We set $\phi_2 = \phi_3 = 90^\circ$ for the TE-wave measurement and $\phi_2 = \phi_3 = 0^\circ$ for the TM-wave measurement. The collimated terahertz waves were focused with the use of a lens with a focal length of 50 mm, and the beam diameter of the focused waves at the sample position was 1.7 mm, which was much smaller than the areas of the reference mirror and metasurface.

The sample was mounted on a translation stage along the z' direction. The reflection signals from the reference mirror $r_{\text{ref}}(t)$ and those from the metasurface $r_{\text{meta}}(t)$ were acquired by moving the sample vertically. The reflection coefficients were obtained as $\tilde{r}(\omega) = -\tilde{r}_{\text{meta}}(\omega)/\tilde{r}_{\text{ref}}(\omega)$, where $\tilde{r}_{\text{ref}}(\omega)$ and $\tilde{r}_{\text{meta}}(\omega)$ represent the Fourier transforms of $r_{\text{ref}}(t)$ and $r_{\text{meta}}(t)$, respectively. The negative sign was required because the reference signals $r_{\text{ref}}(t)$ include the effect of electric field inversion at the mirror. The experimental results for the reflection amplitudes for the TE waves $|\tilde{r}_{\text{TE}}|$ and the TM waves $|\tilde{r}_{\text{TM}}|$ are represented by solid lines in Figs. 4(a) and 4(b), respectively. The reflection amplitudes obtained at the operating frequency of 0.40 THz were $|\tilde{r}_{\text{TE}}| = 0.89$ and $|\tilde{r}_{\text{TM}}| = 0.95$. The experimental results exhibited resonance features above 0.5 THz, which led to a significant reduction in the reflection. The dissipation was caused mainly by the dielectric loss of the adhesive layers, which were not considered in the simulations. The solid line in Fig. 4(c) shows the phase difference $\Delta\theta = \arg(\tilde{r}_{\text{TE}}/\tilde{r}_{\text{TM}})$ derived from the experimental results. The experimental results (solid line) and simulation results (dashed line) are in good agreement except at the resonant frequencies above 0.5 THz. As expected, the phase difference at the operating frequency of 0.40 THz approached π . The discrepancy around the resonance is attributed mainly to dielectric losses, as discussed above.

To estimate the performance of the metasurface as a half-wave plate, we analyzed the amplitude and polarization of output waves for incident waves with 45° polarization ($\phi_i = 45^\circ$). Polarization can be characterized by the Stokes parameters [41]; we considered two Stokes parameters, S_0 and S_2 , which can be calculated as

$$S_0 = \frac{|\tilde{r}_{\text{TE}}|^2 + |\tilde{r}_{\text{TM}}|^2}{2}, \quad S_2 = \text{Re}(\tilde{r}_{\text{TE}}^* \tilde{r}_{\text{TM}}), \quad (9)$$

where S_0 denotes the total flux of the output waves and S_2 represents the flux difference between 45° polarization and 135° polarization. The normalized Stokes parameter defined by S_2/S_0 represents the polarization purity, which becomes -1 when the polarization of the output waves is oriented at 135° —that is, $\phi_r = 135^\circ$. Figure 6(a) shows the normalized Stokes parameter S_2/S_0 calculated with the experimental results. At the operating frequency denoted

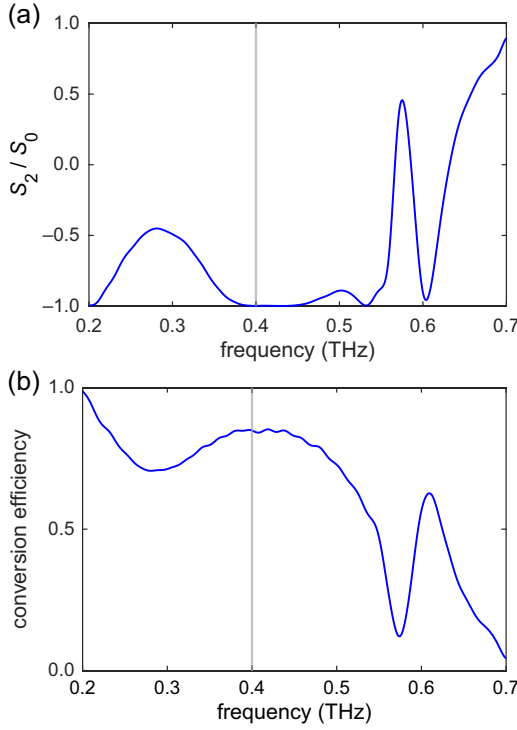


FIG. 6. Polarization property of waves reflected from the reflective metasurface with $w_x = 30 \mu\text{m}$ and $w_y = 50 \mu\text{m}$ for incident waves with 45° polarization. (a) Normalized Stokes parameter S_2/S_0 . (b) Efficiency of polarization conversion from $\phi_i = 45^\circ$ to $\phi_r = 135^\circ$.

by the gray line, S_2/S_0 becomes -1 as expected, which validates that the metasurface functions as a half-wave plate with an ideal phase difference.

Next we consider polarization-conversion efficiency η from $\phi_i = 45^\circ$ to $\phi_r = 135^\circ$. Reflection coefficients are introduced as $\tilde{r}_{\text{co}}$ for $\phi_r = 45^\circ$ and $\tilde{r}_{\text{cr}}$ for $\phi_r = 135^\circ$, and the Stokes parameters of the reflected waves are given as $S_0 = |\tilde{r}_{\text{co}}|^2 + |\tilde{r}_{\text{cr}}|^2$ and $S_2 = |\tilde{r}_{\text{co}}|^2 - |\tilde{r}_{\text{cr}}|^2$ [41]. The polarization-conversion efficiency η can be calculated as

$$\eta \equiv |\tilde{r}_{\text{cr}}|^2 = \frac{S_0 - S_2}{2}. \quad (10)$$

Figure 6(b) shows η derived from the experimental results. The conversion efficiency at the operating frequency, denoted by the gray line, was estimated to be 0.85. It was confirmed that the pure polarization of $\phi_r = 135^\circ$ was achieved with a relatively high conversion efficiency despite the simple implementation of the metasurface using adhesives to bond the substrates. A higher conversion efficiency can be realized with an adhesive-free bonding technique such as room-temperature bonding between metallic films [42].

IV. SIMULTANEOUS CONTROL OF WAVEFRONT AND POLARIZATION

A. Design of phase-gradient metasurface

The extended Babinet's relation expressed by Eq. (8) holds for reflective metasurfaces embedding arbitrary self-complementary structures. The phase difference is always π ; however, either phase $\arg(\tilde{r}_{\text{TE}})$ or phase $\arg(\tilde{r}_{\text{TM}})$ can be freely selected by designing the self-complementary structures. If the self-complementary structures depend on the position x and $\arg(\tilde{r}_{\text{TE}})$ has a constant phase gradient $d[\arg(\tilde{r}_{\text{TE}})]/dx \neq 0$, incident TE waves are reflected at anomalous angles that are different from the normal reflection angle on a flat mirror [43]. The phase gradient of the reflected TM waves is identical to that of the reflected TE waves because of the extended Babinet's relation Eq. (8). Consequently, the TM waves are also reflected at the same anomalous angle. In the process of the anomalous reflection, the function as a half-wave plate—that is, $\arg(\tilde{r}_{\text{TE}}) - \arg(\tilde{r}_{\text{TM}}) = \pi$ —is maintained. Hence, the phase-gradient metasurface simultaneously achieves anomalous reflection and polarization control by tailoring the phase response for either the TE waves or the TM waves. Figure 7(a) shows a unit supercell composed of eight elements labeled as “ M_l ” ($l = 1, 2, \dots, 8$), with the same design as that shown in Fig. 3(a) for various values of design parameters w_x and w_y . If the reflection phase ψ_l for M_l is adjusted to satisfy $\psi_l = \pi l/4 + \psi'$ (ψ' is a constant), which provides a phase difference of 2π at both ends of the supercell, the anomalous reflection angle θ_r for the incident angle θ_i is

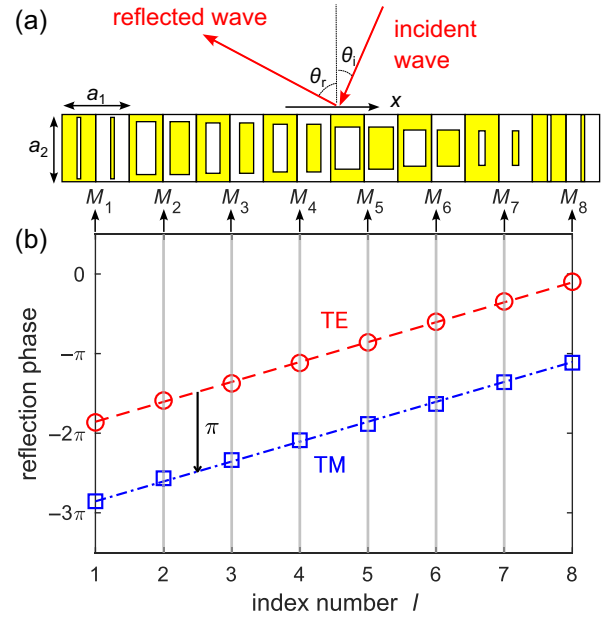


FIG. 7. (a) Unit cell (supercell) of the phase-gradient reflective metasurface composed of eight elements M_l . (b) Reflection phases of each element M_l for TE and TM waves.

TABLE I. Design parameters w_x and w_y for each element M_l constituting the phase-gradient reflective metasurface.

	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
w_x (μm)	44	20	28	28	12	16	40	44
w_y (μm)	8	20	24	28	36	44	48	0

determined with use of the following generalized Snell's law:

$$p(\sin \theta_i - \sin \theta_r) = -\lambda_0, \quad (11)$$

where p is the periodicity of the supercell and λ_0 is the wavelength in a vacuum [18]. For a general periodic structure with periodicity p , the diffraction angles θ_r of m th order are determined by

$$p(\sin \theta_i - \sin \theta_r) = m\lambda_0. \quad (12)$$

For a phase-gradient metasurface satisfying Eq. (11), reflection with diffraction order $m = -1$ is expected. For experimental demonstration using the terahertz-TDS setup shown in Fig. 5(c), which requires $\theta_i + \theta_r = 90^\circ$, the parameters were set as follows: $a_1 = a_2 = 200 \mu\text{m}$, $p = 8a_1 = 1600 \mu\text{m}$, and $d = 100 \mu\text{m}$, which lead to $\theta_i = 24.8^\circ$ and $\theta_r = 65.2^\circ$ for an operating frequency f_0 of 0.384 THz.

Numerical simulations of the metasurfaces were conducted with the same setup as in Fig. 3(b) for incident angle $\theta_i = 24.8^\circ$, with changing w_x from 0 to 48 μm and w_y from 0 to 96 μm in increments of 4 μm , and we singled out eight sets of w_x and w_y from the 325 parameter sets to satisfy the linear phase gradient expressed as $\psi_l = \pi l/4 + \psi'$. The parameter sets are listed in Table I, and the calculated reflection phases for the elements M_l are denoted by circles and squares for the TE and TM waves, respectively, in Fig. 7(b). The dashed line was obtained by fitting the reflection phases ψ_l for the TE waves with $\psi_l = \pi l/4 + \psi'$. The dashed-dotted line was obtained by shifting the dashed line by $-\pi$. The reflection phases denoted by the squares are well aligned with the dashed-dotted line, which implies that each element works as a half-wave plate owing to the self-complementary structures satisfying Eq. (8). Because the phase gradients of the reflected TE and TM waves are identical, waves incident at $\theta_i = 24.8^\circ$ with arbitrary polarization are reflected anomalously at $\theta_r = 65.2^\circ$ while acquiring a π phase difference between the TE and TM wave components.

B. Simulations of anomalous reflection and polarization conversion

Subsequently, we performed numerical simulations for the entire metasurface with the supercell as shown in Fig. 7(a), and obtained all the reflection coefficients for

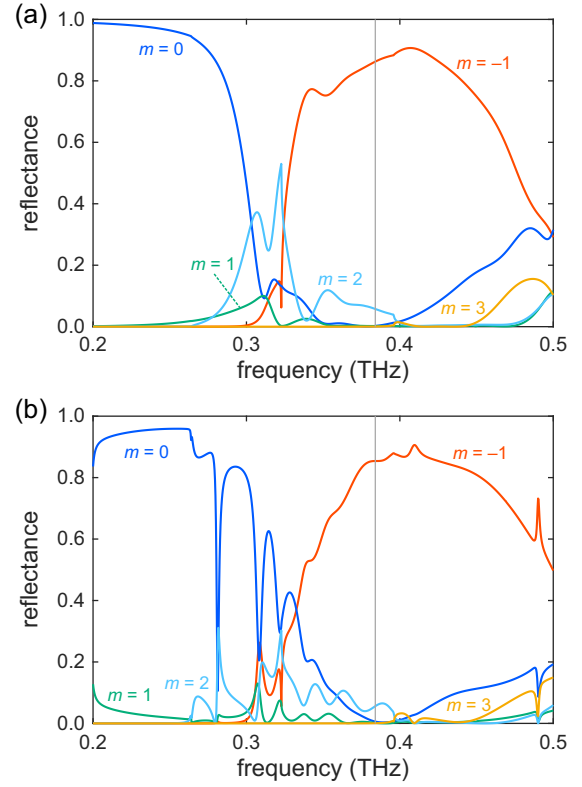


FIG. 8. Reflectances for reflection to diffraction modes $m = -1, 0, 1, 2, 3$ for (a) TE waves and (b) TM waves.

every diffraction mode. Figures 8(a) and 8(b) present the reflectances for reflection to possible diffraction modes, which are limited from $m = -1$ to $m = 3$ below 0.5 THz, for TE and TM waves, respectively. The reflectances for $m = -1$ correspond to the designed anomalous reflection, and the reflection angle θ_r is 65.2° at the operating frequency $f_0 = 0.384$ THz denoted by gray lines. For both polarizations, the reflectances for $m = -1$ were maximized at approximately 0.4 THz, while the reflections of the other modes were significantly suppressed. The difference between the designed frequency $f_0 = 0.384$ THz and the optimal frequency of approximately 0.4 THz is attributed to interactions between neighboring elements that were not considered in the design procedure for obtaining the linear phase gradient in the simulation. The reflections of the other modes can be further suppressed by increase of the number of elements in the supercell to obtain a smoother phase gradient.

We can estimate the polarization of the anomalously reflected waves for the incident waves with 45° polarization using the Stokes parameters defined in Eq. (9) with the reflection coefficients for the $m = -1$ mode. Figure 9(a) shows a normalized Stokes parameter S_2/S_0 for the $m = -1$ mode. The value of S_2/S_0 is -0.97 at the operating frequency f_0 of 0.384 THz denoted by the gray line, which implies that the difference between the reflection

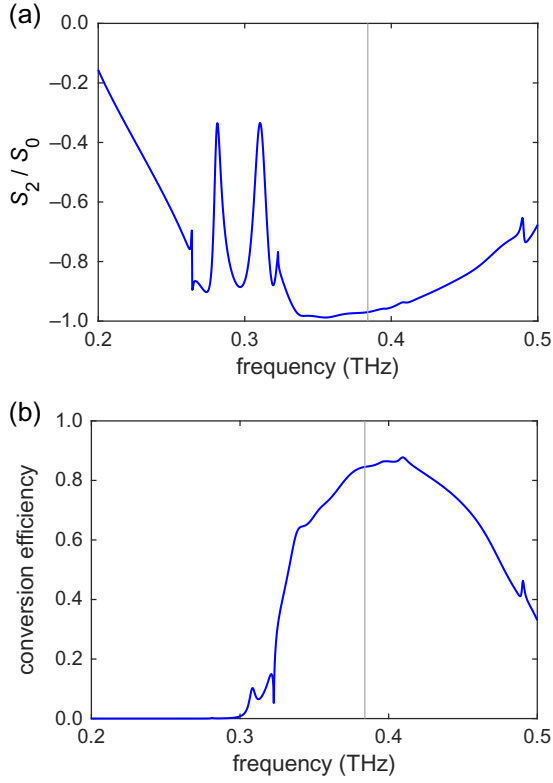


FIG. 9. Polarization property of reflected waves in the $m = -1$ mode for incident waves with illumination angle $\theta_i = 24.8^\circ$ and polarization $\phi_i = 45^\circ$. (a) Normalized Stokes parameter S_2/S_0 . (b) Efficiency of polarization conversion η_s from $\phi_i = 45^\circ$ to $\phi_r = 135^\circ$.

phases for the TE and TM modes was close to π . Figure 9(b) shows the efficiency of conversion η_s from incident waves with 45° polarization to anomalously reflected waves ($m = -1$) with 135° polarization. The conversion efficiency η_s was estimated to be 0.85 at the operating frequency f_0 of 0.384 THz denoted by the gray line. These simulation results validate that the wavefront control could be achieved while its function as a half-wave plate was maintained.

C. Experimental demonstration of anomalous reflection and polarization conversion

We fabricated a reflective metasurface using the parameters listed in Table I using the procedure described in Sec. III. A microphotograph of the fabricated metasurface is presented in Fig. 10(a). We conducted an experiment using the terahertz-TDS system shown in Fig. 5(c) to evaluate the efficiency of polarization conversion η_e from waves incident at $\theta_i = 24.8^\circ$ with polarization 45° to waves anomalously reflected at $\theta_r = 65.2^\circ$ with polarization 135° . The angles of wire grids WG1, WG2, and WG4

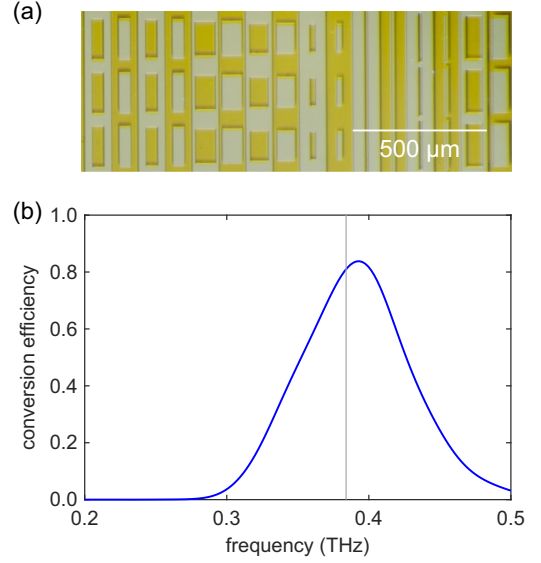


FIG. 10. (a) Microphotograph of the fabricated phase-gradient metasurface. (b) Efficiency of polarization conversion η_e from incident waves with $\theta_i = 24.8^\circ$ and $\phi_i = 45^\circ$ to anomalously reflected waves with $\theta_r = 65.2^\circ$ and $\phi_r = 135^\circ$.

were fixed at $\phi_1 = 45^\circ$, $\phi_2 = 45^\circ$, and $\phi_4 = 90^\circ$, respectively. Under this condition, the polarization of the incident waves was fixed at 45° .

Before measuring the anomalous reflection on the metasurface, we estimated the reference signal $r_{\text{ref}}(t)$ using a flat mirror on the upper half of the sample under the condition where the incident angle θ_i , reflection angle θ_r , and angle of WG3 ϕ_3 were 45° . The flat mirror reflected the incident waves while they maintained a polarization ϕ_r of 45° , and passed through WG3 at $\phi_3 = 45^\circ$. The polarization of the waves from WG3 was projected along $\phi_4 = 90^\circ$ by WG4, and projected at 135° by the detector. As a result, the amplitude decreased by a factor of 2 after WG3. The signal obtained is denoted as $r_{\text{ref}}(t)$.

Next, the angle of WG3 was changed to $\phi_3 = 135^\circ$. We rotated the sample holder to satisfy $\theta_i = 24.8^\circ$ and $\theta_r = 65.2^\circ$. The sample holder was moved vertically until the incident waves interacted with the metasurface. Incident waves with a polarization of 45° interacted with the metasurface at an angle θ_i of 24.8° and were anomalously reflected at an angle θ_r of 65.2° . A component with a polarization of 135° was selected from the anomalously reflected waves when the waves passing through WG3 aligned along $\phi_3 = 135^\circ$. The polarization of the waves from WG3 was projected along $\phi_4 = 90^\circ$ by WG4, and projected at 135° by the detector. The amplitude decreased by a factor of 2 after WG3. The signal obtained is defined as $r_{\text{meta}}(t)$. From $\tilde{r}_{\text{meta}}(\omega)$ and $\tilde{r}_{\text{ref}}(\omega)$, which are Fourier transforms of $r_{\text{meta}}(t)$ and $r_{\text{ref}}(t)$, respectively, the conversion efficiency can be estimated as $\eta_e = |\tilde{r}_{\text{meta}}(\omega)/\tilde{r}_{\text{ref}}(\omega)|^2$, which includes the efficiency of

polarization conversion from $\phi_i = 45^\circ$ to $\phi_r = 135^\circ$ and efficiency of diffraction from $\theta_i = 24.8^\circ$ to $\theta_r = 65.2^\circ$.

The conversion efficiency η_e obtained from the experimental results is shown in Fig. 10(b). The value of η_e at the operating frequency f_0 of 0.384 THz was estimated to be 0.81. The optimal frequency was blueshifted from $f_0 = 0.384$ THz. This deviation can be attributed to interactions between neighboring elements. Note that the definitions of the conversion efficiencies η_s and η_e differ. While η_s in the simulation is defined for the specific diffraction order $m = -1$ given by Eq. (12), thus providing a frequency-dependent diffraction angle, η_e in the experiment is defined for a fixed reflection angle θ_r of 65.2° . The definitions of η_s and η_e coincide only at the operating frequency f_0 of 0.384 THz. Therefore, the shape shown in Fig. 9(b) differs from that shown in Fig. 10(b).

Compared with previous studies on phase-gradient reflective metasurfaces with similar functions that realize a π phase difference between two linear polarizations by adjustment of the shapes of unit structures [21,22], the proposed metasurface always functions as a half-wave plate according to the extended Babinet's relations. Hence, only one of the TE-wave or TM-wave reflection phases needs to be controlled by adjustment of the dimensions of the self-complementary structures.

Previous research based on conventional Babinet's relations demonstrated a phase-gradient transmissive metasurface with self-complementary structures, which separates incident beams into two nondiffracted beams and two anomalously diffracted beams depending on the circular polarizations [44]. In contrast, the phase-gradient reflective metasurface proposed in this paper reflects most incident waves in an anomalous direction with an accompanying π phase difference between the two linear polarizations.

V. CONCLUSION

We extended the conventional Babinet's relations derived for transmissive metasurfaces to those of reflective metasurfaces embedding planar metallic structures in a substrate with a mirror. We proposed two types of reflective metasurface embedding self-complementary structures as applications of the extended Babinet's relations, which guarantee a π phase difference between the two orthogonal linear polarizations. We theoretically and experimentally demonstrated a metasurface-based half-wave plate with a reflective metasurface that included single self-complementary structures in the terahertz region. We also designed a reflective metasurface embedding self-complementary structures with a phase gradient in reflection, and demonstrated simultaneous implementation of anomalous reflection and polarization conversion with high efficiency. The absorption loss caused mainly by the adhesive layers could be reduced by the use of

an adhesive-free bonding technique such as the room-temperature bonding between metallic films [42]. The Joule loss in the metallic structures might be a problem especially in higher-frequency regions, including the optical regime. Deviations from Babinet's relations have been discussed for transmissive metasurfaces [45,46]. A similar discussion might be applied to reflective metasurfaces, because Eq. (4) remains valid even in the presence of the Joule loss. The reflections of the undesired diffraction modes can be further suppressed by increasing the number of elements in the supercell to obtain a smoother phase gradient or by optimizing the entire structure by considering the interactions among the constituting elements.

In our design of the reflective metasurfaces, the metallic structures are assumed to be embedded in the substrate, because the conventional Babinet's relations are rigorously justified for the structures in a uniform medium. Actually, Babinet's relations for transmissive metasurfaces are empirically valid even for metallic structures placed on the top of a substrate. It might be possible to approximately realize the extended Babinet's relations without embedding metallic structures in substrates, but such an implementation is beyond the scope of this work.

In this paper, we focused on static reflective metasurfaces with self-complementary structures to verify the extended Babinet's relation given by Eq. (8), which is a special case satisfying $\tilde{r}_{TE} = \tilde{r}_{TE}^{(c)}$ and $\tilde{r}_{TM} = \tilde{r}_{TM}^{(c)}$ in Eqs. (5) and (6). If the planar structures embedded in the substrate can be converted to their complementary structures, the general Babinet's relations given by Eqs. (5) and (6) can be demonstrated. Reconfigurable-polarization devices with transmissive metasurfaces, whose structures can be switched between the original and complementary structures, have been demonstrated by the use of conventional Babinet's relations given by Eqs. (1) and (2) [29,30]. These approaches can be applied to the reflective metasurfaces proposed in this study. The extended Babinet's relations may pave the way for applications in highly efficient multifunctional metasurfaces such as reconfigurable intelligent surfaces [34–37] and polarization-encoded metasurfaces [38,39]. In addition, the relationship expressed by Eq. (4), which is valid for any planar metallic structure under the derived conditions, could be used for novel design of reflective metasurfaces by incorporation of the design strategy for transmissive metasurfaces.

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APPENDIX A: REFLECTION COEFFICIENT OF A REFLECTIVE METASURFACE FOR NORMAL INCIDENCE

To express $\tilde{r}_i$ as a function of $\tilde{\tau}_i$ and $\tilde{\rho}_i$, we calculated the transfer matrix T_i , defined as

$$\begin{pmatrix} E_2^+ \\ E_2^- \end{pmatrix} = T_i \begin{pmatrix} E_1^+ \\ E_1^- \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} E_1^+ \\ E_1^- \end{pmatrix}, \quad (\text{A1})$$

which connects electric fields E_1^+ and E_1^- at the substrate surface with electric fields E_2^+ and E_2^- immediately before the mirror, as defined in Fig. 11. The transfer matrix can be decomposed into three elements: transfer matrix T_b at the boundary between the vacuum and the substrate; T_d characterizing the propagation in the substrate of length d ; T_m on the metallic structures. T_b and T_d are given as

$$T_b = \frac{1}{2n} \begin{pmatrix} n+1 & n-1 \\ n-1 & n+1 \end{pmatrix}, \quad (\text{A2})$$

$$T_d = \begin{pmatrix} e^{-jkd} & 0 \\ 0 & e^{jkd} \end{pmatrix}, \quad (\text{A3})$$

where n is the substrate refractive index and k is the wave number in the substrate. The scattered electric fields from the metallic structures satisfy $E_{m2}^+ = \tilde{\tau}_i E_{m1}^+ + \tilde{\rho}_i E_{m2}^-$ and $E_{m1}^- = \tilde{\rho}_i E_{m1}^+ + \tilde{\tau}_i E_{m2}^-$, where E_{m1}^+ and E_{m1}^- (E_{m2}^- and E_{m2}^+) represent incoming and outgoing waves on the left (right) side of the structures, respectively, as shown in Fig. 11, and the transfer matrix T_m is given as follows [41]:

$$T_m = \frac{1}{\tilde{\tau}_i} \begin{pmatrix} \tilde{\tau}_i^2 - \tilde{\rho}_i^2 & \tilde{\rho}_i \\ -\tilde{\rho}_i & 1 \end{pmatrix}. \quad (\text{A4})$$

If d is assumed to be one fourth the wavelength in the medium—that is, $kd = \pi/2$ —then the whole transfer

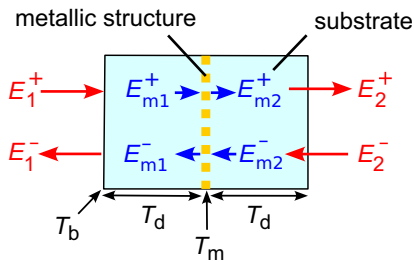


FIG. 11. Transfer-matrix elements from the top surface of the substrate to the bottom end of the substrate.

matrix given by $T_i = T_d T_m T_b$ is expressed as

$$T_i = \frac{1}{2n\tilde{\tau}_i} \begin{pmatrix} -(n+3)\tilde{\tau}_i + 2 & -(n-3)\tilde{\tau}_i - 2 \\ -(n+1)\tilde{\tau}_i + 2 & -(n-1)\tilde{\tau}_i - 2 \end{pmatrix}. \quad (\text{A5})$$

In this calculation, we used $1 + \tilde{\rho}_i = \tilde{\tau}_i$, which is derived from the electric field continuity at the metallic structures. The boundary condition at the mirror surface requires that $E_2^+ + E_2^- = 0$. The reflection coefficient of the metasurface can be derived as

$$\tilde{r}_i = -\frac{T_{11} + T_{21}}{T_{12} + T_{22}} = -\frac{(n+2)\tilde{\tau}_i - 2}{(n-2)\tilde{\tau}_i + 2}. \quad (\text{A6})$$

APPENDIX B: PROOF FOR THE CASE OF OBLIQUE INCIDENCE

Under the condition of no scattering to diffraction modes, the electromagnetic property of metasurfaces is effectively described by surface impedance or surface admittance [13]. For the metasurface with uniform structures, the transmission coefficient for normal incidence is written as

$$\tilde{\tau} = \frac{2\bar{Z}}{2\bar{Z} + Z}, \quad (\text{B1})$$

where Z denotes the wave impedance of the surrounding medium and $\bar{Z}$ is the surface impedance of the metasurface. If the structures are anisotropic along the x and y directions, as shown in Figs. 1(a) and 1(b), the surface impedances in the x and y directions can be defined as $\bar{Z}_{\text{TM}}$ and $\bar{Z}_{\text{TE}}$, respectively. Additionally, if the incident waves are illuminated at an oblique angle θ , the effective wave impedances should be modified to $Z/\cos\theta$ and $Z\cos\theta$ for TE and TM waves, respectively. As a result, the transmission coefficients for the TE and TM waves are, respectively, expressed as

$$\tilde{\tau}_{\text{TE}} = \frac{2\bar{Z}_{\text{TE}} \cos\theta}{2\bar{Z}_{\text{TE}} \cos\theta + Z}, \quad \tilde{\tau}_{\text{TM}} = \frac{2\bar{Z}_{\text{TM}}}{2\bar{Z}_{\text{TM}} + Z \cos\theta}. \quad (\text{B2})$$

Similarly, the surface impedances of the complementary structures are introduced as $\bar{Z}_{\text{TE}}^{(c)}$ and $\bar{Z}_{\text{TM}}^{(c)}$ for TE and TM waves, respectively, and the transmission coefficients can be expressed as

$$\tilde{\tau}_{\text{TE}}^{(c)} = \frac{2\bar{Z}_{\text{TE}}^{(c)} \cos\theta}{2\bar{Z}_{\text{TE}}^{(c)} \cos\theta + Z}, \quad \tilde{\tau}_{\text{TM}}^{(c)} = \frac{2\bar{Z}_{\text{TM}}^{(c)}}{2\bar{Z}_{\text{TM}}^{(c)} + Z \cos\theta}. \quad (\text{B3})$$

By substituting these expressions into Babinet's relations given by Eqs. (1) and (2), we obtained the following

relationship:

$$\bar{Z}_{\text{TE}}\bar{Z}_{\text{TM}}^{(c)} = \frac{Z^2}{4}, \quad \bar{Z}_{\text{TM}}\bar{Z}_{\text{TE}}^{(c)} = \frac{Z^2}{4}. \quad (\text{B4})$$

Next we derived the reflection coefficients $\tilde{r}_i$ ($i = \text{TE, TM}$) for the reflective metasurface shown in Figs. 1(c) and (d) for oblique incidence at angle θ_i . Assuming that the refractive angle is θ in the substrate, Snell's law requires that $\sin \theta_i = n \sin \theta$. The transfer matrices T_b at the substrate surface are expressed as

$$T_b = \frac{1}{2n \cos \theta} \begin{pmatrix} \cos \theta_i + n \cos \theta & -\cos \theta_i + n \cos \theta \\ -\cos \theta_i + n \cos \theta & \cos \theta_i + n \cos \theta \end{pmatrix}, \quad (\text{B5})$$

for TE waves and

$$T_b = \frac{1}{2n \cos \theta} \begin{pmatrix} \cos \theta + n \cos \theta_i & -\cos \theta + n \cos \theta_i \\ -\cos \theta + n \cos \theta_i & \cos \theta + n \cos \theta_i \end{pmatrix}, \quad (\text{B6})$$

for TM waves [41]. The transfer matrix for propagation in the substrate is given by

$$T_d = \begin{pmatrix} e^{-jkd \cos \theta} & 0 \\ 0 & e^{jkd \cos \theta} \end{pmatrix}, \quad (\text{B7})$$

for both TE waves and TM waves. The transfer matrices for the metallic structure T_m can be calculated by substituting Eqs. (B2) and (B3) into Eq. (A4) with $1 + \tilde{\rho}_i = \tilde{\tau}_i$. Subsequently, the whole transfer matrix $T_i = T_d T_m T_d T_b$ can be calculated. Similarly, when Eq. (A6) is derived, the reflection coefficients under a condition of $kd \cos \theta = \pi/2$ can be obtained as

$$\tilde{r}_{\text{TE}} = \frac{-n^2 \bar{Z}_{\text{TE}} \cos^2 \theta + Z_0 \cos \theta_i}{n^2 \bar{Z}_{\text{TE}} \cos^2 \theta + Z_0 \cos \theta_i}, \quad (\text{B8})$$

$$\tilde{r}_{\text{TM}} = \frac{-n^2 \bar{Z}_{\text{TM}} \cos \theta_i + Z_0 \cos^2 \theta}{n^2 \bar{Z}_{\text{TM}} \cos \theta_i + Z_0 \cos^2 \theta}, \quad (\text{B9})$$

for TE and TM waves, respectively. The reflection coefficients of the reflective metasurface embedding the complementary structures with surface impedance $\bar{Z}_{\text{TE}}^{(c)}$ and $\bar{Z}_{\text{TM}}^{(c)}$ are expressed as

$$\tilde{r}_{\text{TE}}^{(c)} = \frac{-n^2 \bar{Z}_{\text{TE}}^{(c)} \cos^2 \theta + Z_0 \cos \theta_i}{n^2 \bar{Z}_{\text{TE}}^{(c)} \cos^2 \theta + Z_0 \cos \theta_i}, \quad (\text{B10})$$

$$\tilde{r}_{\text{TM}}^{(c)} = \frac{-n^2 \bar{Z}_{\text{TM}}^{(c)} \cos \theta_i + Z_0 \cos^2 \theta}{n^2 \bar{Z}_{\text{TM}}^{(c)} \cos \theta_i + Z_0 \cos^2 \theta}. \quad (\text{B11})$$

From Eqs. (B8) and (B11), the sum of $\tilde{r}_{\text{TE}}$ and $\tilde{r}_{\text{TM}}^{(c)}$ yields

$$\begin{aligned} \tilde{r}_{\text{TE}} + \tilde{r}_{\text{TM}}^{(c)} &= \frac{2 \cos^2 \theta \cos \theta_i (Z_0^2 - n^4 \bar{Z}_{\text{TE}} \bar{Z}_{\text{TM}}^{(c)})}{(n^2 \bar{Z}_{\text{TE}} \cos^2 \theta + Z_0 \cos \theta_i)(n^2 \bar{Z}_{\text{TM}}^{(c)} \cos \theta_i + Z_0 \cos^2 \theta)}. \end{aligned} \quad (\text{B12})$$

Using Eq. (B4), which is derived from Babinet's relations and $Z = Z_0/n$, where Z_0 is the wave impedance of a vacuum, we obtain

$$Z_0^2 - n^4 \bar{Z}_{\text{TE}} \bar{Z}_{\text{TM}}^{(c)} = Z_0^2 \left(1 - \frac{n^2}{4}\right), \quad (\text{B13})$$

and this factor becomes zero for $n = 2$, which yields $\tilde{r}_{\text{TE}} + \tilde{r}_{\text{TM}}^{(c)} = 0$. Similarly, $\tilde{r}_{\text{TM}} + \tilde{r}_{\text{TE}}^{(c)} = 0$ can be verified under this condition. In summary, the extended Babinet's relations expressed in Eqs. (5) and (6) are satisfied for $n = 2$ and $2d = \pi/k \cos \theta$.

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