Relativistic damping of laser-beam-driven light sails

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Light sails using Earth-based lasers for propulsion require passive stabilization to stay within the beam. This can be achieved through the scattering properties of the sail, creating optical restoring forces and torques. Undamped restoring forces produce uncontrolled oscillations, which could jeopardize the mission, but it is not obvious how to achieve damping in the vacuum of space. Using a simple two-dimensional model, we show that the Doppler effect and relativistic aberration of the propelling laser beam create damping terms in the optical forces and torques. The effect is similar to the Poynting-Robertson effect causing loss of orbital momentum of dust particles around stars but can be enhanced by design of the geometry of the sail.

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I. INTRODUCTION

Laser-powered light sails [1,2] are one of the few plausible pathways for sending probes to other stars on time scales of a single human generation. With physically realistic but extremely challenging infrastructure [3,4], a light sail of mass $m \simeq 1$ g could be accelerated to a velocity $v = 0.2c$ within a time of approximately 1000 s and acceleration distance $\lesssim 0.1$ A.U., reaching the Proxima Centauri system within 20 years. Such a light sail would be propelled by a powerful laser array based on Earth, the photons of the laser imparting momentum upon reflection on the sail. Because of the finite width of the laser beam, a mechanism is required for the light sail to remain in the center of the beam. Any feedback to adjust the ground-based laser would be too slow as soon as the light sail is a few light-milliseconds away and active impulse or optical feedback mechanisms on board are difficult to achieve within the mass budget and without adding optical absorption that could lead to thermal breakdown of the sail. The most likely implementation of stabilization is thus through passive optical stabilization [5–9], which uses the combination of beam shape and spatial-reflectivity profile to generate a springlike restoring force toward the center of the beam, as well as a restoring torque to keep the sail at the optimal angle. However, the restoring force and torque alone lead to oscillations, which in the absence of damping are maintained throughout the acceleration phase, with an amplitude likely to increase with any perturbations due to, e.g., time-dependent beam misalignment during the acceleration phase.

Any transverse velocity component remaining at the end of the acceleration phase will lead to the craft going off course, with dramatic consequences on the ability to take telemetric measurements of exoplanets. In Fig. 1, we show the deviation from the ideal trajectory after 20 years of cruising at $0.2c$ as a function of the residual transverse velocity at the end of the acceleration phase. Simulations of passive stabilization using optical forces have shown final residual transverse velocities of order approximately 1–150 m/s, depending on the implementation and the level of perturbations [5,7,9], leading to final deviations of up to 0.6 astronomical units after 20 years of travel. Equally, any residual angular velocity is likely to complicate both telemetry and communications with Earth.

Damping is required to reduce these oscillations but is difficult to achieve in space. Srivastava *et al.* [6] have included an arbitrary damping force term without justifying what physical mechanism may cause it. Salary *et al.* [7] have seen a reduction in the spatial amplitude of the oscillations in their simulations, attributed to the shift in frequency from the Doppler effect changing the reflectivity and thus the restoring force. This is akin to changing the stiffness of a spring in a mass-spring system and thus, to lowest order, like changing the slope of the associated parabolic potential well: the spatial extent of the oscillation is reduced but the total energy and thus the maximum kinetic energy during the oscillations remain unaffected. It is thus unclear how much the Doppler effect in that situation is reducing the *velocity* amplitude of the

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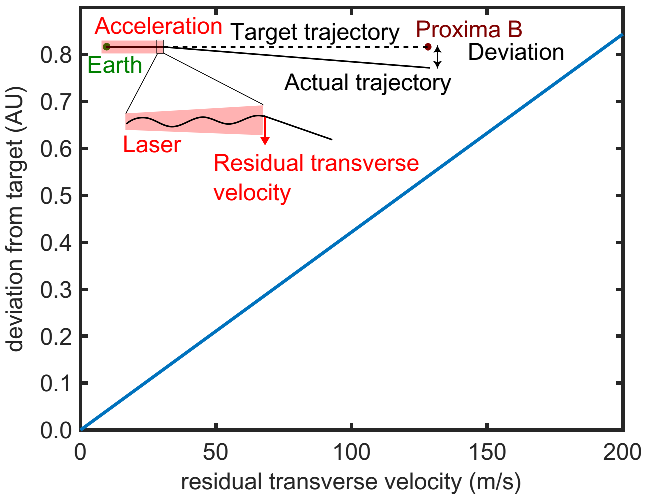

FIG. 1. The deviation from the target course in astronomical units after 20 years of travel as a function of the residual unwanted transverse velocity.

oscillations. Rafat *et al.* [9] have proposed the use of a damped internal degree of freedom, which can indeed effectively reduce the oscillations of the light sail, both in spatial amplitude and in velocity. However, the implementation of a damped internal degree of freedom will be challenging within the mass budget of a light sail only hundreds of nanometers thick.

The Doppler effect has been used as a damping force to slow atoms [10] but only in the direction of propagation of the laser. It is difficult to see how this could be implemented to dampen transverse oscillation of a light sail, as it appears that it would require laser beams propagating orthogonal to the direction of acceleration.

Damping *transverse* to the direction of a light wave is also known: the Poynting-Robertson (PR) effect [11–14] causes dust particles orbiting a star to lose orbital angular momentum, due to relativistic aberration. In the reference frame of the dust, light from the star comes from a direction shifted toward the direction in which the dust is moving [15]. Light is absorbed, leading to a radiation force with a component orthogonal to the radial direction to the star, slowing the dust down. Dust particles eventually fall into the star, over a time roughly proportional to the particle sizes and of order tens of thousands of years [13]. The PR effect has been shown to mildly impact solar-sail dynamics, specifically due to residual absorption by the sail [16–18]. Can a similar effect be used for effective damping of unwanted motion in laser-driven light sails? The situation is quite different from the usual PR effect, even as studied for solar sails: contrary to solar sails or dust, laser-powered light sails have very small transverse velocities—and absorption, which is the basis of most PR studies, must be avoided at all costs.

Here, we show that the angular-reflectivity properties of a light sail, once combined with the Doppler effect and relativistic aberration, indeed provide damping similarly to the PR effect. We derive explicit expressions for the damping forces and torques for a simple two-dimensional two-mirror geometry and show that with appropriate optical design, transverse velocities can be damped to almost arbitrary levels by the end of the acceleration phase, albeit at the cost of increasing the acceleration distance.

II. PRINCIPLE

The origins of the damping forces and torques are illustrated in Fig. 2, using arguably the simplest two-dimensional reflecting object having linear mechanical stability both in translation and rotation in a laser beam:

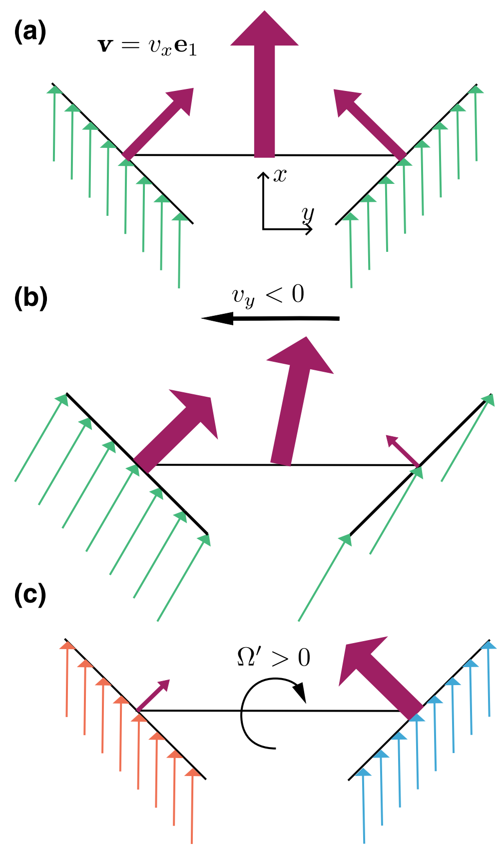

FIG. 2. The principle of the PR damping of laser-driven light sails, in the frame of the sail. (a) For a nonrotating sail moving parallel to the beam direction, the forces are balanced on both mirrors. The net force (central taupe arrow) is purely in the direction of the laser beam. (b) The damping force: for a sail with unwanted transverse velocity component $v_y < 0$, relativistic aberration angles the light of the laser (green arrows), leading to a nonzero transverse force opposing the transverse velocity. (c) The damping torque: for nonzero rotational velocity $\Omega' > 0$, the left mirror has a slightly larger velocity away from the laser source than the right mirror, with the additional red shift reducing the momentum of the photon.

a symmetric set of two angled mirrors connected by a rod [9]. In the figure, the desired direction (the direction of the beam, the x axis) is upward. When moving strictly parallel to x [Fig. 2(a)], the radiation pressure on both mirrors is equal and the sail is accelerated forward only. If the sail has an undesired movement orthogonal to the direction of the laser beam [Fig. 2(b)], relativistic aberration tilts the light in the reference frame of the sail, so that the left mirror intercepts more light. This asymmetry leads to a component of the radiation force perpendicular to the laser beam (in the frame $\mathscr{L}$ of the laser) that is proportional to and opposing the transverse velocity—in effect, a drag force. If the sail is rotating relative to the axis of the laser beam [Fig. 2(c)], the differential Doppler shift leads to different forces on the two mirrors, giving rise to a torque that opposes the rotational motion—a damping torque.

III. NOTATION

A four-vector $\vec{x}$ has, in a specified reference frame, components $(x^0, x^1, x^2, x^3)^T$, where x^0 is the temporal component and the other three are spatial components. A Minkovsky metric diag$(1, -1, -1, -1)$ is implied throughout. We follow the convention of using Roman indices for spatial components (e.g., x^j) and Greek indices for all four spatiotemporal components (x^μ). Spatial three-vectors in any specific frame will be noted as bold Roman letters and unit vectors will be marked with a hat ($\hat{}$) (e.g., $\mathbf{k}$ is the three-vector with components k^i, and $\hat{\mathbf{k}} = \mathbf{k}/|\mathbf{k}|$). Basis four-vectors of a reference frame will be denoted $\hat{e}_\mu$ and we will use the same notation, $\hat{\mathbf{e}}_j$, for spatial unit three-vectors. We will use $\mathbf{v}, v_x, v_y, v_z, v$ for the velocity vector, its components, and the norm, respectively, in the rest frame of the laser. We will use the usual notation $\beta = v/c$, also applicable to individual components, e.g., $\beta_x = v_x/c$, and $\gamma = 1/\sqrt{1-\beta^2}$. The laser accelerating the light sail is in frame $\mathscr{L}$, assumed to be inertial, and the instantaneously comoving inertial frame of the light sail is called $\mathscr{M}$. Frame $\mathscr{M}$ has velocity $\mathbf{v}$ in frame $\mathscr{L}$. Primed quantities refer to quantities in $\mathscr{M}$, while unprimed quantities refer to frame $\mathscr{L}$. The unit four-vectors in $\mathscr{M}$ are related to those in $\mathscr{L}$ through the inverse Lorentz transform $\hat{e}'_\mu = \Lambda(-\mathbf{v})^\nu_\mu \hat{e}_\nu$, with

$$\Lambda(\mathbf{v}) = \begin{pmatrix} \gamma & -\frac{\gamma v_x}{c} & -\frac{\gamma v_y}{c} & -\frac{\gamma v_z}{c} \\ -\frac{\gamma v_x}{c} & 1+\frac{(\gamma-1)v_x^2}{v^2} & \frac{(\gamma-1)v_x v_y}{v^2} & \frac{(\gamma-1)v_x v_z}{v^2} \\ -\frac{\gamma v_y}{c} & \frac{(\gamma-1)v_x v_y}{v^2} & 1+\frac{(\gamma-1)v_y^2}{v^2} & \frac{(\gamma-1)v_y v_z}{v^2} \\ -\frac{\gamma v_z}{c} & \frac{(\gamma-1)v_x v_z}{v^2} & \frac{(\gamma-1)v_y v_z}{v^2} & 1+\frac{(\gamma-1)v_z^2}{v^2} \end{pmatrix}. \tag{1}$$

In $\mathscr{L}$, we choose $\hat{\mathbf{e}}_1$ to point toward the desired star system. Note that because $\mathbf{v}$ is not aligned with either $\hat{\mathbf{e}}_1$ or $\hat{\mathbf{e}}_2$, the spatial parts $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}'_1$ of $\hat{e}_1$ and $\hat{e}'_1$ are not parallel [19] and neither are $\hat{\mathbf{e}}_2$ and $\hat{\mathbf{e}}'_2$.

IV. GEOMETRY

In Fig. 3, we show our light-sail model in the $\mathscr{L}$ and $\mathscr{M}$ frames. The mirrors are at the tips T_1 and T_2 of a massless rod of rest length $2L_0$, each with surface area A_0 (dimensions of length for our two-dimensional treatment). In frame $\mathscr{M}$, the mirrors each make a fixed angle $\alpha'_{1,2} = \pm\alpha_0$ with $\hat{\mathbf{e}}'_1$. In Ref. [9], this geometry has been shown to have a *restoring* force and torque in a parabolic beam, with an equilibrium position in the middle of the beam and with the rod perpendicular to the beam direction.

For simplicity, we consider the laser beam to be a single plane wave, with wave vector $\mathbf{k}$ aligned with $\hat{\mathbf{e}}_1$ (Fig. 3), with intensity I (in power per unit length, for our two-dimensional treatment). With that simplification, there is no restoring force but, as we shall see, there is still a damping force and there are restoring and damping torques. In $\mathscr{L}$, the wave four-vector $\vec{k}$ has coordinates $k^0(1, 1, 0, 0)^T$, with $k^0 = \omega/c$.

We define the normal vector to the mirrors, chosen to point away from the source of light, as follows:

$$\hat{\mathbf{n}}'_{1,2}(\alpha'_{1,2}) = \mathrm{sign}(\sin(\alpha'_{1,2} - \theta'))\left(\sin\alpha'_{1,2}\hat{\mathbf{e}}'_1 - \cos\alpha'_{1,2}\hat{\mathbf{e}}'_2\right), \tag{2}$$

where the sign$(\sin(\alpha'_{1,2} - \theta'))$ ensures the correct orientation away from the light source.

V. LINEAR DAMPING FORCE

We aim to quantify the drag force in the $\hat{\mathbf{e}}_2$ direction in $\mathscr{L}$ that is due to the Doppler shift and relativistic aberration. To do so, we calculate the momentum exchange rate

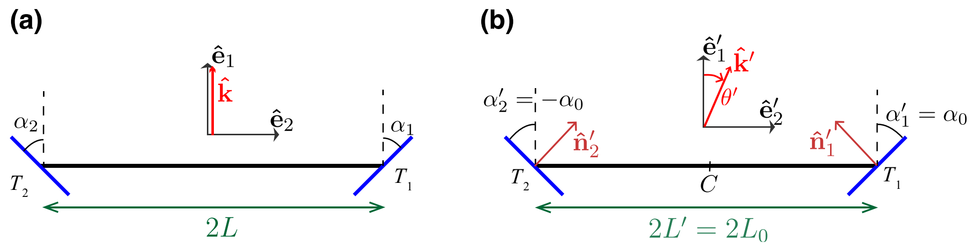

FIG. 3. The geometry of two mirrors in the (a) laser and (b) mirror frames. The mirrors have surface area A_0 in their rest frame, while C is the center of the connecting rod and also the center of mass. The direction of $\mathbf{k}'$ and $\theta' > 0$ on the right correspond to $v_y < 0$, i.e., a sail moving toward the left.

with the sail in $\mathscr{M}$, which gives us an expression of the four-force in $\mathscr{M}$. We then Lorentz boost this back into $\mathscr{L}$ and identify the component of the force in $\hat{\mathbf{e}}_2$. As we are seeking a drag force close to the ideal trajectory, we will assume $v_y \ll v_x < c$, which, to linear order in v_y/v, also implies that

$$v_x/v = \sqrt{1 - v_y^2/v^2} \simeq 1 - \frac{1}{2}\left(\frac{v_y}{v}\right)^2 \simeq 1, \tag{3}$$

$$v_x/c \simeq \beta. \tag{4}$$

A. Derivation

In frame $\mathscr{M}$, in the limit of geometric optics, forces on the mirrors can be calculated from the change of momentum of photons upon reflection. In $\mathscr{M}$, the wave four-vector of the plane wave is given by a Lorentz boost $k'^{\mu} = \Lambda(\mathbf{v})^{\mu}_{\nu} k^{\nu}$, yielding

$$k'^0 = Dk^0, \tag{5}$$

$$k'^1 = k^0\left(1 + (\gamma - 1)v_x^2/v^2 - \gamma v_x/c\right) \simeq Dk^0, \tag{6}$$

$$\begin{aligned} k'^2 &= k^0\left((\gamma - 1)v_x v_y/v^2 - \gamma v_y/c\right) \\ &\simeq k^0 \frac{v_y}{v}\left(\gamma - 1 - \gamma\beta\right), \end{aligned} \tag{7}$$

where D is the Doppler factor

$$D = \gamma\left(1 - \frac{\mathbf{v}\cdot\hat{\mathbf{k}}}{c}\right) \simeq \gamma(1 - \beta) \tag{8}$$

and we have used Eqs. (3) and (4) to simplify Eqs. (6)–(8).

The aberration angle θ' can then be obtained as

$$\theta' \simeq \tan\theta' = \frac{k'^2}{k'^1} \simeq \frac{v_y}{v}\frac{(\gamma - 1 - \gamma\beta)}{D} \simeq -\left(\frac{1}{D} - 1\right)\frac{v_y}{v}. \tag{9}$$

To determine the force on each mirror, we multiply the momentum imparted to the mirror by each photon upon reflection by the rate of arrival of photons. In frame $\mathscr{M}$, each photon imparts a three-momentum

$$\Delta\mathbf{p}'_M = 2\hbar(\mathbf{k}'\cdot\hat{\mathbf{n}}'_{1,2})\hat{\mathbf{n}}'_{1,2}. \tag{10}$$

The rate of photons per surface area in $\mathscr{L}$ is $\Gamma = I/\hbar\omega = I/(\hbar k^0 c)$ and in $\mathscr{M}$ becomes $\Gamma' = D\Gamma$ [20]. The cross section of the mirrors intercepting the laser is given by $A_0|\hat{\mathbf{n}}'_{1,2}\cdot\hat{\mathbf{k}}'|$, so that in $\mathscr{M}$, the total three-force on each mirror is

$$\begin{aligned} \mathbf{F}'_{1,2} &= A_0|\hat{\mathbf{k}}'\cdot\hat{\mathbf{n}}'_{1,2}|\Gamma'\Delta\mathbf{p}'_M \\ &= 2\frac{D^2 I}{c}A_0|\hat{\mathbf{k}}'\cdot\hat{\mathbf{n}}'_{1,2}|(\hat{\mathbf{k}}'\cdot\hat{\mathbf{n}}'_{1,2})\hat{\mathbf{n}}'_{1,2}, \end{aligned} \tag{11}$$

where we have used that, to linear order, $\mathbf{k}' = Dk^0\hat{\mathbf{k}}'$ [Eq. (6)]. Expressing the dot product in terms of θ' and adding the contribution of both mirrors gives, to linear order in θ', the total three-force on the system:

$$\mathbf{F}' \simeq 4\frac{D^2 I}{c}A_0\left(\sin^3\alpha_0\,\hat{\mathbf{e}}'_1 + \theta'\cos\alpha_0\sin 2\alpha_0\,\hat{\mathbf{e}}'_2\right). \tag{12}$$

The general expression for the coordinates of the four-force $\vec{f}$ on an object at velocity $\mathbf{v}$ in a given reference frame in terms of the three-force in that frame is $(\gamma\mathbf{F}\cdot\mathbf{v}/c;\gamma\mathbf{F})$ [21]. In $\mathscr{M}$, the velocity of the sail is zero, with $\gamma' = 1$, so that the four-force $\vec{f}$ has coordinates $(0;\mathbf{F}')$, from which we obtain the four-force in $\mathscr{L}$ through a Lorentz transform. Using Eq. (9) the transverse component of force (along $\hat{\mathbf{e}}_2$) is then, to lowest order in v_y/v,

$$\begin{aligned} f^2 &\equiv \frac{dp^2}{dt'} \\ &\simeq 4A_0\frac{D^2 I}{c}\frac{v_y}{v}\left(-\cos\alpha_0\sin 2\alpha_0(1/D - 1)\right. \\ &\quad \left. + (\gamma - 1)\sin^3\alpha_0\right), \end{aligned} \tag{13}$$

where t' is the time in $\mathscr{M}$ and thus proper time. We find a net transverse force that is proportional to v_y and thus, depending on the overall sign, a possible drag force.

Expressing $1/D - 1$ and $\gamma - 1$ for small β gives some insight into the relative importance of the two terms in brackets in Eq. (13):

$$f^2 \simeq 4A_0 \frac{D^2 I}{c} \frac{v_y}{c} \left(-\cos\alpha_0 \sin 2\alpha_0 \left(1 + \frac{\beta}{2}\right) + \frac{\beta}{2} \sin^3 \alpha_0 \right). \tag{14}$$

At small $\beta < 0.2$, the second and third terms only contribute a correction of the order of 5% to the first term. Importantly, the dominant term is negative (and independent of v_x), so that there is a net drag force that will lead to damping of any transverse motion, from the outset of the acceleration phase, with damping coefficient

$$\zeta \equiv -f^2/v_y \simeq 4A_0 \frac{D^2 I}{c^2} \cos\alpha_0 \sin 2\alpha_0. \tag{15}$$

In Eq. (13), the linear v_y/c dependence comes from the relativistic aberration. The $D^2 I/c$ factor represents the optical power intensity in $\mathscr{M}$ and in Eq. (15) the remaining proportionality constant $4A_0 \cos\alpha_0 \sin 2\alpha_0$ is a geometry-dependent form of the cross section that is not simply the radiation-pressure cross section [14,22] but also depends on the dependence of the radiation pressure on the incident angle. This latter term can be adjusted by geometry, in our simple case by adjusting α_0.

B. Order of magnitude

To assess the importance of the effect, we compare the magnitude of the transverse damping force to the accelerating force along $\hat{\mathbf{e}}_1$. Assuming $\beta < 0.2$, we have $1 \leq \gamma \lesssim 1.02$ and $0.81 \simeq D \leq 1$. Reasonable approximations can thus be obtained assuming D constant and ignoring all other relativistic corrections or β dependencies, which in particular leads to a constant longitudinal acceleration force. The acceleration time to reach a final β_f can then be approximated by $t_f \simeq mc\beta_f/f^1$. During that time, the transverse damping force leads to an exponential decrease of any initial velocity following $v_y = v_{y,0} \exp(-\zeta t/m)$. By the end of the acceleration phase, any initial transverse velocity $v_{y,0}$ is thus attenuated by a factor $\eta = v_{y,\text{final}}/v_{y,0}$, with

$$\ln \eta = -\frac{\zeta}{m} t_f = \frac{-\zeta c}{f^1} \beta_f. \tag{16}$$

From Eqs. (12) and (13), to linear order in v_y/v and zeroth order in v_x/c, we obtain

$$-\frac{\ln \eta}{\beta_f} \simeq \frac{\cos\alpha_0 \sin 2\alpha_0}{\sin^3 \alpha_0} = 2\cot^2\alpha_0. \tag{17}$$

In this last equation, the dependence on mass m, sail size L_0, mirror size A_0, or beam intensity I all cancel between

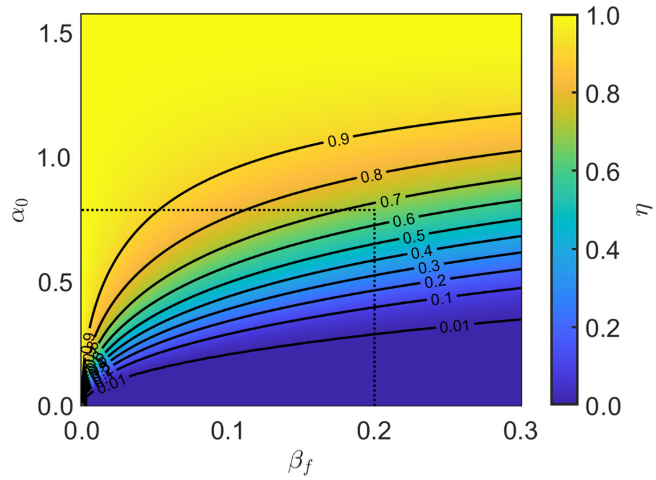

FIG. 4. The attenuation factor $\eta = v_{y,final}/v_{y,0}$ as a function of the mirror angle α_0 and the final longitudinal velocity β_f.

the numerator and denominator, as the longitudinal and transverse acceleration scale identically with those parameters. For the general geometry of Fig. 3, the attenuation factor η thus only depends on α_0 and on the final velocity β_f. We plot the resulting attenuation ratio η as a function of mirror angle α_0 and final velocity β_f in Fig. 4. For $\alpha_0 = \pi/4$ and $\beta_f = 0.2$, any initial velocity is thus reduced to $\eta = \exp(-0.4) \simeq 67\%$ of its initial value (dashed lines in Fig. 4). Within this approximation, valid for relatively small β, the reduction also improves exponentially with β_f. More importantly, this reduction is geometry dependent, with a smaller value of $\alpha_0 = 0.3$ leading to a final transverse velocity of only 1.5% of $v_{y,0}$. However, this improvement comes at the cost of reduced longitudinal acceleration, i.e., increased acceleration time and distance.

Ultimately, the damping force is a function of the angular dependence of the radiation pressure (through the aberration angle). Better optical design may be able to increase the level of damping without compromising acceleration time.

VI. RESTORING AND DAMPING TORQUES

We now allow for rotations of the light sail, with the axis of the sail now making an angle Φ' (Fig. 5) with the $\hat{\mathbf{e}}'_1$ axis, with an angular velocity $\Omega' = d\Phi'/dt'$. The vector $\mathbf{v}$ now denotes the velocity of the center of mass C rather than the velocity of all parts of the sail. For $\Omega' = 0$, using Eq. (11) to calculate the torque about C in $\mathscr{M}$, we obtain, to linear order in θ' and Φ',

$$\tau'_r = -\left(\frac{4L_0 A_0 I D^2}{c} \sin 2\alpha_0\right) (\Phi' - \theta') \hat{\mathbf{e}}'_3, \tag{18}$$

which is a restoring torque toward the direction of the incoming light in $\mathscr{M}$. To obtain a damping term for

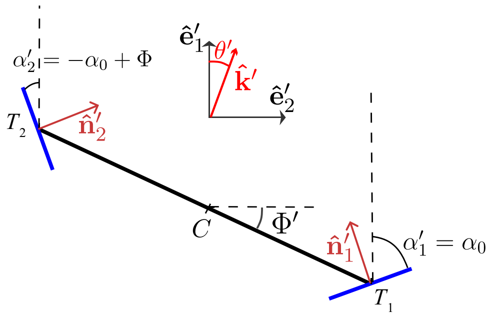

FIG. 5. Rotations of the light sail : geometry in the mirror frame.

$\Omega' \neq 0$, we need to take into account the rotational velocities of the mirrors, which lead to different Doppler shifts and relativistic aberrations for each point of each mirror. We take the further simplifying assumption that the mirrors are small compared to L_0 and can thus be treated as point mirrors, each with unique velocities $\mathbf{v}'_{1,2}$ in $\mathscr{M}$. Equation (11) remains valid but we now have two different values, $\hat{\mathbf{k}}'_{1,2}$ and $D_{1,2}$. Since v_x can reach relativistic values, we need to use relativistic velocity additions $\mathbf{v}_{1,2} = \mathbf{v} \oplus \mathbf{v}'_{1,2}$. Assuming that $v'_{1,2} \ll c$, we obtain, to linear order,

$$\mathbf{v}_{1,2} \simeq \mathbf{v} + \frac{v'_{1,2x}}{\gamma^2}\hat{\mathbf{e}}_1 + \frac{v'_{1,2y}}{\gamma}\hat{\mathbf{e}}_2, \tag{19}$$

from which, again to linear order in $v'_{1,2}/c$ and v_y/v, we obtain

$$D_{1,2} \simeq D\left(1 - \frac{v'_{1,2x}}{c}\right) \tag{20}$$

and

$$\theta'_{1,2} \simeq -\left(\frac{1}{D} - 1\right)\left(\frac{v_y}{v} + \frac{v'_{1,2y}}{\gamma v}\right). \tag{21}$$

The damping torque at "equilibrium" is calculated for $v_y = 0$ and $\Phi' = 0$. In $\mathscr{M}$, the velocities of the mirrors then simplify to $\mathbf{v}'_{1,2} = \Omega'\hat{\mathbf{e}}'_3 \times \mathbf{CT}_{1,2} = \mp\Omega' L_0 \hat{\mathbf{e}}'_1$, so that $\theta'_{1,2} = \theta'$ and $D_{1,2} = D(1 \pm \Omega' L/c)$. The total torque becomes, again to linear order,

$$\tau'_d = -\left(\frac{8L_0^2 A_0 I D^2}{c^2}\sin^3\alpha_0\right)\Omega'\hat{\mathbf{e}}'_3, \tag{22}$$

and is indeed a damping torque, as it is countering and proportional to Ω'.

Equations (18) and (22) lead to a damped oscillating rotational motion, with damping time constant t'_{rd}. Using the total moment of inertia mL_0^2, we have

$$t'_{rd} = \frac{mc^2}{2A_0 I D^2 \sin^3\alpha_0}, \tag{23}$$

which, by a reasoning analogous to that in Sec. V B, leads to a rotational damping ratio η_r, with

$$\ln \eta_r = -\beta_f/2. \tag{24}$$

As for Eq. (17), the final damping ratio does not depend on mass, sail size, mirror size, or beam intensity, as the longitudinal acceleration time t_f and damping time t'_{rd} scale identically with these parameters, so that they cancel in the numerator and denominator. Furthermore, Eq. (24) also does not depend on α_0. However, this is somewhat accidental: the damping ratio depends on the ratio of the longitudinal forces to the moment of inertia, so that it does, in general, depend on the geometry. For the two-mirror system under consideration with a final velocity of $\beta_f = 0.2$, any initial rotational velocity would be reduced by only 10% but this could be enhanced considerably by having a mass distribution closer to the center of gravity (e.g., a payload at the center) or via different optical designs.

VII. CONCLUSIONS

The relativistic transformation of the photon momentum can contribute to significant damping of unwanted transverse and rotational movements of light sails. Both the spacelike and timelike transformations are of importance: for our geometry around equilibrium, transverse *translational* damping comes primarily from the relativistic aberration (spacelike), while *rotational* damping comes primarily from the difference in Doppler terms for both mirrors (timelike). However, for different values of Φ' (or different geometries), both the relativistic aberration and Doppler effects can be at play for the translational and rotational damping, with similar magnitudes. Order-of-magnitude calculations show that over the course of the acceleration phase, unwanted transverse velocity components can be damped to almost arbitrary levels by appropriate optical design (Fig. 4). In our simple geometry, this comes at the cost of slower acceleration but as transverse damping ultimately comes from the angular response of the reflection, it is conceivable that very large damping coefficients could be achieved without loss of longitudinal acceleration in realistic geometries using, e.g., gratings or metasurfaces with sharp angular diffraction patterns [5,7,8].

Our study also highlights that taking the full relativistic transformations of light into account is essential to include damping mechanisms in light-sail dynamics. Studies of light sails that we have encountered so far have included the timelike (Doppler) transformation but neglected the spacelike (relativistic aberration) transformation.

Finally, among our many simplifying assumptions, we have considered the laser to be a single monochromatic plane wave. In practice, the divergence of laser beams will be comparable to the magnitude of the relativistic aberration. When extending this study to more realistic light-sail geometries, it will thus be important to also take into account the angular spread of finite beams, noting that the relativistic transformation of finite beams is a rich field of study in itself, with many surprising phenomena [20,23].

That the Doppler effect and relativistic aberration can, in principle, provide damping should be welcomed by the interstellar light-sail community, as other mechanisms of damping residual motion seem difficult to implement within the extremely narrow mass-budget constraint. Special relativity gifts us a solution that could simplify light-sail designs and considerably improve accuracy of trajectories.

ACKNOWLEDGMENTS

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