Dissipative-coupling-induced steady-state entanglement and one-way steering in a cavity-magnonics system

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In the cavity-magnonics system, the coupling between microwave photon and magnon is of beam-splitter (BS) interaction, which in general does not support steady-state entanglement in a vacuum environment. We propose a scheme to generate and enhance steady-state magnon-photon entanglement and one-way steering in cavity-magnonics system. By coupling microwave cavity mode and magnon mode to a common waveguide, the effective dissipative coupling of the cavity-magnonics system is derived. If there is only nondegenerate parametric (NP) dissipative coupling, entanglement is negatively affected by the effective gain of the cavity mode caused by the common waveguide environment. When the coherent coupling exists, effective gain of cavity mode can cancel the intrinsic dissipation. Consequently, the larger the intrinsic dissipation of cavity the better the entanglement. When the effective dissipative coupling with NP and BS coexist, the effective dissipation can be eliminated and the entanglement is significantly improved. In addition, we find that the asymmetric coupling of the two modes to the common waveguide leads to one-way steering, which means that the cavity mode can guide the magnetic mode, and the magnetic mode can not affect the cavity mode through local measurements. This property facilitates the development of the cavity-magnonics system towards quantum information processing.

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I. INTRODUCTION

The concept of entanglement was proposed by Schrödinger in his replaying to the Einstein-Podolsky-Rosen (EPR) paradox proposed by Einstein *et al.* [1,2]. The generation of a high degree of entanglement is a prerequisite for quantum communication and information processing [3,4]. Therefore, many efforts have been devoted to exploring how to prepare and enhance this quantum resource [5–7]. Using quantum entanglement, many protocols of quantum information, such as quantum teleportation [8], telecloning [9], and quantum cryptography [10] have been realized.

In recent years, a kind of quantum inseparability, called EPR steering as a strict subset of entanglement, has attracted widespread attention [11,12]. EPR steering refers to the nonclassical correlations in a bipartite scenario in which one party remotely affects another's state through local measurements. In general, achieving asymmetric steering is obtained by introducing different losses or noises in the subsystem [13,14], many studies based on different quantum systems have investigated quantum steering, such as atom-mechanical systems [15] and cavity-magnonics systems [16–19]. Due to the asymmetry, it provides additional security, which is useful for various quantum information protocols [20] and is widely used in various quantum information protocols, such as quantum secret sharing [21], one-way quantum computing [22], and no-cloning quantum teleportation [23], subchannel discrimination [24]. It has been experimentally shown that under the proper conditions it can produce a unique asymmetry between two observers [25,26].

Recently, ferrimagnetic systems such as yttrium iron garnet (YIG) with dimensions approximately 100 μm prepared in experiments, provides insights into macroscopic quantum effects, which have drawn extensive interests in various branches of physics over the last decade [27–29]. Magnons can interact with qubits [30,31], phonons [32,33], microwave and optical photons [34], forming different hybrid systems. Due to its high spin density (several orders of magnitude larger than those of previous spin ensembles) and low dissipation rate (as low as 0.01 MHz) [35], the strong [36–38] and even ultrastrong [39,40] coupling between Kittel mode and microwave cavity mode has been achieved. This strong coupling offers the possibility to realize coherent information transfer between different information carriers and therefore may develop potential applications in quantum information processing. This research has enabled the cavity-magnonics system to explore numerous interesting phenomena and applications, such as magnon blockade [41,42], magnetically

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induced transparency [32,37]. Recently, continuous variable entanglement in hybrid magnon-based systems has made great progress, such as tripartite magnon-photon-phonon entanglement [33,43,44], magnon-magnon entanglement [45,46], magnon-mediated optical entanglement [47,48], and magnon-photon entanglement [18,19,49,50].

Magnons can be coupled to microwave photons via magnetic dipoles or the Zeeman effect. Due to the frequency of the microwave photon near to that of magnon, the beam-splitter (BS) interaction can be obtained through the rotation-wave approximation [51]. However, the BS interaction dose not support steady-state entanglement due to the fact that in a vacuum environment, its steady state is a vacuum state. Many studies have been developed to achieve steady-state entanglement by making one of the modes in a squeezing vacuum environment [52–54]. Dissipative magnon-photon coupling [55–58] provide us an alternative way to produce steady-state entanglement. In fact, such indirect interactions mediated by traveling photons are the main object of study in waveguide quantum electrodynamics, and have been demonstrated in various systems, e.g., quantum dots [59,60], atoms [61,62], and superconducting circuits [63,64]. In cavity-magnonics systems, by making the cavity and magnon modes to interact with a common waveguide or damping mode [58], i.e., cooperative external damping [65], it is possible to cause magnon-photon dissipative coupling. Many schemes have been studied based on this system, such as nonreciprocity and unidirectional invisibility [57], nonreciprocal magnon blockade [66], and one-way steering [18,19]. Experimentally, BS dissipative coupling has been realized in cavity-magnonics systems [57]. Most recently, an experimental study of gain-driven polaritons coherent microwave emission has been carried out through the development of a gain-embedded cavity-magnonics platform in which the cavity mode has both negative damping and nonlinear damping [67].

In a closed conserved system, the interactions between subsystems described by Hamiltonian are called coherent coupling. If the effective coupling between two subsystems is obtained by coupling them to a common environment, then this indirect effective coupling is of Lindblad form or non-Hermitian form called dissipative coupling. In this paper, we present a theoretical scheme to prepare and enhance steady magnon-photon entanglement in a cavity-magnonics system. By coupling the microwave cavity and magnon modes to a common waveguide with a time-dependent coupling, we can derive the master equation where the effective dissipative coupling between cavity and magnon modes are clearly presented. The terms $\hat{c}^\dagger\hat{m}^\dagger\rho + \hat{c}\hat{m}\rho$ contained in Lindblad's master equation are nondegenerate parametric (NP) dissipative interaction, where $\hat{c}$ ($\hat{m}$) is the annihilation operator for the cavity (magnon) mode. The terms $\hat{c}^\dagger\hat{m}\rho + \rho\hat{c}\hat{m}^\dagger$ are BS dissipative interaction. Consequently, the steady magnon-photon entanglement can be generated and enhanced. What is more, we derive an additional induced noise resulting from the common waveguide, such that the noise matrix consists of nondiagonal elements, which describe the noise correlation between the cavity and the magnon modes. The induced correlation noises play an essential role in generating quantum phenomena. However, the correlated noise has not been included in dissipative cavity-magnonics researches [18,19,50,57]. If there is only NP dissipative coupling, an effective gainlike dissipation in the cavity mode can be generated due to the common waveguide environment, which has a negative impact on the entanglement. Under this case, coherent coupling channel can balance the net gain of the cavity mode so that entanglement can survive. Meanwhile, we can observe counterintuitive phenomenon, that is to say, the larger the intrinsic dissipation, the better the entanglement at a certain range of parameters. When the dissipative system has NP plus BS coupling, the effective dissipation of the cavity modes can be effectively eliminated. Even without coherent coupling, entanglement and one-way steering still exist and can be enhanced. We also show unidirectional steering in both systems can be reached with the asymmetric coupling of the two modes to the common waveguide. Further, we show that entanglement can survive over a certain range of the intrinsic dissipation of cavity mode $\kappa_{c,i}$, which reduces the requirement for high quality of cavity.

II. THEORETICAL MODEL AND DYNAMICAL EQUATION

A. The system model and total Hamiltonian

As schematically depicted in Fig. 1(a), we consider a cavity-magnonics system where a YIG sphere is placed close to a cross-line microwave circuit. The cross-line circuit supports both the standing wave forming the cavity mode and the traveling wave, and the microwave cavity and the YIG sphere are coupled to the common waveguide. Such a system has been studied in experiment [57]. Microwave cavity coupling to waveguide transmission lines has been widely studied and employed in many theoretical and experimental studies [68–70]. Furthermore, we consider the cavity coupled to the common waveguide with time-dependent coupling, which can be realized by a dc superconducting quantum interference device (SQUID), where the superconducting loop intersected by two Josephson junctions mediating the coupling between the microwave cavity and the waveguide transmission line [71]. Our scheme also can be realized in the configuration shown in Fig. 1(c). A transmission-line resonator supporting the standing wave forming the cavity mode as well as a YIG sphere are simultaneously coupled to the traveling waves in an open waveguide, and the coupling between the cavity mode and the waveguide is modified as time-dependent form [66]. It is worth noting that the

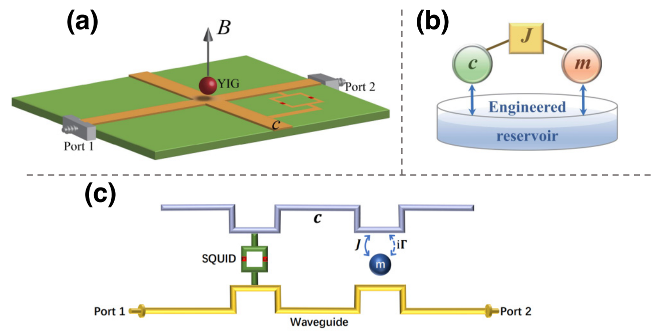

FIG. 1. (a) Schematic of the dissipative photon-magnon coupling system, which consists of a cross-line circuit and a YIG sphere. The cross-line cavity supports both standing cavity modes and traveling waves. A uniform magnetic field acts near the YIG sphere. A dc SQUID is introduced between the cavity mode and waveguide to modulate the time-dependent coupling. (b) The cavity and magnon modes are directly coupled via a coherent interaction J, and are also coupled to the common waveguide environment. (c) A transmission-line resonator supporting the standing wave forming the cavity mode as well as a YIG sphere are simultaneously coupled to the traveling waves in an open waveguide. The cavity mold is connected to the transmission line by a SQUID containing two Josephson junctions.

magnetic flux in the Josephson junctions should not affect the magnon mode of the YIG sphere so as to manipulate them separately, which can be reached if the two magnetic fields are orthogonal [72]. In contrast to Ref. [72], in our scheme the uniform magnetic field B exerted on a YIG sphere may be out of plane relative to the SQUID loop. By using a magnetic shield around the SQUID [73], the two magnetic fields can respond to the YIG sphere and the SQUID separately. The Hamiltonian of the system reads ($\hbar = 1$)

$$H_{\text{tot}} = H_{\text{sys}} + H_V, \tag{1}$$

with

$$H_{\text{sys}} = \widetilde{\omega}_c \hat{c}^\dagger \hat{c} + \widetilde{\omega}_m \hat{m}^\dagger \hat{m} + J(\hat{m}^\dagger \hat{c} + \hat{c}^\dagger \hat{m}), \tag{2}$$

where $\hat{c}$ ($\hat{m}$) is the annihilation operator for the cavity (magnon) mode with the frequency ω_c (ω_m). $\widetilde{\omega}_c$ and $\widetilde{\omega}_m$ are complex and defined as $\widetilde{\omega}_{c(m)} = \omega_{c(m)} - i\kappa_{c(m),i}$, where $\kappa_{c(m),i}$ represent the intrinsic damping rates of the two modes, respectively. The magnon-cavity coupling rate J can be (much) larger than the intrinsic dissipation rates, such that the strong-coupling regime $J > \{\kappa_{m,i}, \kappa_{c,i}\}$ can be achieved [36,37,74]. In this cavity-magnonics system, the waveguide acts as a common reservoir, and both the cavity and magnon mode individually interact with the traveling microwaves in the waveguide [57,75]. The second term of total Hamiltonian have the form

$$\begin{aligned} H_V = {} & \sum_k \omega_k \hat{V}_k^\dagger \hat{V}_k \\ & + \sum_k \lambda_m [(\hat{m} + \hat{m}^\dagger) \hat{V}_k^\dagger e^{-ikx_m} + \text{h.c.}] \\ & + \sum_k \lambda_c(t) [(\hat{c} + \hat{c}^\dagger) \hat{V}_k^\dagger e^{-ikx_c} + \text{h.c.}], \end{aligned} \tag{3}$$

where V_k is boson annihilation operators for the common waveguide with frequency ω_k satisfying $[V_k, V_k^\dagger] = \delta(k - k')$. The second and third lines of the Hamiltonian describe the dipole interactions between traveling photons in waveguide and each mode with individual coupling strengths $\lambda_c(t)$ and λ_m, which arise from the mode overlap between the cavity and magnon mode and the traveling photons, $x_{c(m)}$ represent the position of the cavity (magnon) mode coupling to the waveguide. Here, the time-dependent coupling between a microwave cavity and a waveguide transmission line modulated by Josephson junctions has been studied both theoretically and experimentally [71,76,77]. As mentioned in Ref. [71], the dc SQUID provide a flux-dependent coupling. By using a simple driver, the total flux enclosed by the loop have the form of a time containing. Thus, a time-dependent coupling can be obtained as $\lambda_c(t) = \lambda_c [\mu \cos(2\omega_c t) + \nu]$, ($\mu = 0, 2$ and $\nu = 0, 1$). The cases $\mu = 2, \nu = 0$, and $\mu = 2, \nu = 1$ can all be realized by the above method, more detailed calculations are given in Ref. [71]. The case $\mu = 0, \nu = 1$ means that we do not need time-modulated coupling and the cavity mode can be coupled directly into the waveguide.

B. The effective dissipative coupling

We assume that the intensity of the waveguide power spectrum is concentrated at the cavity frequency ω_c, and the time scales on which system operators evolve are much longer than the correlation time of the bath [77,78]. We assume $\omega_k - \omega_c \gg \lambda_{c(m)}$, using the rotation-wave approximation, we can write the motion equation for the traveling photon V_k in a rotating framework with Hamiltonian $H_0 = \omega_c(\hat{c}^\dagger \hat{c} + \hat{m}^\dagger \hat{m}) + \omega_k \hat{V}_k^\dagger \hat{V}_k$ as

$$\dot{\hat{V}}_k = -i\lambda_c \left(\nu \hat{c} + \frac{\mu}{2} \hat{c}^\dagger\right) e^{-i(\Omega_k t + kx_c)} - i\lambda_m \hat{m} e^{-i(\Omega_k t + kx_m)}, \tag{4}$$

where $\Omega_k = \omega_c - \omega_k$, and the equation above can be formally integrated as

$$\begin{aligned} \hat{V}_k(t) = {} & \hat{V}_k(0) \\ & - i \int_0^t e^{-i\Omega_k s} \left[\lambda_c (\nu \hat{c} + \frac{\mu}{2} \hat{c}^\dagger) e^{-ikx_c} + \lambda_m \hat{m} e^{-ikx_m}\right] ds, \end{aligned} \tag{5}$$

here, $\hat{V}_k(0)$ is $\hat{V}_k$ at the initial time. The motion equations of the cavity mode and the magnon mode can also be solved as

$$\begin{aligned}\dot{\hat{c}} &= -i[\hat{c}, H_{\rm sys}] - \kappa_{c,i}\hat{c} + \sqrt{2\kappa_{c,i}}\hat{c}_{\rm in} \\ &\quad - \int i\lambda_c \left[\nu \hat{V}_k e^{i(\Omega_k t + kx_c)} + \frac{\mu}{2}\hat{V}_k^\dagger e^{-i(\Omega_k t + kx_c)}\right] dk, \\ \dot{\hat{m}} &= -i[\hat{m}, H_{\rm sys}] - \kappa_{m,i}\hat{m} + \sqrt{2\kappa_{m,i}}\hat{m}_{\rm in} \\ &\quad - \int i\lambda_m \hat{V}_k e^{i(\Omega_k t + kx_m)} dk, \end{aligned} \tag{6}$$

where $o_{\rm in}$ $(o = c, m)$ represents input noise operators for cavity and magnon mode, respectively. The noise correlators associated with the input fluctuations are $\langle o_{\rm in}^\dagger(t) o_{\rm in}(t')\rangle = n_o\delta(t-t')$ and $\langle o_{\rm in}(t) o_{\rm in}^\dagger(t')\rangle = (n_o + 1)\delta(t-t')$, where $n_o = 1/[\exp(\hbar\omega_o/k_B T) - 1]$ denotes the mean thermal excitation numbers in the environmental temperature T, with k_B being Boltzmann constant. We substitute Eqs. (5) to (6) and assume that the waveguide has a linear dispersion relation in the vicinity of the ω_c, i.e., $\omega_k - \omega_c = \upsilon = v_g \delta k$, v_g is the group velocity of photons at ω_c. Under the Wigner-Weisskopf approximation, define k_{ω_c} as the wave vector in the vicinity of ω_c, we can obtain the dynamical equations of the system, which include a phase delay of the traveling photon from one mode to the other, for details see Appendix A. Here, we make the phase ϕ equal to an integer multiple of 2π, the Langevin equations of system can be written as

$$\begin{aligned}\dot{\hat{c}} &= -\kappa_{c,i}\hat{c} - \kappa_{c,e}\left(\nu^2 - \frac{\mu^2}{4}\right)\hat{c} - iJ\hat{m} - \Gamma\left(\nu\hat{m} - \frac{\mu}{2}\hat{m}^\dagger\right) \\ &\quad + \sqrt{2\kappa_{c,i}}\hat{c}_{\rm in} + \sqrt{2\kappa_{c,e}}\left(\nu\hat{V}_{\rm in}^{c} - \frac{\mu}{2}\hat{V}_{\rm in}^{c\dagger}\right), \\ \dot{\hat{m}} &= -(i\Delta_m + \kappa_m)\hat{m} - iJ\hat{c} - \Gamma\left(\nu\hat{c} + \frac{\mu}{2}\hat{c}^\dagger\right) \\ &\quad + \sqrt{2\kappa_{m,i}}\hat{m}_{\rm in} + \sqrt{2\kappa_{m,e}}\hat{V}_{\rm in}^{m}, \end{aligned} \tag{7}$$

where $\Delta_m = \omega_m - \omega_c$ is the detuning between the two modes, the effective dissipations are defined as $\kappa_{c(m),e} = 2\pi\lambda_{c(m)}^2(N/v_g)$, and $\kappa_c = \kappa_{c,i} + \kappa_{c,e}(\nu^2 - (\mu^2/4))$, $\kappa_m = \kappa_{m,i} + \kappa_{m,e}$. The effective dissipative coupling becomes $\Gamma = \sqrt{\kappa_{c,e}\kappa_{m,e}}$. It is worth noting that the system has an additional noise term $\hat{V}_{in}^{c(m)}(t) = (1/\sqrt{2\pi})\int ie^{i[\Omega_k t + kx_{c(m)}]} V_k(0)dk$ in Eq. (7), which is resulted from the common waveguide. This was not discussed in the previous schemes for preparing steady entanglement based on dissipative coupled cavity-magnonics system [18,19]. The correlations for the noise operators are given by $\langle \hat{V}_{\rm in}(t)\hat{V}_{\rm in}^\dagger(t')\rangle = \delta(t-t')$, we will show that the noise result in unignorable effect. In order to clearly show the coherent coupling and dissipative coupling of the system, the master equation corresponding to Eq. (7) would be written as

$$\begin{aligned}\dot{\rho} =& -i[H_{om}, \rho] + \frac{\kappa_{c,i}}{2}\mathcal{D}_{c^\dagger,c}\rho + \frac{\kappa_{m,i}}{2}\mathcal{D}_{m^\dagger,m}\rho \\ &+ \frac{\kappa_{c,e}}{2}\nu^2\mathcal{D}_{c^\dagger,c}\rho + \frac{\kappa_{m,e}}{2}\mathcal{D}_{m^\dagger,m}\rho \\ &+ \frac{\kappa_{c,e}}{2}\left\{\frac{\mu^2}{4}\mathcal{D}_{c,c^\dagger}\rho + \frac{\mu\nu}{2}(\mathcal{D}_{c,c}\rho + \mathcal{D}_{c^\dagger,c^\dagger}\rho)\right\} \\ &+ \Gamma\left(\nu\mathcal{D}_{c^\dagger,m}\rho + \frac{\mu}{2}\mathcal{D}_{c,m}\rho + \text{h.c.}\right), \end{aligned} \tag{8}$$

where $H_{om} = \Delta_m\hat{m}^\dagger\hat{m} + J(\hat{c}^\dagger\hat{m} + \hat{m}^\dagger\hat{c})$ and $\mathcal{D}_{o_a,o_b}\rho = 2o_b\rho o_a - o_a o_b\rho - \rho o_a o_b$, detail derivation also can be seen in Appendix A. The second line of Eq. (8) represents the effective dissipation of cavity mode and magnon mode caused by the common waveguide, respectively. The first term of the third line represents the effective gain of the cavity mode, and the second and third terms represent the parametric squeezing of the cavity, all of which are induced by the common waveguide. In the fourth line, the term with a multiple of ν represents the magnon-photon dissipative coupling with a BS interaction, which alone will not produce steady entanglement. While the term with a multiple of μ is a NP interaction, which is the key role for achieving entanglement. It is worthwhile to point out that for $(\mu^2/4) > \nu^2$, see the second term on the right side in the first line of Eq. (7), the common waveguide induce gainlike dissipation of cavity mode. The total damping of the cavity mode has two cases, when $\kappa_{c,i} > \kappa_{c,e}((\mu^2/4) - \nu^2)$, the cavity mode is dissipative. When $\kappa_{c,i} < \kappa_{c,e}((\mu^2/4) - \nu^2)$, the net dissipation of the cavity mode is negative, this gainlike dissipation has not been studied in a dissipative cavity-magnonics system [18,19,50,57]. In recent work [67], a similar gainlike dissipation has been investigated, in which the steady state of the coupled magnon-photon system is reached only when the gain of a van der Pol cavity balanced by a nonlinear damping. However, in our system, although the cavity mode has an effective gain, the linear cavity and the YIG sphere as a whole system can reach a steady state due to the intrinsic dissipation of the magnon mode $\kappa_{m,i}$. And for $(\mu^2/4) = \nu^2$, the effective dissipation can be effectively eliminated.

We can classify effective dissipative coupling as three cases, (i) $\mu = 0$, $\nu = 1$, which means that the cavity interacts with the magnon mode as a BS dissipation coupling. (ii) $\mu = 2$, $\nu = 0$, corresponding to NP dissipative coupling. Meanwhile, the effective dissipation of the cavity mode is of gain effect. (iii) $\mu = 2$, $\nu = 1$, dissipation coupling with BS plus NP. In the next section, we will mainly discuss the later two cases for their abilities to generate and enhance entanglement. In addition, in the setup shown in Fig. 1(a), the cross-line circuit supports both the standing wave forming the cavity mode and the traveling wave inducing the dissipative coupling between the cavity and magnon modes [57,75,79]. So, one can change the coherent and dissipative coupling by moving the position of the YIG sphere as well as modulate the magnetic flux threading the SQUID. Similarly, for the setup shown in Fig. 1(c), the dissipative coupling depends on two quantities that can be adjusted by the distance between a YIG sphere and the waveguide and the magnetic flux in the SQUID, respectively.

III. ENTANGLEMENT GENERATION

Introducing the quadratures of quantum fluctuations with $\delta\hat{X}_c = (\hat{c}+\hat{c}^\dagger)/\sqrt{2}$, $\delta\hat{Y}_c = i(\hat{c}^\dagger - \hat{c})/\sqrt{2}$, $\delta\hat{X}_m = (\hat{m}+\hat{m}^\dagger)/\sqrt{2}$, and $\delta\hat{Y}_m = i(\hat{m}^\dagger - \hat{m})/\sqrt{2}$, from Eq. (7), we derive the linearized dynamic equations in the compact matrix form

$$\dot{u}(t) = Au(t) + n(t), \tag{9}$$

where the vectors $u = [\delta\hat{X}_c, \delta\hat{Y}_c, \delta\hat{X}_m, \delta\hat{Y}_m]^{\mathrm{T}}$ and the noise vector $n = [\sqrt{2\kappa_{c,i}}\hat{X}^c_{\mathrm{in}} + (\nu - (\mu/2))\sqrt{2\kappa_{c,e}}\hat{X}^V_{\mathrm{in}}, \sqrt{2\kappa_{c,i}}\hat{Y}^c_{\mathrm{in}} + (\nu+(\mu/2))\sqrt{2\kappa_{c,e}}\hat{Y}^V_{\mathrm{in}}, \sqrt{2\kappa_{m,i}}\hat{X}^m_{\mathrm{in}} + \sqrt{2\kappa_{m,e}}\hat{X}^V_{\mathrm{in}}, \sqrt{2\kappa_{m,i}}\hat{Y}^m_{\mathrm{in}} + \sqrt{2\kappa_{m,e}}\hat{Y}^V_{\mathrm{in}}]^{\mathrm{T}}$ and the drift matrix A reads

$$A = \begin{bmatrix} -\kappa_c & 0 & \Gamma\left(\frac{\mu}{2}-\nu\right) & J \\ 0 & -\kappa_c & -J & -\Gamma\left(\frac{\mu}{2}+\nu\right) \\ -\Gamma\left(\frac{\mu}{2}+\nu\right) & J & -\kappa_m & \Delta_m \\ -J & \Gamma\left(\frac{\mu}{2}-\nu\right) & -\Delta_m & -\kappa_m \end{bmatrix}. \tag{10}$$

Because of the linearized system dynamics and the Gaussian nature of the quantum noises in our system, the steady state of quantum fluctuations is a continuous variable two-mode Gaussian state. It can be completely characterized via a 4×4 covariance matrix V, which is defined as $V_{ij} = \langle\sigma_i(t)\sigma_j(t') + \sigma_j(t')\sigma_i(t)\rangle/2, (i,j = 1,2,3,4)$. According to the Routh-Hurwitz criterion [80], when all eigenvalues of the matrix A have negative real parts, then the system is stable and reaches its steady state. Since there are an effective gain in the cavity mode and NP dissipative coupling, it is necessary to investigate stability of the system. Since the dynamical equations of the current system are linear, the classical and quantum steady-state equations are consistent, and the stability conditions of the system can be satisfied in both classical and quantum dynamics, which is discussed in detail in Appendix B. Within the stable region of parameters, we can straightforwardly obtain the steady-state covariance matrix V via the Lyapunov equation [81,82]

$$AV + VA^T = -D, \tag{11}$$

with

$$D = \begin{bmatrix} \kappa_{c,i} + U^2\kappa_{c,e} & 0 & \Gamma U & 0 \\ 0 & \kappa_{c,i} + V^2\kappa_{c,e} & 0 & \Gamma\left(\nu+\frac{\mu}{2}\right) \\ \Gamma U & 0 & \kappa_m & 0 \\ 0 & \Gamma V & 0 & \kappa_m \end{bmatrix}, \tag{12}$$

which is defined through $D_{ij}\delta(t-t') = \langle n_i(t)n_j(t') + n_j(t')n_i(t)\rangle/2$, where $U = \nu - (\mu/2), V = \nu + (\mu/2)$. Contrasting with the traditional diagonal form, the matrix D has nonzero nondiagonal elements $D(1,3)$, $D(2,4)$, $D(3,1)$, and $D(4,2)$ resulting from the additional noise V_{in} induced by the common waveguide, which affects the entanglement behavior.

For the continuous-variable two-mode Gaussian state, it is convenient to use the logarithmic negativity E_N [83,84] to quantify the entanglement. For the discussion of the EPR steering, a computable criterion of quantum steering based on quantum coherent information has been introduced [85] for arbitrary bipartite Gaussian states. All measures mentioned above can be computed from the reduced covariance matrix V_s for the magnon and photon modes:

$$V_s = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}, \tag{13}$$

where A, B, and C are 2×2 sub-block matrices. In this case, the logarithmic negativity E_N to quantify the entanglement and the Gaussian quantum steering $S_{cm}(S_{mc})$ between the two modes are expressed as

$$\begin{aligned} E_N &= \max[0, -\ln 2\nu], \\ S_{cm} &= \max[0, \mathcal{R}(2A) - \mathcal{R}(2V_s)], \\ S_{mc} &= \max[0, \mathcal{R}(2B) - \mathcal{R}(2V_s)], \end{aligned} \tag{14}$$

where $\nu = 2^{-1/2}\{\Sigma(V) - [\Sigma(V)^2 - 4\det V_s]^{1/2}\}^{1/2}$, with $\Sigma(V) = \det A + \det B - 2\det C$, and $\mathcal{R}(\omega) = \frac{1}{2}\ln[\det(\omega)]$ is the Renyi-2 entropy.

IV. ENTANGLEMENT ENHANCEMENT

In this section, we numerically show the enhancement of entanglement and the manipulation of one-way steering under the two cases.

A. The system with NP dissipative coupling

For the case $\mu = 0, \nu = 1$, i.e., the dissipative coupling of microwave photons and magnons can only be a BS interaction, it cannot produce any entanglement, as shown by the black-dashed line in Fig. 2(a). However, for $\mu = 2, \nu = 0$, by considering the time-dependent coupling modulation, we can make the cavity mode NP coupling to the waveguide, which results in NP dissipative coupling with the

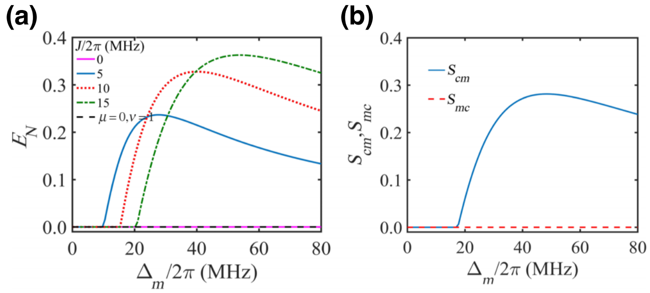

FIG. 2. (a) The magnon-photon entanglement E_N versus the detuning Δ_m for different coherent coupling J. (b) The steering S_{cm} and S_{mc} versus the detuning Δ_m with $J/2\pi = 15$ MHz. The other parameters are $\omega_c/2\pi = 10$ GHz, $\kappa_{c(m),e}/2\pi = \Gamma/2\pi = 5$ MHz, $\kappa_{c,i}/2\pi = 10$ MHz, $\kappa_{m,i}/2\pi = 1$ MHz, $T = 0$.

magnon mode. We plot the magnon-photon entanglement (with different coherent coupling strength J) and steering as functions of detuning Δ_m in Fig. 2(a). If $J = 0$, although we have NP dissipative coupling between optical mode and magnon mode, we still cannot obtain entanglement, see the magenta-solid line. Under this case, the cavity mode has an effective gain, i.e., $(\nu^2 - (\mu^2/4)) < 0$, which is not conducive to generate magnon-photon entanglement. By considering coherent coupling with $J \neq 0$, the other coupling channel is constructed between the cavity mode and the magnon mode. Thus, the coherent coupling can balance the effective gain of the cavity mode resulted from dissipative coupling, allowing the entanglement generation. For a given Γ, the increasing of J leads to an increase in E_N within the range of parameters shown in Fig. 2(a).

As shown in Fig. 2(b), we obtain the one-way steering from cavity mode to magnon mode $c \to m$, which can be described as $S_{cm} \neq 0$, and $S_{mc} = 0$, this means that c can steer m while the steerability disappears in the opposite direction. However, the phenomenon disappears when the two modes interact with the waveguide both in BS form ($\mu = 0, \nu = 1$). In this system, the directional EPR steering between two modes originates from the asymmetric coupling, i.e., cavity mode and magnon mode couple to the waveguide with NP and BS coupling, respectively. Meanwhile, the NP interaction between the cavity and waveguide leads to gain of the cavity mode, this also leads to asymmetric dissipation of the two modes, which means that it is feasible to adjust (steer) the state of the YIG sphere by the cavity mode.

In Fig. 3, we show the effect of the intrinsic dissipation of the two modes on the entanglement. For a given Γ, the entanglement increases as $\kappa_{c,i}$ increasing. However, as shown in the inset, when $\kappa_{c,i}/2\pi \approx 9.899$ MHz, the entanglement reaches its maximum value and does not increase subsequently. Furthermore, we find that we can achieve steady-state entanglement regardless of whether the cavity

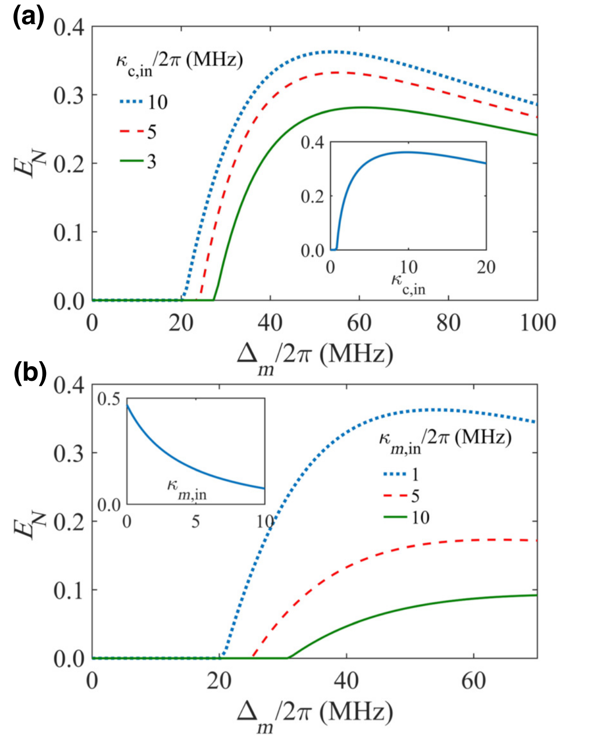

FIG. 3. The magnon-photon entanglement E_N versus the detuning Δ_m for different intrinsic dissipation of the cavity $\kappa_{c,i}$ (a) with $\kappa_{m,i}/2\pi = 1$ MHz and the YIG $\kappa_{m,i}$ (b) with $\kappa_{c,i}/2\pi = 10$ MHz. The inset shows the entanglement E_N versus $\kappa_{c,i}$ (a) and $\kappa_{m,i}$ (b). The other parameters are taken as $\omega_c/2\pi = 10$ GHz, $\kappa_{c(m),e}/2\pi = \Gamma/2\pi = 5$ MHz, $J/2\pi = 15$ MHz, $T = 0$.

mode exhibits a net gain or dissipation. Our scheme is different from Ref. [50] where the magnon-photon entanglement in a dissipative coupled cavity-magnonics system is generated via $\mathcal{PT}$ symmetry, which require a small intrinsic dissipation rate. However, in our system, when the coherent coupling exists, the effective gain of cavity mode can cancel the intrinsic dissipation, which relax the requirement of low intrinsic dissipation of the cavity mode, making it easily feasible in experiment. Furthermore in Figs. 2 and 3, there exists a threshold value of Δ_m beyond which entanglement arises, which is the joint effect of the noise correlation between the magnon and cavity modes induced by the common waveguide environment (nondiagonal elements in matrix D) and the effective gain of the cavity mode.

We now investigate the joint effect of coherent coupling and dissipative coupling on the entanglement. The entanglement E_N versus the detuning Δ_m and coherent coupling J are plotted in Fig. 4(a). For a given dissipative coupling $\Gamma/2\pi = 5$ MHz, entanglement is enhanced with the increase of coherent coupling and of the detuning within an appropriate range, when $J/2\pi > 25$ MHz, the maximum value of E_N will not change significantly. In Fig. 4(b), we show that when $\Gamma = 0$, no entanglement arises regardless

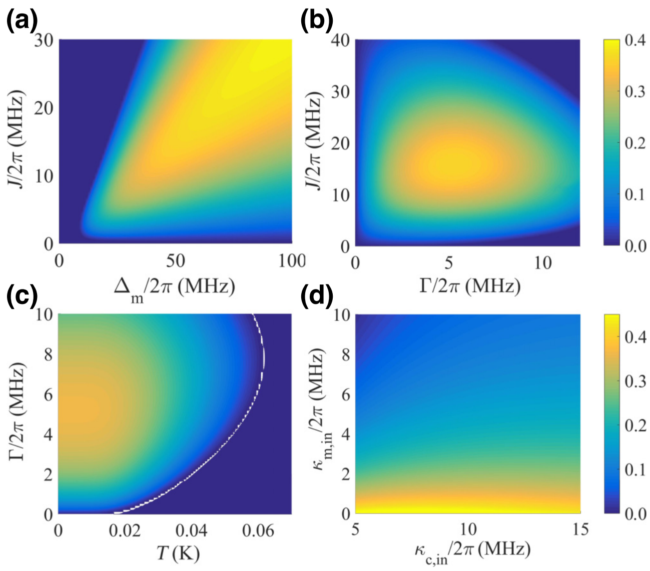

FIG. 4. The magnon-photon entanglement E_N versus (a) the detuning Δ_m and coherent coupling strength J, (b) the dissipative coupling strength Γ and coherent coupling strength J, (c) the environment temperature T and dissipative coupling strength Γ where the white contour lines depict the threshold temperature for entanglement existence or not, (d) the intrinsic dissipation $\kappa_{c(m),i}$. Other parameters are the same as Fig. 2, and $\Delta_m/2\pi = 50$ MHz, $J/2\pi = 15$ MHz.

of the change in coherent coupling J, as we stated above, because the system has only BS coherent coupling. We find that entanglement can be generated and enhanced under the cooperative effect of a NP dissipative coupling and coherent coupling. When the dissipative coupling $\Gamma/2\pi \approx 5$ MHz and the coherent coupling $J/2\pi \approx 15$ MHz, the entanglement achieves its maximum. The large value of NP dissipative coupling Γ also means large external damping $\kappa_{c,e}(\kappa_{m,e})$. The completion between the NP dissipative coupling and the decay rates leads to an optimal value Γ to achieve maximum entanglement. Although a small value of coherent coupling J does not benefit the generation of ideal entanglement, overlarge value of J can bear a large value of detuning Δ_m so that BS interaction dominates in the system and depress the function of NP dissipative coupling. Thus, an optimal value J also favors the entanglement generation.

Next, we discuss the robustness of the present system for some parameters. We plotted the entanglement E_N versus dissipative coupling strength Γ and the environment temperature T in Fig. 4(c). Note that the threshold temperature below which the entanglement E_N appears $T \approx 0.06$ K for $\Gamma/2\pi \approx 8$ MHz, which indicates that the current entanglement scheme still needs a lower temperature environment. Meanwhile, in Fig. 4(d), we show the entanglement affected by $\kappa_{c,i}$ and $\kappa_{m,i}$. We can observe that even when $J < \kappa_{c,i}$, the entanglement still survives with the help of effective dissipative interaction. Although the increasing decay rate of the magnon mode $\kappa_{m,i}$ leads to the decrease of the entanglement effect, however, for a relatively large $\kappa_{m,i}$, a weak photon-magnon entanglement can still be obtained. This is useful for experimental feasibility because it relaxes the requirements for the quality of the cavity and YIG.

B. The system with BS plus NP dissipative coupling

We now study the effect of dissipative coupling with BS plus NP coupling on entanglement, i.e., $\mu = 2, \nu = 1$. Under this case, the effective gain disappears, that is to say, the second term of the right side of the first equation of Eq. (7) becomes zero $[\nu^2 - (\mu^2/4) = 0]$. The entanglement E_N and the steering versus the detuning Δ_m are plotted in Fig. 5(a). The optimal entanglement and one-way steering can occur around $\Delta_m = 0$. The threshold for Δ_m disappears, which is different from Figs. 2 and 3. As mentioned above, the joint effect of the noise correlation between the magnon and cavity modes and the effective gain of the cavity mode leads to the threshold value of Δ_m. Absence of the effective gain of the cavity mode or the nondiagonal elements in matrix D leads to the disappearance of the threshold value for Δ_m, therefore entanglement occurs around $\Delta_m = 0$. The generation of one-way steering in this system is similar to the mechanism we have described above, which essentially arises from the asymmetric coupling of the two modes to the waveguide. The coupling between the cavity mode and waveguide is XX type. In Fig. 5(b), the entanglement E_N versus Γ and Δ_m has been plotted. Even without coherent coupling, entanglement still can be enhanced by increasing the dissipative coupling strength Γ and that the optimal entanglement is always near the resonance point $\Delta_m = 0$. The entanglement in this system has a significant improvement compared with the case $\mu = 2, \nu = 0$.

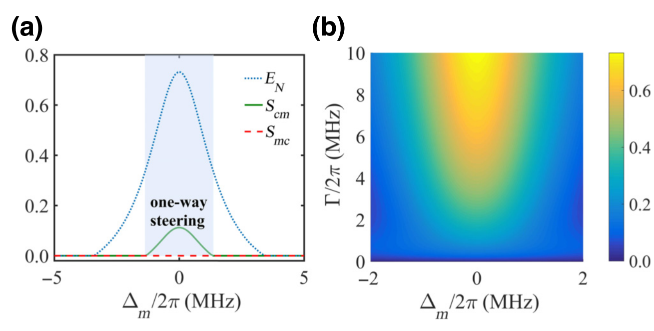

FIG. 5. (a) The magnon-photon entanglement and steering versus the detuning of magnon mode Δ_m. (b) The magnon-photon entanglement E_N as a function of detuning Δ_m and dissipation coupling strength Γ. Other parameters are taken as $\omega_c/2\pi = 10$ GHz, $\kappa_{c(m),e}/2\pi = \Gamma/2\pi = 10$ MHz, $J = 0$, $\kappa_{c(m),i}/2\pi = 1$ MHz, $T = 0$.

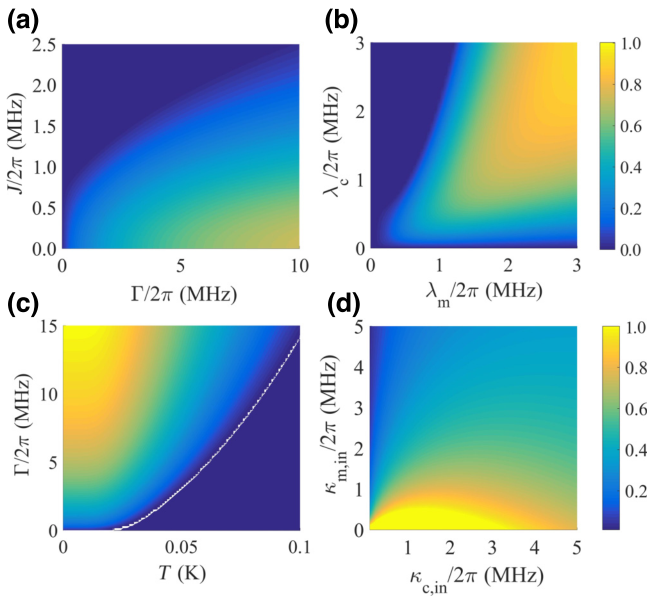

FIG. 6. The magnon-photon entanglement E_N versus (a) dissipative coupling strength Γ and coherent coupling strength J, (b) the coupling strength between the two modes and waveguide $\lambda_{c,(m)}$, (c) the environment temperature T and dissipative coupling strength Γ where the white contour lines depict the threshold temperature for entanglement existence or not, (d) the intrinsic dissipation $\kappa_{c(m),i}$. Other parameters are the same as Fig. 5, and $\Delta_m = 0$.

Under the current case with both NP and BS dissipative coupling, on the one hand, the effective gain of the cavity is wiped off, so that we do not need additional coherent coupling to balance the gain of the cavity mode. This can be seen in Fig. 6(a), for $J = 0$, the entanglement E_N still exists but decreases with the increasing of the coherent coupling. In the experiment, Ref. [58], the coherent coupling between the cavity and magnon modes can be ignored by adjusting the phase delay of the traveling wave from the cavity mode to the magnon mode as an integer multiple of 2π, i.e., the separation between cavity and magnon modes. In all the above simulations, we assumed $\lambda_c = \lambda_m$. It is necessary to verify whether the maximum quantum correlation requires the condition or not. In Fig. 6(b), we plot the entanglement as a function of the two coupling strengths between the cavity (magnon) and waveguide. We find that the optimal entanglement region satisfies exactly $\lambda_m \approx \lambda_c$, which also proves that our previous approximation $\kappa_{c,e} = \kappa_{m,e}$ is reasonable and meaningful. Comparing Figs. 6(c) and 4(c), the threshold temperature has been improved, which shown that the entanglement of the system is robust to environment-induced decoherence compared to the system with only NP dissipative coupling. In addition, the threshold temperature compared to Ref. [86] has also improved. We can still maintain the high degree of entanglement by increasing the dissipative coupling even if the environment temperature increases. Similarly, as can be seen in Fig. 4(d), the current case is robust to the intrinsic dissipation of the two modes. Although the entanglement decreases as the dissipation rate of the two modes increases, for a relatively wide range, the entanglement can still be maintained.

V. EXPERIMENTAL FEASIBILITY

We now discuss the experimental feasibility of the current scheme. The dissipative couplings in the cavity-magnonics system has been implemented in a cross-line microwave circuit [57,70]. Our scheme differs from the above experiments in that we need a time-dependent coupling between the cavity and the waveguide. This can be realized with a superconducting ring intersecting Josephson junction partially sharing the branches with the cavity and the waveguide transmission line [71]. It will provide a flux-dependent coupling. Using an appropriate signal generator, a suitable flux dependence is designed with a resolution of about 0.1 ns, in which only the resonance term with a frequency around $2\omega_c$ affects the coupling. The intrinsic dissipation of the cavity $\kappa_{c,i}/2\pi \approx 1$ MHz can be realized in [87] and the intrinsic dissipation of magnon mode $\kappa_{m,i}/2\pi \approx 1$ MHz can be fitted from the transmission spectra in [57]. Meanwhile, the photon-magnon dissipative coupling and coherent coupling can be modulated by moving the position of the YIG sphere [57], and the dissipative coupling strengths of $\Gamma/2\pi = 19$ MHz or even stronger have been implemented in recent cavity-magnonics experimental systems [57,88]. The magnon-cavity coupling rate J can be (much) larger than the intrinsic dissipation rates, such that the strong-coupling regime $J > \{\kappa_{m,i}, \kappa_{c,i}\}$ can be achieved [36,37,74]. For example, in Ref. [37], the magnon-photon coupling strength can reach $J = 2\pi \times 2.5$ GHz. By placing the YIG sphere 1 mm distance away from the waveguide, the external dissipation rate of the magnon mode $\kappa_{m,e}/2\pi = 0.33$ MHz has been achieved [89]. By reducing the distance between them, the external dissipation can be further increased.

VI. CONCLUSION

In conclusion, we have presented a method to enhance the photon-magnon entanglement in a dissipative coupling cavity-magnonics system, the cavity and magnon modes are coupled simultaneously into the common waveguide, where the cavity mode couples to the waveguide with time-dependent coupling. By choosing a reasonable coupling modulation, we can derive effective dissipative coupling between cavity and magnon modes with two forms: NP and NP plus BS type. This is the key to achieve steady entanglement and unidirectional steering, the only BS interactions between magnons and microwave photons cannot produce steady entanglement in a vacuum environment. For the case of only NP dissipative coupling, we find that there is an effective gain in the cavity mode induced by the common waveguide, and the effective gain in cavity mode is harmful to entanglement. By considering the cooperative effect of coherent coupling and NP dissipative coupling, the effective gain of the cavity mode can cancel the intrinsic dissipation, the entanglement can be successfully generated and increased with the increasing of the intrinsic dissipation. In addition, when the effective dissipative coupling with NP plus BS, the effective dissipation of the cavity mode can be eliminated, so that even without coherent coupling, entanglement and one-way steering remain and increase with the increasing of the dissipative coupling strength. Meanwhile, we find that one-way steering in both cases can be realized with the asymmetric coupling of the two modes to the waveguide. Further, we demonstrate that the entanglement is robust against the dissipation of the cavity mode. The enhanced entanglement strongly supports the cavity-magnonics system as a promising platform for applications of magnon-based quantum information processing.

ACKNOWLEDGMENTS

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APPENDIX A: THE DYNAMICAL EQUATIONS

By substituting Eqs. (5) to (6), we can obtain the dynamical equations of the system as

$$\begin{aligned}\dot{\hat{c}} &= -\kappa_{c,i}\hat{c} - \kappa_{c,e}\left(\nu^2 - \frac{\mu^2}{4}\right)\hat{c} - iJ\hat{m} \\ &\quad - e^{i\phi}\Gamma\left(\nu m - \frac{\mu}{2}m^\dagger\right) \\ &\quad + \sqrt{2\kappa_{c,i}}\hat{c}_{\text{in}} + \sqrt{2\kappa_{c,e}}\left(\nu\hat{V}_{\text{in}}^{c} - \frac{\mu}{2}\hat{V}_{\text{in}}^{c\dagger}\right), \\ \dot{\hat{m}} &= -(i\Delta_m + \kappa_{m,i} + \kappa_{m,e})\hat{m} - iJ\hat{c} - e^{i\phi}\Gamma\left(\nu\hat{c} + \frac{\mu}{2}\hat{c}^\dagger\right) \\ &\quad + \sqrt{2\kappa_{m,i}}\hat{m}_{\text{in}} + \sqrt{2\kappa_{m,e}}\hat{V}_{\text{in}}^{m}, \end{aligned} \tag{A1}$$

where $\phi = k_{\omega_c}|x_m - x_c|$ indicates the phase delay of the traveling photon from one mode to another.

The method of deriving the corresponding master equation from the Langevin equation has been widely investigated and applied [90,91]. The master equation corresponding to Eq. (A1) would be written as

$$\begin{aligned}\dot{\rho} &= -i[H_{om} + H_v, \rho] \\ &\quad + \frac{\kappa_{c,i}}{2}\mathcal{D}_{c^\dagger,c}\rho + \frac{\kappa_{m,i}}{2}\mathcal{D}_{m^\dagger,m}\rho \\ &\quad + \frac{\kappa_{c,e}}{2}\nu^2\mathcal{D}_{c^\dagger,c}\rho + \frac{\kappa_{m,e}}{2}\mathcal{D}_{m^\dagger,m}\rho \\ &\quad + \frac{\kappa_{c,e}}{2}\left\{\frac{\mu^2}{4}\mathcal{D}_{c,c^\dagger}\rho + \frac{\mu\nu}{2}(\mathcal{D}_{c,c}\rho + \mathcal{D}_{c^\dagger,c^\dagger}\rho)\right\} \\ &\quad + \cos\phi\Gamma\left(\nu\mathcal{D}_{c^\dagger,m}\rho + \frac{\mu}{2}\mathcal{D}_{c,m}\rho + \text{h.c.}\right), \end{aligned} \tag{A2}$$

where $H_{om} = \Delta_m\hat{m}^\dagger\hat{m} + J(\hat{c}^\dagger\hat{m} + \hat{m}^\dagger\hat{c})$, $H_v = \sin\phi\Gamma(\nu\hat{c}^\dagger\hat{m} + (\mu/2)\hat{c}\hat{m} + \text{h.c.})$ denotes the cavity-magnon coherent coupling induced by the common waveguide. We also use the method of $\dot{\rho} = -\text{Tr}_R\int_{t_i}^{t}[V(t), [V(t'), \rho_s(t') \otimes \rho_R(t_i)]]dt'$ to obtain the same form of the master equation, where V is the Hamiltonian in the interaction picture. In the main text, we make the phase ϕ equal to an integer multiple of 2π. This induced coherent coupling Hamiltonian can then be neglected and we obtain the master equation in Eq. (8).

APPENDIX B: STABLE CONDITIONS

Since the cavity mode has an effective gain in the current system, it is essential to discuss the stability region of the system. The classical and quantum steady-state equations are consistent since the system dynamics equations are linear, and the stability conditions of the system can be satisfied in both classical and quantum dynamics. Due to us setting $\kappa_{c(m),e} = \Gamma$, as the dissipative coupling changes, both the effective gain and dissipation of the system change. It can be seen from Fig. 7(a) that when the coherent coupling $J = 0$, the system will enter the unstable region as the dissipative coupling increases. However, when coherent coupling is introduced and increased to establish a channel for energy exchange between the cavity mode and the magnon mode, and through the effective dissipation of the magnon mode, the gain of the cavity mode can be effectively balanced and the system brought into the stable region. We also explored the stability of

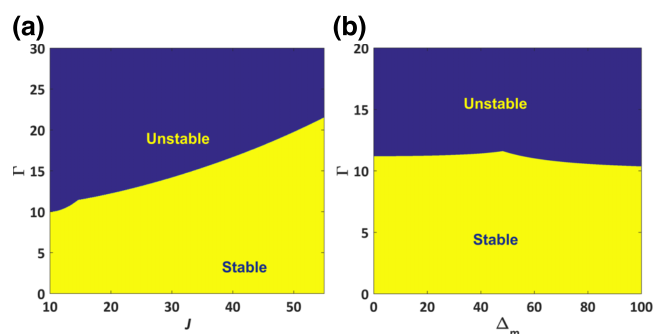

FIG. 7. The stability of the system with $\mu = 2, \nu = 0$, where the system stability is determined by the coherent coupling strength J and effective dissipative coupling Γ with $\Delta_m/2\pi = 50$ MHz in (a) and the detuning Δ_m and effective dissipative coupling Γ with $J/2\pi = 15$ MHz in (b). The yellow part is the stable area, and the blue part is the unstable area. The other parameters are $\omega_c/2\pi = 10$ GHz, $\kappa_{c,i}/2\pi = 10$ MHz, $\kappa_{m,i}/2\pi = 1$ MHz, $T = 0$.

the system determined by the detuning Δ_m and effective dissipative coupling Γ in Fig. 7(b). The parameters of the system studied in the main text satisfy the stability region of the system.

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