


Longitudinal coupling between a Si/Si_{1-x}Ge_x double quantum dot and an off-chip TiN resonator

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Superconducting cavities have emerged as a key tool for measuring the spin states of quantum dots. So far, however, few experiments have explored longitudinal couplings between quantum dots and cavities, and no solid-state qubit experiments have explicitly probed the “adiabatic” regime, where the Purcell decay is strongly suppressed. Here we report measurements of a double-quantum-dot charge qubit coupled to a high-impedance resonator via a “flip-chip” design geometry. By applying an adiabatic ac drive to the qubit through two different channels, and studying the effects of qubit energy detuning, interdot tunneling, and driving strength, we are able to unequivocally confirm the presence of a longitudinal coupling between the qubit and the cavity, while the qubit remains in its ground state. Since this coupling is proportional to the driving amplitude, and is therefore switchable, it has the potential to become a powerful tool in qubit experiments.

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I. INTRODUCTION

Quantum couplings between microwave resonators and quantum dot qubits are expected to form key components in scalable quantum computing architectures [1,2], with potential applications for both readout and couplings between distant qubits. Recent experiments have demonstrated spin-photon interactions [3–7] and photon-mediated spin-spin interactions [8]. Most of these have focused on “transverse” qubit-cavity couplings, which have some disadvantages, such as the required resonance between the qubit and the cavity [9], which can be difficult to engineer and which causes rapid decoherence due to the Purcell effect [10–12]. Other types of coupling are therefore also of interest, including dispersive and dynamic longitudinal couplings. The latter are expected to be strong and tunable, enabling fast gates and quantum nondemolition measurements [13–18]. Both static [19–22] and dynamic [23,24] longitudinal couplings have been demonstrated experimentally in the superconducting-qubit community. Longitudinal-coupling schemes have also

been proposed for quantum dot qubits, and although initial experiments appear promising [25], their full potential is not yet known.

In this paper, we report significant enhancement of transmission through an off-chip microwave cavity, arising from the longitudinal coupling to an ac-driven double quantum dot. The device is tuned to a charge-degeneracy point and driven at the cavity resonance frequency through nonstandard cavity ports. The crucial signature of the longitudinal-coupling mechanism is its change of sign in the resonator response, depending on which of the two quantum dots is driven. The sign change also provides direct proof of longitudinal coupling. We further theoretically describe the longitudinal coupling and its effect on cavity transmission for a range of detuning, tunnel-coupling, and ac driving parameters, and we obtain good agreement with the experimental results.

II. EXPERIMENTAL DEMONSTRATION OF LONGITUDINAL COUPLING

The sample used in this experiment is a 3D-integrated, high-impedance TiN coplanar waveguide resonator on a Si chip coupled to a Si/Si_{1-x}Ge_x quantum dot chip, as depicted schematically in Fig. 1, depicted pictorially in Fig. 2, and explained in further detail in Ref. [7]. The

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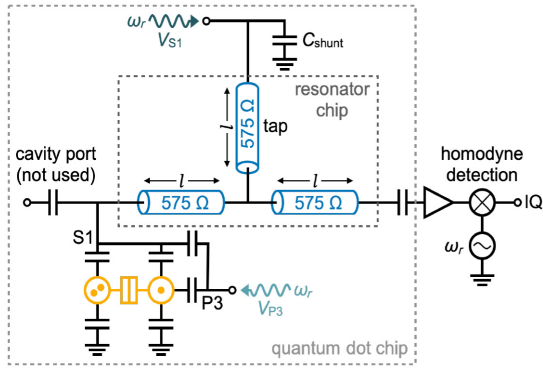


FIG. 1. Simplified circuit model for the qubit-cavity device. A Si/Si_{1-x}Ge_x double quantum dot (yellow) is capacitively coupled to a high-impedance microwave resonator cavity through gate S1. (The circuit element between the two quantum dots represents a tunnel coupling in parallel with a capacitor.) The cavity is made from three TiN coplanar waveguide segments (blue) of length $l = 2.02$ mm and characteristic impedance $Z_{0,r} \approx 575 \Omega$. One of the segments is a voltage-node tap with large shunt capacitance $C_{\text{shunt}} \approx 100$ pF used for voltage biasing S1. The resonator is vertically integrated on a chip separate from the quantum dots, as indicated (see also Fig. 2). During the experiments, a microwave tone at cavity frequency $\omega_r/2\pi = 1.304$ GHz is applied through nonstandard cavity ports, to either S1 (via the tap segment) or plunger gate P3. Transmission through the cavity is measured with use of a homodyne-detection scheme. For further details of the device, see Fig. 5 in Appendix A.

3D nature of the device relieves both wiring and fabrication constraints. Wiring is simplified by allowing more space on the qubit die for qubit gate lines and fan-out; the low-impedance gates reduce cavity losses [26]. Fabrication is simplified because the qubit and cavity chips are no longer constrained to follow the same lithographic processes. Figure 2(d) shows a scanning electron micrograph of a device nominally identical to the one used in the

experiment, in which the quantum dots are capacitively coupled to the resonator through a tap segment on the resonator chip, which is shunted by a large parallel-plate capacitor on the quantum dot chip. Galvanic contacts between the dies are achieved with use of underbump metal pads [Fig. 2(c)] formed at the voltage antinodes near the quantum dots, and at the end of the tap segment. SPICE simulations indicate that the fundamental resonant mode used in this work, with frequency $\omega_r/2\pi = 1.304$ GHz, has weight at both these antinodes and in the tap segment.

Figures 3(a) and 3(b) show interdot charging transitions of a three-electron double quantum dot, formed under plunger gates P2 and P3, tuned to its (2,1) or (1,2) charge occupation, as labeled. The data are obtained as a function of the dc voltages applied to gates P2 and P3, with simultaneous ac modulation of the voltages on gate S1 [Fig. 3(a)] or gate P3 [Fig. 3(b)] at the cavity resonant frequency. As explained in Appendix B, the recorded in-phase and quadrature (IQ) mixer signal is proportional to the microwave signal transmitted through the cavity, which is measured via a homodyne-detection scheme. A dc offset in the homodyne IQ output, which was not calibrated away during experiments, has been subtracted from all data presented in this work.

The resonator response observed in Fig. 3(a) is primarily achieved through a longitudinal curvature coupling ($g_{\parallel}^{\text{dy}} > 0$) induced by our driving the qubit at the cavity resonant frequency. A line cut through the charging transition, shown in Fig. 3(c), provides a clearer view of the suppressed transmission amplitude as a function of the double-quantum-dot detuning ε_0 , as computed with use of the measured lever arm $\alpha_{P2,\varepsilon} = 0.11$ eV/V (see Appendix D 1). A dispersive shift ($\delta\omega$) of the cavity resonant frequency, arising from the energy curvature of the double-quantum-dot energy levels, further contributes to suppression of the signal. However, through the analysis below, we find its effect to be much weaker than that of

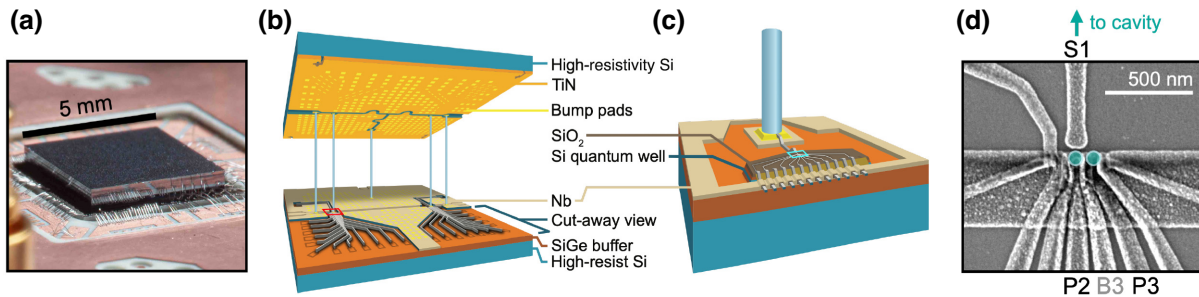


FIG. 2. Vertical integration of a quantum dot qubit and an off-chip microwave cavity. (a) Image of a connected and wirebonded qubit-cavity flip-chip device. (b) A 3D-integrated sample, where a high-impedance TiN resonator die is galvanically connected to a Si/Si_{1-x}Ge_x quantum dot die. (c) Blow-up of the bump-bond pad near a double-quantum-dot mesa. This region is enclosed by the red rectangle in (b). (d) Scanning electron micrograph of a double quantum dot nominally identical to the one used in these experiments. The device is enclosed in the blue rectangle in (c). The cavity is driven by high-frequency lines connected to gate P3 or S1. Plunger gate P2 and barrier gate B3 also are indicated.

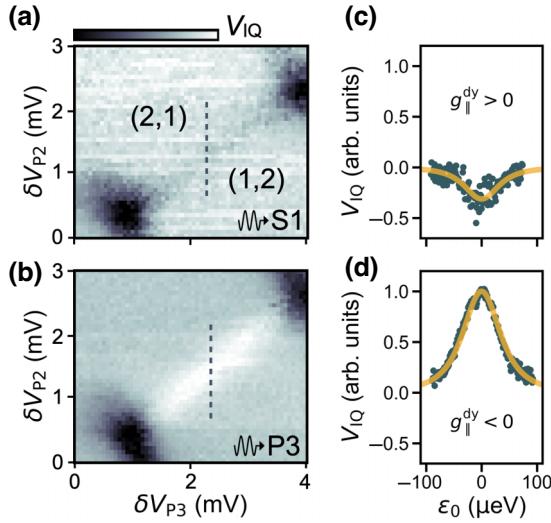


FIG. 3. Double-quantum-dot stability diagrams, as measured via an off-chip microwave cavity. (a),(b) IQ transmission measurements of a double quantum dot tuned near its (2,1)-(1,2) charge-occupation transition. The double-quantum-dot detuning parameter $\varepsilon(t) = \varepsilon_0 + \delta\varepsilon(t)$ is driven by modulation of the voltages on (a) gate S1 or (b) gate P3, with no other changes in device tuning. (c),(d) Line-cuts across the interdot charging transitions, indicated by the dashed lines in (a),(b). Fits are performed with Eq. (3) (solid yellow curves), following the procedure described in Appendix C 1. Arbitrary IQ units have been rescaled, as described in Appendix C 2, with use of the same scaling factor in (a)–(d).

the longitudinal coupling. Note here that by adiabatically sweeping the gate voltages in this stability diagram, we ensure that the qubit remains in its ground state at all times. Thus here, and throughout this work, it is the magnitude of qubit-cavity coupling, not the qubit state itself, that changes near the charging transition.

The behavior observed in Fig. 3(b), and its corresponding line-cut in Fig. 3(d), is extraordinary for being opposite the conventional behavior, and is clearly inconsistent with a dispersive frequency shift. Instead, it indicates an enhancement of the stationary field in the resonator. Remarkably, the observed qubit-cavity coupling ($g_{\parallel}^{\text{dy}} < 0$) not only has sign opposite that of the response in Fig. 3(a) but it is also much larger in magnitude and provides a much-cleaner signal. This is because P3's detuning lever arm $\alpha_{\text{P3},\varepsilon} = 0.09$ eV/V is more than twice as large as S1's, $\alpha_{\text{S1},\varepsilon} = 0.04$ eV/V. Therefore, modulation of the voltage on P3 applies a stronger drive to the qubit, which increases the magnitude of the coupling and the measured resonator response. Aside from our choice of driving gate, there are no other electrostatic changes in the double-quantum-dot detuning or tunnel coupling between Figs. 3(a) and 3(b). Hence, the heightened response in Fig. 3(b) [27], which we show below to be proportional to the amplitude of the qubit drive, has immediate and important consequences for qubit readout.

III. THEORY

To understand these different effects, we construct a Hamiltonian for our qubit system. As argued in Appendix A, since we work in an operating regime very near the charging transition, $\varepsilon_0 = 0$, and since the data in Figs. 3(c) and 3(d) appear symmetric about $\varepsilon_0 = 0$, it should be a good approximation to ignore other nearby level crossings. We therefore treat the system as a simple charge qubit, in which an excess electron moves between two sides of a double quantum dot, atop a fixed (1,1) charge configuration. The resulting qubit Hamiltonian is given by $H_q = (\varepsilon/2)\sigma_z + t_c\sigma_x$, where the double-quantum-dot detuning parameter $\varepsilon = \varepsilon_0 + \varepsilon_q\cos(\omega t)$ includes both static and driving terms, t_c is the tunnel coupling between the two quantum dots, and σ_x and σ_z are Pauli operators acting on the charge basis of left-localized or right-localized states. Since σ_z is proportional to the charge-dipole operator of the double quantum dot and $(a + a^\dagger)$ corresponds to the quantum field of the resonator (a^\dagger and a are photon creation and annihilation operators), the full Hamiltonian of the coupled qubit-cavity system is given by $H = H_q + \hbar\omega_r a^\dagger a + 2\varepsilon_r \cos(\omega_r t)(a + a^\dagger) + \hbar g_0 \sigma_z (a + a^\dagger)$, where g_0 is the bare coupling between the resonator and double quantum dot, and we have included a resonant-cavity-driving term, with amplitude $2\varepsilon_r$. In the absence of coupling ($g_0 = 0$) or qubit driving ($\varepsilon_q = 0$), the bare qubit energy is given by $E_{q0} = \hbar\omega_{q0} = \sqrt{\varepsilon_0^2 + 4t_c^2}$.

In all of the experiments reported here, the device is operated in the adiabatic coupling regime $\omega_r \ll \omega_{q0}$, in which the resonator is unable to excite the qubit. We further consider the weak-driving, dispersive limit, $\varepsilon_q, \hbar g_0 \ll t_c$. As explained in Appendix A, an effective Hamiltonian can be derived under these assumptions. Moving to a frame rotating at the cavity frequency, performing a rotating-wave approximation, and retaining coupling terms up to order $\mathcal{O}[\varepsilon_q g_0, g_0^2]$, we find that the system Hamiltonian reduces to

$$H_\rho \approx \frac{E_{q0}}{2} \tilde{\sigma}_z + \varepsilon_r (\tilde{a} + \tilde{a}^\dagger) + \frac{\hbar g_{\parallel}^{\text{dy}}}{2} \tilde{\sigma}_z (\tilde{a} + \tilde{a}^\dagger) + \hbar \delta\omega \tilde{\sigma}_z \left(\tilde{a}^\dagger \tilde{a} + \frac{1}{2} \right), \quad (1)$$

where the tildes on the operators $\tilde{\sigma}_z$, \tilde{a} , and \tilde{a}^\dagger indicate that they are defined in the rotating frame of the qubit energy-basis states (see Appendix A). The effective coupling terms are given by

$$g_{\parallel}^{\text{dy}} = 4t_c^2 g_0 \varepsilon_q / E_{q0}^3 \quad \text{and} \quad \delta\omega = 8\hbar t_c^2 g_0^2 / E_{q0}^3. \quad (2)$$

Such couplings can be observed as effective shifts in the cavity field or cavity energy that depend on the state of the qubit. They are known as energy-curvature or quantum-capacitance couplings [17,18], and we refer to $g_{\parallel}^{\text{dy}}$ and

$\delta\omega$ here as the dynamic longitudinal and dispersive couplings, respectively. It can be seen that $\delta\omega$ differs from the conventional definition [28] of the dispersive coupling ($\chi = g_{\perp}^2/\omega_q = \delta\omega/2$) by a factor of 2, due to our experiment being performed in the ultradispersive or “adiabatic” regime, $\omega_r \ll \omega_q$ (see Appendix A).

Although the microwave resonator cannot excite the qubit in the present scheme, the qubit has a measurable effect on the resonator through $\delta\omega$ and $g_{\parallel}^{\text{dy}}$, with the latter being proportional to the ac drive applied to the qubit detuning parameter, ε_q . It is crucial to note that an ac drive applied through gate S1 in Fig. 2(d) primarily affects the left quantum dot, while an ac drive on gate P3 primarily affects the right quantum dot. The two drives therefore act on ε_q with opposite sign. On the other hand, the two driving channels act on the cavity (and therefore ε_r) with the same sign. Here, S1 drives the cavity directly, while P3 drives the cavity indirectly through crosstalk between the two gates. As explained in Appendix A, this difference in signs is crucial for observing opposing behaviors in Figs. 3(a) and 3(b). Indeed, for future qubit-readout experiments, it could be preferable to drive the qubit on the same side as the resonator coupling (i.e., through P2 rather than P3), so that $\delta\omega$ and $g_{\parallel}^{\text{dy}}$ affect the resonator in tandem, rather than in opposition, to achieve a stronger total effect.

Finally, we provide an expression for the output of our homodyne detector (see Appendix B), which converts the signal transmitted through the microwave cavity into a dc voltage:

$$V_{\text{IQ}}^- = c \frac{(\varepsilon_r/\hbar) - (g_{\parallel}^{\text{dy}}/2)}{\sqrt{\delta\omega^2 + \kappa^2/4}}, \quad (3)$$

where subscript minus sign indicates that the qubit is always in its ground state, c is a proportionality constant, relating the signal transmitted through the microwave cavity to the output voltage from the IQ mixer, and $\kappa/2\pi$ is the rate of photon loss from the cavity.

IV. RESULTS: ANALYSIS OF COUPLING PARAMETERS

Equations (1)–(3) are used to quantitatively fit the data in Figs. 3(c) and 3(d) to determine the different coupling parameters. The fitting procedure is explained in detail in Appendix C 1, and the results are shown here as yellow curves. After fitting the data, we subtract the asymptotic background signal in V_{IQ}^- , which varies by around 10% between the two drive configurations. Note that the proportionality constant c in Eq. (3) is related to the measurement setup rather than the qubit-cavity coupling, and it cannot be measured directly. Since ε_r and $g_{\parallel}^{\text{dy}}$ are both multiplied by c , their values cannot be determined independently. However, the tunnel coupling t_c can be obtained without ambiguity from the fitting procedure, and the bare qubit-cavity

coupling $g_0/2\pi = 5.5$ MHz can be determined from measured device parameters (see Appendix D 2). Combining these results with knowledge of the applied drive power allows us to estimate ε_r and $g_{\parallel}^{\text{dy}}$ in the current experiments, as described below.

We now further explore the physics of $g_{\parallel}^{\text{dy}}$ by varying different device parameters that affect the coupling strength. (For all remaining measurements reported here, the ac drive is applied to gate P3.) Fig. 4(a) shows ε_0 linecuts labeled “i,” “ii,” and “iii” for three different ac driving amplitudes ε_q . The IQ values for this plot are normalized to the height of the tallest peak (iii), and the plots are shifted vertically for clarity. The reported ε_q values are also relative, since we cannot determine this parameter precisely. We see that the transmission peaks increase significantly in height with increasing ε_q . For comparison, we note that these data were acquired with a similar device tuning as for Fig. 3, where the relative ε_q value was 3.5.

We can extract the quantum couplings $\delta\omega$ and $g_{\parallel}^{\text{dy}}$ at different driving strengths ε_q by fitting the transmission peaks to Eq. (3), following the procedure described in Appendix C. Three such fits are shown in yellow in Fig. 4(a). Figure 4(b) shows the full set of fitting results for $\delta\omega$, and the product parameter $cg_{\parallel}^{\text{dy}}$ (see Appendixes B and C), evaluated at the $\varepsilon_0 = 0$ transmission peak, as a function of ε_q . The points corresponding to the three datasets in Fig. 4(a) are indicated by arrows. Because we are unable to estimate c and $g_{\parallel}^{\text{dy}}$ independently in this fitting procedure, we scale $\delta\omega$ and $cg_{\parallel}^{\text{dy}}$ by their values at $\varepsilon_q = 1$ in the top panel in Fig. 4(b). (In the bottom panel, $\delta\omega$ is reported in kilohertz.) We note that the $cg_{\parallel}^{\text{dy}}$ results in Fig. 4(b) follow a parabolic scaling law with respect to ε_q (dashed gray curve). Since $c \propto \varepsilon_q$ for our homodyne-detection scheme, as explained in Appendix B, we infer that $g_{\parallel}^{\text{dy}} \propto \varepsilon_q$, which is consistent with Eq. (2). This confirms the parametric nature of $g_{\parallel}^{\text{dy}}$, and has important implications for qubit tunability, as we discuss below.

The dispersive-like coupling term $\delta\omega$ is not expected to exhibit ε_q dependence. Unfortunately, the peak-fitting procedure described above does not enable a direct measurement of $\delta\omega$ because of its small size compared with $g_{\parallel}^{\text{dy}}$; however, it does allow us to determine t_c . In Fig. 4(b), we therefore plot $\delta\omega$ computed from Eq. (2). Alternatively, we can also obtain direct measurements of $\delta\omega$ in a slightly different tuning regime, as described in Appendix G, obtaining results very similar to those in Fig. 4(b). These combined results indicate that $\delta\omega$ does not depend on the driving strength ε_q , which is consistent with our theoretical understanding, and they confirm that the dispersive-like coupling is very small in the ultradispersive regime, with $\delta\omega/2\pi \approx 2.6$ – 6.2 kHz.

We can also explore the variation of $g_{\parallel}^{\text{dy}}$ with tunnel coupling. In this case, Fig. 4(c) indicates that the transmission

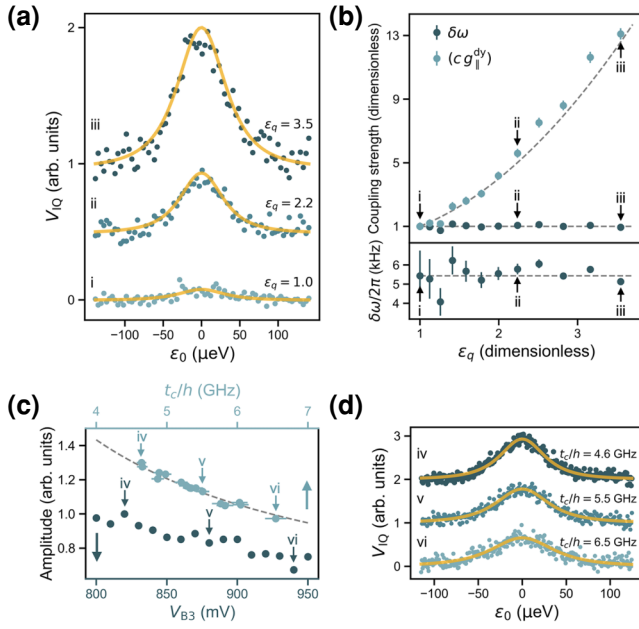


FIG. 4. Dependence of the cavity transmission and cavity-qubit couplings on ac driving amplitude and tunnel coupling for P3 driving near the interdot charging transition. (a) IQ transmission traces for three different values of the ac driving strength (dimensionless unit, see the main text): $\varepsilon_q = 1.0$ (i), $\varepsilon_q = 2.2$ (ii), and $\varepsilon_q = 3.5$ (iii). Data are normalized to the height of the tallest peak (iii), and the traces are offset vertically by 0.5 for clarity, after subtraction of background levels. Yellow curves are fits to Eq. (3). (b) Upper plot: Fitting values for the product $c g_{\parallel}^{\text{dy}}$ and for $\delta\omega$, evaluated at the peak value $\varepsilon_0 = 0$, as a function of ε_q (also dimensionless). Lower plot: Same $\delta\omega$ results, plotted in kilohertz. In both panels, the dashed gray curves correspond to expected scaling behavior: no ε_q dependence for $\delta\omega$ and parabolic dependence for $c g_{\parallel}^{\text{dy}}$. Arrows indicate the traces shown in (a). (c) Transmission peak amplitude versus tunnel-barrier gate voltage (bottom axis) represented by dark-teal data points. Background levels have been subtracted and normalized to the height of the tallest peak. Peak amplitude versus tunnel coupling t_c (top axis), determined from transmission peak fits, represented by light-teal data points. These data points have been offset by 0.3 arbitrary units for clarity. The amplitudes follow a $1/t_c$ trend (dashed gray curve). (Note that the top and bottom axes are not linearly related, as the dependence of t_c on V_{B3} is generally exponential.) (d) IQ transmission traces for the voltage values indicated by arrows in (c): $V_{B3} = 820$ mV (iv), $V_{B3} = 880$ mV (v), or $V_{B3} = 940$ mV (vi). The data are fit to Eq. (3) (yellow curves), yielding t_c values of 4.6, 5.5, and 6.5 GHz, respectively. For clarity, each of the traces is offset by 1 in these arbitrary units.

peak amplitude decreases with t_c . (Here we only vary V_{B3} , keeping all other device parameters fixed.) Note that this experiment was performed on a different date, resulting in device tunings different from those in Figs. 3, 4(a), and 4(b), and a different value of $g_0/2\pi$, 4.1 MHz (see Appendix D 2), due to electrostatic drift.

To analyze the data, we first extract the peak heights. This is done with a Lorentzian fitting form to avoid assumptions about the physical model; however, the results are essentially identical to fits obtained with Eq. (3). The Lorentzian peak heights are plotted in Fig. 4(c) after normalization of the data to the tallest peak, labeled “iv.” We also fit the IQ peaks to Eq. (3). The detailed fitting procedure is described in Appendix C 2. Three example fits are shown in Fig. 4(d), corresponding to the three labeled points iv, v, and vi in Fig. 4(c). The peak heights are replotted in Fig. 4(c) as a function of t_c/h , as determined through this procedure. The behavior is consistent with our expectation from Eqs. (2) and (3) that peak amplitude should be proportional to $g_{\parallel}^{\text{dy}} \propto 1/t_c$.

The global parameter c in Eq. (3) precludes our being able to specifically determine ε_r or ε_q in our fitting procedure. However, by taking into account the full microwave power budget, including input power and all significant attenuation sources, we estimate P3 driving amplitudes ε_q/h as large as approximately 152 MHz in these experiments (see Appendix C). Indeed, we expect P3 to provide the largest longitudinal interaction, due to its strong capacitive coupling to the quantum dot. Using this estimate and the fitting results for t_c , we use Eq. (2) to estimate $|g_{\parallel}^{\text{dy}}/2\pi| \approx 12\text{--}68$ kHz in the current experiments, including the results shown in Figs. 3 and 4. Since $g_{\parallel}^{\text{dy}}$ is proportional to ε_q , we emphasize that it should be possible to further enhance the coupling through stronger driving, while still remaining in the weak-driving regime. Although we did not explore this possibility here, it remains an important direction for future work. To estimate coupling values that could potentially be achieved in the present device, we take $\varepsilon_q/t_c = 0.1$ as an upper bound on weak driving, yielding a maximum value of $|g_{\parallel}^{\text{dy}}/2\pi|$ of 0.28 MHz, which is much larger than our estimates for $\delta\omega$, as described above.

It is crucial to note that the current experiments are performed in the far-detuned regime, $\omega_{q0}/\omega_r \gtrsim 7$, which we refer to as “adiabatic” here, because the resonator cannot excite the qubit. To our knowledge, previous longitudinal-coupling experiments did not access this extreme-detuning regime, either in superconducting implementations [23,24] or in quantum dot qubit implementations [25]. The adiabatic regime has several appealing features: (1) Purcell decoherence is strongly suppressed, since we are far from resonance; (2) there are no engineering constraints related to satisfying a resonance condition; (3) the coupling can be turned on and off via the driving term ε_q . Focusing on this final point, we note that when $\varepsilon_q = 0$, the total coupling is reduced to the small, residual dispersive coupling $\delta\omega$. When larger couplings are desired, they can be achieved by increasing ε_q up to the upper bound on weak driving. A useful figure of merit is therefore given by the tunability ratio, $|g_{\parallel}^{\text{dy}}/\delta\omega| = \varepsilon_q/2\hbar g_0$, which can be large when the

driving amplitude ε_q is much larger than the zero-point fluctuations of the cavity, as felt by the quantum dot. Estimates for the current device indicate a tunability factor of 18.4. However, we expect that tunability factors as large as 79 could be achieved for the tunnel couplings measured here, if the drive were increased to the weak-driving limit. These estimates differ considerably from those obtained in recent experiments in GaAs [25], where much larger values of the dispersive coupling, $\delta\omega/2\pi \sim 0.1\text{--}0.25$ MHz, were obtained [29] due to the large bare resonator coupling g_0 . A larger longitudinal coupling, $g_{\parallel}^{\text{dy}}/2\pi \sim 1$ MHz, was also reported in Ref. [25], yielding figures of merit in the range from 1 to 5, which are much smaller than those reported here. These differences emphasize the inherent competition between the strength (proportional to g_0) and the tunability (proportional to g_0^{-1}) of the longitudinal coupling, where g_0 is an engineered variable.

V. CONCLUSIONS

In conclusion, we have explored the energy-curvature couplings $\delta\omega$ and $g_{\parallel}^{\text{dy}}$ between a Si/Si_{1-x}Ge_x double-quantum-dot charge qubit and an off-chip, high-impedance microwave resonator. By varying the tunnel coupling and applying the drive through different channels and at different strengths, we are able to prove the existence of dynamic longitudinal coupling. We estimate its potential strength $|g_{\parallel}^{\text{dy}}/2\pi|$ here as approximately 68 kHz, which can be increased to 0.28 MHz on stronger driving, even in the far-detuned regime. We compare $g_{\parallel}^{\text{dy}}$ with the dispersive coupling $\delta\omega$, finding $|g_{\parallel}^{\text{dy}}/\delta\omega| > 18$ here, and potentially up to 79 under optimal conditions. Moreover, such strong coupling ($|g_{\parallel}^{\text{dy}}/\delta\omega| \gg 1$) is achieved here in the ultra-dispersive (“adiabatic”) regime. Crucially, such strong, dispersive couplings reduce engineering constraints and suppress Purcell decoherence. We conclude that such longitudinal driving should form an important tool for future quantum dot qubit experiments.

The data that support the findings of this study are openly available in the Zenodo repository at Ref. [30]

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APPENDIX A: DERIVATION OF THE CURVATURE-COUPPLING TERMS

In this work, we study the interactions between an off-chip microwave resonator and a charge qubit. Although we do not explicitly perform qubit operations, the work is motivated by qubit operations, including readout. In some cases, the qubit of interest could be a charge qubit, for which the information is stored in the charge degree of freedom. In other cases, the information is stored as a spin, and is then converted to charge information to facilitate readout. In both cases, the simplest system we could consider comprises a single electron in a double quantum dot. However, the motivation for the current work lies with experiments performed on the quantum dot hybrid qubit, which can be viewed as a cross between a spin and a charge qubit [31,32]. The device incorporates an odd number of three or more electrons in a double quantum dot, and the corresponding gate operations are performed in the vicinity of a charging transition ($\varepsilon \approx 0$), where one of the electrons tunnels between the quantum dots. Below, we first explain why the following theoretical analysis can be framed in terms of the simpler problem, comprising a double quantum dot with one electron.

The relevant basis states for the quantum dot hybrid qubit are fourfold, including two charge configurations and a pair of spin-orbit states, with a characteristic energy splitting approximately equal to the singlet-triplet splitting E_{ST} of the quantum dot containing an even number of electrons. When E_{ST} is somewhat larger than both the detuning parameter ε and the interdot tunnel coupling t_c , we may ignore one or both of the high-energy excited states. For example, in many cases, it is sufficient to consider a three-level description [33]. In the present work, we do not exploit the full hybrid-qubit functionality, and we further make use of the fact that the data in Figs. 3 and 4 appear symmetric with respect to $\varepsilon_0 = 0$ (as consistent with $\varepsilon_q, t_c < E_{ST}$) and do not hint at nearby energy-level

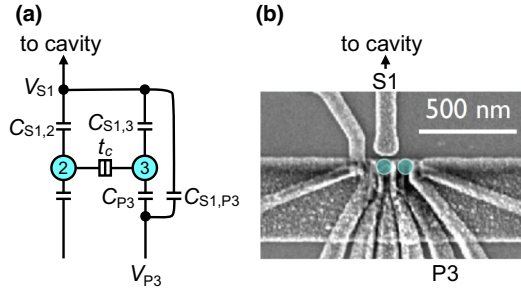


FIG. 5. (a) Simplified circuit diagram of a double quantum dot coupled to an external microwave cavity. Direct capacitances $C_{S1,2}$ and C_{P3} between the quantum dots and gates S1 and P3 are indicated. Cross capacitances $C_{S1,3}$ and $C_{S1,P3}$ are also indicated, as is the double-quantum-dot tunnel coupling t_c . (Note that other capacitances are present in our system, such as couplings to ground, which do not play a direct role in the current discussion and are not shown here.) The voltage V_{S1} on gate S1 also couples to the cavity. Voltages V_{S1} and V_{P3} may both include an ac drive, although only one gate is driven at a time. (b) A triple-quantum-dot device, operated in double-quantum-dot mode, reproduced from Fig. 2(d). In (a), the two activated quantum dots are labeled “2” and “3.”

anticrossings, thus reinforcing such an approximate two-level description. We may therefore restrict our analysis to just the two lowest-energy states. The resulting qubit Hamiltonian is then given by $H_q = (\varepsilon/2)\sigma_z + t_c\sigma_x$.

To construct the full Hamiltonian, we consider the circuit illustrated in Fig. 5, which indicates the relevant voltages and capacitances in our device, formed of a double quantum dot coupled to a microwave cavity. As described in the main text, microwave driving tones are applied in two ways to this circuit.

In the first driving mode, an ac voltage is applied to gate S1, which has two effects on the Hamiltonian. First, the drive causes a modulation of the double-quantum-dot detuning parameter (primarily via quantum dot 2), as described by the Hamiltonian term $(\varepsilon_2/2)\cos(\omega t)\sigma_z$, where ε_2 is the driving amplitude of the detuning parameter via quantum dot 2. There is also a secondary effect of driving gate S1, due to the cross-capacitances labeled “ $C_{S1,3}$ ” and “ $C_{S1,P3}$ ” in Fig. 5(a), which slightly reduces the total detuning drive. Such cross-capacitances modify the primary capacitive couplings; however, their effect is not of leading order, and is therefore modest and is ignored here. A much-more-important contribution to the Hamiltonian, associated with S1 driving, is the cavity coupling, which takes the form $2\varepsilon_r\cos(\omega t)(a + a^\dagger)$. We note that the driving amplitudes ε_2 and ε_r arise from the same voltage oscillations on gate S1, and are therefore proportional. Although it is possible to specify this proportionality constant in terms of lever arms, it is convenient to simply define $\varepsilon_r = \beta_2\varepsilon_2$. In the fitting procedure described

below, β_2 and ε_2 are chosen as fitting parameters, while ε_r becomes a dependent parameter.

The P3 driving mode also modulates the detuning parameter (primarily via quantum dot 3), as described by the Hamiltonian term $(\varepsilon_3/2)\cos(\omega t)\sigma_z$, where ε_3 is the driving amplitude of the detuning parameter via quantum dot 3. Note that although we have ignored cross-capacitances in the S1-driving Hamiltonian, it is very important to include them for the case of P3 driving. This is because the cross-capacitances are solely responsible for driving the cavity in this scenario. To account for this, we introduce the proportionality relation $\varepsilon_r = \beta_3\varepsilon_3$. The cross-capacitance also has a secondary effect on the total detuning through quantum dot 2, of the form $\varepsilon_2 = (\beta_3/\beta_2)\varepsilon_3$. The total driving amplitude for the detuning parameter is therefore slightly reduced to $(1 - \beta_3/\beta_2)\varepsilon_3$ due to crosstalk.

For the experiments described here, it is crucial to note that the S1 and P3 driving terms contribute to the total detuning with opposite sign, because they are applied on opposite sides of the double quantum dot. Taking into account both driving terms, we find the total detuning becomes

$$\varepsilon = \varepsilon_0 + \varepsilon_q \cos(\omega t), \quad (\text{A1})$$

where $\varepsilon_q = \varepsilon_2 - (1 - \beta_3/\beta_2)\varepsilon_3$. On the other hand, the two driving terms contribute to the cavity drive with the same sign:

$$\varepsilon_r = \beta_2\varepsilon_2 + \beta_3\varepsilon_3. \quad (\text{A2})$$

In both these equations, we emphasize that only one of the drives, ε_2 or ε_3 , is active at a time. The full Hamiltonian is finally given by

$$H = \frac{\varepsilon_0 + \varepsilon_q \cos(\omega_r t)}{2} \sigma_z + t_c \sigma_x + 2\varepsilon_r \cos(\omega_r t)(a + a^\dagger) + \hbar\omega_r a^\dagger a + \hbar g_0 \sigma_z (a + a^\dagger), \quad (\text{A3})$$

where, in addition to the terms already discussed, the fourth term describes a microwave cavity with resonant frequency ω_r and the fifth term describes a coupling of strength g_0 between the cavity and the qubit. As a note of clarification, β_2 is used here to describe the relation between the oscillating chemical potential of quantum dot 2 and the cavity drive, while g_0 describes the capacitive coupling between the qubit electron and the cavity. Also note that we have set all driving frequencies equal to the cavity resonant frequency, as consistent with the experiments. We have also ignored any phase difference between the drives applied to S1 and P3, which could arise due to different capacitances on different arms of the quantum dot circuit. This seems justified, to a first approximation,

because the drive responses appear 180° out of phase; however, we would generally expect such a phase difference to suppress the overall magnitude of the P3 response.

The key to deriving an effective curvature Hamiltonian is the following adiabatic approximation: when $\hbar\omega_r \ll 2t_c$, the qubit responds adiabatically to the Hamiltonian driving terms (including the cavity coupling g_0). In other words, the driving terms are unable to excite the qubit. In this adiabatic regime (which is valid for our experiments), the slowly varying terms may be absorbed into the qubit Hamiltonian, giving

$$H = (\tilde{\varepsilon}/2)\sigma_z + t_c\sigma_x + 2\varepsilon_r \cos(\omega_r t)(a + a^\dagger) + \hbar\omega_r a^\dagger a, \quad (\text{A4})$$

where

$$\tilde{\varepsilon} = \varepsilon_0 + \varepsilon_q \cos(\omega_r t) + 2\hbar g_0(a + a^\dagger). \quad (\text{A5})$$

With regard to qubit dynamics, $\tilde{\varepsilon}$ may simply be viewed as a resonator-dependent “constant.” Diagonalizing within the qubit subspace then gives

$$H' = (E_q/2)\tilde{\sigma}_z + 2\varepsilon_r \cos(\omega_r t)(a + a^\dagger) + \hbar\omega_r a^\dagger a, \quad (\text{A6})$$

where

$$E_q = \sqrt{4t_c^2 + [\varepsilon_0 + \varepsilon_q \cos(\omega_r t) + 2\hbar g_0(a + a^\dagger)]^2} \quad (\text{A7})$$

and $\tilde{\sigma}_z$ is a Pauli matrix in the qubit frame.

We now further assume that the experiments are performed in the weak-driving regime, such that $\varepsilon_q, \hbar g_0 \ll t_c$. Taylor expansion of Hamiltonian (A6) up to order $\mathcal{O}[g_0^2, g_0\varepsilon_q]$ then gives

$$\begin{aligned} H' \approx & (E_{q0}/2)\tilde{\sigma}_z + \hbar [g_{\parallel}^{\text{st}} + g_{\parallel}^{\text{dy}} \cos(\omega_r t)] \tilde{\sigma}_z (a + a^\dagger) \\ & + (\hbar\delta\omega/2) \tilde{\sigma}_z (a + a^\dagger)^2 + 2\varepsilon_r \cos(\omega_r t) (a + a^\dagger) \\ & + \hbar\omega_r a^\dagger a, \end{aligned} \quad (\text{A8})$$

where

$$E_{q0} = \sqrt{\varepsilon_0^2 + 4t_c^2} \quad (\text{static qubit energy}), \quad (\text{A9})$$

$$g_{\parallel}^{\text{st}} = \frac{\varepsilon_0 g_0}{E_{q0}} \quad (\text{static longitudinal coupling}), \quad (\text{A10})$$

$$g_{\parallel}^{\text{dy}} = \frac{4t_c^2 g_0 \varepsilon_q}{E_{q0}^3} \quad (\text{dynamic longitudinal coupling}), \quad (\text{A11})$$

$$\delta\omega = \frac{8t_c^2 \hbar g_0^2}{E_{q0}^3} \quad (\text{dispersive coupling}). \quad (\text{A12})$$

We have dropped two inconsequential terms in Eq. (A8). The first is a term proportional to $\varepsilon_q \cos(\omega_r t)\tilde{\sigma}_z$, which adiabatically modifies the qubit energy, but averages to zero

in time-averaged experiments such as the ones described here. The second term is the $\mathcal{O}[\varepsilon_q^2]$ term in the Taylor expansion, which causes weak oscillations proportional to $\varepsilon_q^2 \cos^2(\omega_r t)\tilde{\sigma}_z$ in the laboratory frame and a small, constant shift of the qubit energy splitting in the rotating frame, but does not affect the results described below.

We can simplify Eq. (A8) by noting that “counter-rotating” terms proportional to a^2 or $a^{\dagger 2}$ do not conserve energy and may therefore be eliminated through a rotating-wave approximate, giving

$$\begin{aligned} H' \approx & (E_{q0}/2)\tilde{\sigma}_z + \hbar [g_{\parallel}^{\text{st}} + g_{\parallel}^{\text{dy}} \cos(\omega_r t)] \tilde{\sigma}_z (a + a^\dagger) \\ & + \hbar\delta\omega \tilde{\sigma}_z (a^\dagger a + 1/2) + 2\varepsilon_r \cos(\omega_r t) (a + a^\dagger) \\ & + \hbar\omega_r a^\dagger a. \end{aligned} \quad (\text{A13})$$

Finally, moving to the rotating frame defined by $U = \exp(-i\omega_r t a^\dagger a)$ and applying a rotating-wave approximation, we find that

$$\begin{aligned} H_\rho = U^\dagger H' U - i\hbar U^\dagger \frac{d}{dt} U \approx & \frac{E_{q0}}{2} \tilde{\sigma}_z + \frac{\hbar g_{\parallel}^{\text{dy}}}{2} \tilde{\sigma}_z (\tilde{a} + \tilde{a}^\dagger) \\ & + \hbar\delta\omega \tilde{\sigma}_z \left(\tilde{a}^\dagger \tilde{a} + \frac{1}{2} \right) + \varepsilon_r (\tilde{a} + \tilde{a}^\dagger), \end{aligned} \quad (\text{A14})$$

where \tilde{a}^\dagger and \tilde{a} are photon creation and annihilation operators in the rotating frame. Equation (A14) contains all the curvature couplings.

As noted in the main text, the definition of the dispersive coupling $\delta\omega$ used in the preceding equations differs from the conventional definition of the dispersive coupling χ by a factor of 2. To understand this, we note that Eq. (A14) can also be solved with use of a standard Schrieffer-Wolff perturbation method, yielding

$$\delta\omega \approx g_\perp^2 \left(\frac{1}{\omega_q - \omega_r} + \frac{1}{\omega_q + \omega_r} \right). \quad (\text{A15})$$

In the conventional dispersive regime, where $|\omega_q - \omega_r| \ll \omega_r$, the second term in Eq. (A15) is counter-rotating (small), and it is therefore dropped in many calculations, yielding the conventional dispersive result, $\delta\omega \approx g_\perp^2/\Delta = \chi$, where $\Delta = |\omega_q - \omega_r|$. However, in the ultradispersive regime, $\omega_r \ll \omega_q$, both terms have the same magnitude, and we obtain $\delta\omega \approx 2g_\perp^2/\omega_q \approx 2\chi$.

APPENDIX B: IQ EQUATIONS

Equation (A14) is now used to calculate the dynamics of the density matrix. Expressing the photonic terms of the density matrix in the coherent-state basis $\{|\tilde{\alpha}\rangle\}$, where $a|\tilde{\alpha}\rangle = \tilde{\alpha}|\tilde{\alpha}\rangle$, yields the following dynamical

equations: [17]

$$\frac{d\tilde{\alpha}_{\pm}}{dt} = -i\left(\pm\delta\omega - i\frac{\kappa}{2}\right)\tilde{\alpha}_{\pm} - i\left(\frac{\varepsilon_r}{\hbar} \pm \frac{g_{\parallel}^{\text{dy}}}{2}\right), \quad (\text{B1})$$

where the tilde notation ($\tilde{\alpha}_{\pm}$) indicates that we are working in the rotating frame. Here, the \pm sign corresponds to the eigenvalues of $\tilde{\sigma}_z$ in Eq. (A14); the cavity evolution therefore depends on the state of the qubit. Note that the terms in the first parentheses in Eq. (B1) correspond to diagonal terms in Eq. (A14), with respect to the photon operators, while the terms in the second parentheses correspond to off-diagonal terms. Also note that we have introduced an energy-nonconserving term with strength κ to account for photon loss from the cavity at a rate $\kappa/2\pi$. In the transmission experiments described in the main text, we probe the stationary states of the cavity ($\tilde{\alpha}_{\pm}^{\text{st}}$), which can be obtained from Eq. (B1) by our setting $d\tilde{\alpha}_{\pm}/dt = 0$, giving

$$\tilde{\alpha}_{\pm}^{\text{st}} = -\frac{(\varepsilon_r/\hbar) \pm (g_{\parallel}^{\text{dy}}/2)}{\pm\delta\omega - i\kappa/2}. \quad (\text{B2})$$

The coherent-state amplitudes $\tilde{\alpha}_{\pm}$ are proportional to the output (transmitted) amplitude of the resonator \tilde{A} , which can be measured with a homodyne circuit. The technique converts the rapidly oscillating microwave transmission field to a dc signal. In brief, a copy of the ac input drive (applied to gate S1 or gate P3) is multiplied by the output signal from the cavity, with use of an IQ mixer. A second copy of the input signal, which is phase-shifted by $\pi/2$, is also multiplied by the output signal. Time-averaging of these separate products provides measurement of the I and Q quadratures of the cavity transmission. The cavity input fields in this calculation are defined as

$$a_{\text{in}}^I = A \cos(\omega_r t) \quad \text{and} \quad a_{\text{in}}^Q = -A \sin(\omega_r t), \quad (\text{B3})$$

and the cavity output field is defined as

$$a_{\text{out}} = \tilde{A} \cos(\omega_r t + \phi_{\text{out}}), \quad (\text{B4})$$

where ϕ_{out} represents the phase shift of the transmitted signal. We note that the input field strength A differs for drives applied to gate S1 versus gate P3 at a given microwave generator power, due to their distinct circuit paths and microwave attenuations. The product signals are given by

$$I = \overline{a_{\text{in}}^I a_{\text{out}}} = \frac{A\tilde{A}}{2} \cos(\phi_{\text{out}}), \quad (\text{B5})$$

$$Q = \overline{a_{\text{in}}^Q a_{\text{out}}} = \frac{A\tilde{A}}{2} \sin(\phi_{\text{out}}), \quad (\text{B6})$$

where the bar notation denotes time averaging. The combined IQ output is then given by

$$V_{\text{IQ}}^{\pm} \equiv \sqrt{I^2 + Q^2} = c|\alpha_{\pm}| = c\frac{(\varepsilon_r/\hbar) \pm (g_{\parallel}^{\text{dy}}/2)}{\sqrt{\delta\omega^2 + \kappa^2/4}}, \quad (\text{B7})$$

where the prefactor c accounts for the details of the measurement and room-temperature circuitry, including attenuation, signal amplification, and other effects. This factor prevents us from obtaining an absolute estimate of the various coupling terms directly from V_{IQ}^{\pm} . To interpret the measurements in Figs. 4(a) and 4(b), we point out that $c \propto A \propto \varepsilon_q$, which can be seen by combining Eq. (B7) with the fact that $|\alpha_{\pm}| \propto \tilde{A}$. Finally, we note that the qubit remains in its ground state in all of the experiments described here, so we need to consider only the minus sign in Eq. (B7).

APPENDIX C: FITTING PROCEDURE

In this appendix, we explain the fitting procedure related to Eq. (B7). We note that a dc offset in the homodyne IQ output, which was not calibrated away during experiments, has been subtracted from all data presented in this work.

1. Figure 3 fits

The data in Fig. 3 are fit to Eq. (B7). For the case of S1 driving, we set $\varepsilon_3 = 0$, obtaining

$$V_{\text{IQ}}^- = c\varepsilon_2 \frac{(\beta_2/\hbar) - (2t_c^2 g_0/E_{q_0}^3)}{\sqrt{\delta\omega^2 + \kappa^2/4}} \quad (\text{S1 drive}), \quad (\text{C1})$$

while for P3 driving, we set $\varepsilon_2 = 0$, obtaining

$$V_{\text{IQ}}^- = c\varepsilon_3 \frac{(\beta_3/\hbar) + (1 - \beta_3/\beta_2)(2t_c^2 g_0/E_{q_0}^3)}{\sqrt{\delta\omega^2 + \kappa^2/4}} \quad (\text{P3 drive}). \quad (\text{C2})$$

In these equations, we note that $E_{q_0} = E_{q_0}(\varepsilon_0, t_c)$ and $\delta\omega = \delta\omega(\varepsilon_0, t_c, g_0)$. Since $g_0/2\pi = 5.5$ MHz and $\kappa/2\pi = 125$ kHz are determined independently with use of methods described in Appendix D and the data are obtained as a function of ε_0 , the resulting fitting parameters are given by $(c\varepsilon_2)$, $(c\varepsilon_3)$, β_2 , β_3 , and t_c .

The data in Figs. 3(c) and 3(d) are fit simultaneously (i.e., both datasets are fit with a single call to the fitting function) to Eqs. (C1) and (C2), respectively. This ensures that the common fitting parameters in both equations, β_2 and t_c , are optimized between the two datasets. The best-fit parameters and uncertainties from this fitting procedure are reported in Table I, and are later used in calculating the Hamiltonian terms in Table IV.

After fitting the data, we compute and subtract the asymptotic value $V_{\text{IQ}}^-(\varepsilon_0 \rightarrow \infty) = 2c\varepsilon_r/\hbar\kappa$ from both

TABLE I. Best-fit parameters and uncertainties obtained with use of the fitting procedures described in Appendix C.

	$c \varepsilon_2 / 10^{-5}$ (V GHz)	$c \varepsilon_3 / 10^{-5}$ (V GHz)	$\beta_2 / 10^{-2}$	$\beta_3 / 10^{-3}$	t_c (GHz)
Figures 3(c) and 3(d)	2.52 ± 0.21	12.93 ± 0.60	1.27 ± 0.11	2.89 ± 0.14	6.14 ± 0.21
Figure 4(a), plot i	...	0.92 ± 0.02	...	3.09 ± 0.04	5.57 ± 1.34
Figure 4(a), plot ii	...	4.84 ± 0.06	...	3.09 ± 0.04	5.24 ± 0.26
Figure 4(a), plot iii	...	12.81 ± 0.17	...	3.09 ± 0.04	5.90 ± 0.14
Figure 4(d), plot iv	...	12.77 ± 0.00	...	2.93 ± 0.02	4.65 ± 0.08
Figure 4(d), plot v	...	12.66 ± 0.00	...	2.93 ± 0.02	5.51 ± 0.09
Figure 4(d), plot vi	...	12.57 ± 0.00	...	2.93 ± 0.02	6.55 ± 0.15

line-cuts in Fig. 3. This value varies slightly (approximately 10%) between the two drive configurations, most likely due to a small difference in attenuation on the drive lines. Since c cannot be independently determined, the line-cuts in Fig. 3(d) are rescaled to have a peak value of 1, while the line-cuts in Fig. 3(c) are scaled by the same factor. The parameters ($c \varepsilon_2$) and ($c \varepsilon_3$) have effectively arbitrary units, although we report them here in the unit of volt gigahertz, corresponding to the dc output voltages from the homodyne measurements.

2. Figure 4 fits

The IQ transmission peaks shown in Fig. 4(a) [and the larger set of peaks leading to Fig. 4(b)] are fit to Eq. (C2) at each value of the ac driving amplitude ε_q . Our fitting procedure is as follows. The fitting parameter β_3 is held fixed across all peaks, since we do not expect its value to change as a function of ε_q . We also set β_2 to the value of 1.27×10^{-2} extracted from the Fig. 3 fits, because we are unable to obtain a precise estimate of this quantity without an S1-driven dataset to fit simultaneously. $g_0/2\pi = 5.5$ MHz and $\kappa/2\pi = 125$ kHz are determined independently with use of methods described in Appendix D. This leaves the product $c\varepsilon_3$ and t_c as the remaining fitting parameters for each peak. After fitting the data, we compute and subtract the asymptotic background level $V_{\text{IQ}}^-(\varepsilon_0 \rightarrow \infty) \propto c\varepsilon_q$ from the traces in Fig. 4(a).

The peaks shown in Fig. 4(c) [and the larger set of peaks leading to Fig. 4(d)] are fit to Eq. (C2) at each value of barrier voltage V_{B3} with use of a similar procedure. Once again, we hold β_3 fixed across all fits and set $\beta_2 = 1.27 \times 10^{-2}$; $c\varepsilon_3$ and t_c are again the remaining fitting parameters for each peak. However, in this case we have $g_0/2\pi = 4.1$ MHz, due to a different device tuning for these measurements. After fitting the data, we again determine and subtract the asymptotic background level $V_{\text{IQ}}^-(\varepsilon_0 \rightarrow \infty)$ from the traces in Fig. 4(d). The small (approximately 1%) variation observed in the background is due to time variation in the electrostatic environment and/or change in the lever arm $\alpha_{\text{P3},\varepsilon}$ as a function of V_{B3} .

The best-fit parameters and uncertainties for Figs. 4(a) and 4(d) are reported in Table I and the corresponding Hamiltonian terms are reported in Table IV.

The peak amplitudes plotted in Fig. 4(c) are extracted by our applying Lorentzian fits to the data; these amplitudes are also shown as dark-teal circles in Fig. 8(a). The arbitrary units in Figs. 4(c) and 4(d) are scaled so that the tallest peak (labeled “iv”) has amplitude 1.

APPENDIX D: DEVICE PARAMETER MEASUREMENTS

1. Gate lever arms

We extract gate-to-quantum-dot lever arms by measuring thermal broadening of electron transitions [34]. At finite temperature, thermal fluctuations of the chemical potential lead to charge transitions of finite energy width, which we measure by sweeping the dc plunger voltage V_{P3} through a charging transition of quantum dot 3, which occurs at the voltage V_0 . In the limit $k_B T_e \gg \hbar\omega_r$, the transition linewidth is thermally limited and the resonator’s response to electron tunneling should follow the derivative of the Fermi-Dirac distribution:

$$|V_{\text{IQ}}| = -|A| \operatorname{sech}^2 \left[\frac{\alpha_{\text{P3},3}(V_{\text{P3}} - V_0)}{2k_B T_e} \right] + B, \quad (\text{D1})$$

where k_B is the Boltzmann constant, T_e is the electron temperature, A is the resonator suppression amplitude, B is the blockade transmission signal, and $\alpha_{\text{P3},3}$ is the P3-to-quantum-dot 3 lever arm [35]. As the cryostat temperature is varied, the electron temperature follows the trend

$$T_e = \sqrt{T_{\text{MC}}^2 + T_{e0}^2}, \quad (\text{D2})$$

where T_{MC} is the temperature of the cryostat mixing chamber and T_{e0} is the base electron temperature when T_{MC} is extrapolated to zero [36].

Working at a device tuning comparable to the one used in the measurements for Figs. 3 and 4(d) plot vi, we measure reservoir transition widths at several cryostat temperatures. We then extract a base electron temperature $T_{e0} = 212$ mK and a P3-to-quantum-dot 3 lever arm $\alpha_{\text{P3},3} = 0.149$ eV/V by fitting the data to Eqs. (D1) and (D2). The electron temperature extracted from the fitting is well described by Eq. (D2), as demonstrated in Fig. 6.

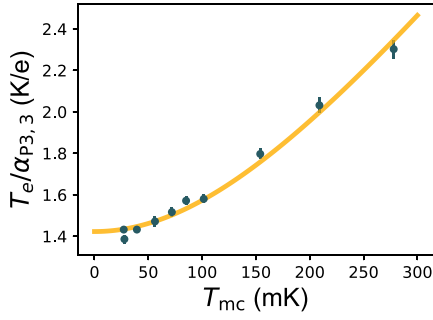


FIG. 6. Data obtained from thermal-broadening measurements for $\alpha_{P3,3}$. Data points extracted from charge-transition line fits are plotted against the mixing-chamber temperature. Fitting these data to Eq. (D2) yields $\alpha_{P3,3} = 0.149$ eV/V and the base electron temperature $T_{e0} = 212$ mK.

The other plunger lever arms are calculated from the slopes of transition lines in stability diagrams where both plungers are swept:

$$\alpha_{P3,\varepsilon} = \alpha_{P3,3} - \alpha_{P3,2} = \alpha_{P3,3} \frac{m_{\text{pol}}}{m_3} \left(\frac{m_3 - m_2}{m_2 + m_{\text{pol}}} \right), \quad (\text{D3})$$

$$\alpha_{P2,\varepsilon} = \alpha_{P2,2} - \alpha_{P2,3} = \frac{\alpha_{P3,\varepsilon}}{m_{\text{pol}}}, \quad (\text{D4})$$

$$\alpha_{P2,2} = \frac{m_3 \alpha_{P2,\varepsilon} + \alpha_{P3,\varepsilon}}{m_3 - m_2}, \quad (\text{D5})$$

where $m_i = (\Delta V_{P2}/\Delta V_{P3})_i$ are the slopes of transition lines of quantum dot 2, transition lines of quantum dot 3, or polarization lines in V_{P2} - V_{P3} space. Similarly, the detuning lever arm for gate S1 is estimated with use of

$$\alpha_{S1,\varepsilon} = \alpha_{S1,2} - \alpha_{S1,3} = \alpha_{P2,2} \left(\frac{\Delta V_{P2}}{\Delta V_{S1}} \right)_2 - \alpha_{P3,3} \left(\frac{\Delta V_{P3}}{\Delta V_{S1}} \right)_3, \quad (\text{D6})$$

where S1 lever arms are calculated from the slopes of transition lines in 2D scans in which both gates are swept.

The lever arms used for each fit are reported in Table II. We measure $\alpha_{P2,2}$ and $\alpha_{P3,3}$ at several additional device tunings using both thermal-broadening estimates and electron-transport bias spectroscopy to further verify that the method described above provides reasonable estimates.

2. Bare qubit-cavity coupling

The bare qubit-cavity coupling g_0 is calculated from device parameters according to [37]

$$g_0 = \frac{\alpha_{S1,\varepsilon} \omega_r}{2e} \sqrt{\frac{2Z_{0,r}}{h/e^2}}, \quad (\text{D7})$$

where the characteristic impedance of the resonator, $Z_{0,r} = 575 \Omega$, is estimated from a quasistatic impedance analysis performed with COMSOL MULTIPHYSICS [7]. The values of $Z_{0,r}$ and g_0 used in each fit are reported in Table II.

3. Microwave resonator parameters

The cavity resonant frequency ω_r and loaded quality factor Q_L are extracted from resonance transmission curves measured with a vector network analyzer. Complex-plane fits to the curves yield $Q_L = 10,470 \pm 32$ for S1 driving and $Q_L = 10,476 \pm 46$ for P3 driving; the two values are very similar, and we use their average in our analysis. The cavity decay rate $\kappa = \omega_r/Q_L$ can then be calculated. Both ω_r and κ are reported in Table II.

APPENDIX E: MICROWAVE DRIVE ENERGY AND CAVITY PHOTON NUMBER

1. Drive-energy upper bound

While the analysis method used in the main text does not provide a precise measure of the qubit detuning drive amplitude ε_q , we can obtain a rough estimate by considering the microwave power applied in experiments. For the measurements reported in Fig. 3, a microwave generator is used to apply a driving power of 6 dBm to the measurement circuit, which is configured with approximately 33 dB of attenuation at room temperature. For the measurements in Figs. 4(a) and 4(b), generator power of -5 to 6 dBm and room-temperature attenuation of 33 dB are used, whereas for the measurements in Figs. 4(c) and 4(d) generator power of 15 dBm and room-temperature attenuation of 42 dB are used. For all measurements, an additional 40 dB of attenuation is incurred inside the cryostat. Furthermore, the quantum dot device is engineered with buried coplanar waveguide bias leads whose characteristic impedance $Z_{0,g}$ is approximately 1Ω [7]. Microwave attenuation at the bias leads has not been fully characterized; however, on the basis of SPICE simulations of our device, we expect a minimum attenuation of 10 dB due to impedance mismatch between the leads and the 50Ω coaxial cabling

TABLE II. Device parameters obtained with use of the methods described in Appendix D.

	$\alpha_{P2,\varepsilon}$ (eV/V)	$\alpha_{P3,\varepsilon}$ (eV/V)	$\alpha_{S1,\varepsilon}$ (eV/V)	$Z_{0,r}$ (Ω)	$\omega_r/2\pi$ (GHz)	$\kappa/2\pi$ (kHz)	$g_0/2\pi$ (MHz)
Figures 3(c) and 3(d)	0.11	0.09	0.04	575	1.3038	124.5	5.5
Figures 4(a) and 4(b)	0.11	0.09	0.04	575	1.3038	124.5	5.5
Figures 4(c) and 4(d)	0.10	0.10	0.03	575	1.3038	124.5	4.1

TABLE III. Upper-bound estimates for the ac detuning drive amplitude and cavity photon number computed in Appendix E.

	Driving gate	P_{in} (pW)	ε_q/h (MHz)	ε_r/h (kHz)	$\langle n \rangle$
Figures 3(a) and 3(c)	S1	20	27	340	30
Figures 3(b) and 3(d)	P3	20	-137	397	41
Figures 4(a) and 4(b)	P3	2–20	-39 to -137	119–424	4–46
Figures 4(c) and 4(d)	P3	20	-152	447	51

inside the cryostat. Summing the above contributions gives an estimated input power $P_{\text{in}} \lesssim -77$ dBm = 20 pW for all measurements.

When ac modulation is applied to plunger gate P3, the detuning drive amplitude is related to the input power by

$$\varepsilon_q \approx \varepsilon_3 = -\alpha_{\text{P3},\varepsilon} \sqrt{2Z_{0,g} P_{\text{in}}}, \quad (\text{E1})$$

where $\alpha_{\text{P3},\varepsilon}$ is the detuning lever arm and the factor of $\sqrt{2}$ converts rms-voltage amplitudes to peak-voltage amplitudes. Note that for this estimate, we ignore the small crosstalk correction proportional to (β_3/β_2) seen in Eq. (A1). Equation (E1) is not applicable for the measurements reported in Figs. 3(a) and 3(c). In the corresponding experiments, gate S1 is driven through the $\lambda/4$ -resonator segment, resulting in additional voltage attenuation. We estimate the resulting ε_q values by multiplying the output of Eq. (E1) by the ratio of best-fit parameters $(c\varepsilon_2)/(c\varepsilon_3)$ reported in Table I. Thus, for all experiments performed here, we estimate drive amplitudes of $|\varepsilon_q/h| \lesssim 152$ MHz, obtaining the full set of results given in Table III. These drive strengths are comparable to those used in similar qubit experiments [38].

We emphasize again that these values should be treated as approximate upper bounds due to the uncertainty in attenuation at the quantum dot bias leads.

2. Photon number

The average number of photons populating the resonator $\langle n \rangle$ is given by the stationary value of the cavity coherent

state $\tilde{\alpha}_{\pm}^{\text{st}}$. Squaring Eq. (B2) gives

$$\langle n \rangle = |\tilde{\alpha}_{\pm}^{\text{st}}|^2 \approx \frac{4\varepsilon_r^2}{\hbar^2 \kappa^2}. \quad (\text{E2})$$

Note that this quantity is dominated by the direct resonator drive, and we ignore small contributions from the couplings $g_{\parallel}^{\text{dy}}$ and $\delta\omega$. Using Eq. (A2) and values from Tables I and III to compute ε_r , we estimate photon populations $\langle n \rangle \lesssim 50$ for all measurements reported here. The full set of results is given in Table III.

The number of thermal photons in the resonator due to finite system temperature and blackbody radiation from resistive loads is on the order of 1 [39]. This residual population should be negligible for most measurements compared with the number of photons produced by direct resonator drive.

We have considered the impact of higher-order corrections to the curvature couplings that may become significant at large photon number [17]. For our system, we expect such corrections to be unimportant when $\langle n \rangle \lesssim 10^3$ – 10^4 , due to the quantum nondemolition nature of the curvature interactions and the fact that we are working in the adiabatic coupling regime.

APPENDIX F: CALCULATED ENERGY COUPLINGS

Using methods described here and in the main text, we obtain estimates for t_c , g_0 , etc. In Table IV, we report these t_c values and the corresponding energy couplings, which are computed with Eqs. (A11) and (A12).

TABLE IV. Estimated energy coupling parameters for experiments reported in the main text. The tunnel coupling t_c is obtained from fitting; quantum-dot–cavity couplings $\delta\omega$ and $g_{\parallel}^{\text{dy}}$ are computed according to Eqs. (A11) and (A12) with use of t_c and parameters from Tables II and III. The values for $g_{\parallel}^{\text{dy}}$ are upper-bound approximations, based on the ε_q values reported in Table III. The reported values all correspond to the peaks, centered at $\varepsilon_0 = 0$.

	t_c/h (GHz)	$\delta\omega/2\pi$ (kHz)	$g_{\parallel}^{\text{dy}}/2\pi$ (kHz, approximate)	$ g_{\parallel}^{\text{dy}}/\delta\omega $
Figure 3(c)	6.1	4.9	12.1	2.5
Figure 3(d)	6.1	4.9	-61.4	12.4
Figure 4(a), plot i	5.6	5.4	-19.3	3.5
Figure 4(a), plot ii	5.2	5.8	-45.7	7.9
Figure 4(a), plot iii	5.9	5.1	-63.9	12.4
Figure 4(d), plot iv	4.6	3.7	-67.6	18.4
Figure 4(d), plot v	5.5	3.1	-57.0	18.4
Figure 4(d), plot vi	6.5	2.6	-47.9	18.4

APPENDIX G: ADDITIONAL DATA: DIRECT MEASUREMENT OF DISPERSIVELIKE COUPLING

For the figures in the main text, the dispersive coupling $\delta\omega$ is estimated from Eq. (A12), where t_c values are obtained from the fitting analysis described in the main text and g_0 is estimated with use of the geometrical parameters of the device. However, $\delta\omega$ can also be measured directly from the shift of the cavity resonant frequency when the cavity is coupled to the double quantum dot. Figure 7(a) shows a typical analysis, where the ac drive is applied to gate P3. Here, the dark-blue cavity resonance peaks are obtained deep in the Coulomb-blockade regime ($\varepsilon_0 \gg 0$), where the qubit-cavity coupling is tiny. The lighter-blue curve is obtained at the interdot charging transition ($\varepsilon_0 = 0$), where the coupling is strongest. Equation (A12) gives a prediction for the state-dependent frequency shift, which is analogous to the effect of χ in the conventional dispersive regime. The blown-up peaks (inset) show the frequency shift caused by the dispersivelike coupling, with $\delta\omega/2\pi \approx 3.3$ kHz. In the ultradispersive limit, such shifts are found to be much smaller than the resonant linewidth. The double quantum dot remains in its ground state for all experiments in this work, so the direction of the resonance shift caused by the coupling is always towards lower frequencies.

This peak-shift analysis is repeated for different detuning values about the interdot charging transition, as shown in Fig. 7(b). Here, the experimental tuning is the same tuning as used for Figs. 3(b) and 3(d). (Note that although the measured frequency shifts are actually negative, we plot their absolute value here, since $\delta\omega$ is defined as a positive quantity in Appendix A.) The yellow curve in Fig. 7(b) is not a fit; rather, it shows the *predicted* frequency shift obtained from Eq. (A12), with use of the t_c values obtained from the fit shown in Fig. 3(d). The lighter-yellow shading

corresponds to the 1σ confidence interval, estimated from parameter uncertainties of the fitting algorithm. The good agreement between the direct measurement and the theoretical prediction supports our theoretical interpretation of the results and validates our fitting methods.

Finally, in Fig. 7(c), we show the experimentally measured frequency shifts as a function of the ac driving strength ε_q . Here the specific data point corresponding to Fig. 7(a) is indicated by the arrow. While some spread is observed in the data, we observe no systematic variation of $\delta\omega$, which agrees with the conclusions we draw from Fig. 4(b). We note, however, that the two datasets were acquired at different device tunings, and therefore they are not directly related.

APPENDIX H: ADDITIONAL DATA: LOW-BARRIER VOLTAGE BEHAVIOR

The fitting results reported in Table IV confirm that smaller t_c values yield larger absolute values of $g_{\parallel}^{\text{dy}}$, as desired for strong qubit-resonator couplings. In this appendix, we explore the possibility of extending the data in Fig. 4(c) to lower t_c values.

Figure 8(a) shows results of such an analysis, which includes all the data shown in Fig. 4(c), but also includes data obtained at lower values of V_{B3} , where we expect to observe lower t_c values. Using the same IQ peak-fitting procedure as for Fig. 4(d), we also determine the corresponding t_c values for these data, as reported in Fig. 8(b). The data reveal surprising behavior that is inconsistent with a continuous downward trend of t_c , in the low- V_{B3} regime. Indeed, for V_{B3} values below those reported in Fig. 4(c), the t_c results remain roughly constant, but exhibit distinct nonmonotonic features.

To explain this behavior, we note that lower V_{B3} values should yield larger tunnel barriers between quantum dots

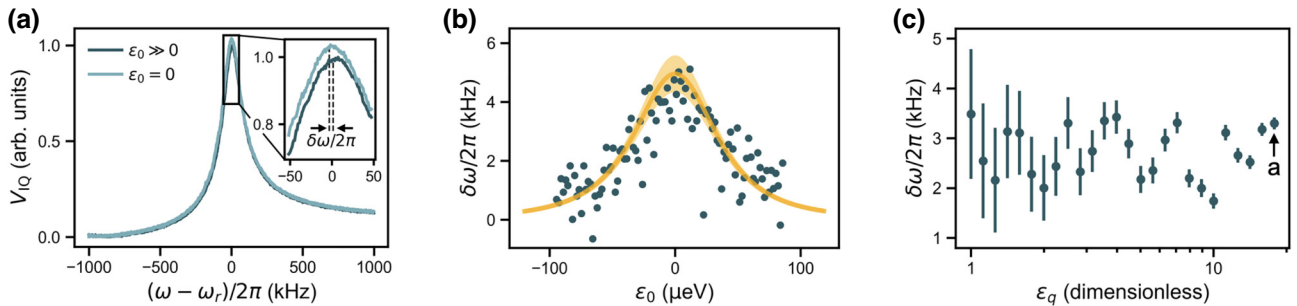


FIG. 7. Direct measurements of the dispersivelike coupling from cavity resonance shifts for an ac drive applied to gate P3. (a) Cavity transmission as a function of driving frequency $\omega/2\pi$ measured at the interdot charging transition ($\varepsilon_0 = 0$, light blue) and deep within the Coulomb-blockade regime ($\varepsilon_0 \gg 0$, dark blue). The inset shows an enlarged view showing the small shift of the resonant peak due to the dispersivelike coupling $\delta\omega$. (b) Resonant peak shifts are obtained as a function of ε_0 . The dark-yellow curve and light-yellow 1σ confidence interval show the prediction for $\delta\omega$ obtained from Eq. (A12) with use of fitting results for t_c shown in Fig. 3(d). (c) Directly measured resonant-frequency shifts as a function of ac driving amplitude ε_q in relative units. The data point corresponding to (a) is indicated with an arrow.

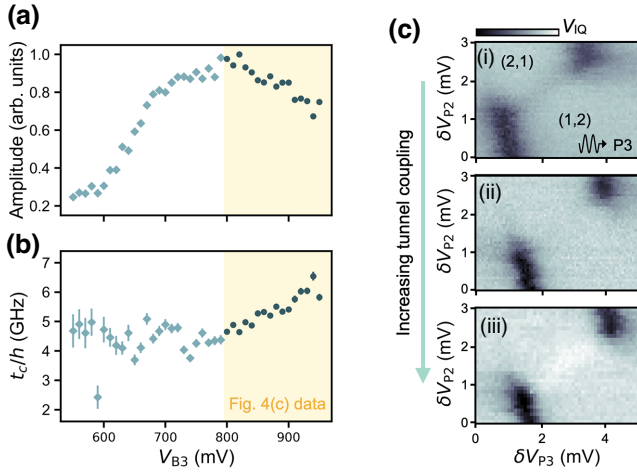


FIG. 8. Cavity transmission responses measured for P3 driving near the interdot charging transition, extending the dataset in Fig. 4(c) to lower V_{B3} values. (a) Cavity transmission peak amplitudes, extracted by our fitting transmission peaks to Lorentzian forms as a function of dc barrier voltage V_{B3} . The data have been normalized so that the largest measured amplitude corresponds to 1 arbitrary unit. Error bars are smaller than the markers. (b) Tunnel coupling t_c as a function of V_{B3} estimated from transmission peak fits to Eq. (C2). The low- t_c outlier at $V_{B3} = 590$ mV is believed to be the result of electrostatic instability during the measurement of that trace. In (a),(b), the dark-teal circles at $V_{B3} \geq 800$ mV correspond to data shown in Fig. 4(c); light-teal diamonds at $V_{B3} < 800$ mV show an extended dataset discussed in Appendix H. (c) Three other device tunings, measured in sequence, and measured separately from the data in (a),(b). Moving from panel (i) to panel (iii), one sees the response changes from a dip in panel (i) to a peak in panel (iii), passing through a region of no response [panel (ii)].

2 and 3. Such behavior is not observed in Fig. 8(b). Instead, the data are consistent with raising the tunnel barrier so far that the quantum dots are squeezed out of the top side of the gated region in the device shown in Fig. 2(d). This can occur here because no screening gate is present in the top portion of the device, and the quantum dots are therefore more weakly confined in this region. The distinct features observed in Fig. 8(b) are likely caused by first one quantum dot and then the other quantum dot being squeezed out of its intended location. The sudden drop in amplitude in Fig. 8(a), at $V_{B3} \approx 700$ mV, correlates with the onset of the rightmost bump feature in Fig. 8(b). We speculate that this occurs when quantum dot 3 leaves its intended confinement location, resulting in a sudden reduction of the lever arm between quantum dot 3 and gate P3; in turn, this causes a sudden drop in the driving amplitude, ϵ_3 .

Figure 8(c) shows a sequence of transmission responses measured at three additional device tunings, also in the low- V_{B3} regime. These scans are obtained by our simultaneously varying V_{B3} and the right-side reservoir-accumulation-gate voltage [see the right-hand side of

Fig. 2(d)], causing t_c to increase on movement from panel (i) to panel (iii). For these measurements, we also observe a sign change of the IQ peak height. However, in contrast with Fig. 3, we do not change driving ports in this case. Interestingly, we are again able to fully extinguish the transmission response, as shown in panel (ii), for a particular tuning. The progression illustrates an interplay between the longitudinal ($g_{\parallel}^{\text{dy}}$) and dispersive ($\delta\omega$) couplings, showing that they can be independently tuned. We speculate that the effect can be explained by lateral quantum dot motion, which modifies both the gate lever arms and the bare coupling g_0 . The possible dynamics are likely more complicated than those in Figs. 8(a) and 8(b) however, because in Fig. 8(c), the changes in V_{B3} are at least partially compensated by changes in the reservoir accumulation gate.

To summarize, reducing V_{B3} below 800 mV in this device does not appear to cause the intended effect of lowering t_c . The IQ peak-fitting results in this regime are therefore meaningless, because the fitting equations do not include the relevant physics. We emphasize, however, that such behavior occurs here only because of the device geometry, and would not be expected to occur in a device with an additional, top screening gate. It is therefore important to explore the possibility of lowering t_c values in future experiments. We also note that intentionally shifting the lateral positions of the quantum dots may provide additional means for tuning $g_{\parallel}^{\text{dy}}$ and $\delta\omega$ independently.

APPENDIX I: ADDITIONAL DATA: CAVITY DRIVE VIA A SECOND ON-CHIP DEVICE

Charge sensing in the main text is performed by our applying an ac drive to device gate S1 or P3. We are also able to charge sense by driving the plunger gate of a second, unaccumulated quantum dot device on the qubit die. The second device is nominally identical to the qubit device used in the experiments reported in the main text with an upper screening gate connected to the same microwave resonator, as shown in Fig. 9(a). The ac drive excites the cavity and modulates the voltage of gate S1. The charge-stability diagrams acquired with this technique, therefore, look very similar to those acquired when S1 is driven, with a dip in the IQ signal at the polarization line. Figure 9(b) shows a stability diagram acquired at the same tuning as for the measurements in Fig. 3. The line-cut in Fig. 9(c) shows the cavity response near the charge-degeneracy point.

APPENDIX J: ADDITIONAL DATA: TRANSMISSION ENHANCEMENT AT OTHER CHARGE OCCUPATIONS

The experiments reported in the main text are performed at the (2,1)-(1,2) charge configuration. However, we have

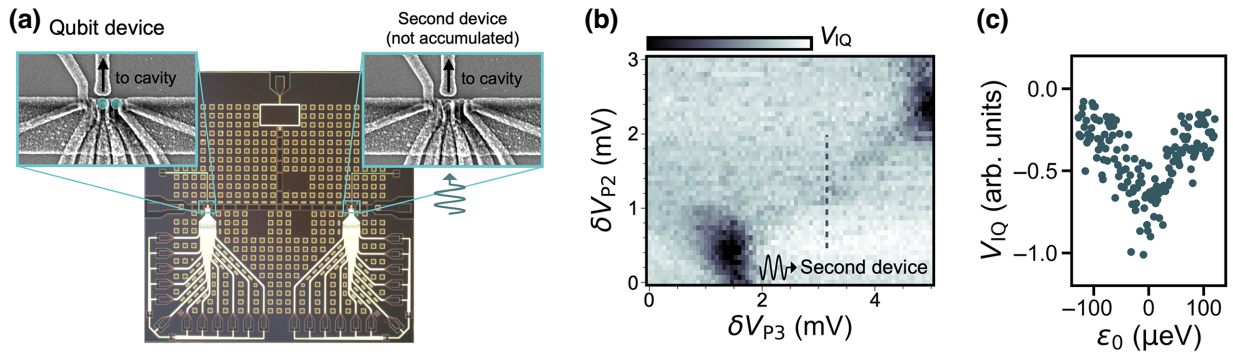


FIG. 9. Cavity drive via a second on-chip device. (a) Dark-field optical micrograph of the qubit die with two quantum dot devices located in the blue-boxed regions. Scanning electron micrographs of the nominally identical devices are also shown. A 1.3-GHz microwave drive applied to the center plunger gate of the right-side device excites the resonator and can be used for charge sensing. (b) IQ transmission measurement of the double quantum dot tuned to its (2,1)-(1,2) charge-occupation transition while the cavity is being driven via the second device plunger. (c) Line-cut across the interdot charge transition indicated in (b). Resonator transmission is suppressed at the charge-degeneracy point. The scaling used for the arbitrary units here is the same as used in Fig. 3.

observed resonator transmission enhancement at several additional interdot transitions when longitudinal coupling is activated via the P3 drive. The charge-stability diagram in Fig. 10(a) shows enhanced transmission for at least four transitions: (2,1)-(1,2); (3,1)-(2,2); (2,2)-(1,3); and (3,2)-(2,3). Higher-resolution scans at the latter three transitions are shown in Fig. 10(b). Some transitions in Fig. 10(a) appear to show suppressed transmission [e.g., (4,0)-(3,1)], while others have no visible polarization line [e.g., (1,1)-(0,2)]. We believe this variation in resonator response is

strongly dependent on tunnel coupling for each configuration, as discussed in the main text. It may further depend on multielectron states for some charge occupations; this topic lies beyond the scope of our analysis.

APPENDIX K: DEVICE FABRICATION AND EXPERIMENTAL SETUP

Detailed information regarding fabrication of the flip-chip device and the measurement circuit used for experiments performed in this work can be found in Ref. [7].

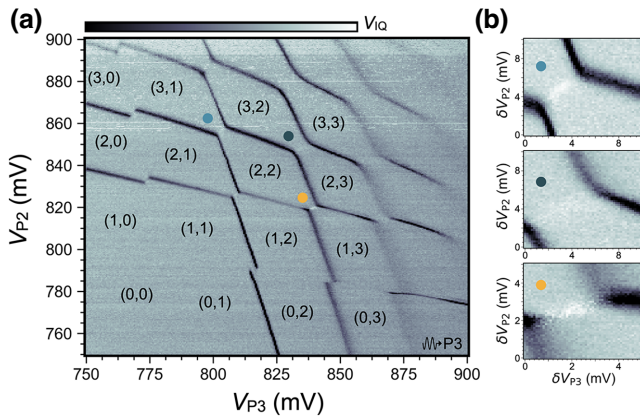


FIG. 10. Enhanced transmission at several charge occupations. (a) Large-scale charge-stability diagram for the double quantum dot. Experiments reported in the main text are performed at the (2,1)-(1,2) charge-occupation transition. Enhanced resonator transmission is also visible at several polarization lines for the same tuning. (b) Enlarged stability diagrams showing enhanced resonator transmission at the polarization lines. Colored circles correspond to matching circles in (a), which indicate the voltage-space location for each scan. All plots are acquired at the same barrier and screening-gate voltage tunings. The color scale is the same as in (a).

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