Decoherence-Protected Implementation of Quantum Gates

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We present a scheme to implement a universal set of quantum gates based on achievable interactions, and the gates can be protected against decoherence through a dynamical-decoupling approach without encoding. By properly designing system evolution, the desired system interactions commute with the elements forming dynamical decoupling pulses. Thus, the effect of decoherence can be eliminated by repeatedly applying the pulses, without noticeably affecting the system evolution governed by the desired system interactions given a small enough time interval between pulses. Moreover, due to the commutation between the elements forming the pulses and the desired system interactions, our scheme is resistant to different types of decoherence, and so not limited to specific decoherence. Our scheme also works well in the case that the desired system interactions cannot be achieved ideally due to imperfect control of system parameters, through the action of dynamical-decoupling pulses.

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I. INTRODUCTION

Realizing noise-resistant quantum operations at the level of physical qubits is essential for further development of quantum computation. This is one of the main challenges faced by researchers to achieve fault-tolerant quantum computation, even though quantum error-correction codes (QECCs) have been extensively explored for actively detecting and correcting errors [1–16]. On the negative side, the success of QECCs replies on the requirements of physical-qubit resource and small enough errors [6–9]. Because of the requirements, fault-tolerant quantum computer is beyond our reach with state-of-the-art technologies, still awaiting future developments in both theories and experiments.

Besides the QECCs, dynamical decoupling (DD) is also an active approach to fight against decoherence due to the interaction between system and environment [17–20]. Specifically, DD fights against errors by using external pulse sequences to eliminate unwanted couplings, without observably causing impacts on the system dependent on the time interval between the pulses. Compared with QECCs, the DD approach does not require a large number of physical qubits for encoding or measurement qubits for detecting errors, but needs a relatively acceptable number of control pulses, which are achievable in different quantum systems [21–25].

DD makes it possible to decouple physical qubits from the environment with comparably modest resource, ensuring the protection of the coherence of the qubits. However, applying the DD technique to the implementation of quantum gates cannot be achieved with ease since the DD pulses may aimlessly remove both the interaction between the system and environment, and the desired system interactions for executing quantum gates. How to make the DD technique compatible with quantum operations is the key task in achieving decoherence-protected quantum gates. The issue can be resolved through properly proposed system interactions [26], or by encoding [27–31] or by specially designed DD pulses [32–34] to average out undesired terms and keep the terms required for implementing quantum gates. The general idea in the schemes is based on the harmony of gate Hamiltonians and DD pulses, for ensuring excellent performance of different gates in the presence of noises.

In this work, we present a scheme for executing decoherence-protected quantum gates formulated on properly designed system interactions, without encoding or specific DD pulses. Both single-qubit and two-qubit gates in our scheme can be actively preserved by different DD pulses. Our scheme is based on two types of qubit-qubit interactions, and the desired interactions are achievable in superconducting systems with controllable system parameters [35–39]. Two types of auxiliary qubits are needed for realizing a universal set of quantum gates. The use of auxiliary qubits preserves single-qubit gate operations

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from the disturbance caused by DD pulses. Meanwhile, the auxiliary qubits remain in their initial states after the evolution of system, and their coherence will be protected by the DD pulses as well. Neither measurement of auxiliary qubits nor replacement of used auxiliary qubits with fresh qubits is required. The desired system interactions commute with the elementary operations forming DD pulses. Thus, the effect of decoherence on both data qubits and auxiliary qubits can be eliminated by repeatedly applying the pulses, without evidently affecting quantum gate operations given small enough interval between the DD pulses. Moreover, due to the commutation between the decoupling elementary operations and the desired system interactions, our scheme is resistant to different types of decoherence and any DD approach constructed from the decoupling elementary operations is workable. Our scheme performs excellently even with some residuals in free system Hamiltonian, which may be caused by imperfect control of system parameters, due to its compatibility with the DD approach. The merit eases the requirement of strictly adjusting system parameters in actual experiments. Another useful merit possessed by our scheme is that we do not need to change to the interaction picture to get desired gate Hamiltonians and as a result, the quantum gates in our scheme are preserved from decoherence in every step. Or else, further exploration is required for the protection of the system at the step of changing it back.

II. DECOHERENCE-PROTECTED QUANTUM GATES

Our scheme is based on two types of system interactions:

$$H_1 = \frac{\epsilon}{2}(\sigma_z^i + \sigma_z^j) + \frac{\Delta}{2}(\sigma_x^i + \sigma_x^j) + J_z\sigma_z^i\sigma_z^j, \quad (1)$$

and

$$H_2 = \frac{\epsilon}{2}(\sigma_z^i + \sigma_z^j) + \frac{\Delta}{2}(\sigma_x^i + \sigma_x^j) + J_x\sigma_x^i\sigma_x^j, \quad (2)$$

where ϵ and Δ describe the energy of qubits, J_z is the coupling strength in z direction, and J_x is the coupling strength in x direction. The two types of interactions can be implemented in superconducting qubits with controllable parameters ϵ, Δ, J_x, and J_z [35–39].

To implement single-qubit rotations about the z axis, we employ H_1 as

$$H_1^{1\&A} = \frac{\epsilon}{2}(\sigma_z^1 + \sigma_z^A) + \frac{\Delta}{2}(\sigma_x^1 + \sigma_x^A) + J_z\sigma_z^1\sigma_z^A. \quad (3)$$

The Hamiltonian describes the coupling between data qubit 1 and auxiliary qubit A. We adjust the values of ϵ and Δ such that $\epsilon, \Delta \ll J_z$ during the system evolution. The evolution operator according to $H_1^{1\&A}$ is then of the form, $e^{-i\theta_1\sigma_z^1\sigma_z^A}$ with $\theta_1 = J_z t$, where t is the evolution time. If initially the auxiliary qubit A is in its ground state $|1\rangle_A$, we get single-qubit rotations about the z axis on data qubit 1 as $U_1 = e^{i\theta_1\sigma_z^1}$.

Single-qubit rotations about the x axis on data qubit 1 can be achieved by utilizing another auxiliary qubit B from the evolution governed by H_2 of the following form:

$$H_2^{1\&B} = \frac{\epsilon}{2}(\sigma_z^1 + \sigma_z^B) + \frac{\Delta}{2}(\sigma_x^1 + \sigma_x^B) + J_x\sigma_x^1\sigma_x^B, \quad (4)$$

which represents the interaction connecting data qubit 1 to auxiliary qubit B. Similarly we make the values of ϵ and Δ small enough to neglect the terms according to the condition $\epsilon, \Delta \ll J_x$. The resultant evolution operator is $e^{-i\theta_2\sigma_x^1\sigma_x^B}$ with $\theta_2 = J_x t$. If we set the auxiliary qubit B to be in its initial state $|-\rangle_B = 1/\sqrt{2}(|0\rangle_B - |1\rangle_B)$, we obtain single-qubit rotations about the x axis on data qubit 1 and that is $U_2 = e^{i\theta_2\sigma_x^1}$.

Two-qubit quantum gates can also be realized on the basis of H_1 expressing the interaction between two data qubits,

$$H_1^{1\&2} = \frac{\epsilon}{2}(\sigma_z^1 + \sigma_z^2) + \frac{\Delta}{2}(\sigma_x^1 + \sigma_x^2) + J_z\sigma_z^1\sigma_z^2. \quad (5)$$

Again, we control the parameters ϵ and Δ properly to get an evolution operator $U_3 = e^{-i\theta_3\sigma_z^1\sigma_z^2}$ with $\theta_3 = J_z t$, without using any auxiliary qubit. The U_3 is locally equivalent to control-Z gate, namely $U_{CZ} = e^{-i(\pi)/4}(S \otimes S)U_3|_{\theta_3=-\pi/4}$ with $S = e^{i(\pi)/4}U_1|_{\theta_1=-\pi/4}$. Moreover, the Hadamard gate can be achieved through the combination of the rotations about x and z axes in sequence, $H = -iU_2|_{\theta_2=\pi/2}U_1|_{\theta_1=-\pi/4}U_2|_{\theta_2=-\pi/4}U_1|_{\theta_1=\pi/4}$. The control-NOT (CNOT) gate is hence executable given the control-Z gate and the Hadamard gate, $U_{\mathrm{CNOT}} = (I_2 \otimes H)U_{CZ}(I_2 \otimes H)$ (where I_2 is the 2×2 identity matrix).

Moreover, another type of two-qubit quantum gate is achievable according to H_2 with the below coupling between qubits 1 and 2,

$$H_2^{1\&2} = \frac{\epsilon}{2}(\sigma_z^1 + \sigma_z^2) + \frac{\Delta}{2}(\sigma_x^1 + \sigma_x^2) + J_x\sigma_x^1\sigma_x^2. \quad (6)$$

In the case that ϵ and Δ are adjusted to be small enough, we obtain an evolution operator $U_4 = e^{-i\theta_4\sigma_x^1\sigma_x^2}$ with $\theta_4 = J_x t$. Therefore, we can implement a universal set of quantum gates on data qubits, including both Clifford and non-Clifford gates.

The reason that we employ auxiliary qubits is to make the DD approach compatible with the universal set of quantum gates. In other words, the auxiliary qubits are required to ensure that the desired coupling terms for implementing single-qubit gates will not be canceled out by the DD pulses. Given the auxiliary qubits, the system evolution to realize single-qubit gates on the data qubit is governed either by Hamiltonian (3) or (4). The two types of Hamiltonians approximately commute with the decoupling group $\mathcal{G} = \{\mathbb{1}^{\otimes N}, \sigma_x^{\otimes N}, \sigma_y^{\otimes N}, \sigma_z^{\otimes N}\}$ [28,29], when ϵ and Δ are small enough. Even in the case that the two parameters cannot be effectively neglected, our scheme is still workable with the DD techniques. This is because the unwanted terms with the two parameters can be diligently eliminated during the system evolution with the application of DD pulses, while the desired terms for quantum gates commute with the above decoupling group. Similar reasoning goes to two-qubit quantum gates, which are based on Hamiltonians (5) and (6). Therefore, desired coupling terms in Hamiltonians [(3), (4), (5), (6)] will not be eliminated by the DD pulses and meanwhile undesired terms or decoherence will be effectively removed, leading to decoherence-protected quantum gates.

From the above explanations, it is easy to see that our quantum gates are achieved in the Schrödinger picture and we control only experimentally accessible system parameters to engineer the quantum gates. We do not need to change to the interaction picture to get desired gate Hamiltonians. This merit is helpful to protect quantum gates as the additional step of evolution to change the system back to the Schrödinger picture is not required. Otherwise, we need to additionally explore how to protect the quantum gates when we change the system back to the Schrödinger picture in order to fully preserve the quantum gates from noises.

In the literature, different methods of encoding logical qubits have been proposed to make the DD techniques adaptable with implementing quantum gates. Minimally, two physical qubits are required for encoding one logical qubit [30]. While in our scheme, the required computational resource can be reduced by removing encoding, but relying on auxiliary qubits. This is because one auxiliary qubit can be shared by different data qubits, as illustrated in Fig. 1. There are two types of auxiliary qubits required in our scheme. We need to ensure each data qubit is connected to both types of auxiliary qubits, in order to achieve arbitrary decoherence-protected single-qubit gate on any data qubit. As shown in Fig. 1, eight auxiliary qubits are needed for a quantum network with 12 data qubits, and hence a total of 20 physical qubits are needed. With the encoding protocol [30], a set of 24 physical qubits are required to achieve a quantum network with 12 logical qubits. Moreover, without the need of encoding, the complexity of measuring data qubits can also be mitigated.

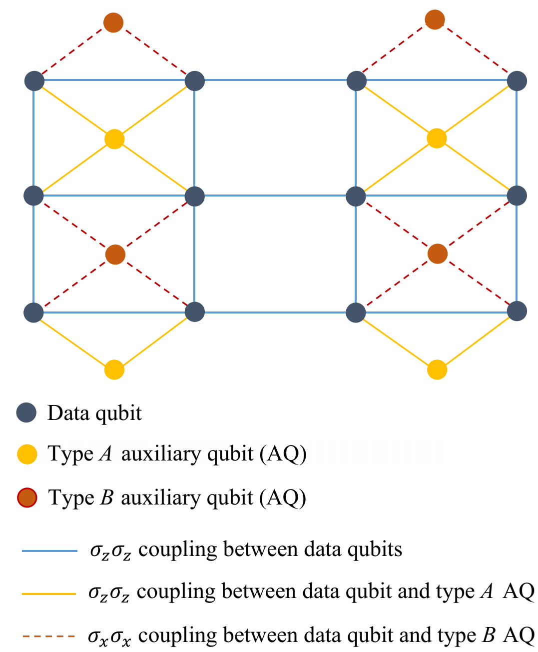

FIG. 1. Illustration of the couplings among data qubits and the two types of auxiliary qubits. Each data qubit is connected to one type A auxiliary qubit and one type B auxiliary qubit, and as a result, the arbitrary single-qubit gate can be achieved on each data qubit. Two neighbor data qubits are connected to each other in the form of $\sigma_z\sigma_z$ interaction or $\sigma_x\sigma_x$ interaction (this interaction is not demonstrated in the figure), in order to execute two-qubit gates on them.

III. NUMERICAL RESULTS

In this section, we explore the performance of our decoherence-protected quantum gates by randomly choosing 50 initial states to find average fidelities, where $|\psi\rangle_0 = 1/(\sqrt{|a_0|^2 + |a_1|^2})(a_0\,|0\rangle_1 + a_1\,|1\rangle_1)\,|1\rangle_A$ for U_1, $|\psi\rangle_0 = 1/(\sqrt{|a_0|^2 + |a_1|^2})(a_0\,|0\rangle_1 + a_1\,|1\rangle_1)\,|-\rangle_B$ for U_2, and $|\psi\rangle_0 = 1/(\sqrt{\sum |a_{ij}|^2})\sum_{i,j=0,1} a_{ij}\,|i\rangle_1\,|j\rangle_2$ for U_3 and U_4 with $a_{0,1}$ or $a_{ij} \in \mathbb{C}$. In our numerical codes, partial trace function is from the online resource [40]. Specifically, we numerically calculate four types of gate fidelities, in the absence of decoherence, in the presence of decoherence but without applying DD pulses, in the presence of decoherence by applying periodic DD (PDD) pulses. PDD can be expressed by $\mathrm{PDD}_{n_p} = (U_{\mathrm{Blk}})^{n_p}$, where n_p is the number of times repeating U_{Blk} and $U_{\mathrm{Blk}} = \sigma_z^{\otimes N} U(T_0)\sigma_x^{\otimes N} U(T_0)\sigma_z^{\otimes N} U(T_0)\sigma_x^{\otimes N} U(T_0)$ formed by $U(T_0)$ (describes the evolution of the system with decoherence over T_0 with $T_0 = t/(4n_p)$, where t is desired evolution time) and the elements in the decoupling group [31]. We choose $n_p = 3$ in the calculations, and round the fidelities up to four decimal places.

In the presence of system-environment coupling, we assume the total Hamiltonian is written as $H_T = H_S + H_e$, where H_S and H_e are system Hamiltonian and the stochastic error term. In our calculations, H_S is $H_1^{1\&A}$, $H_2^{1\&B}$, $H_1^{1\&2}$,

TABLE I. The fidelities of a universal set of quantum gates in the absence (presence) of decoherence in case 1, with ideal initial states of auxiliary qubits. Some fidelity is approximately equal to 1 as we round the numbers up to four decimal places.

Gate	Without decoherence	With decoherence		
	Fidelity	Fidelity without DD	Fidelity with ideal DD	Fidelity with nonideal DD
$\sigma_x = -iU_2\|_{\theta_2=\frac{\pi}{2}}$	0.9970	0.9491	0.9997	0.9992
$S = e^{i\frac{\pi}{4}}U_1\|_{\theta_1=-\pi/4}$	0.9988	0.9779	0.9998	0.9995
$T = e^{i\frac{\pi}{8}}U_1\|_{\theta_1=-\pi/8}$	0.9997	0.9951	≈ 1	0.9996
$U_3\|_{\theta_3=-\pi/4}$	0.9972	0.9657	0.9997	0.9993
$U_4\|_{\theta_4=-\pi/4}$	0.9976	0.9575	0.9999	0.9994

or $H_2^{1\&2}$, and

$$H_e = \sum_{k=1}^{N} \delta_k^x(t)\sigma_x^k + \delta_k^y(t)\sigma_y^k + \delta_k^z(t)\sigma_z^k, \tag{7}$$

where N is the number of physical qubits and $\delta_k^{x,y,z}(t)$ describe the time-dependent strength of stochastic errors in different directions. In our calculations, we numerically solve the dynamics governed by H_T and $\delta_k^{x,y,z}(t)$ are randomly selected from a uniform distribution $[J_0, 10J_0]$ where $J_0 = 2\pi \times 2$ MHz to describe stochastic time-dependent broadband errors. The time dependence is modeled by generating the random numbers at different moments. According to MATLAB, ode45 solver solves a differential equation by choosing N steps in numerical integration (where N is dependent on specified precision). When we add random numbers in odefun, random numbers are created at every step with respect to time, indicating the random numbers $\delta_k^{x,y,z}(t)$ are dependent on time. By this way, we approximate the errors that are varying at any moment.

A. Ideal initial states of auxiliary qubits and DD pulses

In the following, we study two cases of selecting different system parameters with ideal initial states of auxiliary qubits and DD pulses to show that quantum gates with desired fidelities are accomplishable in our scheme with applying DD pulses, no matter the term with ϵ or Δ can be neglected or not when DD techniques are employed.

Case 1. In this case, we choose the following parameters to investigate the performance of different gates, $\epsilon = 2\pi \times 10$ MHz, and $\Delta = 0$, $J_x = J_z = 2\pi \times 100$ MHz. Here, we purposely set the value of ϵ to be small compared with $J_x = J_z$ and Δ to be zero. The numerical results are summarized in Table I. With small value of ϵ and zero value of Δ, we can ignore the effect of the free Hamiltonian and obtain very good performance of the gates when decoherence is not taken into account. With decoherence, the performance of the gates is negatively affected without applying DD pulses. With DD pulses, we observe that PDD preferably protect the quantum gates from the decoherence caused by the system-environment interaction in all x, y, and z directions. The results are understandable since our desired interaction terms commute with the decoupling group. Any DD technique based on the decoupling group elements should be accomplishable in our scheme to eliminate the effect of the decoherence. Dependent on the values of $\delta_k^{x,y,z}(t)$, we can correspondingly adjust the number of the DD pulses to attain excellent gate fidelities in practical experiments. Moreover, we observe that the gate fidelities with enough DD pulses are even better than those without the decoherence. This is because the nonzero free Hamiltonian term can be removed by the DD pulses too and hence the gate fidelities are further improved.

Case 2. We then revise $\epsilon = 2\pi \times 100$ MHz and keep the other parameters unchanged in the second case. Now the value of ϵ is not small enough to be ignored, comparable to $J_x = J_z$. We similarly find the average gate fidelities and the numerical results are summarized in Table II. It is

TABLE II. The fidelities of a universal set of quantum gates in the absence (presence) of decoherence in case 2, with ideal initial states of auxiliary qubits. Some fidelity is approximately equal to 1 as we round the numbers up to four decimal places.

Gate	Without decoherence	With decoherence		
	Fidelity	Fidelity without DD	Fidelity with ideal DD	Fidelity with nonideal DD
$\sigma_x = -iU_2\|_{\theta_2=\frac{\pi}{2}}$	0.7534	0.6723	0.9984	0.9978
$S = e^{i\frac{\pi}{4}}U_1\|_{\theta_1=-\pi/4}$	0.8884	0.8342	0.9998	0.9994
$T = e^{i\frac{\pi}{8}}U_1\|_{\theta_1=-\pi/8}$	0.9708	0.9541	≈ 1	0.9995
$U_3\|_{\theta_3=-\pi/4}$	0.7677	0.7063	0.9997	0.9993
$U_4\|_{\theta_4=-\pi/4}$	0.8124	0.7204	0.9999	0.9993

TABLE III. The fidelities of a universal set of quantum gates in the absence (presence) of decoherence in case 2, provided nonideal initial states ($F_g = F_e = 0.995$) with ideal or nonideal DD pulses.

Gate	Without decoherence	With decoherence		
	Fidelity	Fidelity without DD	Fidelity with ideal DD	Fidelity with nonideal DD
$\sigma_x = -iU_2\|_{\theta_2=\frac{\pi}{2}}$	0.7524	0.6690	0.9986	0.9978
$S = e^{i\frac{\pi}{4}}U_1\|_{\theta_1=-\pi/4}$	0.8914	0.8413	0.9962	0.9968
$T = e^{i\frac{\pi}{8}}U_1\|_{\theta_1=-\pi/8}$	0.9676	0.9513	0.9980	0.9981

understandable that the performance of the quantum gates is not as good as that in the first case when decoherence is not considered, due to the effect of the term with parameter ϵ. In this case, our scheme is still workable to achieve desirable gate fidelities with applying DD pulses since the undesired term with parameter ϵ can be canceled out during the system evolution by the DD pulses. Repeat the base pulse sequence of PDD, it is shown clearly that the effect of the undesired term is almost not visible given sufficient DD pulses for gates S, T, U_3, and U_4. While for σ_x gate, the negative influence by the undesired term is noticeable and this may be because a longer evolution time is needed to achieve the gate. Therefore, the DD pulses can eliminate the effects due to the term with parameter ϵ (and similarly Δ) and decoherence. With increasing value of ϵ or Δ, a greater number of times of repeating base PDD sequence will be required to eliminate the unwanted terms. The result is friendly and useful in practical experiments if there are some uncontrollable residuals even though ϵ and Δ are adjustable.

B. Ideal initial states of auxiliary qubits but nonideal DD pulses

In actual experiments, DD pulses are implemented via evolution operators over certain time intervals. Therefore, we may not have ideal DD pulses due to unexpected errors occurred in the evolution, and we then explore the performance of our scheme with nonideal DD pulses applied.

The required DD pulses can be realized by $\sigma_z \propto \exp(-iH_z t)$ and $\sigma_x \propto \exp(-iH_x t)$ up to a constant of $-i$, where $H_z = 1/2(\omega_z + \delta\omega_z)\sigma_z$ and $H_x = 1/2(\omega_x + \delta\omega_x)\sigma_x$ with $\delta\omega_{z,x}$ describing possible errors, which may be caused by the control of parameters or decoherence, when $\omega_z t = \omega_x t = \pi$. We choose $\omega_{z,x} = 2\pi \times 5$ GHz and $\delta\omega_{z,x} = 0.002\omega_{z,x}$ to numerically explore the performance of the quantum gates with the parameters illustrated in both case 1 and case 2, and summarize the results in Tables I and II as well. As expected, the nonideal DD pulses cause negative impacts on the fidelities of the quantum gates. When the errors $\delta\omega_{z,x}$ are sufficiently small, the negative impacts are not that noticeable. Thus, the success of our scheme depends on nearly ideal DD pulses.

C. Nonideal initial states of auxiliary qubits

One more point about the feasiblity of our scheme in practical experiments is dependent on the precision in preparing initial states of auxiliary qubits. As initial states may not be perfectly prepared, the fidelity of the initial states would adversely influence the performance of single-qubit gates in our scheme. In this subsection, we assume the initial states of auxiliary qubits are not well prepared and they are described by $|g\rangle_{\text{im}} = \sqrt{F_g}\,|g\rangle + \sqrt{1-F_g}e^{-i\theta_e}\,|e\rangle$ and $|-\rangle_{\text{im}} = \sqrt{F_-}\,|-\rangle + \sqrt{1-F_-}e^{-i\theta_+}\,|+\rangle$, where $F_{g,-}$ indicate the fidelities of the initial states and $\theta_{e,+}$ are random phases. The single-qubit gates are then investigated in numerical ways with the parameters illustrated in case 2, given the nonideal initial states. The numerical results are shown in Table III, where $F_g = F_e = 0.995$. Without doubt, we observe unfavorable influences by the nonideal initial states on the performance of the quantum gates. The gate fidelities saved by DD pulses are comparable to F_g and F_e, given enough ideal or nonideal DD pulses. Thus, our scheme requires high-fidelity initiallization of the auxiliary qubits in order to achieve desired single-qubit gate fidelities. The high-fidelity initialization of the auxiliary qubits may be achievable by resorting to machine-learning technique [41,42].

IV. DISCUSSION AND CONCLUSION

We explore the implementation of a universal set of quantum gates that can be protected against decoherence by the DD techniques, based on achievable interactions. The compatibility of implementing quantum gates and the DD techniques is on the basis of employing auxiliary qubits, rather than encoding. For single-qubit gates, we utilize auxiliary qubits and proper system interactions to execute decoherence-protected rotations about x and z axes, ensured by the fact that the desired system interactions commute with the elements forming DD pulses. For two-qubit gates, no auxiliary qubit is needed for implementing decoherence-protected nontrivial quantum gates. Because of the commutation between the elements forming DD pulses and the desired system interactions, the effect of different types of decoherence can be eliminated by repeatedly applying DD pulses, without manifestly affecting the system evolutions. Moreover, compared with the protocols with encoding, our scheme relaxes the need of qubit resource and reduces the complexity of measuring data qubits. In practical experiments due to control capabilities, there may be some residuals in free system Hamiltonian besides the desired interactions. We illustrate that our scheme is not sensitive to the residual terms, with DD pulses applied properly. On another positive side, our scheme is based on the Hamiltonians in the Schrödinger picture rather than in the interaction picture and thus, there is no need of an additional step of evolution back to the Schrödinger picture. This merit simplifies the process of protecting the quantum gates from decoherence. Otherwise, we need to explore the protection of the quantum gates in the additional step of evolving back to the Schrödinger picture as well, in order to achieve full protection of the quantum gates.

Given the decoherence-protected Clifford and non-Clifford gates in our scheme, arbitrary quantum operation can be protected actively against decoherence. Thus our scheme is of practical relevance in resolving various quantum computation tasks. Here as an example, we would like to discuss the application of our protected quantum gates in the implementation of QECCs. As mentioned in Ref. [43], the integration of the DD and QECCs leads to more powerful platform for preserving quantum information from errors due to the (dis)advantages of either method.

The first example to discuss is surface codes that are widely explored for achieving fault-tolerant quantum computation [10,11]. The syndrome measurements are crucial for surface codes since the implementation of surface codes depends on the syndrome measurements aided by desired quantum operations acting on computational qubits [11]. The syndrome measurements can be executed by acting Hadamard and CNOT gates in sequence on measurement and data qubits, together with certain measurements [10]. Therefore, based on our protected quantum gates, it is possible to achieve protected implementation of surface codes.

Besides the possible application in surface codes, our scheme is also beneficial for protecting the implementation of other types of QECCs. For most of the QECCs, the implementation of the codes conventionally relies on single- and two-qubit quantum gates acting on physical qubits [3]. Specifically, 12 control-Z gates are needed for generating the Steane code, and five control-phase gates plus several single-qubit gates are employed in the encoding circuit of the five-qubit code [44]. Consider the number of quantum gates required, it is highly essential to preserve quantum information from decoherence during the multistep system evolution. The decoherence-protected quantum gates presented in our scheme can enhance the success probability of implementing the QECCs.

Our scheme is also helpful for executing quantum machine learning in physical systems. In Ref. [45], the authors explored the creation of handwritten digits with high resolution on the basis of quantum circuits formed by different gates, showing the power of quantum computers in enhancing the performance of machine-learning algorithms. The desired quantum gates in the quantum circuit are preservable from decoherence according to our scheme. Therefore, the quantum machine-learning protocol can be protected by DD techniques.

To conclude, our scheme plays favorable roles in boosting the success rate of performing different quantum-computation tasks in the presence of decoherence. The robust merit and the achievability of desired system interactions in superconducting systems signify the scheme as an useful step forward towards the development of fault-tolerant quantum computation.

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